WORKSHOP ON BIRATIONAL GEOMETRY
MOSCOW, 22–24 NOVEMBER 2017
ABSTRACTS

Florin Ambro (Simion Stoilow Institute)
An injectivity theorem

I will present a generalization of the injectivity theorem of Esnault–Viehweg to a class of non-normal algebraic varieties.

Paolo Cascini (Imperial College)
Polarized endomorphisms on projective varieties

I will survey about some recent progress towards a characterisation of projective varieties which admit a polarised endomorphism. I will focus, in particular, on normal projective threefolds defined over an algebraically closed field of positive characteristic. Joint work with S. Meng and D.-Q. Zhang.

Grzegorz Kapustka (Jagiellonian University)
On the Morin problem

We will study the Morin problem and present a method of classification of finite complete families of incident planes in $\mathbb{P}^5$. As a result we prove that there is exactly one, up to $\mathrm{Aut}(\mathbb{P}^5)$, configuration of maximal cardinality 20 and a unique one parameter family of 19 planes. Finally, I will discuss the Morin problem in higher dimensions, and in particular study configurations of $\mathbb{P}^3$ in $\mathbb{P}^9$ and their relations with polarized hyper-Kähler fourfolds of $K3^{[2]}$ type and Beauville–Bogomolov degree 4.

Alexandra Kuznetsova (HSE)
3-*subgroups in Cremona group*

One possible approach to studying Cremona groups is to investigate their finite subgroups. Dolgachev and Iskovskikh have classified all finite subgroups of Cremona group of rank 2. For Cremona group of rank 3 we can estimate the number of generators of its finite subgroups. For example, Prokhorov considered abelian p-subgroups of this group and showed some bounds for different prime numbers p. The aim of my talk is to describe how to estimate the number of generators of abelian and also non-abelian p-groups in Cremona groups of ranks 2 and 3 and to show how to get a good bound for $p = 3$.

Konstantin Loginov (HSE)
Standard models of degree 1 del Pezzo fibrations

We discuss the construction of some good birational models for three-dimensional del Pezzo fibrations over a curve. These fibrations (along with conic bundles and Fano varieties) occur when the Minimal Model Program is applied to a three-dimensional rationally connected variety. It is natural to ask if one can obtain a model whose canonical divisor is Cartier (a Gorenstein model), and with as good singularities as possible. We explain how to construct Gorenstein model with canonical singularities for a degree 1 del Pezzo fibration that admits an action of a finite group G.

Miles Reid (University of Warwick)
The Tate–Oort group scheme of order p and 5-torsion Godeaux surfaces

Over an algebraically closed field of characteristic p, there are three group schemes of order p, namely $\mathbf{F}_p^+ = \mathbb{Z}/p$, the multiplicative subgroup scheme μ_p in $\mathbb{G}_\mathrm{m}$, and the additive subgroup scheme α_p in $\mathbb{G}_\mathrm{a}$. The Tate–Ort group scheme is a construction in mixed characteristic that puts these three into one happy family, together with the ordinary cyclic group $\mathbb{Z}/p$ in characteristic zero. The applications of this construction to moduli theory include the Eichler–Shimura description of the moduli spaces $Ga_0(p)$ and $Ga_1(p)$ of elliptic curves plus an order p subgroup near the prime p.

Godeaux surfaces or Godeaux CY 3-folds with 5-torsion are quotients of quintics in $\mathbb{P}^3$ or $\mathbb{P}^4$ by a group scheme of order 5. We use the Tate–Oort group C_5 to construct an irreducible family of Godeaux surfaces that includes the usual characteristic zero surfaces together with the surfaces with Pic^{tau} of order 5 in characteristic 5 due to Lang, Miranda amd Liedtke. The main computational difficulty involves understanding the representation theory of C_p. This is joint work with Kim Soonyoung.

Vadim Vologodsky (HSE and University of Oregon)
Non-commutative crystalline cohomology

This is a joint work with Alexander Petrov.

Let X be a smooth scheme over a field $\mathbf{F}_p$. Though X may have many liftings over $\mathbb{Z}/p^n\mathbb{Z}$ or have no liftings at all, the algebraic de Rham cohomology $\mathrm{H}_{dR}(X/\mathbf{F}_p)$ admits a canonical lifting (strictly speaking, the de Rham complex admits a lifting and cohomology behaves as prescribed by the universal coefficients formula) $\mathrm{H}_{cris}(X/\mathbb{Z}/p^n)$ which is called the crystalline cohomology of X. If $\tilde{X}$ is a smooth lifting of X over $\mathbb{Z}/p^n$ then there exists a canonical isomorphism $\mathrm{H}_{dR}(\tilde{X}/\mathbb{Z}/p^n) \cong \mathrm{H}_{cris}(X)$.

I will explain a non-commutative generalization of this construction. Namely, we show that the periodic cyclic homology of a DG category over $\mathbf{F}_p$ admits a canonical lifting over $\mathbb{Z}/p^n\mathbb{Z}$ and that the latter coincides with the the periodic cyclic homology of a lifting of the DG category $\mathbb{Z}/p^n\mathbb{Z}$.

Jarosław Wiśniewski (University of Warsaw)
Combinatorics of torus action and low dimensional contact manifolds

I will report on the ongoing project whose task is to use combinatorics arising from algebraic torus action to understand complex contact manifolds which arise in the context of LeBrun–Salamon conjecture about quaternionic-Kähler manifods. The project is developed in collaboration with Jarosław Buczyński and Andrzej Weber.

Jakub Witaszek (Imperial College)
Liftability of the Frobenius morphism and images of toric varieties

The celebrated proof of the Hartshorne conjecture by Shigefumi Mori allowed for the study of the geometry of higher dimensional varieties through the analysis of deformations of rational curves. One of the many applications of Mori's results was Lazarsfeld's positive answer to the conjecture of Remmert and Van de Ven which states that the only smooth variety that the projective space can map surjectively onto, is the projective space itself. Motivated by this result, a similar problem has been considered for other kinds of manifolds such as abelian varieties (Demailly–Hwang–Mok–Peternell) or toric varieties (Occhetta–Wiśniewski). In my talk, I would like to present a completely new perspective on the problem coming from the study of Frobenius lifts in positive characteristic. This is based on a joint project with Piotr Achinger and Maciej Zdanowicz.