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Abstracts

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TABLE OF CONTENTS

RELATING STRUCTURE AND POWER: A JUNCTION BETWEEN CATEGORICAL SEMANTICS, MODEL THEORY AND DESCRIPTIVE COMPLEXITY

SAMSON ABRAMSKY

There is a remarkable divide in the field of logic in Computer Science, between two distinct strands: one focussing on semantics and compositionality ("Structure"), the other on expressiveness and complexity ("Power"). It is remarkable because these two fundamental aspects are studied using almost disjoint technical languages and methods, by almost disjoint research communities. We believe that bridging this divide is a major issue in Computer Science, and may hold the key to fundamental advances in the field.

In this talk, we describe a novel approach to relating categorical semantics, which exemplifies the first strand, to finite model theory, which exemplifies the second. This was introduced in [1], and substantially extended in [2, 3].

Combinatorial games such as Ehrenfeucht–Fraïssé games, pebble games, and bisimulation games are widely used in finite model theory, constraint satisfaction, modal logic and concurrency theory We show how each of these types of games can be described in terms of an indexed family of comonads on the category of relational structures and homomorphisms. The index k is a resource parameter which bounds the degree of access to the underlying structure. The coKleisli categories for these comonads can be used to give syntax-free characterizations of a wide range of important logical equivalences. Moreover, the coalgebras for these indexed comonads can be used to characterize key combinatorial parameters: tree-depth for the Ehrenfeucht-Fraïssé comonad, treewidth for the pebbling comonad, and synchronization-tree depth for the modal unfolding comonad.

This approach has been extended to guarded fragments [4] and generalized quantifiers [5]. Applications to homomorphism preservation theorems are described in [6, 7], and to Lovász-type theorems on isomorphisms of relational structures in [8]. An axiomatic framework in terms of *arboreal covers* of extensional categories is developed in [9].

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DEFINABILITY IN THE TURING DEGREE STRUCTURES

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Among the most difficult and valuable problems of the computability theory definability problems occupy the most significant place.

The most important achievements in this field include the proof of the definability of the jump operator in the global Turing degree theory by Shore and Slaman [5], of the set of computably enumerable (c. e.) degrees $\mathcal{R}$ in the local Turing degree theory ($\mathcal{D}\,(\leq_T \emptyset')$) by Slaman and Woodin [6], the existence of an infinite definable set of c. e. degrees in the finite levels of the Ershov difference hierarchy by Arslanov, Kalimullin and Lempp [1], the proof of the definability of the e-jump in the enumeration degrees by Kalimullin [3], the definability of the all jump classes Low_n and $\mathrm{High}_{n-1}(n \geq 2)$ in the $\mathcal{R}$ degrees by Nies, Shore and Slaman [4], the definability of the total enumeration (e-) degrees by Cai, Ganchev, Lempp, Miller and Soskova [2].

In recent years, an intensive search for the "natural" definition for the jump operator, in particular for the degree $\mathbf{0}'$ in the global Turing degree theory, a search for natural definitions for classes of c. e. degrees, for jump classes Low_n and High_n, for the degree classes within different levels of the Ershov hierarchy in the local Turing degree theory was carried out. These studies have produced a number of encouraging results.

These problems are closely related to some other major open problems of computability theory, such as the existence of nontrivial automorphisms of structures of degrees of unsolvability.

In my talk I will provide an overview of these studies.

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HYPERSTATIONARY SETS

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For κ a regular uncountable cardinal, the unbounded subsets of κ are the positive sets with respect to the Fréchet filter on κ (i.e., the set of subsets of κ whose complement has cardinality less than κ), whereas the stationary sets are the positive sets with respect to the closed unbounded (club) filter on κ. There is a potential hierarchy of filters, extending the club filter, whose positive sets give rise to the notion of hyperstationary set (i.e. ξ-stationary for some $\xi > 1$): We say that a subset A of some limit ordinal κ is *0-stationary* if it is unbounded. For $\xi > 0$, we say that A is *ξ-stationary* if and only if for every $\zeta < \xi$, every pair of subsets S and T of κ that are ζ-stationary *simultaneously ζ-reflect* to some $\alpha \in A$, i.e., $S \cap \alpha$ and $T \cap \alpha$ are both ζ-stationary in α.

Thus, $A \subseteq \kappa$ is 0-stationary iff it is unbounded, it is 1-stationary iff it is stationary, and it is 2-stationary iff every stationary $S \subseteq \kappa$ reflects to some $\alpha \in A$, i.e., $S \cap \alpha$ is stationary in α. Writing $\mathcal{F}_\kappa^\xi$ for the set $\{X \subseteq \kappa : \kappa - X \text{ is not } \xi\text{-stationary}\}$, we have that $\mathcal{F}_\kappa^0$ is the Fréchet filter on κ, and $\mathcal{F}_\kappa^1$ is the club filter. In general, $\mathcal{F}_\kappa^\xi$, for $\xi \geq 2$, is a filter iff κ is ξ-stationary ([1, 3]).

Now it turns out that for the filters $\mathcal{F}_\kappa^\xi$, $\xi \geq 2$, to be non-trivial, large cardinals are needed. Indeed, the existence of a 2-stationary cardinal κ is equiconsistent with the existence of a weakly compact cardinal ([10]). Moreover, in the constructible universe, L, a regular cardinal κ is $(\xi+1)$-stationary iff it is Π_ξ^1-indescribable ([2,3]).

The original motivation for the introduction and study of ξ-stationary sets was the still open problem of the ordinal topological completeness of Generalized Provability Logics $\mathbf{GLP}_\xi$, for $\xi > 2$ ([4,8,9], [5,6]). The ordinal topologies $\langle \tau_\zeta : \zeta < \xi \rangle$ involved in any proof of completeness of $\mathbf{GLP}_\xi$ must be non-discrete, and the non-isolated points of the τ_ζ topology are exactly the ordinals that are ζ-stationary ([2]).

We shall discuss the intriguing connections between hyperstationary sets, large cardinals, the normality of the $\mathcal{F}_\kappa^\xi$ filters, their corresponding ordinal topologies, and the key combinatorial issues involved in the (possible) proof of ordinal topological completeness of $\mathbf{GLP}_\xi$. New interesting set-theoretical notions, such as *hypercofinalities* or *hypersquares* ([7]) come naturally out of these connections.

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ПОЛУРЕШЁТКИ ЛАХЛАНА И ПОЛУРЕШЁТКИ РОДЖЕРСА

Ю. Л. ЕРШОВ

В работах [1, 2] автор обсуждает некоторые полезные взаимоотношения между алгеброй и логикой. Здесь будет рассмотрен ещё один интересный случай. В работах Лахлана [3, 4, 5] установлены важные свойства семейств m-степеней. В работе [3] определено довольно громоздкое алгебраическое понятие, которое оказалось эквивалентным понятию дистрибутивной верхней полурешётки. В работе [5] для описания главных идеалов полурешётки рекурсивно-перечислимых m-степеней был введён объект, который позже был назван полурешёткой Лахлана. В работах С. Ю. Подзорова [11, 12] было произведено дальнейшее исследование понятия полурешётки Лахлана. В работе [4] была установлена такая

Теорема. *Для любой m-степени $\underline{a}$ существует m-степень $\underline{b}$ такая, что $\underline{a} < \underline{b}$ и для любой m-степени $\underline{c}$ такой, что $\underline{c} < \underline{b}$, справедливо $\underline{c} \leqslant \underline{a}$.*

Также аналогичная теорема была установлена о полурешётке рекурсивно перечислимых m-степеней для $\underline{a}$, не являющегося наибольшим элементом.

В работе автора [6] было установлено следующее расширение этой теоремы.

Теорема. *Для любой счётной дистрибутивной полурешётки D и её идеала I любое изоморфное вложение I на идеал полурешётки L_m всех m-степеней продолжается до изоморфного вложения D на идеал из L_m.*

Оказалось, что это свойство инъективности полурешётки L_m справедливо и для полурешётки L_F всех нумераций любого неодноэлементного конечного множества F.

Отсюда легко следует, что в предположении континуум-гипотезы любая такая полурешётка L_F изоморфна L_m.

В работе [7] Е. А. Палютина установлена справедливость этого утверждения без предположения континуум-гипотезы.

Пусть S — некоторое семейство рекурсивно-перечислимых множеств. Полурешётка L^0_S всех вычислимых нумераций семейства S называется *полурешёткой Роджерса семейства S*.

Если S — конечное семейство, то полурешётка Роджерса L^0_S имеет наибольший элемент и является полурешёткой Лахлана.

Пусть n — натуральное число, $S_n \leftrightharpoons \{\varnothing, \{0\}, \ldots, \{n\}\}$. С. Д. Денисов [10] установил следующий замечательный результат:

Теорема. *Пусть L — полурешётка Лахлана, $a \in L$. Тогда любое эффективное вложение идеала $\hat{a}$ на идеал $L^0_{S_n}$ продолжается до эффективного вложения L на идеал $L^0_{S_n}$.*

Отсюда вытекает, что для любых натуральных n и m полурешётки $L^0_{S_n}$ и $L^0_{S_m}$ изоморфны.

Полурешётки Лахлана, обладающие этим свойством, называются *универсальными.*

Пусть S и T — два конечных семейства рекурсивно перечислимых множеств. Когда их полурешётки Роджерса L^0_S и L^0_T изоморфны?

Нетрудно установить, что если частично упорядоченные множества $\langle S, \subseteq\rangle$ и $\langle T, \subseteq\rangle$ изоморфны, то и их полурешётки Роджерса также изоморфны.

Теорема Денисова показывает, что это достаточное условие не является необходимым.

Для каждого семейства рекурсивно перечислимых множеств S через S^0 обозначим семейство всех немаксимальных (по включению) множеств из S. (Так, $S^0_n = \{\varnothing\}$).

В работе [8] установлено, что изоморфизм частично упорядоченных множеств $\langle S^0, \subseteq\rangle$ и $\langle T^0, \subseteq\rangle$ является необходимым для изоморфизма их полурешёток Роджерса.

О. В. Кудинов предположил, что эти условия являются и достаточными.

До настоящего времени эта гипотеза не доказана.

Другим открытым вопросом является: Будут ли (изоморфные) полурешётки Роджерса $L^0_{S_n}$ и $L^0_{S_m}$ изоморфны и как нумерованные множества?

В работе автора [9] было получено некоторое расширение результата Денисова. Используя технику из этой работы, можно установить следующую теорему:

Теорема. *Полурешётка Роджерса $L^0_{S_\omega}$ семейства $S_\omega \leftrightharpoons \{\varnothing, \{0\}, \{1\}, \ldots\}$ является универсальной решёткой Лахлана.*

Почему работы Лахлана [4, 5] оказались опубликованы в журнале «Алгебра и логика»? Это сейчас наш журнал попал в Q1 базы Scopus. А тогда (1972) «Алгебра и логика» даже не являлся журналом (а был трудом одноимённого семинара). Думаю, что статьи

[4, 5], технически довольно сложные, не представляли широкого интереса для исследователей, которые не обладали достаточным опытом одновременно в теории вычислимости и в алгебре.

Поэтому «Алгебра и логика» оказалась наиболее подходящим местом для публикации этих замечательных статей, которые оказали огромное влияние на дальнейшие исследования.

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COMPARING Π^1_2-PROBLEMS IN COMPUTABILITY THEORY AND REVERSE MATHEMATICS

DENIS R. HIRSCHFELDT

Reverse mathematics gives us a way to compare the relative strength of theorems by establishing implications and nonimplications over a weak subsystem of second-order arithmetic, typically RCA_0, which corresponds roughly to computable mathematics. In many cases, nonimplications over RCA_0 are proved using ω-models, i.e., models of RCA_0 with standard first-order part. Implication over RCA_0 and over ω-models are not fine enough for some purposes, however, so other notions of computability-theoretic reduction between theorems have been extensively studied. These are particularly well-adapted to a class of theorems that includes a large proportion of those that have been studied in reverse mathematics: A Π^1_2-*problem* is a sentence

$$\forall X\,[\Theta(X) \rightarrow \exists Y\,\Psi(X,Y)]$$

of second-order arithmetic such that Θ and Ψ are arithmetic. The term "problem" reflects a computability-theoretic view that sees such a sentence as a process of finding a suitable Y given an X satisfying certain conditions.

This talk will discuss some of these approaches to studying the relative strength of Π^1_2-problems, focusing in particular on combinatorial examples, including work in [2, 1].

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ADVENTURES BEYOND POSSIBLE WORLDS

WESLEY H. HOLLIDAY

I will survey a recent line of research [16] on generalizations of possible world semantics, building on the "possibility semantics" of Humberstone [17] and on general algebraic semantics for nonclassical logic. So far these investigations have focused on the following areas of logic:

Boolean algebra. The standard Stone duality for Boolean algebras is non-constructive, relying on the ultrafilter principle. Inspired by work on possibility semantics [11], a choice-free analogue of Stone duality for Boolean algebras has been developed in [6].

Modal logic. Humberstone [17] originally gave possibility semantics for propositional modal logic. His framework has been generalized and systematically investigated in [10, 11, 2, 19, 20]. Possibility semantics for first-order modal logic has also been developed in [1, 9, 16].

One motivation for going beyond possible world semantics is the existence of modal logics that cannot be characterized by any class of possible world frames (Kripke incompleteness). Modal incompleteness results for even more general semantics are covered in [14].

Modal logic with propositional quantifiers. Kripke incompleteness arises in an especially natural way with more expressive languages, such as the language of modal logic with propositional quantifiers. Algebraic semantics for this language are investigated in [13, 12, 7, 8].

Intuitionistic logic. Classical possibility semantics has been generalized to "nuclear semantics" for intuitionistic logic in [3, 4, 18].

Inquisitive logic. Inquisitive logic aims to expand the purview of logic beyond the logic of statements to include the logic of questions. In [5], the choice-free Stone duality of [6] is used to give semantics for inquisitive logic on a classical base. In [15], the nuclear perspective of [4] is used to give semantics for inquisitive logic on an intuitionistic base.

In addition to an overview of this work, I will discuss several open problems and directions for future research.

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ON REFLEXIVE SUBJECTIVE PROBABILITY

LEON HORSTEN

A *reflexive* subjective probability statement is a statement in which the notion of subjective probability occurs in the scope of an occurrence of the subjective probability predicate. For example:

It seems highly likely to me that
if I am very confident that I will get the job,
I will perform well at the interview and will be offered the job.

Questions about such statements play an increasingly important role in formal epistemology.

An analogy with reflexive (or typefree) truth suggests itself. Reflexive truth has been studied intensively over the past decades, both from a proof-theoretic and from a model-theoretic perspective. In this context it is clear that we must somehow deal with the semantic paradoxes, but we have been relatively (albeit not totally!) successful with doing so. For starters, on the proof theoretic side, we have a fairly good idea of what should count as incontrovertible basic principles of typefree truth.[1]

In the case of reflexive subjective probability, analogues of the semantic paradoxes have to be confronted. But surprisingly little work has been done in this area.[2] It is at present not even very clear what the axiomatic core of a theory of reflexive subjective probability, i.e., the analogue of Kolmogorov's axioms for typefree probability, looks like. To address this question is a primary aim of my talk. Against a resulting background core system, I will then consider less elementary principles such as infinite additivity principles and probabilistic reflection principles.

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[1] See for instance [2].

[2] Exceptions are [3] and [1].

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STRICT-Π^1_1 REFLECTION: A PROOF-THEORETIC PERSPECTIVE

GERHARD JÄGER

Strict-Π^1_1 reflection is a truly remarkable principle. It has been discussed in detail, for example, in Barwise [1]. However, the focus there is on the consequences of strict-Π^1_1 reflection for generalized recursion theory, definability theory, and the model theory of infinitary languages.

In this lecture we change the perspective and look at strict-Π^1_1 reflection from the point of view of proof theory. And for doing that we introduce two environments for sets and classes:

- A "tamed" version in which the interaction between sets and classes is severely limited and, as a consequence, quantification over classes can be considered as a sort of bounded quantification.
- The "full" version in which sets and classes interact as in von Neumann-Bernays-Gödel set theory.

In both cases we identify the least Σ_1 and Π_2 models of the respective theories and clarify their relationship to Kripke-Platek set theory and power Kripke-Platek. The ordinal analysis we need to achieve that builds on and extends methods recently developed in [2].

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APPLICATIONS OF PROOF THEORY TO CORE MATHEMATICS: RECENT DEVELOPMENTS

ULRICH KOHLENBACH

In this talk we survey some recent developments in the project of applying proof-theoretic transformations to obtain new quantitative and qualitative information from given proofs in areas of core mathematics such as nonlinear analysis, convex optimization and geodesic geometry ([2, 3]). We will discuss some of the following items:

(1) The recent extraction of uniform rates of convergence for the ε-capture in the Lion-Man game in the context metric spaces X satisfying a suitable 'betweennes property' ([7]). Here a low complexity rate of convergence is extracted from a proof that made iterated use of sequential compactness arguments (i.e. arithmetical comprehension). The extraction also replaced the strong assumption of the compactness of X by its boundedness.

(2) In [1], we extracted the first explicit moduli of uniform continuity from noneffective continuity proofs for concepts of proximal maps in uniformly convex Banach spaces. It turned out, that in order to get a modulus which (as desired) is independent of the scalar involved one has to modify the previously suggested definition of such maps giving rise to a new definition.

(3) Most recently, in [8] together with P. Pinto, we analyzed proofs due to T. Suzuki which reduce viscosity generalizations of convergence proofs in optimization to the usual versions in terms of rates of convergence and metastability.

(4) In [6], it is shown how the assumption of metric regularity gives low complexity rates of convergence for algorithms which in general do not have computable rates of convergence. In the particular case of the uniqueness of the solution such an approach is used in [9] for a wide range of algorithms computing zeros of accretive set-valued operators as used for abstract Cauchy problems.

(5) An arithmetization of a highly noneffective convergence proof for the computation of so-called sunny nonexpansive retractions has recently led to effective bounds in [10] and is used in [5] to obtain metastability of a strongly convergent Halpern-type form of the proximal point algorithm in Banach spaces.

(6) A polynomial rate of convergence in Bauschke's celebrated solution of the 'zero displacement conjecture' has been extracted

from Bauschke's proof which heavily uses the machinery of maximal monotone operators ([4]). Very recently, this was much generalized to cover so-called averaged mappings by Sipoş [11].

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POTENTIALISM AND CRITICAL PLURAL LOGIC

ØYSTEIN LINNEBO

Potentialism is the view that certain types of entity are successively generated, in such a way that it is impossible to complete the process of generation. What is the correct logic for reasoning about all entities of some such type? Under some plausible assumptions, classical first-order logic has been shown to remain valid, whereas the traditional logic of plurals needs to be restricted. In this talk, I answer the open question of what is the correct plural logic for reasoning about such domains. The answer takes the form of a critical plural logic. An unexpected benefit of this new logic is that it paves the way for an alternative analysis of potentialism, which is simpler and more user-friendly than the extant modal analysis.

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ADVENTURES IN LAMBEK CALCULUS

ANDRE SCEDROV

Language and relational models, or L-models and R-models, are two natural classes of models for the Lambek calculus. Completeness w.r.t. L-models was proved by Pentus and w.r.t. R-models by Andreka and Mikulas. It is well known that adding both additive conjunction and disjunction together yields incompleteness, because of the distributive law. The product-free Lambek calculus enriched with conjunction only, however, is complete w.r.t. L-models (Buszkowski) as well as R-models (Andreka and Mikulas). The situation with disjunction turns out to be the opposite: we prove that the product-free Lambek calculus enriched with disjunction only is incomplete w.r.t. L-models as well as R-models, in the non-commutative as well as the commutative (linear) case. The derivability problem for the Lambek calculus with conjunction and disjunction is known to be decidable. Adding the explicit multiplicative unit constant changes things drastically. Namely, if we extend Lambek calculus with conjunction by certain simple rules for the multiplicative unit, sound in L-models, then the system becomes undecidable, even in the small fragment with only one implication, conjunction, and unit. In the language with the unit, the algebraic logic of all L-models is strictly included in (does not coincide with) the algebraic logic of regular L-models. This is joint work with Max Kanovich and Stepan L. Kuznetsov.

In the second part of the talk we discuss structural restrictions of linear logic modalities. Examples of such refinements are subexponentials, light linear logic, and soft linear logic. We bring together these refinements of linear logic in a non-commutative setting. We introduce a non-commutative substructural system with subexponential modalities controlled by a minimalistic set of rules. Namely, we disallow the contraction and weakening rules for the exponential modality and introduce two primitive subexponentials. One of the subexponentials allows the multiplexing rule in the style of soft linear logic and light linear logic. The second subexponential provides the exchange rule. For this system, we construct a sequent calculus, establish cut elimination, and also provide a complete focused proof system. We illustrate the expressive power of this system by simulating Turing computations and categorial grammar parsing for compound sentences. Using the former, we prove undecidability results. The new system employs Lambek's non-emptiness restriction, which is incompatible with the standard (sub)exponential setting. Lambek's restriction is crucial for applications in linguistics: without this restriction, categorial grammars incorrectly mark some ungrammatical phrases as being correct. This is joint work with Max Kanovich, Stepan L. Kuznetsov, and Vivek Nigam.

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REASONING ABOUT EPISTEMIC SUPERIORITY

SONJA SMETS

Dynamic-epistemic logics and temporal-epistemic logics have been used as a fruitful basis to model various interactive scenarios that involve the change in epistemic attitudes of communicating agents. While these systems are excellent for the purpose of modelling several communication-based scenarios, the downside is that they require us to make explicit all the specific sentences that are being communicated. This level of specification can be too strong for several applications. In particular when we aim to model scenarios in which agents communicate 'all they know' (by e.g. giving access to one's information database to all or some of the other agents), as well as more complex informational events, such as hacking. In these cases we assume that some agent(s) instantly 'read' all the information stored at a specific source.

Modelling such scenarios requires us to extend the framework of epistemic logics to one in which we abstract away from the specific announcement and formalize directly the action of sharing 'all you know' (with some or all of the other agents). In order to do this, we introduce these sharing 'all you know'-actions and formalize their effect, i.e. the state of affairs in which one agent (or group of agents) has *epistemic superiority* over another agent (or group). Concrete we capture the epistemic superiority of agents by enriching the language with comparative epistemic assertions for individual and groups of agents (as such extending the comparison-types considered in [5]).

Another ingredient that we add to our logical system, is a new modal operator for 'common distributed knowledge', used to model situations in which we achieve common knowledge in a larger group of agents by information-sharing only within each of the subgroups. This new concept of 'common distributed knowledge' combines features of both common knowledge and distributed knowledge. We position this work in the context of other known work related to: the problem of converting distributed knowledge into common knowledge via acts of sharing [4]; the more semantic approach in [2] on communication protocols requiring agents to "tell everybody all they know"; the work on public sharing events with a version of common distributed knowledge in [1]; and the work on resolution actions in [6].

In this presentation I will focus on the above described tools to reason about epistemic superiority and common distributed knowledge, which have led to completely axiomatized and decidable logical systems. This work is fully based on recent joint work with A. Baltag in [3].

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AUTOMORPHISM GROUPS OF HOMOGENEOUS STRUCTURES

KATRIN TENT

The relationship between geometric structures and their automorphism groups has been the focus of Klein's *Erlanger Programm* postulated in 1872. In the meantime there has been a wide range of research in the spirit of the program, not just in geometric structures but also with respect to the automorphism groups of other structures. The relationship between structures and their automorphism groups has also given rise to many interesting model theoretic questions.

There are a number of important examples where the automorphism group of a structure and the structure itself carry exactly the same information, in the sense that one can be recovered from the other without any loss of detail. This is, for example, the case for projective spaces and their automorphism groups, but can also be detected in many other settings. In the model theoretic sense this can often be expressed as a bi-interpretation between the automorphism group and the underlying structure, see e.g. [7].

Of particular interest are the automorphism groups of homogeneous structures, which often arise from the model theoretic construction known as the Fraïssé limit. These constructions often lead to ω-categorical structures, i.e. structures which have a unique countable model up to isomorphism. In this case, the connection between the structure and its automorphism group is also reflected in the well-known result due to Coquand stating that two ω-categorical structures are bi-interpretable if and only if their automorphism groups are isomorphic as topological groups, where a basis of the topology for such an automorphism group is given by pointwise stabilizers of finite sets, turning these automorphism groups into polish groups.

This characterization raises the question how difficult it is (in the sense of Borel reducibility) to detect whether two such structures have isomorphic automorphism groups. In joint work with Nies and Schlicht [6] we use the concept of a coarse group to show that the isomorphism relation for oligomorphic subgroups of S_∞ is Borel reducible to a Borel equivalence relation with all classes countable.

In a different direction it can be noted that the automorphism groups of very homogeneous structures are often simple groups or have very few (natural) normal subgroups, see e.g. [5, 8, 9, 1, 3, 2]. From the model theoretic perspective the simplicity of an automorphism group can often be deduced from the existence of a notion of independence, very similar to the one studied in stability theory. While many of the structures to which this setting applies are far from stable, the existence of a stationary independence relation often sheds new light on its automorphism group. This is particularly visible in the case of the Urysohn space, or variations thereof, random graphs, etc.

In my talk I will give a survey of some results relating to these questions.

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SYMMETRIC PROPERTIES AND BOOLEAN COMPLEXITY

ALASDAIR URQUHART

A property of sets of CNF formulas is *symmetric* if it is preserved under complementation of variables. Let CNF^{Δ} be a formulation of CNF where distinct formulas have disjoint sets of variables, and SAT^{Δ} the satisfiable formulas in CNF^{Δ}. The main theorem says that no symmetric property of sets of CNF formulas can force an algorithm for SAT^{Δ} to take superlinear time. This result is based on observations of Ryan Williams from 2010.

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LÖB'S PRINCIPLE FOR PAIR THEORIES

ALBERT VISSER

Can we eliminate the various design choices from the statement of the Second Incompleteness Theorem? What makes a coordinate-free version indeed a version of Second Incompleteness Theorem?

There are various approaches to the coordinate-free treatment of the Second Incompleteness Theorem. In this talk, we will zoom in on one such approach. (Some other, but closely related approaches are pursued in [4] and [5].)

Consider a recursively enumerable sequential theory U with full induction for a designated interpretation of number theory N. We can define a big Kripke model (a 'Kripke Universe') $\mathfrak{M}$ with as nodes models of U, such that necessity in this model coincides with arithmetised provability in U relativised to N. (See [2], [3], and [1] for some of the ingredients of the result.) The definition of the accessibility relation of $\mathfrak{M}$ is coordinate-free in the sense that it does not require design choices connected to arithmetisation. We call necessity in $\mathfrak{M}$: *p-validity*. Thus, the equivalence of p-validity with arithmetised provability can be considered as *p-validity elimination*.

If we drop the demand that we have full induction on some interpretation of number theory N, we loose p-validity elimination. So, it would seem that, sadly, the idea of using the big model for a coordinate-free treatment of the Second Incompleteness Theorem goes down the drain. But let's not be hasty. I will argue that, as long as we are aiming at the Second Incompleteness Theorem, there is a modified result that may still count as a coordinate-free treatment. What is more, I will argue that we can view the apparent bug as a feature.

Using an idea of Fedor Pakhomov, we can employ p-validity to prove a version of the Second Incompleteness Theorem for Pair Theories —a place where (full) arithmetisation cannot go. We will sketch an argument that shows that we even get Löb's Principle for non-modal sentences in the case of pair theories.

The full logic of p-validity is currently unknown both for pair theories and for sequential theories.

Acknowledgements. I am grateful to Fedor Pakhomov for sharing his insights on these matters.

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