STEKLOV MATHEMATICAL INSTITUTE OF RUSSIAN ACADEMY OF SCIENCES
SKOLKOVO INSTITUTE OF SCIENCE AND TECHNOLOGY

**INTERNATIONAL CONFERENCE
ON OCCASION OF VICTOR BUCHSTABER'S 75TH BIRTHDAY**

ALGEBRAIC TOPOLOGY, COMBINATORICS AND MATHEMATICAL PHYSICS

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ABSTRACTS

Steklov Mathematical Institute of RAS
Skolkovo Institute of Science and Technology

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АЛГЕБРАИЧЕСКАЯ ТОПОЛОГИЯ, КОМБИНАТОРИКА И МАТЕМАТИЧЕСКАЯ ФИЗИКА

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Plenary lectures

On the integral cohomology of orbifolds

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A variety of tools, including specialized cohomology theories, have been crafted in recent years to study orbifolds arising in a variety of different areas of topology and geometry, [1], [2] and [3]. Strangely enough however, the singular integral cohomology ring remains somewhat intractable in most cases.

The example of weighted projective space, does succumb to the traditional methods of algebraic topology, [5]. Additively, its cohomology agrees with that for ordinary projective space but the ring structure is saturated with divisibility arising from the weights. Essential to the computation is the observation that weighted projective spaces can be constructed by a sequence of canonical cofibrations, in a manner not unlike that for CW complexes.

Motivated by this, we identify classes of orbifolds which can be built in this way using "orbifold cells" or "**q**-cells". Some of these ideas, originated by Goresky in [4], were developed in

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a toric framework, by Poddar and Sarkar in [7], though our results are not restricted to *toric* orbifolds.

We derive conditions on the **q**-cell structure of an orbifold which ensure that the integral cohomology is free of torsion and concentrated in even degree. In particular, the toric setting allows for a translation into conditions on the fan or characteristic map, which suffice for a complete calculation of the integral cohomology rings. The constructions in [6] allow for an extension of the results to *torus* orbifolds as well.

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Locally Isometric Delone Sets

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The atomic structure of crystalline materials exhibits a very high symmetry. One of the central problems of the local theory for regular systems is to explain the genesis of the symmetry of crystalline structures through geometric features of their fragments of relatively small size.

An ideal crystal is a discrete set X in $\mathbb{R}^d$ which is a finite union of translates of a full-rank lattice Λ. However, generally saying, $\mathrm{Sym}(X)$ contains not only pure translations. Therefore, the crystalline structure X can be described in another way: X is a finite union of several $\mathrm{Sym}(X)$-orbits. An each orbit $\mathrm{Sym}(X)\cdot x$ is a regular system, i.e., a Delone set whose symmetry group acts point-transitively. The concept of the regular system generalizes the concept of the lattice. Though regular systems are arranged more complicate than lattices, by the Schoenflis-Bieberbach theorem, any regular system is a union of several translates of a lattice.

The regularity properties and conditions can be described in terms of ρ-clusters. Given $x \in X$ and $\rho > 0$, a subset of all points $x' \in X$ with distance $|xx'| \leq \rho$ is a *ρ-cluster* $C_x(\rho)$ of the point x. Given x and y from X, two ρ-clusters $C_x(\rho)$ and $C_y(\rho)$ are said to be *equivalent* if for some Euclidean isometry g $g(x) = y$ and $g(C_x(\rho)) = C_y(\rho)$. For given ρ the number of all classes of ρ-clusters in X is denoted by $N(\rho)$. The *cluster counting function* $N(\rho)$ is a positive, integer-valued,

This work is supported by the Russian Science Foundation under grant 14-11-00414

non-decreasing function. For any regular system $N(\rho) = 1$ for all $\rho > 0$.

The *regularity radius* is defined by the two conditions. First, equivalence of all $\hat{\rho}_d$-clusters in a Delone set $X \subset \mathbb{R}^d$, i.e., condition $N(\hat{\rho}_d) = 1$, implies the regularity of the X. Second, $\hat{\rho}_d$ is the minimal value in the sense that for $\forall \varepsilon > 0$ there is a Delone set X with $N(\hat{\rho}_d - \varepsilon) = 1$, which is not a regular system. As proved recently, $\hat{\rho}_d \geq d2R$.

From now on we denote by X a Delone set in which all $2R$-clusters are assumed to be equivalent, i.e. $N(2R) = 1$. We call a set X with $N(2R) = 1$ *locally isometric*. From $\hat{\rho}_d \geq d2R$ it follows that a locally isometric Delone set, generally saying, is not a regular systems. However, for some classes of Delone sets, e.g., for locally antipodal sets, as we showed recently for any d, condition $N(2R) = 1$ implies X to be a regular system.

Now we will focus on locally isometric Delone sets in $\mathbb{R}^3$. Due to long ago proven Stogrin's lemma, for such sets the groups $S_x(2R)$ of the $2R$-clusters are unable to contain a rotation axis of the order to exceed 6. The list of finite groups with this restriction is finite. Assuming each group from this list as a group $S_x(2R)$ for a Delone set, by means of numerous non-trivial geometric arguments one can prove that $\hat{\rho}_3 \leq 10R$.

On the background of the estimates $6R \leq \hat{\rho}_3 \leq 10R$ it is particularly interesting that for most groups, that are possible as $S_x(2R)$, the condition $N(2R) = 1$ is sufficient for regularity of a Delone set. Moreover, on this way we get another proof of the estimate $\hat{\rho}_3 \leq 10R$. This result represents also notable progress towards improving the upper bound for $\hat{\rho}_3$.

Geometry and Topology of non-negatively curved manifolds

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This is a very brief survey on the important theme in riemannian geometry, of manifolds with positive/non-negative curvatures.

Newton polygon method and solvability of equations by quadratures

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Consider a homogeneous linear differential equation

$$y^n + a_1(t)y^{n-1} + \cdots + a_n(t)y = 0 \tag{1}$$

whose coefficients a_i belong to a differential field K.

Theorem 1 *The equation* (1) *can be solved by quadratures over K if and only if the following conditions hold: 1) the equation* (1) *has a solution $y_1 = \int f(t)dt$ where f is an algebraic function over K, 2) the linear differential equation of order $(n-1)$ obtained from* (1) *by the reduction of order using the solution y_1 is solvable by quadratures over the differential field $K(y_1)$.*

The standard proof (E. Picard and E. Vessiot, 1910) of Theorem 1 uses the differential Galois theory and is rather involved. In the talk I will discuss an elementary proof of Theorem 1 based on old arguments suggested by J. Liouville, J. Ritt and M. Rosenlicht.

J. Liouville in 1839 proved Theorem 1 for $n = 2$. J. Ritt in 1948 simplified his proof [1]. He used expansion of solutions (as functions of a parameter) into converging Puiseux series. J. Ritt studied algebraic properties of such series using the Newton polygon method.

M. Rosenlicht in 1973 proved [2] the following theorem.

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Theorem 2 *Let n be a positive integer, and let f be a polynomial in several variables with coefficients in a differential field K and of total degree less than n. Then if the differential equation*

$$y^{(n)} = f(y, y', y'', \dots) \tag{2}$$

has a solution representable by quadratures over K, it has a solution algebraic over K.

A homogeneous linear differential equation (1) of second order can be reduced to the nonlinear Riccati equation

$$u' + a_1(t)u + a_2(t) + u^2 = 0 \tag{3}$$

which is a particular case of (2) for $n = 2$. To prove Theorem 1 for $n = 2$ Liouville and Ritt proved first Theorem 2 for the Riccati equation (3). To prove Theorem 1 in general case M. Rosenlicht proved first Theorem 2 for a generalized Riccati equation of order $n-1$. The reduction of Theorem 1 to Theorem 2 for the generalized Riccati equation is straightforward. But Rosenlich's proof of Theorem 2 is rather involved. It is applicable to abstract differential fields of characteristic zero and makes use of the valuation theory.

In the talk I will discuss a proof of Theorem 2 which does not rely on the valuation theory. It generalizes Ritt's arguments (makes use of the Puiseux expansion and Newton polyhedron method) and provides an elementary proof of the classical Theorem 1.

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Linear operators with self-consistent potential

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Many systems of nonlinear equations of mathematical physics can be regarded as "linear equations with self-consistent potentials". Among them are non-linear Schrödinger equation, two-dimensional sigma-models, including the equations of n-field. In the talk a general algebraic-geometrical scheme of constructions of their solutions will be presented.

Integrability property of polynomial graph invariants

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The *symmetric chromatic polynomial*, which generalizes the conventional chromatic polynomial, was discovered in middle 90'ies independently by R. Stanley [3] and S. Chmutov, S. Duzhin, and S. Lando [1] (under the name of *weighted chromatic polynomial*). We show that the generating function for the symmetric chromatic polynomial of all connected graphs satisfies (after appropriate scaling change of variables) the Kadomtsev–Petviashvili integrable hierarchy of mathematical physics. Moreover, we describe a large family of polynomial graph invariants giving the same solution of the KP. The key point here is a Hopf algebra structure on the space spanned by graphs and the behavior of the invariants on its primitive space.

There is at least one more Hopf algebra possessing the same property, but at the moment we are unable to predict how to find or construct such Hopf algebras.

The talk is based on a joint work with S. Chmutov and M. Kazarian [2].

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Generic torus orbit closures in Schubert varieties

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The standard action of a torus $(\mathbb{C}^*)^n$ on $\mathbb{C}^n$ induces an action of $(\mathbb{C}^*)^n$ on the flag variety $\mathcal{F}l(\mathbb{C}^n)$. As is well-known, if a torus orbit is *generic* in $\mathcal{F}l(\mathbb{C}^n)$, then its closure is a smooth toric variety called a permutohedral variety. If a torus orbit is not generic, its closure is not necessarily smooth but normal ([1]); so any torus orbit closure in $\mathcal{F}l(\mathbb{C}^n)$ is a toric variety. Therefore we are naturally led to study toric varieties which appear as torus orbit closures in $\mathcal{F}l(\mathbb{C}^n)$.

The Schubert variety X_w associated to a permutation w on n letters is a torus invariant subvariety of $\mathcal{F}l(\mathbb{C}^n)$. In this talk, I will define a *generic* torus orbit in X_w and discuss the fan associated to its closure. It turns out that the generic torus orbit closure in X_w is not necessarily smooth and the smoothness at the fixed point w is equivalent to acyclicity of a graph associated to w. As a result, we will see that the smoothness of the generic torus orbit closure in X_w is closely related to (but not necessarily same as) the smoothness of X_w. This is joint work with Eunjeong Lee.

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New aspects of complexity theory for 3-manifolds

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We outline some points and resent results of classical complexity theory for 3-manifolds. This includes description of special spine theory and the theory of virtual manifolds.

Integrable Hamiltonian systems, naturally defined by symmetric powers of algebraic curves

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We have found k integrable Hamiltonian systems on $\mathbb{C}^{2k}$(or on $\mathbb{R}^{2k}$, if the base field is $\mathbb{R}$), naturally defined by a symmetric power $\mathrm{Sym}^k(V_g)$ of a plain hyperelliptic curve V_g of genus g. When $k = g$ the symmetric power $\mathrm{Sym}^k(V_g)$ is bi-rationally isomorphic to the Jacobian of the curve V_g and our system is equivalent to a well known Dubrovin's system which has been derived and studied in the theory of finite gap solutions (algebra-geometric integration) of the Korteweg-de-Vrise equation. In the case $k = 2$ and $g \geq 1$ we have found the coordinates in which the systems obtained and their Hamiltonians are polynomial [1]. For $k = 2,\ g = 1, 2, 3$ we present these systems explicitly as well as we discuss the problem of their integration [2]. In particular, if $k = 2,\ g > 2$ the solution of the systems is not a $2g$ periodic Abelian function.

Most of the results obtained can be easily extended to a wider class of curves, such as non-hyperelliptic and non-plain.

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We are also sure, but have not proved yet, that for $k > 2$ there are natural variables in which the systems obtained and their Hamiltonians are all polynomial.

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Algebraic and geometric properties of flag Bott-Samelson varieties and applications to representations

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Eunjeong Lee *(Korea Advanced Institute of Science and Technology),* EunjeongLee@kaist.ac.kr

The notion of flag Bott manifolds is introduced in [1] as a generalization of Bott manifold and flag variety. In this talk, we introduce the notion of flag Bott–Samelson variety as a generalization of Bott–Samelson variety and flag variety. Using a birational morphism from an appropriate Bott–Samelson variety to a flag Bott–Samelson variety, we compute Newton–Okounkov bodies of flag Bott–Samelson varieties as generalized string polytopes, which are applied to give polyhedral expressions for irreducible decompositions of tensor products of G-modules. Furthermore, we show that flag Bott–Samelson varieties are degenerated into flag Bott manifolds with higher rank torus actions which generalizes the toric degeneration result of Grossberg and Karshon of Bott-Samelson varieties to Bott manifolds. We also find the Duistermaat–Heckman measures of the moment map images of flag Bott–Samelson varieties with the torus action together with invariant closed 2-forms. This talk is based on the authors preprint [2].

The work was partially supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry os Science, ICT and Future Planning (N0. 2016R1A2B4010823).

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On the cohomology rings of Riemannian manifolds with special holonomy

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We demonstrate how by using the intersection theory to calculate the cohomology of G2-manifolds constructed by using the generalized Kummer construction. For one example we find the generators of the rational cohomology ring and describe the product structure.

Dense sphere packings: state of the art and algebraic geometry constructions

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How dense can we pack equal spheres in the Euclidean space $\mathbb{R}^N$? The question looks natural and is treated by humanity at least since the end of 16th century. The first four hundred years of research gave us the answers only in dimensions 1, 2, and 3. Quite recently, the answers for $N = 8$ and $N = 24$ — that we always presumed to be true — were proved by an elegant technique using modular forms [1], [2].

If we restrict ourselves to the easier situation when the centers of the spheres form a lattice (an additive subgroup of $\mathbb{R}^N$) the answer is known for N from 1 to 8, and, of course, for $N = 24$. Not too much either ...

We have to ask easier questions. Can we bound the density and how? Which constructions give us packings that, if not being the best, are however dense enough?

Number fields and curves over finite fields provide lovely constructions [3]. To find out their densities we need to know a lot about our algebraic geometry objects. In particular, we study their zeta-functions.

As usual, when we do not know the answer for a given N we try to look at what happens when $N \to \infty$. This time we need to understand the asymptotic behaviour of zeta-functions when the genus tends to ∞, cf. [4], [5], [6], [7].

My dream is a nice theory of limit objects such as projective limits of curves or infinite extensions of $\mathbb{Q}$, as yet we are very far from it.

Another great challenge is to construct lattice sphere packings that are denser that those given by a random construction (so-called Minkowski bound).

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Asymptotic of RSK-Correspondence and new kind of Laws of Large Numbers for Bernoulli Scheme

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The Robinson-Schensted-Knuth-correspondence (RSK - correspondence) is widely used in combinatorics, theory of symmetric functions and in finite mathematics. We will discuss the analog of RSK-correspondence for infinite schemes which was defined by Vershik-Kerov (1987) and recently studied by Sniady-Romic (2016). In particular, we proved the new type of LLN (Law of Large Numbers) for Knuth equivalence and for numeration of the lattices.

From combinatorics to geometry of polyhedra via spectral graph theory

Alexander P. Veselov *(Loughborough, UK and Moscow, Russia),* `A.P.Veselov@lboro.ac.uk`

For an arbitrary polygon consider a new one by joining the centres of the consecutive edges. Iteration of this procedure leads to a shape, which is affine equivalent to a regular polygon. This regularisation effect is usually ascribed to Comte de Buffon (1707-1788), but allegedly was known already to the Roman mosaics craftsmen.

A natural analogue of this procedure for 3-dimensional polyhedra in general degenerates, so in order to have a sensible shape we should make some assumptions about combinatorial structure of the initial polyhedron. I will explain some results in this direction from [1], using the deep results from spectral graph theory due to Colin de Verdière [2] and Lovász [3].

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On free algebras of automorphic forms

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The famous theorem of Shephard–Todd–Chevalley says that the algebra of polynomial invariants of a finite linear group Γ acting in a complex vector space V is free (i.e. is generated by algebraically independent polynomials) if an only if the group Γ is generated by (complex) reflections.

A natural infinite analogue of a finite linear group is a discrete group Γ of holomorphic transformations of a complex symmetric domain $\mathcal{D}$ with the quotient $\mathcal{D}/\Gamma$ of finite volume, acting in some homogeneous $\mathbb{C}^*$-bundle over $\mathcal{D}$. Here the domain $\mathcal{D}$ serves as an analogue of the projective space PV, while the total space of the $\mathbb{C}^*$-bundle serves as an analogue of the punctured vector space V. The analogues of polynomial invariants are automorphic forms.

A simple topological consideration due to O. Shvartsman, which is equally applicable to finite linear groups and discrete groups of holomorphic transformations, shows that the algebra of automorphic forms may be free only if the group Γ is generated by reflections. It is not difficult to see that reflections exist only in two series of symmetric domains: in the complex balls $\mathcal{B}_n = U_{1,n}/(U_1 \times U_n)$ and in $\mathcal{D}_n = O^+_{2,n}/(SO_2 \times O_n)$, the symmetric domains of type IV. Moreover, if even the group Γ is generated by reflections, the algebra of automorphic forms need not be free. In particular, for a discrete group Γ acting

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in $\mathcal{D}_n$ with non-compact quotient, the algebra of automorphic forms may be free only if $n \leq 10$ [1]; meanwhile there are many reflection groups Γ acting in $\mathcal{D}_n$ with non-compact quotient of finite volume for any n.

Leaving aside the particular case $\dim \mathcal{D} = 1$, only few examples of free algebras of automorphic forms were known until recently. Moreover, just one such example due to J. Igusa (1962) was known in dimension 3 and no examples in bigger dimensions. In [2], the speaker proved that the natural algebra of automorphic forms for the group $\Gamma_n = O^+_{2,n}(\mathbb{Z})$ is free for $n = 4, 5, 6, 7$ and determined the weights of its generators. In the present talk, these results are extended to $n = 8, 9, 10$. The weights of generators in all these cases are given in the following table.

n	Weights
4	$4, 6, 8, 10, 12$
5	$4, 6, 8, 10, 12, 18$
6	$4, 6, 8, 10, 12, 16, 18$
7	$4, 6, 8, 10, 12, 14, 16, 18$
8	$4, 6, 8, 10, 12, 12, 14, 16, 18$
9	$4, 6, 8, 10, 10, 12, 12, 14, 16, 18$
10	$4, 6, 8, 8, 10, 10, 12, 12, 14, 16, 18$

These results were obtained by means of an interpretation of the quotient $\mathcal{D}_n/\Gamma_n$ as the moduli space of a suitable family of multipolarized $K3$ surfaces.

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Simplicial James-Hopf map and decompositions of the unstable Adams spectral sequence for suspensions

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The project was carried out in the PhD thesis of Fedor Pavutnitskiy. We use combinatorial group theory methods to extend the definition of a classical James-Hopf invariant to a simplicial group setting. This allows us to realize certain coalgebra idempotents at sSet_*-level and obtain a functorial decomposition of the spectral sequence, associated with the lower p-central series filtration on the free simplicial group.

The talk will aim to general audience, starting from the introduction of basic notions and techniques on the topic.

Cyclohedron and Kantorovich-Rubinstein polytopes

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Theorem 1 ([2, Theorem 31]) *There exists a quasi-metric (asymmetric distance function) ρ on the set $[n]$ such that the associated Kantorovich-Rubinstein polytope (introduced in [3]),*

$$KR(\rho) = \mathrm{Conv}\left\{\frac{e_i - e_j}{\rho(i,j)} \mid 1 \leqslant i \neq j \leqslant n\right\}$$

is affinely isomorphic *to the dual W_n° of the cyclohedron W_n.*

A close relative of Theorem 1 is the following theorem. .

Theorem 2 *There exists a triangulation of the boundary of the $(n-1)$-dimensional type A root polytope $Root_n$ parameterized by proper faces of the $(n-1)$-dimensional cyclohedron. More explictly there exists a map $\phi_n : \partial(W_n^\circ) \to \partial(Root_n)$, inducing a piecewise linear homeomorphism of boundary spheres of polytopes W_n° and $Root_n$. The map ϕ_n sends bijectively vertices of $\partial(W_n^\circ)$ to vertices of the polytope $Root_n$, while higher dimensional faces of $Root_n$ are triangulated by images of simplices from $\partial(W_n^\circ)$.*

In the lecture we will explore topological and combinatorial consequences of these results, for example the map described in Theorem 2 defines a 'canonical' quasi-toric manifold over a cyclohedron W_n.

The work was partially supported by the Ministry of Education, Science and Technological Development of Republic of Serbia, Grant 174034.

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Invited lectures

Log terminal singularities, platonic tuples and iteration of Cox rings

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Looking at the well understood case of log terminal surface singularities, one observes that each of them is the quotient of a factorial one by a finite solvable group. The derived series of this group reflects an iteration of Cox rings of surface singularities. We extend this picture to log terminal singularities in any dimension coming with a torus action of complexity one. In this setting, the previously finite groups become solvable torus extensions, and Cox rings are defined by trinomials corresponding to platonic tuples.

This is a joint work with Lukas Braun, Jürgen Hausen, and Milena Wrobel.

All extensions of C_2 by $C_{2^n} \times C_{2^n}$ are good for the Morava K-theory

Malkhaz Bakuradze *(Faculty of Exact and Natural Sciences. Iv. Javakhishvili Tbilisi State University, Georgia),*
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This talk is concerned with analyzing the 2-primary Morava K-theory of the classifying spaces BG of the groups G in the title. In particular it answers affirmatively the question whether transfers of Euler classes of complex representations of subgroups of G suffice to generate $K(s)^*(BG)$. Here $K(s)$ denotes Morava K-theory at prime $p = 2$ and natural number $s > 1$. The coefficient ring $K(s)^*(pt)$ is the Laurent polynomial ring in one variable, $\mathbb{F}_2[v_s, v_s^{-1}]$, where $\mathbb{F}_2$ is the field of 2 elements and $deg(v_s) = -2(2^s - 1)$.

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Hirzebruch Functional Equations

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The Hirzebruch functional equation is

$$\sum_{i=1}^{n} \prod_{j \neq i} \frac{1}{f(z_j - z_i)} = c \qquad (1)$$

with constant c and initial conditions $f(0) = 0, f'(0) = 1$. It originates in the theory of Hirzebruch genera.

The Hirzebruch genus is one of the most important classes of invariants of manifolds. A series $f(z) = z + \sum_{k=1}^{\infty} f_k z^{k+1}$ with f_k in a ring R determines a Hirzebruch genus of stably complex manifolds [2]. The condition for a complex genus to be fiber multiplicative with respect to $\mathbb{C}P^{n-1}$ is given by (1).

Well-known series of solutions of (1) include the function determining the $\chi_{a,b}$ genus and the elliptic genus of level N for n divisible by N.

In the talk we present classification results for solutions of equation (1) leading to a complete classification of complex genera that are fiber multiplicative with respect to $\mathbb{C}P^{n-1}$ for $n \leqslant 6$. A topological application is effective calculation of coefficients of elliptic genera of level N for $N = 2, 3, 4, 5, 6$ in terms of solutions of a differential equation with parameters in an irreducible algebraic manifold in $\mathbb{C}^4$. This equation is

$$\begin{aligned} f(z)f'''(z) - 3f'(z)f''(z) = \\ = 6q_1 f'(z)^2 + 12q_2 f(z)f'(z) + 12q_3 f(z)^2. \quad (2) \end{aligned}$$

It is a corollary of the functional equation from [3].

This work is supported by the Russian Science Foundation, grant 14-50-00005.

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Cohomology formulae of real toric spaces

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For a simplicial complex K on $[m]$ and a mod 2 simplicial complex $\lambda\colon \mathbb{Z}_2^m \to \mathbb{Z}_2^n$, we have the associated real toric space $M^{\mathbb{R}}(K,\lambda) := \mathbb{R}\mathcal{Z}_K/\ker\lambda$.

In this talk, we provide an explicit R-cohomology ring formula of a real toric space in terms of K and Λ, where R is a commutative ring with unity in which 2 is a unit. Interestingly, it has a natural $(\mathbb{Z}\oplus\operatorname{row}\Lambda)$-grading.

Theorem 1 *There are* $(\mathbb{Z}\oplus\operatorname{row}\Lambda)$*-graded* R*-algebra isomorphisms*

$$H^*(M) \cong \bigoplus_{\omega\in\operatorname{row}\Lambda} \widetilde{H}^{*-1}(K_\omega),$$

where the product structure on $\bigoplus_{\omega\in\operatorname{row}\Lambda}\widetilde{H}^{*-1}(K_\omega)$ *is given by the canonical maps*

$$\widetilde{H}^{k-1}(K_{\omega_1})\otimes\widetilde{H}^{\ell-1}(K_{\omega_2})\to\widetilde{H}^{k+\ell-1}(K_{\omega_1+\omega_2})$$

which are induced by simplicial maps $K_{\omega_1+\omega_2}\to K_{\omega_1}\star K_{\omega_2}$ *when* $\star$ *denotes the simplicial join.*

If time allows, we also discuss about the integral cohomology of real toric space. This work is partially jointly with Hanchul Park [1] and Li Cai [2].

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Billiards within quadrics and Chebyshev type polynomials

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A comprehensive study of periodic trajectories of billiards within ellipsoids in d-dimensional Euclidean space is presented. The novelty of the approach is based on a relationship established between periodic billiard trajectories and extremal polynomials on the systems of d intervals on the real line. By leveraging deep, but yet not widely known results of the theory of generalized Chebyshev polynomials, fundamental properties of billiard dynamics are proven for any d, viz., the monotonicity of sequences of winding numbers and the injectivity of frequency maps. As a byproduct, for $d = 2$ a new proof of the monotonicity of the rotation number is obtained and the result is generalized for any d. The case study of trajectories of small periods T, $d \leq T \leq 2d$ is given. It is proven that all d-periodic trajectories are contained in a coordinate-hyperplane and that for a given ellipsoid, there is a unique set of caustics which generates $d+1$-periodic trajectories. A complete catalog of trajectories with small periods is provided for $d = 3$. This is a joint work with M. Radnović [1].

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Surgery and cell-like maps

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The main goal of surgery theory is the classification of manifolds and manifold structures. The structure set $\mathcal{S}^{CAT}(X)$ of the Poincare complex X measures the number of distinct CAT-manifolds in the simple homotopy class of X where CAT is the category. The surgery theory was initiated for $CAT = DIFF$ but it works better for $CAT = TOP$. In this talk we consider the later but then we apply our results to differentiable manifolds.

Contrary to the DIFF, in the case of topological manifolds $\mathcal{S}^{TOP}(M)$ is a group. We define a subset $\mathcal{S}^{CE}(M) \subset \mathcal{S}^{TOP}(M)$ generated by homotopy equivalences $h : N \to M$ that come as homotopy lifts of g in the diagram

$$\begin{array}{ccc} N & \xrightarrow{h} & M \\ g\downarrow & & f\downarrow \\ X & \xrightarrow{=} & X \end{array} \tag{1}$$

where f and g are cell-like maps. Quinn's theorem implies that if X is finite dimensional then h is homotopic to a homeomorphism and hence h defines a trivial element $[h] = 0 \in \mathcal{S}^{TOP}(M)$. Thus, to have $\mathcal{S}^{CE}(M) \neq \emptyset$ one needs use cell-like maps that raise dimension to infinity. Such maps were constructed in the 80s [2]. The construction is based on Edwards' theorem and results of Anderson-Hodgkin and Buchstaber-Mishchenko [1]. The important feature of the construction is that a cell-like map of a manifold can kill a K-theory class [3].

We give a complete description of $\mathcal{S}^{CE}(M)$ which is a bit technical. A special case of that is

Theorem 1 *For any manifold M the set $\mathcal{S}^{CE}(M)$ is a group. For a simply connected manifold M with finite $\pi_2(M)$ the group $\mathcal{S}^{CE}(M)$ equals the odd torsion subgroup of $\mathcal{S}^{TOP}(M)$.*

As a corollary we construct two smooth nonhomeomorphic manifolds that admit cell-like maps with the same image. We use this result to construct exotic convergence of Reimannian manifolds in the Gromov-Hausdorff moduli space.

This is a joint work with Steve Ferry and Shmuel Weinberger [4].

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Exchange classes of rectangular diagrams, Legendrian knots, and the knot symmetry group

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Rectangular diagrams are a particularly nice way to represent knots and links in the three-space. The crucial property of this presentation is the existence of a monotonic simplification algorithm for recognizing the unknot [I.D., 2006]. The present research (joint with M. Prasolov) is motivated by an attempt to extend the monotonic simplification procedure to arbitrary knot types.

Another nice feature of rectangular diagrams is their relation to Legendrian knots. Namely, each rectangular diagram defines, in a very natural way, two Legendrian knots, one with respect to the standard contact structure, and the other with respect to the mirror image of the standard contact structure. These two Legendrian knots always have an important mutual independence property [I.D., M.Prasolov, 2013], which is roughly this: any Legendrian stabilization and destabilization of each of the two Legendrian types can be done without altering the other, by applying elementary moves to the rectangular diagram.

Among elementary moves defined for rectangular diagrams, there are those that preserve both Legendrian knot types associated with the diagram. These are exchange moves. *An exchange class* is a set of rectangular diagrams that can be obtained from a fixed diagram by exchange moves.

Let K be a topological knot type, and let L_1 (respectively, L_2) be a ξ_+-Legendrian (respectively, ξ_--Legendrian) knot types of topological type K, where ξ_+ and ξ_- are the standard contact structure and its mirror image, respectively. There are *symmetry groups* G, H_1, H_2 naturally associated with K, L_1, and L_2, respectively.

Theorem 1 *In the above settings, the set of exchange classes representing* L_1 *and* L_2 *simultaneously, is in one-to-one correspondence with the set* $H_1 \backslash G / H_2$ *of double cosests.*

The proof uses, among other things, a trick from a joint work of I.Dynnikov and V.Shastin (in preparation).

The necessary definitions will be given in the talk.

On the Matveev's complexity of knot complements in thickened surfaces

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Recently, A. Akimova, S. Matveev and L. Nabeeva tabulated all prime knots in a thickened torus, presented by diagrams with up to 5 crossings, and also all prime knots in a thickened Klein bottle, presented by diagrams with up to 3 crossings. In the talk we will discuss the Matveev's complexity values for the complements of these knots and the upper bound of the complexity for complements of some infinite series of knots in a thickened torus.

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On model of Josephson effect, constrictions and transition matrix of double confluent Heun equation

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In 1973 B.Josephson received Nobel Prize for discovering a new fundamental effect concerning a *Josephson junction,* – a system of two superconductors separated by a very narrow dielectric: there could exist a supercurrent tunneling through this junction. We will discuss the model of the overdamped Josephson junction, which is given by a family of first order non-linear ordinary differential equations on two-torus:

$$\begin{cases} \dot{\phi} = -\frac{\sin\phi}{\omega} + \frac{B}{\omega} + \frac{A}{\omega}\cos\tau \\ \dot{\tau} = 1 \end{cases} \quad (1)$$

The *frequency* parameter ω is fixed; the parameters B and A are called respectively the *abscissa,* and the *ordinate.*

It is important to study the rotation number of system (1) as a function $\rho = \rho(B, A)$ and to describe the *phase-lock areas:* its level sets $L_r = \{\rho = r\}$ with non-empty interiors. They were studied by V.M.Buchstaber, O.V.Karpov, S.I.Tertychnyi. In 2010 they observed in [1] the following *quantization effect:* phase-lock areas exist only for integer values of the rotation number. It is known that each phase-lock area is a garland of infinitely many bounded domains going to infinity in the

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vertical direction; each two subsequent domains are separated by one point called *constriction*.

Conjecture 1. *All the constrictions of every phase-lock area L_r lie in its axis $\Lambda_r = \{B = r\omega\}$.*

It was proved in [2] that Conjecture 1 holds for $\omega \geq 1$, and in general, for every $\omega > 0$ all the constrictions in L_r have abscissas $B = \omega l$, $l \in \mathbb{Z}$, $l \equiv r(mod 2)$, $l \in [0, r]$.

V.M.Buchstaber and S.I.Tertychnyi observed that family (1) is equivalent to a special family of second order linear complex differential equations on the Riemann sphere with two irregular nonresonant singularities at zero and at infinity, the well-known *double confluent Heun equations*.

We present the following new result of the speaker.

Theorem 2. *Consider the non-constriction point $\mathcal{P}_r$ of intersection of the boundary of the phase-lock area L_r with its axis Λ_r with the biggest ordinate. The phase-lock area L_r contains the ray in Λ_r bounded from below by the point $\mathcal{P}_r$.*

Theorem 2 is proved via studying the Heun equation: its Stokes matrices and the transition matrix between its appropriate canonical solution bases "at zero"and "at infinity".

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Schubert calculus and quantum integrable systems

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In the talk we will describe a new feature of the classical, equivariant and quantum Schubert calculus which holds for all types of the classical Lie groups. As the main example we will use the type A Grassmanians. The usual definition of the Schubert cycles involves a choice of a parameter, namely a choice of a full flag. Studying the dependence of the construction of the Schubert cycles on these parameters in the equivariant cohomology leads to an interesting solution to the quantum Yang Baxter equation and hence connects the Schubert calculus to the theory of quantum integrable systems. In this talk we will describe the corresponding quantum integrable systems, who turn out to be two 5 vertex lattice models, in geometric representation theory terms and outline some unexpected consequences of this connection for Schubert calculus. We will also explain how the above is connected to the recent developments of modern theory of quantum groups developed by Nekrasov Shatashvilli Okounkov and Maulik.

Polytopal realizations of cluster, subword and accordion complexes and representation theory

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Associahedra are a family of polytopes appearing in different branches of mathematics. The were first introduced by D. Tamari and rediscovered by J. Stasheff in the early 1960s in the context of the associativity. The vertices of the $n-$dimensional associahedron bijectively correspond to the triangulations of the $(n+3)-$dimensional regular polygon, and the edges correspond to flips. This is one of many combinatorial descriptions of the structure of the associahedron. Later, there were introduced a lot of generalizations of this family of polytopes. I will discuss three of them, their different geometric realizations and their relation to the representation theory.

In early 2000s, S. Fomin and A. Zelevinsky introduced the notion of *cluster algebras* in order to study dual canonical bases in double Bruhat cells and the phenomenon of total positivity. A cluster algebra is defined by a *quiver*, or an oriented graph, without loops or 2-cycles.It has a set of generators, called *cluster variables*, that are grouped in overlapping subsets of a fixed cardinality, called *clusters*. Relations correspond to the operation of mutation between clusters different only in one cluster variable. One may thus study abstract simplicial complexes whose vertices correspond to cluster variables, maximal simplices correspond to clusters, and edges correspond to mutations. These are called *cluster complexes*. Fomin showed that for quivers of finite Dynkin type, these complexes are polytopal, and in type A_n the dual polytope is the $n-$dimensional associahedron. More generally, the polytopes dual to cluster complexes are called *generalized associahedra*. For any initial cluster, one may construct 2 different geometric realizations of a generalized associahedron. Their dual fans are the $d-$fan and the $g-$fan of the algebra, encoding all its algebraic structure. They also encode the so-called *wall and chamber structure* of the path algebra of the corresponing quiver. The toric variety associated to the $g-$fan is the toric degeneration of the corresponding cluster variety.

Let W be a finite Coxeter group, $S = \{s_1, \ldots, s_n\}$ be a set of simple reflections generating W. Consider a word $\mathbf{Q} := \mathbf{Q}_1 \ldots \mathbf{Q}_m$ in the alphabet of simple reflections ($\mathbf{Q}_i \in S$ $\forall\, i = 1, \ldots, m$) and an element π of the group W. *The subword complex* $\Delta(\mathbf{Q}; \pi)$ is a pure simplicial complex on the set of vertices $\{\mathbf{Q}_1, \ldots, \mathbf{Q}_m\}$ corresponding to the letters (more precisely, to their positions) in the word Q. A set of vertices yields a simplex if the complement in Q to the corresponding subword contains a reduced expression of π. The maximal simplices correspond to the complements of reduced expressions of π in the word $\mathbf{Q}$. Subword complexes were introduced by A. Knutson and E. Miller in the article [5]. They showed in [6] that $\Delta(\mathbf{Q}; \pi)$ is spherical if and only if the Demazure product of the word Q equals π; otherwise, $\Delta(\mathbf{Q}; \pi)$ is a triangulated ball. For spherical subword complexes, there arise natural questions of the existence, of the combinatorial description and of geometric realizations of their polar dual polytopes. In the group W there exists the unique longest element denoted by w_o. We will consider subword complexes of the form $\Delta(\mathbf{c}\, \mathbf{w}_o; w_o)$, where $\mathbf{c}$ is a reduced expression of a Coxeter element, $\mathbf{w}_o$ is an arbitrary reduced expression of w_o. Such complexes admit a realization by *brick polytopes* of V.Pilaud–C. Stump [7] that we will denote by $\mathbf{B}(\mathbf{c}\,\mathbf{w}_o; w_o)$. C. Ceballos, J.-P. Labbé and C. Stump [1] proved that the complexes $\Delta(\mathbf{c}\,\mathbf{w}_o(\mathbf{c}); w_o)$, where $\mathbf{w}_o(\mathbf{c})$ is the so-called $\mathbf{c}$-*sorting word* for w_o, are the generalized cluster complexes of type W. Therefore, the polytopes $\mathbf{B}(\mathbf{c}\,\mathbf{w}_o(\mathbf{c}); w_o)$ realize the $c-$associahedra of type W. The choice of a Coxeter element c is equivalent to the choice of a quiver Q being an orientation of the Coxeter diagram of the group W. The dual fan to the brick polytope realizing the generalized associahedron is the $g-$fan of the corresponding cluster algebra.

The choice of an arbitrary reduced expression $\mathbf{w}_o$ of the element w_o is equivalent to the choice of a Dyer total order on the set of positive roots of the corresponding root system Φ, which in turn is equivalent to the choice maximal green sequence of mutations of the quiver Q of a (non-necessarily linear) stability condition on the category of representations of Q over some ground field. Thus, one can introduce the notion of $(\mathbf{c}, \mathbf{w}_o)$-*stable* positive roots forming the set $\mathrm{Stab}(\mathbf{c}, \mathbf{w}_o)$, resp. stable representations. In [2] (see also [3] for more details), i prove the following theorem.

Theorem 1 (i) *The vertices of* $\Delta(\mathbf{c}\,\mathbf{w}_o; w_o)$ *and, equivalently, the facets of* $\mathbf{B}(\mathbf{c}\,\mathbf{w}_o; w_o)$ *are in a one-to-one correspondence with the simple negative and the* $(\mathbf{c}, \mathbf{w}_o)$*-stable positive roots in the system* Φ.

(ii) *Let expressions* $\mathbf{w}_o$, $\mathbf{w}'_o$ *be such that* $\mathrm{Stab}(\mathbf{c}, \mathbf{w}_o) \subset \mathrm{Stab}(\mathbf{c}, \mathbf{w}'_o)$. *Then the complex* $\Delta(\mathbf{c}\,\mathbf{w}'_o; w_o)$ *can be obtained from the complex* $\Delta(\mathbf{c}\,\mathbf{w}_o; w_o)$ *by a sequence of edge subdivisions. Similarly, a ceratin geometric realization of the polytope*

$\mathbf{B}(\mathbf{c}\,\mathbf{w}'_o; w_o)$ can be obtained from the polytope realizing $\mathbf{B}(\mathbf{c}\,\mathbf{w}_o; w_o)$ by a sequence of truncations of faces of codimension 2. In particular, for any expression, $\mathbf{B}(\mathbf{c}\,\mathbf{w}_o; w_o)$ is combinatorially equivalent to a 2−truncated cube.

The polytopes for words related by an elementary braid move of order m either coincide, or are related by a truncation of one face of codimension 2 $(m-2)$ times. This comes from the fact such an operation might change the set of stable positive roots only in one root subsystem of rank 2. We call the polytopes $\mathbf{B}(\mathbf{c}\,\mathbf{w}_o; w_o)$ *stability associahedra*. Theorem 1 implies that all the stability associahedra and, in particular, all generalized associahedra are 2−truncated cubes. The dual fans of this realization of generalized associahedra are the d−fans of cluster algebras. In a joint work in progress with Vincent Pilaud and Salvatore Stella, we work on the definition algebras whose structure is naturally encoded by these d−fans and by the g−fans given by the brick polytopes.

Theorem 1 provides a partial order on the set of reduced expressions of w_o given by the inclusion of sets of stable positive roots. In type A_n with the linear orientation, the resulting poset is isomorphic with the poset of triangulations of the cyclic polytope of dimension 3 with the 2nd (higher) Tamari-Stasheff order. In spirit of Reading's Cambrian lattices, we call it *the 2nd higher Cambrian order* of type (W, c). In [4] i show that in terms of complexes, or corresponding fans, Theorem 1 can be generalized to any acyclic quiver Q, and we get a semi-lattice of maximal green sequences. For the corresponding complexes, the order is given by the order of edge subdivisions. The choice of a maximal green sequence is equivalent to the choice of a (non-necessarily linear) stability condition with finitely many stable objects.

Another generalization of associahedra is given by polytopes dual to so-called *accordion complexes*. Their vertices correspond to certain dissections of regular polytopes, instead of triangulations. Recently, their combinatorics and geometric realizations were linked to the wall and chamber structure of gentle algebras. I will overview the progress in this area.

Theorem 1 provides a partial order on the set of maximal green sequences given by the inclusion of sets of stable positive roots. In type A_n with the linear orientation, the resulting poset is isomorphic with the poset of triangulations of the cyclic polytope of dimension 3 with the 2nd (higher) Tamari-Stasheff order. In spirit of Reading's Cambrian lattices, we call it *the 2nd higher Cambrian order* of type (W, c). In [4] i show that in terms of complexes, or corresponding fans, Theorem 1 can be generalized to any acyclic quiver Q, and we get a semi-lattice of maximal green sequences. For the corresponding complexes, the order is given by the order of edge subdivisions. The choice of a maximal green sequence is equivalent to the choice of a (non-necessarily linear) stability condition with finitely many stable objects.

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LS-category of moment-angle manifolds and Massey products

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We give various bounds for the Lusternik-Schnirelmann category of moment-angle complexes $\mathcal{Z}_K$ and show how this relates to vanishing of Massey products in $H^*(\mathcal{Z}_K)$. In particular, we characterise the Lusternik-Schnirelmann category of moment-angle manifolds $\mathcal{Z}_K$ over triangulated d-spheres K for $d \leq 2$, as well as higher dimension spheres built up via connected sum, join, and vertex doubling operations. This characterisation is given in terms of the combinatorics of K, the cup product length of $H^*(\mathcal{Z}_K)$, as well as a certain Massey products. Some of the applications include calculations of the Lusternik – Schnirelmann category and the description of conditions for vanishing of Massey products for moment-angle complexes over fullerenes and k-neighbourly complexes.

From real regular multisoliton solutions of KP-II to finite-gap solutions

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The real regular mutlisoliton solutions of the Kadomtsev-Petviashvili-II equation are parametrized by the points of totally non-negative Grassmannians. Using the Postnikov's classification of the of totally non-negative Grassmannians in terms of Le-networks we associate to each positroid cell a canonical rational reductive M-curve, and the points of the positroid cell are parametrized by real divisors on these curves satisfying the regularity conditions.

By perturbing these curves one naturally obtains real regular finite-gap solution of KP-II, which are quasipariodic structures formed by solitons. The first nontrivial example is $Gr^{\mathrm{TP}}(2,4)$, i.e. the set of points in $Gr(2,4)$ with all Plücker coordinates positive. We explicitly construct the corresponding spectral curve and its regular perturbation. The last one is a regular M-curve of genus 4.

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Combinatorial Hopf algebras and generalized permutohedra

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Hopf algebra structures naturally arise if we know how to compose and decompose combinatorial objects. In addition, a multiplicative functional gives rice to a combinatorial Hopf algebra. If we employ the formalism of combinatorial Hopf algebras we can reconstruct various old and obtain some new algebraic, enumerative combinatorial invariants.

The generalized Dehn-Sommerville relations are defined in an arbitrary combinatorial Hopf algebra and we solve these relations in the case of hypergraphs [1]. This is the first non-standard solution different from the classical that is given by eulerian posets.

The universal morphism of combinatorial Hopf algebras produces a quasisymmetric function invariant. To a variety of combinatorial objects we can associate convex polytopes which are generalized permutohedra. In general, this quasisymmetric function invariant has a geometric meaning as the enumerator function of lattice points associated to generalized permutohedra. The prominent example is the Stanley chromatic symmetric function of simple graphs which is interpreted as the enumerator function of lattice points associated to graphic-zonotopes. We studied the cases of nestohedra and matroid base polytopes [2], [3].

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Lower Bounds for the Degree of a Branched Covering of a Manifold

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Recall, that an n-fold PL branched covering $f\colon X^N \to Y^N$ of closed connected PL manifolds is any Open and PL map $X^N \to Y^N$, which is, a fortiori, finite-to-one, and, moreover, $n = \max_{y\in Y} |f^{-1}(y)| < \infty$. We consider the following

Problem (A). *Suppose X^N and Y^N are closed connected oriented PL manifolds satisfying the following 4 conditions:*

(1) For both manifolds their integral homology have no torsion;

(2) The target Y is simply-connected;

(3) There exist maps $f_m\colon X \to Y$ with $\deg(f_m) = m$ for all $m \in \mathbb{N}$;

(4) There exists a map $f\colon X \to Y$ which is a PL n-fold branched covering for some $n \geq 2$.

The problem is to find any lower bounds for the minimal n in item 4, expressed in terms of topology (cohomology or homotopy) of the manifolds X and Y.

Recall, that the rational *cup-length* $L(X)$ of a space X is defined as the greatest $m \in \mathbb{N}$ for which there exist homogeneous elements $a_1, a_2, \dots, a_m \in H^{*\geq 1}(X;\mathbb{Q})$ with nonzero product $a_1 a_2 \dots a_m \neq 0$.

Theorem 1 (I.Berstein-A.L.Edmonds'1978) *Suppose that $f\colon X^N \to Y^N$ is a PL (or TOP) n-fold branched covering of*

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closed connected orientable PL (or TOP) manifolds. Then the following estimate holds:

$$n \geq \frac{L(X)}{L(Y)}.$$

Theorem 2 (J.W.Alexander'1920) *For any closed connected oriented PL manifold X^N there exists a PL branched covering $X^N \to S^N$.*

Note that, in Alexander's construction, the degree n of such a branched covering $X^N \to S^N$ is always greater than $N!$ and increases with the number of maximal simplices of the manifold X^N.

Let us introduce the crucial notion of the *gt_n-property* (from words "group transfer"). Let $n \geq 2$ be any fixed integer. By a *tower* of graded rings (algebras) we mean a pair consisting of a graded commutative ring $A^* = \oplus_{i=0}^{\infty} A^i$ without 2-torsion and any subring B^* of A^*.

Definition 1 *We say that a tower (A^*, B^*) has the gt_n-property, if given any homogeneous elements $a_1, a_2, \ldots, a_n \in A^{*\geq 1}$, there exist homogeneous elements $b_I \in B^{*\geq 1}, I \subset \{1, 2, \ldots, n\}$, of appropriate degrees, for which the following relation holds:*

$$a_1 a_2 \ldots a_n = b_{1,2,\ldots,n} + b_{2,3,\ldots,n} a_1 + b_{1,3,4,\ldots,n} a_2 + \ldots + b_{1,2,\ldots,n-1} a_n$$

$$+ \sum_{i<j} b_{1,2,\ldots,\hat{i},\ldots,\hat{j},\ldots,n} a_i a_j + \ldots + b_1 a_2 a_3 \ldots a_n + b_2 a_1 a_3 a_4 \ldots a_n$$

$$+ \ldots + b_n a_1 a_2 \ldots a_{n-1}.$$

The following lemma is our key result (see [4]).

Lemma 1 (D.G.'2018) *Let $n \geq 2$ be any fixed integer. Suppose a graded commutative $\mathbb{Z}[1/n!]$-algebra C^* with an action of any group G (finite or infinite) and a subgroup $H \subset G$ of index n are given. Let $A^* := C^H$, and let $B^* := C^G$, so that $B^* \subset A^*$ is a tower of $\mathbb{Z}[1/n!]$-algebras. Then this tower (A^*, B^*) has the gt_n-property.*

The most nontrivial part of the work under consideration is this purely algebraic Lemma 1. Its proof requires new technics, — so called *Frobenius n-homomorphisms* of graded algebras, which was introduced and developed by the author in [3]. The theory of Frobenius n-homomorphisms of commutative ungraded algebras was created by V.M.Buchstaber and E.G.Rees in several papers starting from 1996.

Let us note, that the proof of our Lemma 1 cannot be simplified (up to the author's knowledge), even when the group G is finite and the algebra C^* is a finite-dimensional $\mathbb{Q}$-algebra.

Suppose G is a finite group, $H \subset G$ is a subgroup of index n, and W is a compact polyhedron with the simplicial action of the group G. Then the prominent **Transfer Theorem** states that the corresponding PL map $f := \pi_{G,H} \colon W/H \to W/G$ induces the monomorphism

$$f^* \colon H^*(W/G; \mathbb{Q}) \to H^*(W/H; \mathbb{Q})$$

which is, moreover, equivalent to the inclusion $H^*(W; \mathbb{Q})^G \subset H^*(W; \mathbb{Q})^H$.

For the n-fold PL branched covering $f \colon X^N \to Y^N$ of closed connected PL manifolds it can be rather easily constructed a compact polyhedron W^N with a simplicial action of the symmetric group S_n such that $X = W/S_{n-1}$, $Y = W/S_n$ and $f = \pi_{S_n, S_{n-1}}$.

The Berstein-Edmonds' estimate can be easily derived from Lemma 1. Moreover, in many cases our Lemma 1 gives much better lower bound for the n from Problem (A). For example, the following theorem holds (see [4]).

Theorem 3 (D.G.'2018) *Let $k \geq 1$ and $N \geq 4k+2$ be any fixed integers. Let $X^N := T^N$ be the N-torus, and $Y^N := S^2 \times \ldots \times S^2 \times S^{N-2k}$. Then any n-fold branched covering $f\colon X^N \to Y^N$ satisfies the condition $n \geq N - 2k$.*

Here, Berstein-Edmonds' estimate gives only $n \geq N/(k+1)$. Moreover, if we consider branched coverings in Problem (A), THE BEST lower bound for n, known before our Lemma 1, was the Berstein-Edmonds' estimate.

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Algebraic links in the Poincaré sphere and the Alexander polynomials

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The Alexander polynomial in several variables is defined for links in three-dimensional homology spheres, in particular, in the Poincaré sphere: the intersection of the surface $S = \{(z_1, z_2, z_3) \in \mathbb{C}^3 : z_1^5 + z_2^3 + z_3^2 = 0\}$ (the E_8 surface singularity) with the 5-dimensional sphere $\mathbb{S}^5_\varepsilon = \{(z_1, z_2, z_3) \in \mathbb{C}^3 : |z_1|^2 + |z_2|^2 + |z_3|^2 = \varepsilon^2\}$. An algebraic link in the Poincaré sphere is the intersection of a germ of a complex analytic curve in $(S, 0)$ with the sphere $\mathbb{S}^5_\varepsilon$ of radius ε small enough. It is well known that the Alexander polynomial in several variables of an algebraic link in the usual 3-sphere $\mathbb{S}^3_\varepsilon = \{(z_1, z_2) \in \mathbb{C}^2 : |z_1|^2 + |z_2|^2 = \varepsilon^2\}$ (that is of the intersection of a plane curve singularity $(C, 0) \subset (\mathbb{C}^2, 0)$ with $\mathbb{S}^3_\varepsilon$) determines the topological type of the link. We discuss to which extend the Alexander polynomial in several variables of an algebraic link in the Poincaré sphere determines the topology of the link. There exist analytic curves in $(S, 0)$ such that the Alexander polynomials of the corresponding links coincide, but the curves have combinatorially different resolutions. (In this case it is not clear whether or not the links are topologically equivalent.) The dual graph of the minimal resolution of the E_8 surface singularity has the standard E_8-form. We show that, if the strict transform of a curve in $(S, 0)$ does not intersect the component of the exceptional divisor of the minimal resolution

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corresponding to the end of the longest tail in the corresponding E_8-diagram, then its Alexander polynomial determines the combinatorial type of the minimal resolution of the curve and therefore the topology of the corresponding link.

Alexander polynomial of an algebraic link in the Poincaré sphere coincides with the Poincaré series of the filtration defined by the corresponding curve valuations. We show that, under conditions similar for those for curves, the Poincaré series of a collection of divisorial valuations determines the combinatorial type of the minimal resolution of the collection.

The talk is based on joint results with A. Campillo and F. Delgado

Topology of complexity one quotients

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With Susan Tolman, in the context of our classification of complexity one Hamiltonian torus actions, we describe the topology of the quotients of such actions.

Homotopy poisson brackets and thick morphisms

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For an arbitrary manifold M, consider the supermanifolds ΠTM and ΠT^*M, where Π is the parity reversion functor. The supermanifold ΠTM has an odd vector field that can be identified with the de Rham differential d; functions on it can be identified with differential forms on M. The supermanifold ΠT^*M has a canonical odd Poisson bracket $[\,,\,]$ (the antibracket); functions on it can be identified with multivector fields on M. An arbitrary even function P on ΠT^*M which obeys the master equation $[P, P] = 0$ defines an even homotopy Poisson structure on the manifold M and an odd homotopy Poisson structure (the "higher Koszul brackets") on differential forms on M. We construct a nonlinear transformation from differential forms endowed with the higher Koszul brackets to multivector fields considered with the antibracket by using the new notion of a thick morphism of supermanifolds, a notion recently introduced.
(Based on joint work with Th. Voronov.)

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Relative phantom maps

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De Bruijn-Erdős theorem [1] states that the chromatic number of an infinite graph equals the maximum of the chromatic numbers of its finite subgraphs. To a graph one associates a simplicial complex called the box complex, and the chromatic number of a graph is related with a homotopy invariant of its box complex called the index. Then one may ask whether the index of a box complex has the same property as the chromatic number stated in de Bruijn-Erdős theorem. This leads one to the relative version of a phantom map, where a phantom map [3] is a map from a CW-complex such that its restriction to any finite subcomplex is trivial.

In this talk, the triviality of a relative phantom map will be discussed, and criteria for triviality in terms of rational cohomology will be given. Then a problem on a relative phantom map coming from combinatorics will be partially solved.

This is joint work with K. Iriye and T. Matsushita [2].

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Moment-angle manifolds and linear programming

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Moment-angle manifolds have originated in [1] and studied in a greater detail in [2]. Our motivation starts with the following statement:

Lemma 1 *Any linear function on the convex polytope gives rise to a Morse function on the corresponding real moment-angle manifold (Morse-Bott in the complex case).*

So maximizing a linear function over a simple convex polytope is equivalent to maximizing some Morse function on the corresponding real moment-angle manifold.

Optimizing a linear function on a convex polytope is a very well-known problem, known as linear programming. One can use gradient descent on the moment-angle manifold to optimize a linear function on the original polytope, thus solving a linear programming problem. Convex optimization methods operating in the polytope interior are known as interior-point (or path-following) methods. They started to gain popularity with [3]; see [4].

Moment-angle manifolds are well-defined for simple polytopes which are dense in the space of all convex polytopes. Since the method is interior-point, it can be used to tackle generic convex linear problems as well.

An alternative formulation of the method is running a gradient flow on a polytope itself but using a pushforward of Riemannian metric from the moment-angle manifold to form a gradient. Riemannian metrics in the context of convex optimization have been discussed in [5].

The code for the method is available online:

- https://github.com/kustarev/malp-python (Python version);
- https://github.com/kustarev/malp-cpp (C++ version).

The code also contains examples of solving actual optimization problems: optimizing linear function on a high-dimensional simplex and solving a portfolio optimization task.

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Algebras from oriented triangulated manifolds

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First, by extracting the idea and techniques of classical cluster algebras, a class of algebras, named as *bistellar cluster algebras*, are constructed from closed oriented triangulated even-dimensional manifolds by performing middle-dimensional bistellar moves. This class of algebras exhibit the algebraic behaviour of middle-dimensional bistellar moves and do not satisfy the classical cluster algebra axiom: "every cluster variable in every cluster is exchangeable". Thus the construction of a bistellar cluster algebras has a quite difference from one of a classical cluster algebra. Next, using the bistellar cluster algebras and the techniques of combinatorial topology, we construct a direct system associated with a set of PL homeomorphic PL manifolds of dimension 2 or 4, and we then show that the limit of this direct system is a PL invariant. This is a joint work with Alastair Darby and Fang Li.

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Poincaré's rotation number in dynamics and knot theory

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Let $\varphi\colon S^1 \to S^1$ be an orientation preserving homeomorphism of the circle $S^1 = \mathbb{R}/\mathbb{Z}$, and let $\tilde{\varphi}\colon \mathbb{R} \to \mathbb{R}$ be a lift of φ. Then, for each $x \in \mathbb{R}$, the limit

$$\tau(\tilde{\varphi}) := \lim_{n\to\infty} \frac{\tilde{\varphi}^n(x)}{n}$$

exists and does not depend on the choice of x. This limit is called the *translation number* of $\tilde{\varphi}$. Considered modulo integers, it is called the *rotation number* of φ.

These invariants were first defined by Poincaré and play a significant rôle in modern dynamics [1]–[3].

It turns out that Poincaré's rotation and translation numbers have useful applications in knot theory, braid theory, the theory of mapping class groups of surfaces [4]–[9]. We will overview main concepts and results in this research area.

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Lim colim versus colim lim

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The use of $\mathbf{lim}^1$ (and in extreme cases also $\mathbf{lim}^2, \mathbf{lim}^3, \dots$) provides a reasonable description of any limiting behaviour in homology and cohomology for simplicial complexes (or CW complexes) and, on the other hand, for compact spaces. In contrast, homology and cohomology (even ordinary) of non-triangulable non-compact spaces have been very poorly understood until recently, due to the lack of any clues on how direct limits (**colim**) interact with inverse limits (**lim**). I will talk about a few first steps in this direction.

1) Milnor proved two uniqueness theorems for axiomatic (co)homology: for compacta (1960) and for infinite simplicial complexes (1961). We obtain their common generalization: the Eilenberg–Steenrod axioms along with Milnor's map excision axiom and a (non-obvious) common generalization of Milnor's two additivity axioms suffice to uniquely characterize (co)homology of Polish spaces (=separable complete metric spaces). The proof provides a combinatorial description of the (co)homology of a Polish space in terms of a simplicial (co)chain complex satisfying a symmetry of the form **lim colim** = **colim lim**.

2) A situation in which **lim** and **colim** do not commute, but their "commutator" can be computed in terms of $\mathbf{lim}^1$ and a new functor $\mathbf{lim}^1_{\mathrm{fg}}$. Namely, there are two well-known approximations of the homology of a Polish space X (which themselves do not satisfy even the Eilenberg–Steenrod axioms): "Čech homology" $qH_n(x)$ and "Čech homology with compact

supports" $pH_n(X)$. The homomorphism $pH_n(X) \to qH_n(X)$, which is a special case of the natural map **colim lim** $\to$ **lim colim**, need not be either injective (P. S. Alexandrov, 1947) or surjective (E. F. Mishchenko, 1953), but it is still unknown whether it is surjective for locally compact X. It turns out that for locally compact X, the dual map in cohomology $pH^n(X) \to qH^n(X)$ is surjective and we are able to compute its kernel. This computation has applications to embeddability of compacta in $\mathbb{R}^m$. The original map $pH_n(X) \to qH_n(X)$ is surjective and its kernel is computed if X is a "compactohedron", i.e. contains a compactum whose complement is a polyhedron.

3) A combinatorial homotopy theory associated with the axiomatic homology and cohomology. Namely, "fine shape" — a common correction of strong shape and compactly generated strong shape (which differ from each other essentially by permuting a **lim** with a **colim**) for Polish spaces, obtained by taking into account the topology on the indexing sets. For compacta, fine shape coincides with strong shape, and in general, its definition can be said to reconcile Borsuk's and Fox's approaches to shape. Both Steenrod–Sitnikov homology and Čech cohomology (the ones satisfying the axioms) are proved to be invariant under fine shape, which cannot be said of any of the previously known shape theories. In fact, for a (co)homology theory, fine shape invariance is a strong form of homotopy invariance which implies the map excision axiom.

Polynomial Lie algebras and growth rates of their subalgebras

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The notion of Lie-Reinhart algebra [1] generalises the properties of the Lie algebra $Vect^{\infty}(M)$ of vector fields on a smooth manifold M viewed as a module over the commutative algebra $C^{\infty}(M)$ of smooth functions on M.

Definition 1 *Let R be a commutative unital ring and A a commutative R-algebra. A pair $(A, \mathcal{L})$ is called a Lie-Reinhart algebra if*

1) $\mathcal{L}$ is a Lie algebra over R which acts on (the left of) A (by derivations), i.e.

$$X(ab) = X(a)b + aX(b), \forall a, b \in A, \forall X \in \mathcal{L};$$

2) $\mathfrak{g}$ is an A-module.

The pair $(A, \mathcal{L})$ must satisfy the compatibility conditions that are

$$\begin{aligned} [X, aY] = X(a)Y + a[X, Y], \forall X, Y \in \mathcal{L}, \forall a \in A; \\ (aX)(b) = a(X(b)),\ \forall a, b \in A, \forall X \in \mathcal{L}. \end{aligned} \qquad (1)$$

Victor Buchstaber proposed in [2, 3] to study a very important special case of Lie-Reinhart algebras $(A, \mathcal{L})$ that he called *polynomial Lie algebras*. In this case

1) $A = \oplus_{i \in \mathbb{Z}} A_i = R[t_1, t_2, \ldots, t_p]$ is the $\mathbb{Z}$-graded polynomial algebra over R on p variables $t_1, \ldots, t_p$;

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2) the Lie algebra $\mathcal{L} = \oplus_{i\in\mathbb{Z}}\mathcal{L}_i$ is $\mathbb{Z}$-graded;

3) $\mathcal{L}$ is a free left module of rang N over $A = R[t_1, t_2, \ldots, t_p]$ with the fixed basis $L_1, \ldots, L_N, L_i \in \mathcal{L}_{n_i}, i = 1, \ldots, N$;

4) the gradings are compatible with each other

$$A_i\mathcal{L}_j \subset \mathcal{L}_{i+j},\ \mathcal{L}_i(A_j) \subset A_{i+j},\ i, j \in \mathbb{Z}.$$

Theorem 1 *The Lie subalgebra* $\mathcal{L}ie_R(L_1, \ldots, L_N) \subset \mathcal{L}$ *generated by* $L_1, \ldots, L_N$*, grows at most polynomially.*

The slow growth of $\mathcal{L}ie_R(L_1, \ldots, L_N)$ for some important examples of polynomial Lie algebras is related to the integrabilty of the corresponding hyperbolic systems of PDE [3, 4].

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On one–point commuting difference operators of rank one

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One-point commuting difference operators of rank 1 are considered. The coefficients in such operators depend on one functional parameter, and the degrees of shift operators in difference operators are positive. These operators are studied in the case of hyperelliptic spectral curves, where the base point coincides with a point of branching. Examples of operators with polynomial and trigonometric coefficients are constructed. Operators with polynomial coefficients are embedded in differential operators with polynomial coefficients. This construction provides a new method for constructing commutative subalgebras in the first Weyl algebra. A relationship between one-point commuting difference operators of rank 1 and one-dimensional finite-gap Schrödinger operators is investigated. In particular, a discretization of the finite-gap Lamé operators is obtained.

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Circle actions on 4–manifolds

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We consider an equivariant classification of smooth actions of the circle group S^1 on oriented 4–manifolds. The problem of classification of torus and circle actions on 4-manifolds was considered in 1970s by Orlik, Raymond, Pao, Fintushel, Melvin, Parker and Yoshida. In particular, Fintushel proved that a simply connected S^1–manifold must be a connected sum of copies of S^4, $\pm CP^2$, and $S^2 \times S^2$.

Let $S(p, q)$ denote an S^1–rotation on S^4 with weights p and q. Consider an action of S^1 on CP^2:

$$[z_0 : z_1 : z_2] \to [z_0 : e^{ia\varphi} z_1 : e^{ib\varphi} z_2],\ a, b \in \mathbb{Z},\ \varphi \in [0, 2\pi].$$

Denote CP^2 with this action by $P(a, b)$ and $-CP^2$ by $Q(a, b)$.

Let R_n denote the orbit space of the following free S^1 action on $S^3 \times CP^1$:

$$((u, v), [z_0 : z_1]) \to ((e^{i\varphi} u, e^{i\varphi} v), [z_0 : e^{in\varphi} z_1]),\ |u|^2 + |v|^2 = 1.$$

Note that R_n is $S^2 \times S^2$ if n is even and the twisted S^2–bundle over S^2 if n is odd.

Denote by $R_n(k, \ell)$ the following action on R_n:

$$((u, v), [z_0 : z_1]) \to ((u, e^{ik\varphi} v), [z_0 : e^{i\ell\varphi} z_1]).$$

Now we extend Fintushel's theorem.

Theorem 1 *Any simply connected oriented 4–dimensional S^1–manifold can be represented as an equivariant connected sum of $P(a, b)$, $Q(c, d)$, $R_n(k, \ell)$, and $S(p, q)$.*

Let M^4 be an oriented manifold with a circle action. Then $F(M,S^1)$ (the fixed point set of the action) consists of isolated points $p_1,\dots,p_m$ and 2-dimensional oriented manifolds $F_1,\dots,F_k$. Denote by w_{i1},w_{i2} the S^1–representation weights at p_i and by ε_i its sign. Let n_j denote the Euler number of the normal bundle of F_j in M. Denote these numbers by W.

There are strong relations between numbers in W. The rigidity of L–genus (signature) implies the following equation:

$$\sum_{i=1}^{m}\varepsilon_i\,\frac{(z^{w_{i1}}+1)(z^{w_{i2}}+1)}{(z^{w_{i1}}-1)(z^{w_{i2}}-1)}-\sum_{j=1}^{k}\frac{4zn_j}{(z-1)^2}=\sum_{i=1}^{m}\varepsilon_i. \qquad (1)$$

(Here we assume that weights w_{i1} and w_{i2} are relatively prime.)

Let us assign to each F_j a set of $2n_j$ unit weights $\{w_{jr}\}$, $r=1,\dots,2n_j$, $w_{jr}=1$. Using L–rigidity and some simple topology, we can proved that the set of weights $\{w_{ij}\}$ can be divided into pairs $(w_{ij},w_{k\ell})$ such that $w_{ij}=w_{k\ell}$ with $i\neq k$. So we have a graph that we call a *graph of weights.*

We have proved that to every edge e of the graph of weights G_W we can associate a 2-sphere S_e in M. We denote by n_e the Euler number of the normal bundle of S_e in M. Then

$$\sum_{e\in E(G)}n_e+\sum_{j=1}^{k}n_j=3\,L(M)=p_1(M).$$

Theorem 2 *Suppose a graph G_W satisfies (1). Then there is a 4-dimensional S^1–manifold with this graph of weights.*

Taking this opportunity, I wish to thank V.M. Buchstaber, I.M. Krichever and S.P. Novikov for introducing me to this interesting area of topology.

Diagonal complexes

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(The talk is based on a joint work with J. Gordon)

Given an n-gon, the poset of all collections of pairwise non-crossing diagonals is isomorphic to the face poset of some convex polytope called *associahedron*. We replace in this setting the n-gon (viewed as a disc with n marked points on the boundary) with an arbitrary oriented surface equipped with a number of labeled marked points ("vertices"). The surface is not necessarily closed, and may contain a number of punctures. With appropriate definitions (in a sense, we mimic the construction of associahedron) we arrive at cell complexes $\mathcal{D}$ and its barycentric subdivision $\mathcal{BD}$. If the surface is closed, the complex $\mathcal{D}$ (as well as $\mathcal{BD}$) is homotopy equivalent to the space of metric ribbon graphs $RG_{g,n}^{met}$, or, equivalently, to the decorated moduli space $\widetilde{\mathcal{M}}_{g,n}$ [2], [1]. For bordered surfaces, we prove the following:

(1) Contraction of a boundary edge does not change the homotopy type of the complex.

(2) Contraction of a boundary component to a new marked point yields a forgetful map between two diagonal complexes which is homotopy equivalent to the Kontsevich's tautological circle bundle [3]. Thus, contraction of a boundary component gives a natural simplicial model for the tautological bundle. As an application, we compute the first Chern class (also its powers) in combinatorial terms. The latter result is an application of the Mnev-Sharygin local combinatorial formula [4].

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(3) In the same way, contraction of several boundary components corresponds to Whitney sum of the tautological bundles.

(4) Eliminating of a puncture gives rise to a bundle which equals to a surgery on the universal curve. In particular, the bundle incorporates at a time all the M. Kontsevich's tautological S^1-bundles.

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Smooth actions of compact Lie groups on complex projective spaces

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The goal of the talk is to present constructions of smooth actions of compact Lie groups G on complex projective spaces such that the manifold of points fixed under the action of G on the complex projective space $\mathbb{C}P^n$ in question –

$$M = \{x \in \mathbb{C}P^n \mid g \cdot x = x \text{ for all } g \in G\}$$

– is not stably almost complex,

– is stably almost complex but not almost complex,

– is almost complex but not homotopically symplectic,

– is homotopically symplectic but not symplectic,

– is symplectic but not Kähler.

We give examples of manifolds M with specific properties listed above, and we prove that the manifolds M can occur as the fixed point sets of smooth actions of compact Lie groups on complex projective spaces.

In particular, following arguments in [4], we prove that for every compact Lie group G, there exists a smooth action of G on a complex projective space $\mathbb{C}P^n$ such that the fixed point set is not a symplectic manifold and therefore, the action of G on $\mathbb{C}P^n$ is not symplectic with respect to any symplectic structure on $\mathbb{C}P^n$.

Examples of non-symplectic smooth actions on symplectic manifolds were obtained for the first time in [2], for actions of the circle S^1 on products $\mathbb{C}P^1 \times \cdots \times \mathbb{C}P^1$ and $N \times \mathbb{C}P^1$ for some 4-dimensional closed symplectic manifold N.

The article [3] presents a description of manifolds M which can occur as the fixed point sets of smooth actions of finite Oliver groups G on complex projective spaces.

The recent work [1] (now in progress) focuses on answering the question which symplectic manifolds M can occur as the fixed point sets of symplectic actions of a given compact Lie group G on specific symplectic manifolds.

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Isoperimetric inequalities for Laplace eigenvalues on the sphere and the real projective plane

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This talk contains two recent results concerning isoperimetric inequalities on the sphere and the real projective plain.

The first result (joint work with Nadirashvili) is an isoperimetric inequality for the second non-zero eigenvalue of the Laplace-Beltrami operator on the real projective plane. For a metric of unit area this eigenvalue is not greater than 20π. This value is attained in the limit by a sequence of metrics of area one on the projective plane. The limiting metric is singular and could be realized as a union of the projective plane and the sphere touching at a point, with standard metrics and the ratio of the areas $3:2$.

The second result (joint work with Karpukhin, Nadirashvili and I. Polterovich) is an isoperimetric inequality for all eigenvalues of the Laplace-Beltrami operator on the sphere. It is shown that for any positive integer k, the k-th nonzero eigenvalue of the Laplace-Beltrami operator on the two-dimensional sphere endowed with a Riemannian metric of unit area, is maximized in the limit by a sequence of metrics converging to a union of k touching identical round spheres. This proves a conjecture posed by Nadirashvili in 2002 and yields a sharp isoperimetric inequality for all nonzero eigenvalues of the Laplacian on a sphere. Earlier, the result was known only for $k = 1$ (J. Hersch, 1970), $k = 2$ (N. Nadirashvili, 2002; R. Petrides, 2014) and $k = 3$ (N. Nadirashvili and Y. Sire, 2017). In particular, it is proven that for any $k \geqslant 2$, the supremum of the k-th nonzero eigenvalue on a sphere of unit area is not attained in the class of Riemannian metrics which are smooth outsitde a finite set of conical singularities.

Path integrals on real, p-adic, and adelic spaces

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We study path integrals in ordinary, p-adic and adelic quantum mechanics for systems determined by quadratic Lagrangians. The corresponding probability amplitudes $\mathcal{K}(x'', t''; x', t')$ for two-dimensional systems with quadratic Lagrangians are found. The obtained expressions are generalized to any finite-dimensional spaces. These exact general formulas are presented in the form which is invariant under interchange of the number fields $\mathbb{R} \longleftrightarrow \mathbb{Q}_p$ and $\mathbb{Q}_p \longleftrightarrow \mathbb{Q}_{p'}$, $p \neq p'$. This invariance shows the fundamental rôle of adelic path integral in mathematical physics of quantum phenomena.

This is joint work with Branko Dragović.

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Towards integrability structure in 3D Ising model

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The Ising model is an amazing area of interaction between algebraic and geometric methods, topology and exactly solved models in statistical physics, describing among others critical phenomena.

The integrability of the 3D system is still hypothetical. In the talk we develop an algebraic interpretation [1] based on the Zamolodchikov tetrahedron equation.

The main part of the work is related to the combinatorics of the n-simplicial complex [2]. We first construct some recursion procedure on the spaces of solutions for the n-simplex equation. Then we propose such a weight matrix in 3D Ising model which satisfies an analog of the tetrahedron equation with spectral parameter. Our analog does not provide the same simple integrability property as the original one. The principal goal of this talk is to draw attention of the experts community to this phenomenon.

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Toric topology of the complex Grassmann manifolds and $(2n, k)$-manifolds

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The family of the complex Grassmann manifolds $G_{n,k}$ with the canonical action of the algebraic torus $(\mathbb{C}^*)^n$ and, consequently, the compact torus $T^n = \mathbb{T}^n$ is well known. The interest in description of these actions is motivated by the classical and modern problems of algebraic geometry, algebraic and equivariant topology, symplectic geometry and enumerative combinatorics. In the well known papers of Gel'fand-Serganova, Goresky-MacPherson, Kapranov etc, it is studied the $(\mathbb{C}^*)^n$-equivariant topology of $G_{n,k}$. In this context, there is the analogous of the moment map $\mu : G_{n,k} \to \Delta_{n,k}$ for the hypersimplex $\Delta_{n,k}$. In the case $k = 1$ we have the complex projective space $\mathbb{C}P^{n-1}$, which is the fundamental example of a toric manifold and $\Delta_{n,1}$ is a simplex. For $k \geq 2, k \neq n-1$ the combinatorics of $\Delta_{n,k}$ does not determine the structure of the orbit space $G_{n,k}/T^n$ any more.

We study the action of the compact torus T^n on $G_{n,k}$ by developing the methods of the toric geometry and the toric topology and propose the method for the description of the orbit space $G_{n,k}/T^n$. In the talk it will be presented our approach and the results related to this problem. The first non-trivial example is the orbit space $G_{4,2}/T^4$, which is the space of complexity 1. We earlier proved that $G_{4,2}/T^4$ is homeomorphic to $\partial\Delta_{4,2} * \mathbb{C}P^1$ and the proof contains the foundations of our approach. The space $G_{5,2}/T^5$, which is the non-trivial example of the space of complexity 2, is much more complicated. In this case we demonstrate our methods in more details and prove that $G_{5,2}/T^5$ is homotopy equivalent to $\partial\Delta_{5,2} * \mathbb{C}P^2$. Our methods allow to describe the orbits spaces of the other compact homogeneous spaces of positive Euler characteristic as well. We demonstrate it, in particular, for the flag manifold F_3 and prove that F_3/T^3 is homeomorphic to S^4.

The methods and the results, which aim to be discussed, represent the fundaments for our theory of $(2l, q)$-manifolds. This theory is concerned with M^{2l}-manifolds with an effective action of the torus T^q, $q \leq l$ which have the finite number of isolated fixed points, and an analogous of the moment map $\mu : M^{2l} \to P^q$, where P^q is a q-dimensional convex polytope. The theory is axiomatized by the data which generalize the structural results in the case of complex Grassmann manifolds and they allow to describe the equivariant topology of M^{2n} as well as the orbit stace M^{2n}/T^k.

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The homotopy theory of polyhedral products associated with flag complexes

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This is joint work with Taras Panov.

Polyhedral products have received considerable attention recently as they unify diverse constructions from several seemingly separate areas of mathematics: toric topology (moment-angle complexes), combinatorics (complements of complex coordinate subspace arrangements), commutative algebra (the Golod property of monomial rings), complex geometry (intersections of quadrics), and geometric group theory (Bestvina-Brady groups). In this talk we investigate the homotopy theory of polyhedral products associated to flag complexes.

Let K be a simplicial complex on the vertex set $[m] = \{1, 2, \ldots, m\}$. For $1 \leq i \leq m$, let (X_i, A_i) be a pair of pointed CW-complexes, where A_i is a pointed subspace of X_i. Let $(\underline{X}, \underline{A}) = \{(X_i, A_i)\}_{i=1}^m$ be the sequence of pairs. For each simplex $\sigma \in K$, let $(\underline{X}, \underline{A})^\sigma$ be the subspace of $\prod_{i=1}^m X_i$ defined by

$$(\underline{X}, \underline{A})^\sigma = \prod_{i=1}^{m} Y_i \qquad \text{where} \qquad Y_i = \begin{cases} X_i & \text{if } i \in \sigma \\ A_i & \text{if } i \notin \sigma. \end{cases}$$

The *polyhedral product* determined by $(\underline{X}, \underline{A})$ and K is

$$(\underline{X}, \underline{A})^K = \bigcup_{\sigma \in K} (\underline{X}, \underline{A})^\sigma \subseteq \prod_{i=1}^{m} X_i.$$

A simplicial complex K is *flag* if any set of vertices of K which are pairwise connected by edges spans a simplex.

The *flagification* of K, denoted K^f, is the minimal flag complex on the same set $[m]$ that contains K. We therefore have a simplicial inclusion $K \to K^f$. We prove the following.

Theorem 1 *Let K be a simplicial complex on the vertex set $[m]$, let K^f be the flagification of K, and let L be the simplicial complex given by m disjoint points. Let $(\underline{X}, \underline{A})^L \xrightarrow{g} (\underline{X}, \underline{A})^K \xrightarrow{f} (\underline{X}, \underline{A})^{K^f}$ be the maps of polyhedral products induced by the maps of simplicial complexes $L \longrightarrow K \longrightarrow K^f$. Then the following hold:*

(a) the map Ωf has a right homotopy inverse;

(b) the composite $\Omega f \circ \Omega g$ has a right homotopy inverse.

In particular, consider the special case when each A_i is a point. Write $(\underline{X}, \underline{*})$ for $(\underline{X}, \underline{A})$ and notice that $(\underline{X}, \underline{*})^L = X_1 \vee \cdots \vee X_m$. If K is a flag complex on the vertex set $[m]$ then the simplicial map $L \longrightarrow K$ induces a map

$$f \colon X_1 \vee \cdots \vee X_m = (\underline{X}, \underline{*})^L \longrightarrow (\underline{X}, \underline{*})^K.$$

By Theorem 1, Ωf has a right homotopy inverse. That is, $\Omega(\underline{X}, \underline{*})^K$ is a retract of $\Omega(X_1 \vee \cdots \vee X_m)$. This informs greatly on the homotopy theory of $\Omega(\underline{X}, \underline{*})^K$ since the homotopy type of $\Omega(X_1 \vee \cdots \vee X_m)$ has been well studied; in particular, when each X_i is a suspension the Hilton-Milnor Theorem gives an explicit homotopy decomposition of the loops on the wedge.

Orbits in real loci of spherical homogeneous spaces

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The talk is based on a joint work in progress with S. Cupit-Foutou.

As everyone knows, all non-degenerate quadratic forms in n variables are equivalent over $\mathbb{C}$, whereas over $\mathbb{R}$ they are distributed among $n+1$ equivalence classes, by the inertia law. This classical result is a particular case of a general phenomenon: given a homogeneous variety X for a complex algebraic group G defined over real numbers, the group of complex points $G(\mathbb{C})$ acts on $X(\mathbb{C})$ transitively, while the real Lie group $G(\mathbb{R})$ may have several (finitely many) orbits in the real locus $X(\mathbb{R})$. The problem is to classify these real orbits.

We address this problem for a particular class of homogeneous varieties, namely *spherical* homogeneous spaces. Here G is a connected reductive group. Sphericality means that a Borel subgroup $B \subset G$ acts on X with a dense open orbit. Spherical spaces compose a nice and important class of homogeneous varieties including symmetric spaces, flag varieties, etc. In our talk, we concentrate on two cases: (A) X is a symmetric space; (B) G is split over $\mathbb{R}$. Note that the space $X = GL_n/O_n$ of quadratic forms belongs to both cases.

Our approach to solving the problem in case (A) is an algebraic one based on Galois cohomology. It is quite standard that the set of $G(\mathbb{R})$-orbits in $X(\mathbb{R})$ is in a bijection with the kernel of the natural Galois cohomology map $\mathrm{H}^1(\mathbb{R}, H) \to$

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$\mathrm{H}^1(\mathbb{R}, G)$, where $H \subset G$ is the stabilizer of a base point $x_0 \in X(\mathbb{R})$. For a symmetric subgroup H, this kernel can be effectively computed.

In case (B), our approach is more geometric. We observe that the open Borel orbit $Bx_0 \subset X$ intersects each $G(\mathbb{R})$-orbit in $X(\mathbb{R})$ and the intersection is a union of finitely many open $B(\mathbb{R})$-orbits in $X(\mathbb{R})$. The open $B(\mathbb{R})$-orbits in the real locus of Bx_0 are easy to describe: they are in a bijection with the $T(\mathbb{R})$-orbits in $Z(\mathbb{R})$, where $T \subset B$ is a split maximal torus and $Z = Tx_0 \subset Bx_0$ is a so-called *Brion–Luna–Vust slice*. To understand which of these orbits glue together, we introduce an action of the Weyl group W on the set of $B(\mathbb{R})$-orbits compatible with the W-action on the set of ambient B-orbits defined by F. Knop. This construction involves the actions of minimal parabolic subgroups $P_\alpha \supset B$ and the symplectic geometry of the cotangent bundle T^*X.

The stabilizer of Bx_0 under this W-action is a semidirect product $W_X \ltimes W_L$, where the first factor is a certain crystallographic reflection group (the *little Weyl group* of X) and the second factor is a parabolic subgroup in W acting trivially on the set of open $B(\mathbb{R})$-orbits. We now come to our main result.

Theorem 1 *The orbit set $X(\mathbb{R})/G(\mathbb{R})$ is in a bijection with the set of W_X-orbits on $Z(\mathbb{R})/T(\mathbb{R})$.*

The latter set and the W_X-action can be described combinatorially.

A similar theorem holds in case (A) (with a bit different meaning of W_X and Z). This gives us a hope that Theorem 1 can be extended to arbitrary spherical homogeneous spaces defined over real numbers.

An elementary approach to Somos-4 sequences

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A sequence Somos-4 is defined by initial data s_0, s_1, s_2, s_3 and fourth-order recurrence

$$s_{n+2}s_{n-2} = \alpha s_{n+1}s_{n-1} + \beta s_n^2.$$

Usually properties of this sequence are studied by means of elliptic functions. The talk will be devoted to the new elementary approach to Somos-4 sequences. Hopefully it will be suitable for higher-rank Somos sequences corresponding to curves of higher genus.

On face numbers of flag simplicial complexes

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Let $\mathcal{K}$ be an n-dimensional simplicial complex. Denote by f_i the number of i-dimensional simplices of $\mathcal{K}$. Characterization of possible f-vectors $(f_0, \dots, f_n)$ of various classes of simplicial complexes is a classical problem of enumerative combinatorics.

Among the most well-known results in this direction are: (1) the Kruskal-Katona theorem describing all possible f-vectors of general simplicial complexes; (2) Analogue of the Kruskal-Katona theorem for Cohen-Macaulay simplicial complexes; (3) The upper bound theorem due to McMullen, which gives necessary conditions for a tuple of integers to be the f-vector of a triangulation of an n-dimensional sphere; (4) g-Theorem, characterizing the f-vectors of simplicial polytopes.

The proofs of these results led to numerous constructions, associating certain algebraic and topological objects to combinatorial objects (simplicial complexes, triangulations of spheres, polytopes, etc). These constructions allow to employ methods of homological algebra, algebraic geometry and algebraic topology in purely combinatorial problems.

Following a similar path, we derive a series of inequalities on the f-vectors of flag simplicial complexes. Our talk is built upon the results of Denham, Suciu [1] and Panov, Ray [2], where the authors relate the Poincaré series of a face ring of a flag simplicial complex to the Poincaré series of a free graded algebra. The main result can be formulated as follows.

Theorem 1 ([3, Thm. 1.1]) *Let $\mathcal{K}$ be a flag simplicial complex with f-vector $(f_0, \dots, f_n)$. Then for any $N \geq 1$ we have*

$$(-1)^N \sum_{d|N} \mu(N/d)(-1)^d p_d(\underline{\alpha}) \geq 0, \tag{1}$$

where p_d is d-th Newton polynomial expressed in elementary symmetric polynomials $\underline{\alpha} = (\alpha_1, \alpha_2, \dots)$ with

$$\alpha_n := \sum_{i=0}^{n-1} f_i \binom{n-1}{i},$$

$\mu(n)$ is the Möbius function

$$\mu(n) = \begin{cases} (-1)^k, & \text{if } n \text{ is a product of } k \text{ distinct prime factors;} \\ 0, & \text{otherwise,} \end{cases}$$

and the summation is taken over all positive divisors of N.

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Twisted Homology and Twisted Simplicial Groups

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We investigate simplicial complexes with twisted structure on vertices and small categories with twisted structure on objects. Then we introduce a twisted construction of simplicial groups for such simplicial complexes and small categories by varying faces and degeneracies from twisted data in the free product construction of simplicial groups, which gives a new construction of simplicial groups. The homotopy type of the resulting twisted simplicial groups is different from the untwisted case because of variation on faces and degeneracies. The main result determines the homotopy type of the twisted simplicial groups.

This is a joint work with Jingyan Li and Jie Wu.

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On Zeros of Yamada Polynomial for Spatial Graphs

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Polynomial invariants of knots, links, and spatial graphs are studied from various points of views. One of interesting problems is to describe the distribution of zeros of the polynomial invariant.

The mostly investigated case is the Jones polynomial for a knot. The roots of the Jones polynomial for all prime knots with at most ten crossings were computed numerically in [1], that lead to interesting observations. In [2] was shown that zeros of Jones polynomials of (pretzel) links are dense in the whole complex plane.

In [3] S. Yamada introduced a polynomial invariant for spatial graphs which is known now as Yamada polynomial. The behaviour of Yamada polynomial under replacing of an edge by a sub-diagram of a link was described in [4] for some classes of graph. Using the exact formulae of Yamada polynomial for some classes of spatial graphs we get the following result.

Theorem 1 [4] *Zeros of the Yamada polynomial for spatial graphs are dense in the following region:*

$$\Omega = \Big\{ z \in \mathbb{C} \, : \, |z + 1 + z^{-1}| \geq \min\{1, |z^3 + 2z^2 + z + 1|, |1 + z^{-1} + 2z^{-2} + z^{-3}|\} \Big\}.$$

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We will also discuss the extension of the method from [4] which leads to the proof of density of Yamada polynomial for spatial graphs in the whole complex plane.

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Microformal geometry and homotopy algebras

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I will speak about the notion of a “thick morphism”, which generalizes ordinary smooth maps. Like ordinary maps, a thick morphism induces an action on functions (pullback), but unlike the familiar case, such pullbacks are, in general, non-linear transformations. They have the form of formal non-linear differential operators and are constructed by some perturbative procedure. (Thick morphisms themselves are defined as formal canonical relations between the cotangent bundles specified by generating functions of particular type.) Being non-linear, these pullbacks cannot be algebra homomorphisms; however, their derivatives at each point turn out to be homomorphisms.

The non-linearity is a feature essential for application to homotopy bracket structures on manifolds. Roughly, “non-linearity” = “homotopy”. A thick morphism intertwining odd master Hamiltonians of two S_∞-structures (which is practically described by a Hamilton-Jacobi type equation for the generating function) induces an L_∞-morphism of the corresponding homotopy Poisson algebras. Application to homotopy Poisson structures was our primary motivation; but there are also applications to vector bundles and Lie algebroids.

There are two parallel versions: “bosonic” (for even functions) and “fermionic” (for odd functions). The bosonic version has a quantum counterpart. “Quantum pullbacks” have the form of particular Fourier integral operators. There is also an application to “quantum brackets” induced by BV-type operators.

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Converse of Smith Theory

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In 1942, P. A. Smith [3] showed that the fixed point of a p-group action on a finite $\mathbb{Z}_p$-acyclic complex is still $\mathbb{Z}_p$-acyclic. In 1971, Lowell Jones studied the converse problem and showed that any $\mathbb{Z}_p$-acyclic finite CW-complex is the fixed point of a $\mathbb{Z}_p$-action on a finite contractible CW-complex. In 1974, Robert Oliver [2] extended Jones' work to the problem that, for a given finite group G and a finite CW-complex F, whether F is the fixed point of a (semi-free or general) action of G on a finite contractible CW-complex.

We study the following problem. Suppose G is a finite group, and $f\colon F \to Y$ is a map between finite CW-complexes. Is it possible to extend F to a finite G-CW complex X satisfying $X^G = F$, and extend f to a G-map $g\colon X \to Y$ (G acts trivially on Y), such that g is a homotopy equivalence after forgetting the G-action? The work of Jones and Oliver can be regarded as the special case that Y is a point.

In case of general G-action, we find that Oliver's theory largely remains true. In case of semi-free G-action, the problem has an obstruction in K_0, and we calculate some examples.

This is a joint work with Sylvain Cappell of New York University, and Shmuel Weinbeger of University of Chicago.

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International Seminar on Toric Topology and Homotopy Theory

Iterated higher Whitehead products in topology of moment-angle complexes

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The moment-angle complex $\mathcal{Z}_{\mathcal{K}}$ is a cell complex built of produts of polydiscs and tori parametrised by simplices in a finite simplicial complex $\mathcal{K}$. The moment-angle complex is a special case of polyhedral products that are interesting in themselves. Polyhedral products provide a wonderful basis for applying the unstable homotopy theory methods.

In this talk we will study the topological structure of moment-angle complexes $\mathcal{Z}_{\mathcal{K}}$ from the point of view of iterated higher Whitehead products. Higher Whitehead products in the homotopy groups of moment-angle complexes and polyhedral products were first studied by T. Panov and N. Ray in [6]. They obtained structural results and proposed several problems, some of which will be discussed in the talk. Further important results on the structure of higher Whitehead products for special classes of

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simplicial complexes were obtained in the works of Grbic and Therialut [4], Iriye and Kishimoto [5].

Consider two classes of simplicial complexes. The first class B_Δ consists of simplicial complexes $\mathcal{K}$ for which $\mathcal{Z}_\mathcal{K}$ is homotopy equivalent to a wedge spheres. The second class W_Δ consists of $\mathcal{K} \in B_\Delta$ such that all spheres in the wedge are realized by iterated higher Whitehead products. Buchstaber and Panov asked in [2, Problem 8.4.5] if it is true that $B_\Delta = W_\Delta$.

In this talk we will show that this is not the case.

Theorem 1 *Let $\mathcal{K}$ be the simplicial complex $(\partial\Delta^2 * \partial\Delta^2) \cup \Delta^2 \cup \Delta^2$. The moment-angle complex $\mathcal{Z}_\mathcal{K}$ is homotopy equivalent to a wedge of spheres $(S^7)^{\vee 6} \vee (S^8)^{\vee 6} \vee (S^9)^{\vee 2} \vee S^{10}$, but the sphere $S^{10} \subset \mathcal{Z}_\mathcal{K}$ cannot be realized by a linear combination of iterated higher Whitehead products.*

On the other hand, we show that the class W_Δ is large enough.

Theorem 2 *Let $\mathcal{K} \in W_\Delta$. Then the simplicial complex $\mathcal{J}_n(\mathcal{K}) = (\partial\Delta^n * \mathcal{K}) \cup \Delta^n$ also belongs to W_Δ.*

Theorem 3 *If $\mathcal{K}_1, \mathcal{K}_2 \in W_\Delta$ then $\mathcal{K} = \mathcal{K}_1 \cup_I \mathcal{K}_2 \in W_\Delta$ for any common face I.*

Then using these operations we prove that there exists a simplicial complex that realizes any given iterated higher Whitehead product. Also, we describe the smallest simplicial complex that realizes the given product.

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Toric topology of balanced simplicial complex

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Stanley introduced in [4] an important class of simplicial complexes that arise often in combinatorics, topology and algebra so called balanced simplicial complexes. A n-dimensional simplicial complex K is called balanced if its set of vertices can be splitted into n disjoint subsets such that there is no two vertices spanning the edge of K and belonging to the same subset. One of central questions in combinatorics is description of all integer vectors that may appear as the face vectors of convex polytopes. The most celebrated result in this problematic is the famous g-theorem.

In the paper [2] Klee and Novik conjectured a stronger bound concerning the face numbers of balanced simplicial n-polytopes. The conjecture known as the Balanced Lower Bound Theorem is proved in [1, Theorem 1.3] ("if" part) and in [2, Theorem 5.8] ("only if" part).

Theorem 1 (Balanced Lower Bound Theorem) *Let P be a balanced simplicial n-polytope. Then*

$$\frac{h_0(P)}{\binom{n}{0}} \leq \frac{h_1(P)}{\binom{n}{1}} \leq \cdots \leq \frac{h_{[\frac{n}{2}]}}{\binom{n}{[\frac{n}{2}]}}. \tag{1}$$

The equality $\frac{h_{i-1}(P)}{\binom{n}{i-1}} = \frac{h_i(P)}{\binom{n}{i}}$ for some $i \leq \frac{n}{2}$ if and only if P has the balanced $(i-1)$-stacked property.

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The proof of the Balanced Generalized Lower Bound Theorem given by Juhnke-Kubitzke and Murai in [1] is based on the existence of so called linear system of parameters and a Lefschetz element of the Stanley-Reisner ring of simplicial polytopes. Corollaries have insightful topological meanings for canonical quasitoric manifolds which we explain in this contribution.

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The equivariant cohomology and K-theory of a cohomogeneity-one action

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We compute the Borel equivariant cohomology and equivariant K-theory of a cohomogeneity-one action of a connected, compact Lie group on a topological space M, obtaining more explicit expressions in the event M is a manifold.

The K-theoretic result requires the principal isotropy groups be connected with torsion-free fundamental group, but does not require extension to rational coefficients. Along the way we are forced to something close to a classification of the permissible systems of isotropies of such an action. We also unexpectedly obtain results regarding the Mayer–Vietoris sequence and cohomology of a mapping torus in an arbitrary multiplicative equivariant cohomology theory.

The cohomological portion of this work is joint with Oliver Goertsches, Chen He, and Liviu Mare.

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Simplicial G-Complexes and Representation Stability of Polyhedral Products

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In [2], Church and Farb introduced the theory of representation stability which generalises the classical homology stability to situations when each vector space V_m has a Σ_m-action. In this talk, we study this notion arising in toric topology. For a simplicial G-complex K, a polyhedral product $(X, A)^K$ is a G-invariant subspace of a product space. We show that the stable homotopy decomposition of $\Sigma(X, A)^K$, due to [1], is homotopy G-equivariant. Considering a sequence of simplicial Σ_m-complexes $\{K_m\}$, we study the representation stability of polyhedral products $(X, A)^{K_m}$ and state criteria on Σ_m-complexes K_m which imply the representation stability of $\{H_i((X, A)^{K_m})\}$. This is a joint work with Jelena Grbić.

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On the quasitoric bundles

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Looking back on the partial classification results of quasitoric manifolds, we notice that "bundle-type" ones are especially standing out. For example, in the classification of quasitoric manifolds with the second Betti number 2 [1], most of them are fiber bundles whose the base spaces and fibers are complex projective spaces.

In [2], I introduced a new notion called quasitoric bundle to give a precise definition of the term "bundle-type quasitoric manifold." In the more recent study of them, it is proved that the classifying space BT of a compact torus T works as the classifying space of quasitoric bundles in a certain sense. Moreover, there are some applications of this fact to the topological classification of bundle-type quasitoric manifolds.

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The cohomology rings of Hessenberg varieties and Schubert polynomials

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Let n be a positive integer. The **(full) flag variety** $\mathcal{F}\ell(\mathbb{C}^n)$ in $\mathbb{C}^n$ is the collection of nested linear subspaces $V_\bullet := (V_1 \subset V_2 \subset \ldots \subset V_n = \mathbb{C}^n)$ where each V_i is an i-dimensional subspace in $\mathbb{C}^n$. Considering a linear map $X : \mathbb{C}^n \to \mathbb{C}^n$ and a weakly increasing function $h : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\}$ satisfying $h(j) \geq j$ for $j = 1, \ldots, n$, called a **Hessenberg function**, a **Hessenberg variety** is defined by

$$\mathrm{Hess}(X, h) := \{V_\bullet \in \mathcal{F}\ell(\mathbb{C}^n) \mid XV_i \subseteq V_{h(i)} \text{ for } i = 1, \ldots, n\}.$$

Here we concentrate on Hessenberg varieties $\mathrm{Hess}(N, h)$ when $X = N$ a nilpotent matrix whose Jordan form consists of exactly one Jordan block. We define a polynomial

$$f_{i,j} := \sum_{k=1}^{j} \Big(\prod_{\ell=j+1}^{i} (x_k - x_\ell) \Big) x_k \tag{1}$$

for $1 \leq j \leq i \leq n$. Here, we take by convention $\prod_{\ell=j+1}^{i}(x_k - x_\ell) = 1$ whenever $i = j$. From the result of [1], the following isomorphism as $\mathbb{Q}$-algebras holds

$$H^*(\mathrm{Hess}(N, h); \mathbb{Q}) \cong \mathbb{Q}[x_1, \ldots, x_n]/(f_{h(1),1}, f_{h(2),2}, \ldots, f_{h(n),n}).$$

Moreover, there is a surprising connection that this presentation can be obtained from a hyperplane arrangement ([2]). The main theorem is as follows.

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Theorem 1 ([3]) *Let i, j be positive integers with $1 \leq j < i \leq n$. Then the polynomial $f_{i-1,j}$ in (1) can be written as an alternating sum of certain Schubert polynomials $\mathfrak{S}_{w_k^{(i,j)}}$:*

$$f_{i-1,j} = \sum_{k=1}^{i-j} (-1)^{k-1} \mathfrak{S}_{w_k^{(i,j)}} \tag{2}$$

where $w_k^{(i,j)}$ $(1 \leq k \leq i-j)$ is a permutation on n letters $\{1, 2, \ldots, n\}$ defined by $(s_{i-k}s_{i-k-1} \ldots s_j)(s_{i-k+1}s_{i-k+2} \ldots s_{i-1})$ using the transpositions s_r of r and $r+1$. Here, we take by convention $(s_{i-k+1}s_{i-k+2} \ldots s_{i-1}) = \mathrm{id}$ whenever $k = 1$.

We can interpret the equality (2) in Theorem 1 from a geometric viewpoint under the circumstances of having a codimension one Hessenberg variety $\mathrm{Hess}(N, h')$ in the original Hessenberg variety $\mathrm{Hess}(N, h)$.

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Homotopy types of gauge groups over high dimensional manifolds

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The homotopy types of gauge groups have been investigated by many experts in the latest twenty years. In particular, Kishimoto, Kono, Theriault, So and others have studied the gauge groups over 4-dimensional manifolds. In this talk, we will use So's decomposition methods to study the homotopy theory of gauge groups over higher dimensional manifolds. For instance, we will study the E-type gauge groups over $(n-1)$-connected $2n$-manifolds. We will further investigate other $2n$-manifolds and sphere bundles as well. A particular interesting case is about a family of 5-dimensional manifolds as the total spaces of S^1-principal bundles overs simply-connected four manifolds. We will give many homotopy decompositions of gauge groups under the mentioned cases.

The Gromov width of generalized Bott manifolds

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The Gromov width is an invariant of the symplectic manifold measuring the size of the standard ball that can be symplectically embedded. The moment polytope determines the symplectic structure on a toric manifold, hence the Gromov width. We describe an explicit formula for the Gromov width of generalized Bott manifolds in terms of the defining equations of the moment polytope. This is based on the joint work with Eunjeong Lee and Dong Youp Suh.

Khovanov homology via immersed curves in the 4-punctured sphere

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Khovanov homology $Kh^{i,j}(K)$ is a bi-graded $\mathbb{F}_2$-vector space, which is a combinatorially defined invariant of a knot. The key properties and applications of $Kh^{i,j}(K)$ are the following:

1. Graded Euler characteristic of Khovanov homology is the Jones polynomial of a knot, i.e. $\frac{1}{(t^{1/2}+t^{-1/2})}\sum_{i,j}(-1)^{i+j+1}t^{j/2}dim_{\mathbb{F}_2}(Kh^{i,j}(K)) = J_K(t)$.
2. Khovanov homology was used in the first combinatorial proof of Milnor conjecture, which states that the unknotting number of the (p,q)-torus knot is equal to $\frac{(p-1)(q-1)}{2}$.
3. Khovanov homology detects the unknot, whereas whether or not the Jones polynomial detects the unknot is an old open question.

Suppose we have a decomposition of a knot into two 4-ended tangles along a 2-sphere: $(S^3, K) = (D^3, Q) \cup_{(S^2, 4pts)} (D^3, T)$, where Q is a trivial tangle. We will describe a geometric interpretation of $dim_{\mathbb{F}_2}(Kh^{i,j}(K))$ as a minimal number of intersections of two immersed curves $L(Q), L(T)$ in the 4-punctured sphere. For 2-bridge knots we will show that the curves are natural in the sense that they are $SU(2)$ traceless representation varieties of rational tangles. Our work is aimed towards understanding of what geometric and topological information $Kh^{i,j}(K)$ contains.

Toric manifolds over an n-cube with one vertex cut

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Toric manifolds (= compact smooth toric varieties) over an n-cube are known as Bott manifolds (or Bott towers) and their topology is well studied. The blow up of Bott manifolds at a fixed point provides toric manifolds over an n-cube with one vertex and they are all projective since so are Bott manifolds. On the other hand, Oda's 3-fold, which is known as the simplest non-projective toric manifold, is over a 3-cube with one vertex cut. In this talk, we classify toric manifolds over an n-cube with one vertex cut as varieties and also as smooth manifolds. It turns out that there are many non-projective toric manifolds over an n-cube with one vertex cut (we can even count them in each dimension) but surprisingly they are all diffeomorphic.

If time permits, I will talk about toric manifolds over a product of simplices with a face cut, which is a generalization of an n-cube with one vertex cut. This work is ongoing.

This is joint work with Sho Hasui, Mikiya Masuda and Seonjeong Park.

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On the new families of flag nestohedra arising in toric topology

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It is well known that polyhedral products provide us with examples of manifolds and cellular spaces with highly nontrivial topological structure, see [2]. In particular, it was shown in [3] that for any $n \geq 2$ there exists a 2-truncated cube Q^n such that a Massey product of order n is defined and nontrivial in $H^*(\mathcal{Z}_Q)$.

In this talk we introduce a family of flag nestohedra P, one for each dimension $n \geq 2$, that has the same property and show that there exists a family $\mathcal{F}$ of moment-angle-manifolds over flag nestohedra, such that for any set $\{n_1, \ldots, n_r\}$ with $r \geq 1$ and $n_i \geq 2$ for $1 \leq i \leq r$ there is a manifold $M \in \mathcal{F}$ with the property that $H^*(M)$ has a strictly defined (i.e., it contains a unique element) nontrivial Massey product of order n_i for $1 \leq i \leq r$.

An operator d which maps a convex polytope to the disjoint union of its facets was introduced in [1]. Simple polytopes are characterized by the following formula which holds only for them:

$$F(dP) = \frac{\partial}{\partial t} F(P),$$

where $F(P) = \alpha^n + f_{n-1}\alpha^{n-1}t + \ldots + f_0 t^n$. Here $n = \dim P$ and f_k denotes the number of k-dimensional faces. Operator

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d allows us to get differential equations for generating series of families of polytopes.

Using the theory of the differential ring of simple polytopes developed in [1], we obtain the value of the boundary operator d on the family P. It turns out that d is determined by four families of flag nestohedra: P itself, permutohedra Pe, stellahedra St, and graph-associahedra P_Γ, where Γ_n consists of a complete graph on $[n] = \{1, \ldots, n\}$ vertices and the vertices $\{n\}$ and $\{n+1\}$ are linked by an edge.

Finally, we introduce an operation of duplication on building sets and determine generating series and the corresponding systems of differential equations for the result of this operation when taken on the flag nestohedra families above.

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Massey products in toric topology

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Massey products are higher cohomology operations that are often important to the study of formality of spaces, among many other applications. Moment-angle complexes, one of the main notable objects of study in toric topology, have a natural underlying combinatorial structure and it is this structure that allows us to study combinatorial obstructions to Massey products in the cohomology of the moment-angle complex. This talk will present frameworks of combinatorial operations on simplicial complexes that create non-trivial Massey products on classes of any given degree.

Homotopy decompositions of gauge groups

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Let X be a path-connected pointed topological space and G be a topological group. Given a principal G-bundle $P \to X$, the group of bundle automorphisms covering the identity on X is called the gauge group of $P \to X$. Endowed with the compact-open topology, the gauge group of $P \to X$ is homotopy equivalent to the loop space of the path component of $\mathrm{Map}(X, BG)$ containing the map that classifies the bundle [1]. Although the set of isomorphism classes of principal G-bundles over a finite CW-complex X might be infinite, there exist only finitely many distinct homotopy types among the gauge groups [2]. One approach to the homotopy classification problem of gauge groups is to obtain decompositions of the gauge groups or their loop spaces. In this talk I will present some results on homotopy decompositions of gauge groups when G is a compact connected simple Lie group.

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Projective toric manifolds and wedge operation

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The wedge operation is a classical operation defined on the set of simplicial complexes. When we know all toric manifolds over a simplicial complex K, there is a way to find all toric manifolds over simplicial complexes $K(J)$ obtained by a sequence of wedges from K, for all positive integer tuple J. It was introduced in [1] and [2].

In this talk, we are interested in the simplicial complex $\mathcal{C}(J)$, where $\mathcal{C}$ is the face complex of the dual of 3-cube with one vertex cut. We completely classify toric manifolds over $\mathcal{C}(J)$ by using the classification of toric manifolds over $\mathcal{C}$ due to [3].

We note that $\mathcal{C}$ supports the simplest non projective toric manifold known as Oda's example. We also show that, for each J, there is only one non-projective toric manifold over $\mathcal{C}(J)$. This talk is jointly with Suyoung Choi and Hanchul Park.

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Betti numbers of real toric manifolds arising from a graph

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For a graph G, a graph cubeahedron $\Box_G$ and a graph associahedron $\triangle_G$ are simple convex polytopes which admit projective smooth toric varieties. In this talk, we introduce a graph invariant, called the b-number, which computes the Betti numbers of the real toric manifold corresponding to a graph cubeahedron. The b-number is a counterpart of the notion of a-number, introduced by S. Choi and H. Park, which computes the Betti numbers of the real toric manifold corresponding to a graph associahedron, see [1]. We also show that for a forest G and its line graph $L(G)$, the real toric manifold $X^{\mathbb{R}}(\triangle_G)$ over $\triangle_G$ and the real toric manifold $X^{\mathbb{R}}(\Box_{L(G)})$ over $\Box_{L(G)}$ have the same Betti numbers. This talk is based on a joint work with B. Park and H. Park.

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Homotopical approach to group extensions and homology of morphisms

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In this talk we will review the well known result on classification of group extensions with abelian kernels by (equivalence classes of) 2-cocycles from point of view of homotopy theory of simplicial groups. This approach will allow us to extend the classical theory to homology and cohomology of group homomorphisms and obtain analogues of the above-mentioned classification theorem, together with some facts about universal extensions.

On the numbers of fullerenes

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A mathematical fullerene is a 3-dimensional simple convex polyhedron where all faces are pentagons and hexagons. The topic of the work deals with the asymptotics of the number $\Phi(n)$ of fullerenes with less than n hexagons up to combinatorial equivalence.

The dual graph of the fullerene has all its faces triangle and vertices of degree 5 or 6. The Euler formula implies that there are exactly 12 vertices of degree 5. Let us take a set of unit equilateral triangles in one-to-one correspondence with faces of the dual graph and glue some pairs of triangles in the correspondence with edges of the graph. We obtain some metric space equivalent to a sphere with a flat metric with 12 cone singularities of angle defect $\frac{\pi}{3}$. This construction gives us a map from the set of fullerenes to the space of flat metrics with cone singularities.

W.P. Thurston ([1]) proved that for every $\alpha_1, \ldots, \alpha_n \in (0, 2\pi)$ such that $\alpha_1 + \cdots + \alpha_n = 4\pi$ the moduli space $C_{\alpha_1,\ldots,\alpha_n}$ of flat metrics with cone singularities of defects $\alpha_1, \ldots, \alpha_n$ is a manifold of dimension $2(n-3)$ with a natural complex hyperbolic metric.

It is shown that the asymptotics of the number of fullerenes is $\Phi(k) \sim ck^{10}$. The result of P. Engel and P. Smillie [2] claims that the weighted sum of oriented triangulations of a sphere with $2k$ triangles and vertices of degree less or equal than 6

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is exactly $\frac{809}{2^{15}\cdot 3^{13}\cdot 5^2}\sum_{d|k} d^9$. The trivial consequence is that the constant in the asymptotics is $c = \frac{809\pi^{10}}{2^{17}\cdot 3^{18}\cdot 5^4\cdot 7\cdot 11}$.

We present another proof of the formula

$$\Phi(k) = \frac{809\pi^{10}}{2^{17}\cdot 3^{18}\cdot 5^4\cdot 7\cdot 11}k^{10} + O(k^9).$$

The approach is based on the representation of fullerenes as points in moduli space of flat metrics with cone singularities and the result of C.T. McMullen [3] counting the volume of this moduli space. Our approach gives also the asymptotics $\widehat{\Phi}_k = \frac{809\sqrt{3}\pi^9}{2^{14}.3^{18}.5^3.7}k^9 + O(k^8)$ of the number of fullerenes with two adjacent pentagons.

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Equivariant K-theory of toric orbifolds

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Toric orbifolds are topological generalization of projrctive toric varieties. We introduce some sufficient conditions on the combinatorial data associated to a toric orbifold to ensure an invariant CW-structure of the toric orbifold. In this talk I will discuss 3 different equivariant cohomology theories of toric orbifolds. This is a joint work with V. Uma.

On the coinvariant rings of pseudo-reflection groups

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Let G be a compact connected Lie group, T a maximal torus of G, and W the Weyl group of G. Then the cohomology ring (with $\mathbb{R}$-coefficients) of the flag manifold G/T is isomorphic to the coinvariant ring of W, that is, $H^*(BT)/(H^{>0}(BT)^W)$.

Kyoji Saito ([1], see also [2] and [3]) gave an alternative way to obtain the coinvariant ring. Let S denote $H^*(BT)$ ($\cong \mathrm{Sym}(\mathrm{Lie}(T)^*)$) and $\mathrm{Der}(S)$ denote the $\mathbb{R}$-module of all derivations of S (i.e. linear maps $\theta\colon S \to S$ satisfying the Leibniz's rule.) Recall that W acts on $\mathrm{Lie}(T)$ as an orthogonal reflection group, and then W determines a central hyperplane arrangement $\mathcal{A}_W$ in $\mathrm{Lie}(T)$ as the set of the kernels of reflections. Let α_H denote the (positive) root whose kernel is H. The logarithmic derivation module $D(\mathcal{A}_W)$ is defined as follows:

$$D(\mathcal{A}_W) = \{\theta \in \mathrm{Der}(S) \mid \theta(\alpha_H) \in \alpha_H S, \forall H \in \mathcal{A}_W\}.$$

Saito proved that the image of $D(\mathcal{A}_W)$ through the natural map $\mathrm{Der}(S) \to S$ coincides with the ideal of S generated by W-invariant elements of positive degrees.

When we consider unitary reflection groups W on a vector space V over $\mathbb{C}$, we also obtain a hyperplane arrangement $\mathcal{A}_W$. The most natural definition of a module $D'(\mathcal{A}_W)$ analogous to the logarithmic derivation modules for Weyl groups seems to be as follows:

$$D'(\mathcal{A}_W) = \{\theta \in \mathrm{Der}(S) \mid \theta(\alpha_H) \in \alpha_H^{r_H - 1} S, \forall H \in \mathcal{A}_W\},$$

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where α_H is a fixed linear form on V whose kernel is H and r_H is the order of the stabilizer of H. Even though researchers of hyperplane arrangements employ this definition, the image of $D'(\mathcal{A}_W)$ does not coincide with the coinvariant ring of W.

I will give a "correct" definition of $D'(\mathcal{A}_W)$ for a unitary reflection group W and prove its image is the ideal of $\mathrm{Sym}(V^*)$ generated by W-invariant elements of positive degrees.

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Stringy invariants, lattice polytopes, and combinatorial identities

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We give a combinatorial interpretation of the stringy Libgober-Wood identity in terms of generalized stringy Hodge numbers and intersection products of stringy Chern classes for arbitrary projective $\mathbb{Q}$-Gorenstein toric varieties.

As a first application we derive a novel combinatorial identity relating arbitrary-dimensional reflexive polytopes to the number 24. In an equivalent way, we extend the well-known formula for reflexive polygons including the number 12 to LDP-polygons and toric log del Pezzo surfaces, respectively. Our further application is motivated by computations of stringy invariants of nondegenerate hypersurfaces in 3-dimensional algebraic tori whose minimal models are K3-surfaces, giving rise to a combinatorial identity for the Euler number 24. Using combinatorial interpretations of the stringy E-function and the stringy Libgober-Wood identity, we show with purely combinatorial methods that this identity holds for any 3-dimensional convex lattice polytope containing exactly one interior lattice point. This talk is based on joint work with Victor Batyrev.

Obstructions to factorization of differential operators on the algebra of densities on the line

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Algebra of densities was introduced in 2004 by H. Khudaverdian and Th. Voronov in connection with Batalin-Vilkovisky geometry. It is a commutative algebra with unit and an invariant scalar product naturally associated with every manifold (and containing the algebra of functions). It gives a convenient framework to consider differential operators acting on densities of different weights simultaneously. We shall show that factorization of differential operators acting on densities on the line is different from what we know for the classical case, where factorizations always exist and their structure is known due to Frobenius's theorem. We explicitly describe the obstruction to factorization of the generalized Sturm-Liouville operator in terms of a solution of the corresponding classical Sturm-Liouville equation. See [1].

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Classification of gauge groups over 4-manifolds

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Let M be an orientable, smooth, closed 4-manifold and let G be a simple, simply-connected, compact Lie group. Given a principal G-bundle P over M with its second Chern class k, the associated *gauge group* $\mathcal{G}_k(M)$ is defined to be the group of G-equivariant automorphisms of P which fix M. Although there are infinitely many classes of principal G-bundles over M, there are only finitely many homotopy types of gauge groups over M. Over the last twenty years, topologists have been studying the homotopy types of gauge groups over 4-manifolds for many cases, especially when M is a simply-connected spin 4-manifold. In this seminar I will talk about the classification of gauge groups over simply-connected 4-manifolds and introduce my work on the cases where M is a simply-connected non-spin 4-manifold or a non-simply-connected 4-manifold.

Totally normally split quasitoric manifolds

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A closed stably complex manifold M is called a totally normally split, or TNS-manifold for short, if its stably normal vector bundle $NM \to M$ is stably isomorphic to a Whitney sum of some complex linear vector bundles. (Only topological locally trivial vector bundles will be discussed here.)

Theorem 1 ([1]) *Let M^4 be a stably complex simply connected closed 4-manifold. Then M^4 is a TNS-manifold iff the intersection form of 2-cycles of M^4 is non-definite.*

Quasitoric TNS-manifolds are simply connected. There are many diverse examples of such a family of manifolds. Among the smooth projective toric TNS-manifolds one has: any toric surface not isomorphic to $\mathbb{C}P^2$; Bott towers (towers of $\mathbb{C}P^1$-bundles); equivariant blow-up of an invariant submanifold of (complex) codimension 2 of any toric TNS-manifold. A remarkable property of quasitoric manifolds is given by the following

Theorem 2 ([2]) *Let M^{2n} be a quasitoric TNS-manifold. Then any complex vector bundle $\xi \to M$ is is stably isomorphic to a Whitney sum of complex linear vector bundles.*

There is a criterion for a quasitoric manifold M^{2n} to be TNS. For any element $\alpha \in H^{2(n-k)}(M;\mathbb{R})$ of the cohomology ring of M consider the homogeneous real k-form

$$Q_\alpha : \ H^2(M;\mathbb{R}) \to \mathbb{R}, \ x \mapsto \langle \alpha x^k, [M] \rangle,$$

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where $\langle \alpha x^k, [M] \rangle$ is the evaluation at the fundamental class of M, and $k = 1, \ldots, n$. We say that Q_α is admissible if it takes values of opposite signs as a real-valued function.

Theorem 3 ([2]) *Let M^{2n} be a quasitoric manifold. Then it is TNS iff the form Q_α is admissible for any $\alpha \in H^{2(n-k)}(M; \mathbb{R})$, $k = 1, \ldots, n$.*

Theorem 3 generalises Theorem 1 in the family of quasitoric manifolds. Using Theorem 3 one deduces

Theorem 4 ([2]) *Let M^6 be a smooth projective toric TNS-manifold. Then the respective moment polytope $P^3 \subset \mathbb{R}^3$ is a flag polytope.*

In the talk we will discuss different versions of the above TNS-criterion for a quasitoric manifold M: in terms of K-theory of M and the volume polynomial of the respective multifan of M.

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GKM-orbifolds and their equivariant cohomology rings

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If a topological space X with torus action satisfies certain conditions, then one can make use of GKM theory to compute its equivariant cohomology ring. When X is an orbifold, GKM-theory is restricted to field coefficients. In this talk, we discuss GKM theory over integer coefficients and apply this to a certain class of GKM-orbifolds. In particular, for the case when X is equipped with an half dimensional torus action, which we call them *torus orbifolds*, we introduce the notion of *weighted face ring*. It encodes the orbit space of torus action on X together with singularities. Applying GKM-theory to this class of orbifolds, we get a description of the integral equivariant cohomology rings of torus orbifolds in terms of weighted face rings. This is a joint work with Alastair Darby and Shintaro Kuroki.

Polyhedral products and commutator subgroups of right-angled Artin and Coxeter groups

Yakov A. Veryovkin *(Lomonosov Moscow State University),* verevkin_j.a@mail.com

We construct and study polyhedral product models for classifying spaces of right-angled Artin and Coxeter groups, general graph product groups and their commutator subgroups. By way of application, we give a criterion of freeness for the commutator subgroup of a graph product group, and provide an explicit minimal set of generators for the commutator subgroup of a right-angled Coxeter group.

This is a joint work with Taras Panov.

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On the geometry of regular Hessenberg varieties

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Regular Hessenberg varieties are a family of subvarieties of the full flag variety G/B. This family contains the full flag variety, Peterson variety and perutohedral variety. This subject makes connections between representation theory, combinatorics, algebraic geometry and algebraic topology. In this talk we discuss the cohomology groups of structure sheaves on regular Hessenberg varieties and the degree of regular Hessenberg varieties under their Kodaira embeddings.

Theorem 1 *Let $X = Hess(\mathcal{A}, h)$ be a regular Hessenberg variety and $h(i) \geq (i+1)$ for $1 \leq i \leq n-1$. Then*

$$H^i(X, \mathcal{O}_X) = 0, \quad \forall i \geq 1.$$

Theorem 2 *Let λ be a regular dominat weight and assume that $h(i) \geq (i+1)$ for $1 \leq i \leq n-1$. Then the Hilbert polynomial of the embedding $Hess(\mathcal{A}, h) \hookrightarrow \mathbb{P}(V_\lambda)$ does not depend on the regular matrix $\mathcal{A}$.*

This is a joint work with Hiraku Abe and Naoki Fujita.

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