

Bilateral conference

*Integrable Systems and Geometry*

December 20–22, 2021

Web-site: <http://shanghai-moscow-2021.mi-ras.ru/>

Organizers

- Steklov Mathematical Institute of Russian Academy of Sciences
- Steklov International Mathematical Center
- N.N.Bogolyubov Laboratory of Geometric Methods in Mathematical Physics, Lomonosov Moscow State University
- Wu Wen-Tsun Center of Mathematical Sciences at Shanghai Jiao Tong University

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How to connect

The conference will be held online via Zoom. To receive the link to the Zoom meeting, please, register at [https://zoom.us/meeting/register/tJArf--hqDMjHdI\\_wqrR0wcH9tSFwb1Dj11T](https://zoom.us/meeting/register/tJArf--hqDMjHdI_wqrR0wcH9tSFwb1Dj11T).

## INVITED TALKS

- Gregory Chechkin (Lomonosov Moscow State University)  
*On the Bojarskii–Meyers estimates*  
Scheduled for Dec. 21, 13:10 (Moscow) / 18:10 (Shanghai)
- Petr Grinevich (Steklov Mathematical Institute of Russian Academy of Sciences), Paolo Santini (University of Roma “La Sapienza”)  
*The linear instability of the Akhmediev breather, the “missed” modes and the regular approach*  
Scheduled for Dec. 20, 13:10 (Moscow) / 18:10 (Shanghai)
- Yuan Lou (Shanghai Jiao Tong University)  
*On principal eigenvalues for time-periodic parabolic operators*  
Scheduled for Dec. 20, 10:00 (Moscow) / 15:00 (Shanghai)
- Oleg Mokhov (Lomonosov Moscow State University)  
*Non-flat Frobenius manifolds: geometry and integrability*  
Scheduled for Dec. 22, 11:00 (Moscow) / 16:00 (Shanghai)
- Maxim Pavlov (P. N. Lebedev Physical Institute of the Russian Academy of Sciences)  
*Nondiagonalizable hydrodynamic type systems integrable by Tsarev’s generalised hodograph method*  
Scheduled for Dec. 21, 11:00 (Moscow) / 16:00 (Shanghai)
- Oleg Sheinman (Steklov Mathematical Institute of Russian Academy of Sciences)  
*Hitchin systems on hyperelliptic curves: separation of variables*  
Scheduled for Dec. 22, 10:00 (Moscow) / 15:00 (Shanghai)
- Dmitry Treshev (Steklov Mathematical Institute of Russian Academy of Sciences)  
*On isochronous dynamics*  
Scheduled for Dec. 20, 11:00 (Moscow) / 16:00 (Shanghai)
- Kaizhi Wang (Shanghai Jiao Tong University)  
*Weak KAM approach to first-order mean field games*  
Scheduled for Dec. 20, 12:10 (Moscow) / 17:10 (Shanghai)
- Yaguang Wang (Shanghai Jiao Tong University)  
*On Prandtl Boundary Layers*  
Scheduled for Dec. 22, 13:10 (Moscow) / 18:10 (Shanghai)
- Chunjing Xie (Shanghai Jiao Tong University)  
*Analysis of steady solutions for the incompressible Euler system in an infinitely long nozzle*  
Scheduled for Dec. 22, 12:10 (Moscow) / 17:10 (Shanghai)
- Guofu Yu (Shanghai Jiao Tong University)  
*Christoffel transformations for (partial-)skew-orthogonal polynomials and applications in integrable systems*  
Scheduled for Dec. 21, 10:00 (Moscow) / 15:00 (Shanghai)
- Xiang Zhang (Shanghai Jiao Tong University)  
*Regularity and generic divergence of local first integrals for analytic systems*  
Scheduled for Dec. 21, 10:00 (Moscow) / 17:10 (Shanghai)

## SCHEDULE

Moscow time	Talk	Shanghai time	Chairman
Monday, Dec. 20			
9:45–10:00	Opening	14:45–15:00	
10:00–10:50	Yuan Lou <i>On principal eigenvalues for time-periodic parabolic operators</i>	15:00–15:50	T. Ratiu
11:00–11:50	Dmitry Treschev <i>On isochronous dynamics</i>	16:00–16:50	
12:10–13:00	Kaizhi Wang <i>Weak KAM approach to first-order mean field games</i>	17:10–18:00	D. Treschev
13:10–14:00	Petr Grinevich, Paolo Santini <i>The linear instability of the Akhmediev breather, the “missed” modes and the regular approach</i>	18:10–19:00	
Tuesday, Dec. 21			
10:00–10:50	Guofu Yu <i>Christoffel transformations for (partial-)skew-orthogonal polynomials and applications in integrable systems</i>	15:00–15:50	Yuan Lou
11:00–11:50	Maxim Pavlov <i>Nondiagonalizable hydrodynamic type systems integrable by Tsarev’s generalised hodograph method</i>	16:00–16:50	
12:10–13:00	Xiang Zhang <i>Regularity and generic divergence of local first integrals for analytic systems</i>	17:10–18:00	D. Millionschikov
13:10–14:00	Gregory Chechkin <i>On the Bojarskii–Meyers estimates</i>	18:10–19:00	
Wednesday, Dec. 22			
10:00–10:50	Oleg Sheinman <i>Hitchin systems on hyperelliptic curves: separation of variables</i>	15:00–15:50	V. Buchstaber
11:00–11:50	Oleg Mokhov <i>Non-flat Frobenius manifolds: geometry and integrability</i>	16:00–16:50	
12:10–13:00	Chunjing Xie <i>Analysis of steady solutions for the incompressible Euler system in an infinitely long nozzle</i>	17:10–18:00	Congming Li
13:10–14:00	Yaguang Wang <i>On Prandtl Boundary Layers</i>	18:10–19:00	

## ABSTRACTS

*On the Bojarskii–Meyers estimates*

**Gregory Chechkin**

Lomonosov Moscow State University

In the talk, we consider boundary value problems for second-order elliptic equations in domains with micro-inhomogeneities depending on a small parameter, in the vicinity of the boundary. For the solutions of these problems, it has been proved the higher integrability of the gradient of the solution. Such inequalities will be used for a priori estimates of solutions and for estimations of the rate of convergence as a small parameter tends to zero.

Scheduled for Dec. 21, 13:10 (Moscow) / 18:10 (Shanghai)

*The linear instability of the Akhmediev breather, the “missed” modes and the regular approach*

**Petr Grinevich, Paolo Santini**

Steklov Mathematical Institute of Russian Academy of Sciences

University of Roma “La Sapienza”

The Akhmediev breathers are used in the literature as a mathematical model of anomalous (rogue) waves. For physical applications it is important to know if they are stable. Until recently, the common opinion in the literature was that these solutions are neutrally stable due to “saturation of instabilities” mechanism. We show that these solutions are unstable by explicitly constructing the “missed modes”, i.e. exponentially growing in time solutions of the linearized equation. We also demonstrate how the unstable modes can be derived using a regular procedure.

Scheduled for Dec. 20, 13:10 (Moscow) / 18:10 (Shanghai)

*On principal eigenvalues for time-periodic parabolic operators*

**Yuan Lou**

Shanghai Jiao Tong University

We will discuss some recent progress on the asymptotic behaviors of principal eigenvalues for time-periodic parabolic operators, with applications to infectious disease models. The talk is based on joint works with Shuang Liu (Beijing Institute of Technology), Rui Peng (Jiangsu Normal University) and Maolin Zhou (Chern Institute).

Scheduled for Dec. 20, 10:00 (Moscow) / 15:00 (Shanghai)

*Non-flat Frobenius manifolds: geometry and integrability***Oleg Mokhov**

Lomonosov Moscow State University

We introduce the notion of a non-flat Frobenius manifold, which generalizes the notion of a flat Dubrovin–Frobenius manifold. Locally, non-flat Frobenius manifolds are described by the curved Witten–Dijkgraaf–Verlinde–Verlinde equations (the curved WDVV equations) naturally arising in some physical models and in the theory of submanifolds with potential of normals in pseudo-Euclidean spaces that was developed by the author. Earlier it was proved by the author that the WDVV equations are natural special reductions of the fundamental equations of the theory of submanifolds in pseudo-Euclidean spaces and any Dubrovin–Frobenius manifold can be realized as a special flat submanifold with flat normal bundle in a pseudo-Euclidean space. In this talk we prove that the curved WDVV equations are also natural special reductions of the fundamental equations of the theory of submanifolds in pseudo-Euclidean spaces and any non-flat Frobenius manifold can be realized as a special submanifold with potential of normals in a pseudo-Euclidean space. Besides, we propose a Lax pair for the curved WDVV equations and claim that the curved WDVV equations are integrable. This research was supported by the Russian Science Foundation under grant 20-11-20214.

Scheduled for Dec. 22, 11:00 (Moscow) / 16:00 (Shanghai)

*Nondiagonalizable hydrodynamic type systems integrable by Tsarev’s generalised hodograph method***Maxim Pavlov**

P. N. Lebedev Physical Institute of the Russian Academy of Sciences

We investigate integrability of nondiagonalizable hydrodynamic type systems. In the simplest nontrivial case ( $2 \times 2$  Jordan blocks) we derive integrability conditions. Two distinguish classes of solutions are selected:

- (1) linearly-degenerate case;
- (2) strongly-nonlinear case.

The first case belongs to the so-called Darboux integrable class. The second case belongs to MKP hierarchy.

Scheduled for Dec. 21, 11:00 (Moscow) / 16:00 (Shanghai)

*Hitchin systems on hyperelliptic curves: separation of variables***Oleg Sheinman**

Steklov Mathematical Institute of Russian Academy of Sciences

We present separating variables and relations for hyperelliptic Hitchin systems corresponding to classical groups, and give explicit expressions for the Hamiltonians, symplectic form and action-angle coordinates via these variables.

Scheduled for Dec. 22, 10:00 (Moscow) / 15:00 (Shanghai)

*On isochronous dynamics***Dmitry Treschev**

Steklov Mathematical Institute of Russian Academy of Sciences

We propose a systematic approach to the problem of isochronicity in Hamiltonian dynamics. In particular, we present a necessary and sufficient conditions for isochronicity in the case of 1 degree of freedom in terms of Taylor expansion of the Hamiltonian function.

Scheduled for Dec. 20, 11:00 (Moscow) / 16:00 (Shanghai)

*Weak KAM approach to first-order mean field games***Kaizhi Wang**

Shanghai Jiao Tong University

I will give a short introduction to mean field games first, and then talk about our two recent works on first-order mean field games: existence of solutions of contact mean field games (with X. Hu); a semi-discrete approximation scheme for mean field games (with R. Iturriaga). Our methods come from weak KAM theory for Hamiltonian and contact Hamiltonian systems.

Scheduled for Dec. 20, 12:10 (Moscow) / 17:10 (Shanghai)

*On Prandtl Boundary Layers***Yaguang Wang**

Shanghai Jiao Tong University

In this talk, we shall review recent progress on mathematical theory of the Prandtl boundary layers in incompressible flow. The stability, back flow and blowup in finite time will be addressed. Moreover, we shall present our recent study of boundary layers in geophysical flow.

Scheduled for Dec. 22, 13:10 (Moscow) / 18:10 (Shanghai)

*Analysis of steady solutions for the incompressible Euler system in an infinitely long nozzle***Chunjing Xie**

Shanghai Jiao Tong University

Stagnation point in flows is an interesting phenomenon in fluid mechanics. It induces many challenging problems in analysis. We first derive a Liouville type theorem for Poiseuille flows in the class of incompressible steady inviscid flows in an infinitely long strip, where the flows can have stagnation points. With the aid of this Liouville type theorem, we show the uniqueness of solutions with positive horizontal velocity for steady Euler system in a general nozzle when the flows tend to the horizontal velocity of Poiseuille flows at the upstream. Finally, this kind of flows are proved to exist in a large class of nozzles. This is a joint work with Congming Li and Yingshu Lv.

Scheduled for Dec. 22, 12:10 (Moscow) / 17:10 (Shanghai)

*Christoffel transformations for (partial-)skew-orthogonal polynomials and applications in integrable systems*

**Guofu Yu**

Shanghai Jiao Tong University

In this talk, we consider the Christoffel transformations for skew-orthogonal polynomials and partial-skew-orthogonal polynomials. We demonstrate that the Christoffel transformations can act as spectral problems for discrete integrable hierarchies, and therefore we derive certain integrable hierarchies from these transformations. Some reductional cases are also considered.

Scheduled for Dec. 21, 10:00 (Moscow) / 15:00 (Shanghai)

*Regularity and generic divergence of local first integrals for analytic systems*

**Xiang Zhang**

Shanghai Jiao Tong University

In this talk we first introduce background on local integrability of analytic differential systems, and then present our recent results about regularity and generic divergence of analytic differential systems near a singularity with one zero eigenvalue and others nonresonant. These results answer the related problems partially existing from 2003.

Scheduled for Dec. 21, 12:10 (Moscow) / 17:10 (Shanghai)