Workshop

“New Trends in Mathematical Physics”

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Organizers

Steklov Mathematical Institute of Russian Academy of Sciences, Moscow

Steklov International Mathematical Center, Moscow

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Titles and Abstracts of Talks

Luigi Accardi

University of Rome Tor Vergata

The n-dimensional quadratic Heisenberg algebra as a “non-commutative” sl(2, C)

We prove that the commutation relations among the generators of the n--dimensional quadratic Heisenberg algebra ($n\in\mathbb{N}$), give rise to a kind of \textit{non--commutative extension} of $\hbox{sl}(2, \mathbb{C})$ (more precisely of its unique 1--dimensional central extension). This \textit{non-commutativity} has a different nature from the one considered in quantum groups. In particular we prove that, for most values of n, this Lie algebra cannot be isomorphic to $\hbox{sl}(N, \mathbb{C})$ for almost any value of N. %The elements of this algebra are parametrized by matrices and we find a necessary and sufficient %condition for on the matrices the Hermitean We study under which conditions the vacuum distribution of an Hermitean element of this algebra can be reduced to a product measure by a change of coordinates in the test function space, as in the Gaussian case, and we find a necessary and sufficient condition for this. From this we deduce that vacuum distributions of generic Hermitean elements \textbf{cannot} be reduced to product measures. In particular this is true for the truncated Virasoro fields for which we give some examples where this reduction is possible.

Luigi Accardi

University of Rome Tor Vergata

The Stochastic Limit as mathematical theory of quantum transport, dissipation and decays

In the first part of my talk I will briefly sketch the philosophy and some achievements of the stochastic limit of quantum theory (\textbf{from now on: SLQT}). This will be based on a decades collaboration with \textbf{Yun Gang Lu} and \textbf{Igor Volovich}, enriched by fundamental contributions coming from the collaboration with: {Alberto Frigerio, Irina Arefeva, Sergei Kozyrev,}} {Alexander Pechen, Roman Roshin, Kentaro Imafuku, }} {Shuichi Tasaki, Masanori Ohya}} (in chronological order of my interaction with each of them). The second part of my talk can be considered as an introduction to Roberto Quezada's talk in this conference for reasons that will be clear later on. This will also be based on a decades collaboration with all the persons mentioned above and in addition with: Franco Fagnola, Roberto Quezada, Skander Hachicha, Julio Garcia, Fernando Guerrero--Poblete. In it I will restrict my attention to Markov generators arising from SLQT. I will try to explain why this very special class of generators contains much more information than the GKSL form which, just because of its generality, cannot contain the finer information required for real physical applications. The main idea of this part of the talk is that the old picture according to which an environment drives a system interacting with it to a dynamical equilibrium (stationary) state must be modified because, if the system Hamiltonian has degeneracies in its spectrum, the equilibrium states can be many and each of them has a domain of attraction that in several cases can be explicitly described. This leads to the idea of \textbf{physical control of quantum systems}. The information contained in the master equation is only a small part of the full information included in the stochastic limit. If time allows, I will give some examples of this additional information.

Daniel Afanasev

GBOU "School №1561"

Global properties of sperically symmetric solutions in General Relativity with an Electromagnetic field and a Cosmological Constant

We will discuss general relativity with cosmological constant minimally coupled to electromagnetic field and assume that four-dimensional space-time manifold is the warped product of two surfaces with Lorentzian and Euclidean signature metrics. We will see, that field equations imply that at least one of the surfaces must be of constant curvature. It leads to the symmetry of the metric, although any symmetry of solutions is not assumed from the very beginning. This effect is called «spontaneous symmetry emergence». We will give classification of global solutions in two cases: (i) both surfaces are of constant curvature and (ii) the Riemannian surface is a sphere (spherically symmetric solutions). We will discuss one new global spherically symmetric solution, which is interesting from physical viewpoint. It describes changing topology of space sections in time.

Grigori Amosov

Steklov Mathematical Institute of RAS

On classical capacity of quantum Weyl channels

The additivity of minimal output entropy is proved for the Weyl channel obtained by the deformation of a q-c Weyl channel. The classical capacity of channel is calculated. This work is supported by the Russian Science Foundation under grant 19-11-00086.

Alexander Aptekarev

Keldysh Institute of Applied Mathematics

Multiple Orthogonal Polynomials with respect to Hermite weights: Applications and asymptotics

We start with the definition of the Hermite multiple orthogonal polynomials by means of orthogonality relations. Then we present several recent applications, such as eigenvalues distribution of random matrices ensembles with external field and Brownian bridges. The main goal of the talk will be the asymptotics of this polynomial sequence when the degree of the polynomial is growing in the scale corresponding to its variable (so called Plancherel – Rotach type asymptotics). The starting point for our asymptotical analysis is the recurrence relations for multiple orthogonal polynomials. We will present an approach based on the construction of decompositions of bases of homogeneous difference equations. Another approach, based on the semiclassical asymptotics in the case of complex-valued phases will be presented in S. Yu. Dobrokhotov’s talk.

Irina Aref’eva

Steklov Mathematical Institute of RAS

Holography and random matrices

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Alberto Barchielli

Polytechnic University of Milan

A quantum optomechanical system in a Mach-Zehnder interferometer

Quantum stochastic calculus (QSC) is a very flexible tool in the quantum theory of open systems and in quantum optics. In particular propagation in optical circuits can be modeled. In this use QSC to model the propagation of the light quantum field in a Mach-Zender interferometer, where one of the classical mirrors is substituted by an oscillating quantum mirror. Usually, such optomechanical systems are studied only when operating in optical cavities; here the interaction fields/quantum system is pure scattering and no cavity is involved. In spite of the simplicity of the circuit and of the interaction, the light at the outputs ports shows quantum effects. Under some choices of the adjustable parameters, we can obtain a negative Mandel Q-parameter in the case of direct detection of the output light or, in case of homodyne detection, the light can present a spectrum with a negative contribution (a sub-shot-noise spectrum); these two quantum effects are usually attributed to squeezed light.

Felipe Barra

University of Chile

Equilibrium quantum batteries and their nonequilibrium operations

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Vladimir Belokurov

Lomonosov Moscow State University, Institute of Nuclear Research RAS

Schwarzian functional integrals calculus

We propose a universal approach to functional integration over the group of diffeomorphisms of the circle.

Denis Borisov

Institute of Mathematics, Ufa Federal Research Center RAS

Resolvents of graphs with small edges

We consider a Schroedinger operator on a graph with small edges subject to arbitrary boundary conditions. The potential and the matrices in the boundary conditions can depend on a small parameter governing the lengths of the small edges. We discuss the structure of the resolvent of such operator. The main result provides a limiting operator in the sense of the norm resolvent convergence and states that the perturbed resolvent depends holomorphically on the small parameter in an appropriate sense.

Jean-Bernard Bru

University of the Basque Country

Large Deviations for Fermions at Equilibrium - An Approach to Macroscopic Behavior at Nanoscales

In 2012, experimental measurements of electric resistance of nanowires in Si doped with phosphorus atoms demonstrate that quantum effects on charge transport almost disappear for nanowires of lengths larger than a few nanometers, even at very low temperature (4.2K). Such experiments suggest an exponentially fast convergence of microscopic quantities towards macroscopic ones in real fermion systems. In Mathematics, it corresponds to the existence of so-called large deviations for the corresponding quantities. We will discuss this mathematical issue for fermion systems on the lattice.

Marco Cattaneo

IFISC (CSIC-UIB, Spain) and University of Turku (Finland)

Symmetry and block structure of the Liouvillian superoperator in partial secular approximation

The partial secular approximation, consisting in keeping only the slowly-rotating terms of the Bloch-Redfield master equation, has received a considerable amount of attention in the recent past, especially due to its ability to solve the local vs global problem for master equations of multipartite open quantum systems [1]. In this talk we show that the accurate application of the partial secular approximation induces a symmetry on the superoperator level, yielding a phase-covariant system dynamics [2]. This may be employed to greatly reduce the complexity of the master equation by decomposing the Liouvillian superoperator into independent blocks. Moreover, we prove that, if the stationary solution of the master equation is unique, i.e. the dynamics relaxes toward an equilibrium steady state, then one single block contains all the information about it, and only some steady-state coherences may emerge [2]. Finally, we provide some examples of phase-cova riant master equations for both fermionic and bosonic systems. [1] Cattaneo et al., New J. Phys. 21, 113045 (2019) [2] Cattaneo et al., Phys. Rev. A 98, 052347 (2020)

Dariusz Chruściński

Nicolaus Copernicus University

Universal Spectra of Random Lindblad Operators

We introduce an ensemble of random Lindblad operators, which generate completely positive Markovian evolution in the space of the density matrices. The spectral properties of these operators, including the shape of the eigenvalue distribution in the complex plane, are evaluated by using methods of free probabilities and explained with non-Hermitian random matrix models. We also demonstrate the universality of the spectral features.

Nikolay Chuprikov

Tomsk State Pedagogical University

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On the optical-mechanical analogy of the Dirac theory

It is shown that the quantum ensemble of a Dirac electron with a given energy and right-handed (left-handed) helicity, moving under the action of an external scalar electric field inhomogeneous in the direction of particle motion, is an analogue of an electromagnetic TE (TM) wave that propagates in a dispersive medium inhomogeneous in the direction of wave motion. The ensemble can only consist of a sub-ensemble of spin-up particles or only of a sub-ensemble of spin-down particles, but it always contains the sub-ensembles of heavy and light quasiparticles together - they form a single whole, and their separation has no physical meaning. The sub-ensemble of heavy (light) quasiparticles, which is described with the large (small) component of the Dirac bispinor in the standard representation, is an analogue of the electric (magnetic) component of the electromagnetic TE(TM)-wave. The relative effective mass of the light (heavy) quasiparticle is an analogue of the relative magnetic permeability (dielectric permittivity). A (one-particle) resolution of the Klein paradox is proposed.

Vasily Denisov

Moscow State University

Theorems on stabilization of solutions of parabolic equations

We consider difference of Stielties means and solutions of Cauchy problem for Heat equation for initial functions, which have pawer growth. We give sufficient conditions for stabilization this difference to zero, when t tends to infinity.

Goran Djordjevic

University of Nis

Classical and Quantum Dynamics of DBI Type Lagrangians in p-Adic Context

Dynamics of a class of Dirac-Born-Infeld (DBI) type Lagrangians is considered. Motivation comes from string and D-brane theory with possible application in inflation theory. Our consideration is done in the context of classical and quantum mechanics with tachyon like potentials. In this lecture an accent is on p-adic aspects, quantization is done in the form of the path integrals on real and p-adic spaces, followed by discussion on conditions for their ``adelization``.

Sergey Dobrokhotov

Institute for Problems in Mechanics RAS

Asymptotics of Hermitian type orthogonal polynomials: real semiclassical approximation for the asymptotics with complex-valued phases

We consider orthogonal polynomials determined by the recurrence relations. We obtain a uniform asymptotics of diagonal polynomials (z,a) in the form of an Airy function for n>>1, which is a far-reaching generalization of the Plancherel-Rotach asymptotic formulas for Hermitian polynomials. We discuss one of the possible approach which we call "real semiclassics for asymptotics with complex-valued phases”, another approach based on the construction of decompositions of bases of homogeneous difference equations is discussed in A.I.Aptekarev’s talk. Introducing an artificial small parameter h=O(1/n) and a continuous function φ(x,z,a) such that (z,a) = φ(kh,z,a), we reduce the described to a pseudo – differential equation for φ, where x is a variable and (z,a) are parameters. Seeking its solution in the WKB- form, one obtains the Hamilton-Jacobi equations with complex Hamiltonians connected with a third-order algebraic curve. This circumstance is the main difficulty of solving the problem and, as a rule, leads to the transition from the real variable x to the complex one. In this problem, we propose a different approach. We divide the pseudodifferential equation in question into two with the following properties. The symbol of the first equation is real, the corresponding phase is real and is defined globally for all x. The operator defining the second equation has a complex-valued symbol, (complex Hamiltonian). However, this equation can be approximated by two equations, one of which has asymptotics with a purely imaginary phase, and the symbol of the second is pure real and has the form cosp+V_0 (x)+hV_1 (x)+O(h^2). This ultimately allows us to represent the desired asymptotic uniformly through the Airy function of the complex argument.

Mikhail Dolgopolov

Samara State Technical University & LMP

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Scanning and compaction of discrete ion fluxes by the magnetic field system and the ion-emission quantum energy converter

The processes in the electronically controlled plasma power system with the capacity from 4 to 25 kW are considered. The installation is based on the physical principles of the compaction of ion and plasma flows with discrete changes in the magneto-optical systems control parameters. Energy generation is carried out by the converting the synthesis products energy into the quantum energy converter, which is the ion-emission electro-vacuum device or the span klystron, operating on high-energy ions at the frequency of 50 kHz and higher.

Branko Dragovich

Institute of Physics Belgrade

COSMOLOGY OF NONLOCAL GRAVITY

Despite numerous significant phenomenological confirmations and many nice theoretical properties, General Relativity (GR) is not final theory of gravity. Problems mainly come from quantum gravity, cosmology and astrophysics. Therefore many attempts are under consideration to adequately extend GR. In this talk, we consider some models of nonlocal modified GR, where nonlocality is presented by an analytic function of the d'Alembert-Beltrami operator. We obtained some exact vacuum cosmological solutions of the corresponding equations of motion. We pay special attention to the model which exact cosmological solutions contain effects that mimic dark matter and dark energy. Here, dark energy is produced by the cosmological constant Λ. For this solution, computed cosmological parameters are in good agreement with cosmological observations. Details can be found in our recent papers, see [1, 2] and references therein.

Tatiana Dudnikova

Keldysh Institute of Applied Mathematics

Convergence to stationary nonequilibrium states for Hamiltonian dynamical systems

In the talk, various Hamiltonian systems described by partial differential equations and by discrete equations are considered. The long-time convergence of statistical solutions to an equilibrium distribution is proved. These results are applied to the case when the initial state of the system in some infinite ``parts" of the phase space has the Gibbs distribution with different temperatures. In this case, the states of the system are found in which the limiting energy current density does not vanish. Thus, a class of the stationary nonequilibrium states is constructed for the considered models.

Fridrikh Dzheparov

National Research Center "Kurchatov Institute”

Pressure dependence of phonon populations and non-standard quasiadditive Integrals of motion

The existing equilibrium statistical physics is based on application of standard quasiadditive integrals of motion, which include energy, momentum, rotation momentum, and number of particles. It is shown that this list is far from complete and that any quasiadditive dynamic variable can be mapped to corresponding quasiadditive integral of motion. As a result an ensemble with a given external pressure is constructed. It provides the first example of the distribution in which phonon populations depend on pressure differently than in the canonical Gibbs ensemble.

Lyudmila Efremova

Lobachevsky State University of Nizhny Novgorod; Moscow Institute of Physics and Technology

On the partial integrability property of maps obtained by small smooth perturbations of skew products

The concept of the partial integrability is introduced for discrete dynamical systems in the plane. Sufficient conditions of the partial integrability are proved for maps obtained by small smooth perturbations of skew products. Dynamical properties of the partially integrable maps are investigated. Examples of the partially integrable maps are given.

Pavel Exner

Doppler Institute for Mathematical Physics and Applied Mathematics in Prague

On the discrete spectrum of soft quantum waveguides

The talk deals with soft quantum waveguides described by a two-dimensional Schr\"odinger operators with an attractive potential in the form of a channel of a fixed profile built along a smooth curve in $\mathbb{R}^2$. If the latter is infinite and not straight, but asymptotically straight in a suitable sense, we show using Birman-Schwinger principle that the discrete spectrum of such an operator is nonempty if the potential well defining the channel profile is deep and narrow enough. We also address the question about ground state optimization in the situation when the generating curve has a for of a loop. Some related results and problems are also mentioned.

Franco Fagnola

Polytechnic University of Milan

Supercritical Poincaré-Andronov-Hopf bifurcation in a mean field quantum laser equation

In this joint work with Carlos M. Mora (Concepción, Chile) we study the dynamical system properties of a Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) equation with mean-field Hamiltonian that models a simple laser with a mean field approximation of a quantum system describing a single-mode optical cavity and two-level atoms coupled to a reservoir. We prove that the quantum master equation has a unique regular stationary solution. In case a relevant parameter Cb, i.e., the cavity cooperative parameter, is less than 1, we prove that any regular solution converges exponentially fast to the equilibrium, and so the stationary state is a globally asymptotically stable equilibrium solution. We obtain that a locally exponential stable limit cycle is born at the regular stationary state as Cb passes through the critical value 1. Thus, the mean-field laser equation has a Poincaré-Andronov-Hopf bifurcation at Cb = 1 at the level of density matrices from a global attractor quantum state, where the light is not emitted, to a locally stable set of coherent quantum states producing coherent light. Moreover, we establish the local exponential stability of the limit cycle in case a relevant parameter is between the first and second laser thresholds appearing in the semiclassical laser theory. In order to prove the exponential convergence of the quantum state, as the time goes to infinity, we develop a new technique for proving the exponential convergence in open quantum systems that is based on a variation of constant formula, which is obtained by combining probabilistic techniques with classical arguments from the semigroup theory. Furthermore, applying our main results we find the long-time behaviour of the von Neumann entropy, the photon-number statistics, and variances of quadratures.

Valeri Frolov

University of Alberta, Canada

Spinoptics in a curved spacetime

High frequency approximation for Maxwell equations in a curved spacetime is developed. An effect of spin on photons motion in an external gravitational field is discussed.

Anatolii Gushchin

Steklov Mathematical Institute of RAS

Extensions of the space of continuous functions and its application to the Dirichlet problem for elliptic equations.

Рассматриваются слабые решения задачи Дирихле, обобщающие как классические, так и обобщенные решения. Дается обзор результатов в этом направлении. В частности, приводятся теоремы о разрешимости задачи Дирихле при граничной функции из пространства L_p и правых частей уравнения из достаточно широкого класса. Обсуждаются нерешенные задачи.

Alexander Holevo

Steklov Mathematical Institute of RAS

Multimode quantum Gaussian observables: structure and capacities

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Oleg Inozemcev

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Steklov Mathematical Institute of RAS

On formulation of the Eigenstate Thermalization Hypothesis

Eigenstate thermalization hypothesis (ETH) is discussed. We show that one common formulation of ETH does not necessarily imply thermalization of an observable of isolated many body quantum system. To get thermalization one has to postulate the canonical or microcanonical distribution in the ETH-ansatz. More generally, any other average can be postulated in the generalized ETH-ansatz which leads to a corresponding equilibration condition.

Mikhail Katanaev

Steklov Mathematical Institute of RAS

Point disclinations in the geometric theory of defects

We use the Chern--Simons action for a SO(3)-connection for the description of point disclinations in the geometric theory of defects. The most general spherically symmetric SO(3)-connection with zero curvature is found. The corresponding orthogonal spherically symmetric SO(3)-matrix and n-field are computed. Two examples of point disclinations are described.

Richard Kerner

Sorbonne University

Unifying colour SU(3) with Z_3-graded Lorentz-Poincaré Algebra

A generalization of Dirac's equation is presented, incorporating the three-valued colour variable in a way which makes it intertwine with the Lorentz transformations. We show how the Lorentz-Poincaré group must be extended to accomodate both SU(3) and the Lorentz transformations. Both symmetries become intertwined, so that the system can be diagonalized only after the sixth iteration, leading to a six-order characteristic equation with complex masses similar to those of the Lee-Wick model. The spinorial representation of the Z_3-graded Lorentz algebra is presented, and its vectorial counterpart acting on a Z_3-graded extension of the Minkowski space-time is also constucted. Application to new formulation of the QCD and its gauge-field content is briefly evoked.

Khachatur Khachatryan

Yerevan State University, Institute of Mathematics NAS, Armenian

О знакопеременных решениях одного класса многомерных интегральных уравнений с выпуклой нелинейностью

Доклад посвящен вопросу разрешимости одного класса нелинейных 2 двумерных интегральных уравнений сверточного типа на R^2. Указанный класс уравнений имеет приложение в теории p-адических открыто-замкнутых струн, в математической теории пространственно-временного распространения эпидемии. Доказывается существование знакопеременного и ограниченного решения. В одном частном случае исследуется также асимптотическое поведение построенного решения. В конце приводятся конкретные прикладные примеры указанных уравнений для иллюстрации полученных результатов.

Andrei Khrennikov

Linnaeus University

Quantum-like models: decision making and social laser

This talk is about applications of the mathematical formalism of quantum theory outside of physics.

Yana Kinderknecht (Butko)

Technical University of Braunschweig

Stochastic representations for solutions of a class of integro-differential evolution equations

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Evolution equation with time-fractional derivatives are actively discussed in the literature nowadays. Such equations are used in particular in models of anomalous diffusion. Different stochastic processes are used for stochastic representations of solutions of such equations, e.g. the so-called "generalized grey Brownian motion" and Markov processes slowed down by so-called "inverse subordinators". In this talk, we consider a general class of integro-differential equations which generalize time- and space-fractional heat equation. We showin which way and which classes of stochastic processes (generalizing the above mentioned processes) can be used for stochastic representations for solutions of this class of evolution equations. This is a joint work with Christian Bender, Saarland University.

Vladimir Korepin

CN Yang Institue for Theoretical Physics of Stony Brook

Lattice nonlinear Schroedinger equation: history and open problems

The model has many names: Bose gas with delta interaction, Lieb-Liniger, Tonks-Girardeau and nonlinear Schroedinger. It is solvable by algebraic Bethe ansatz. We shall use notations of quantum inverse scattering method. We shall mention applications.

Alexey Koshelev

Universidade da Beira Interior, Covilh

Analytic infinite derivative field theories: classical and quantum aspects

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Valery Kozlov

Steklov Mathematical Institute of RAS

Linear system with quadratic invariant as Schrodinger equation

We consider linear systems of differential equations in a real Hilbert space admitting an invariant as a positive definite quadratic form. It is assumed that the system has a simple discrete spectrum and that the eigenvectors form a complete orthonormal system. Under these conditions, the linear system is reduced to the form of the Schrödinger equation by introducing a suitable complex structure. As an example, such a reduction was carried out for the system of Maxwell's equations in space without currents. These observations allow us to consider the dynamics determined by some linear differential equations of mathematical physics from the point of view of the basic principles and methods of quantum mechanics.

Sergey Kozyrev

Steklov Mathematical Institute of RAS

Genome as a functional program

Functional programming provides control of errors and high level of parallelism in execution of programs. Both these properties are important for description of functioning and evolution of biological systems. In this talk we discuss the point of view of genome as a functional program and Darwinian evolution as a problem of machine learning for functional programs.

Edward Kurianovich

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Relativistic Brownian motion – 2

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Dmitry Levkov

Institute for Nuclear Research RAS, Institute for Theoretical and Mathematical Physics MSU

Semiclassical S-matrix in dilaton gravity with a boundary

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We consider black hole formation and evaporation in dilaton gravity with a boundary. First, we point out that two-dimensional Russo-Susskind-Thorlacius (RST) model for evaporating black holes is locally equivalent - at the full quantum level - to flat-space limit of the Jackiw-Teitelboim (JT) gravity that was recently shown to be unitary. Globally, the two models differ by a reflective spacetime boundary added in the RST model. Second, we use complex semiclassical method to compute scattering amplitudes of a point particle in dilaton gravity. The corresponding semiclassical solution is interpreted as formation of the intermediate black hole that decays into a final-state particle. The probability of this transition is suppressed by the number of black hole states.

Vadim Malyshev

Moscow State University

Structure of Classical Mathematical Physics as new Project and new Journal

Besides very popular now quantum panorama of physics, it could be useful also to come back to classical mathematical physics with few simple and common sense axioms based on Newton's laws and on a part of classical electrodynamics. Exact axioms can be very useful. In particular, the explosive development of equilibrium statistical physics in the second half of 20th century occured due to the only axiom — Gibbs distribution. Moreover, new fields of mathematics appeared (e.g. cluster expansion theory). I want also to discuss what axioms could be for non equilibrium case and for micro models of continuum mechanics. Certainly, Gibbs distribution will not play so important role. There are also many new results in this direction which I would like also to present.

Nikolay Marchuk and Dmitriy Shirokov

Steklov Mathematical Institute of RAS, HSE University, IITP RAS

On some equations modeling the Yang-Mills equations

We considers plane-wave solutions of the Yang–Mills equations, which allow one to write out three systems of equations modeling the Yang–Mills system. An explicit form of all plane-wave solutions of the Yang–Mills equations with the SU(2) gauge symmetry and zero current in a (pseudo)Euclidean space of arbitrary finite dimension is presented.

Sergey Mayburov

Lebedev institute of physics RAS

Oscillations of nucleus decay parameters in nonlinear quantum mechanics

Nucleus radioactivity law is fundamental law of physics; in accordance with it, decay parameters for any unstable nuclei are constant and independent of environment. Recently, several experiments reported annual and daily decay rate variations for nucleus α- and β-decay of the order .05 % [1,2]. These results suppose that observed rate oscillations follows to variation of Sun gravitation potential U resulting from elliptic form of Earth orbit and its daily rotation [1]. We argue that such effects can be explained by the presence of additional nonlinear terms for interaction of quantum systems with gravitation field, proposed by Kibble [3]. In our approach, modified Doebner-Goldin nonlinear term [4] used for description of gravity influence on decay rate. In Gamow theory, α-decay described as quantum tunneling of α-particle through potential wall on nucleus edge. Its shown that such nonlinear Hamiltonian induces variations of tunneling rate which result in nucleus decay rate variations which parameters agree with experiment. In particular, best fit to experimental data for Po-214 α-decay gives nonlinear term proportional to U time derivative [5].

Andrey Mikhailov

Russian Federal Research Institute of Fisheries and Oceanography

Relativistic Brownian motion – 1

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Farrukh Mukhamedov

United Arab Emirates University

A Quantum Markov Chain approach to Phase Transitions for quantum Ising model with competing XY-interactions on a Cayley tree

The main aim of the present talk by means of the quantum Markov chain (QMC) approach is to establish the existence of a phase transition for the quantum Ising model with competing XY interaction. In this scheme, the C^*-algebraic approach is employed to the phase transition problem. Note that these kinds of models do not have one-dimensional analogues, i.e. the considered model persists only on trees. It turns out that if the Ising part interactions vanishes, then the model with only competing XY-interactions on the Cayley tree of order two does not have a phase transition. By phase transition we mean the existence of two distinct QMC which are not quasi-equivalent and their supports do not overlap. Moreover, it is also shown that the QMC associated with the model have clustering property which implies that the von Neumann algebras corresponding to the states are factors.

Arseny Mironov

Prokhorov General Physics Institute of the RAS

The Ritus-Narozhny conjecture and resummation of radiative corrections in QED in a constant crossed field

By considering radiative corrections in a constant crossed field of up to 3rd-loop order, Ritus and Narozhny observed their dramatic enhancement with the field, and conjectured that the proper expansion parameter for QED in a strong constant crossed field is $g=\alpha\chi^{2/3}$, where the dynamical quantum parameter $\chi=e\sqrt{-(Fp)^2}/m^3$ combines the particle momentum p with the external field strength tensor F. According to the conjecture, the main contribution comes from the bubble-type corrections obtained by insertion of the 1-loop polarization operators to the photon propagators. At $g>1$ the perturbation theory breaks down, and all such contributions should be resummed. In this talk, we will present and discuss the first nonperturbative result in this context, the resummed bubble-type polarization corrections to the electron self-energy in a constant crossed field. Our analysis confirms the relevance of the scaling parameter g to the enhancement of bubble-type radiative corrections. This parameter actually represents the characteristic value of the ratio of the 1-loop polarization operator eigenvalue to the photon virtuality. After an all-order resummation we identify and discuss main contributions to the self-energy with different formation regions and asymptotic behaviour for $g\gg1$.

Mukadas Missarov

Kazan Federal University

Generalization of the hierarchical model on the two-dimensional lattice

We describe this field in the Gibbsian form in the finite volume of the hierarchical lattice. We consider the non-Gaussian model in which the Gaussian part is given by the self-similar field and non-Gaussian part is given by the self-interaction $(\bar\psi\psi)^{2}$. We show that the action of the renormalization group is local and is defined by the finite-dimensional superintegral.

Vladimir Nazaikinskii

Ishlinsky Institute for Problems in Mechanics RAS

Partial spectral flow and the Aharonov–Bohm effect in graphene

We study the Aharonov–Bohm effect in an open-ended tube made of a graphene sheet whose dimensions are much larger than the interatomic distance in graphene. An external magnetic field vanishes on and in the vicinity of the graphene sheet, and its flux through the tube is adiabatically switched on. It is shown that, in the process, the energy levels of the tight-binding Hamiltonian of π-electrons unavoidably cross the Fermi level, which results in the creation of electron–hole pairs. The number of pairs is proven to be equal to the number of magnetic flux quanta of the external field. The proof is based on the new notion of partial spectral flow, which generalizes the ordinary spectral flow already having well-known applications (such as the Kopnin forces in superconductors and superfluids) in condensed matter physics.

Sergey Postnov

Trapeznikov Institute of Control Sciences

Optimal control problems investigation for fractional diffusion and diffusion-wave equations

We consider a problems of optimal control for a model system, which defined by a one-dimensional non-homogeneous diffusion or diffusion-wave equation with a time derivative of fractional-order. For the first equation the optimal control problem investigated with final condition on system state. For the second equation we study the optimal damping problem itself and with additional condition on system state. In general case we consider both of boundary and distributed controls which are p-integrable functions ($p>1$). Two types of optimal control problem considered: the problem of control norm minimization at given control time and the problem of time-optimal control at given restriction on control norm. Optimal control problems reduced to an infinite-dimensional l-problem of moments. This problem studied on the base of approximate solution of the equation, which allow us to construct a finite-dimensional l-problem of moments.

Ekaterina Pozdeeva

Skobeltsyn Institute of Nuclear Physics MSU

Cosmological attractor in Einstein-Gauss-Bonnet gravity

We use the equations of the Einstein-Gauss-Bonnet gravity in the Friedmann universe and inflationary parameters in term of e-folding number for the slow-roll regime. With help of this formulation, we obtain gravity models with the Gauss-Bonnet term leading to analytical expressions of inflationary parameters coinciding with inflationary parameters of cosmological attractor models in the leading order approximation. The model is a generalization to the cosmological attractor of exponential form initially proposed for general relativity. We consider the possible expanding of our models for a large field. We calculate and compare the inflationary parameters for the both models estimate order of accuracies for large field expansion.

Roberto Quezada

Metropolitan Autonomous University, Mexico

Breaking of the similarity principle in Markov generators of low density limit.

We construct examples of β-equilibrium Low Density Limit (LDL) generators which admit non-(β, H_{S})-equilibrium states. We prove that in some cases, the attraction domain of the (β, H_{S})-equilibrium state is empty, meaning that \textbf{the similarity principle is broken in the LDL limit}.

Zoran Rakic

University of Belgrade, Serbia

On non-local modified gravity. On the square root model and its cosmological solutions.

Despite to all significant gravitational phenomena discovered and predicted by general theory of relativity, it is not a complete theory. One of actual approaches towards more complete theory of gravity is its non-local modification. We consider non-local modification of the Einstein theory of gravity in framework of the pseudo-Riemannian geometry, with the non-local term of the form $\,\mathcal{H}(R)\mathcal{F} (\Box)\mathcal{G}(R),$ where $\mathcal{H}$ and $\mathcal{G}$ are differentiable functions of the scalar curvature R, and $\mathcal{F} (\Box) = \sum_{n=0}^{\infty}\, f_n\Box^n$ where f_n are is an analytic function of the d'Alambert operator $\Box$. Using calculus of variations of the action induced by the metric tensor $\,g_{\mu\nu},$ we derived the corresponding equations of motion. Firstly, we consider several models of the above mentioned type, as well as the case when the scalar curvature is constant. Specially, we are paid our attention to the case where $\mathcal{H}(R)=\mathcal{G}(R)=\displaystyle{\sqrt{R-2\,\Lambda}},$ and find some new cosmological solutions and we test validity of obtained solutions with experimental data. This is joint work with I. Dimitrijević, B. Dragovich, A. Koshelev and Jelena Stanković.

Vsevolod Sakbaev

Moscow Institute of Physics and Technology

Dynamics of quantum states generated by Schrodinger equation admitting Blow up phenomenon

We study the blow up phenomenon for the nonlinear Schrodinger equation and for the linear Schroedinger equation with degeneration and singularities. The relationships between the blow up, self-focusing and destruction of quantum state are obtained. The continuation of the solution of Schrodinger equation through the blow up time by means of the curve in the set of mixed singular state is constructed.

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Vsevolod Sakbaev

Moscow Institute of Physics and Technology

On the operator approach to the weak convergence of measures and limit theorems

We show that the weak convergence of the sequence of probability measures is the pointwise convergence of the convolution with measures operators in the topological vector space of bounded continuous functions with the topology of pointwise convergence. The generalized convergence of the sequence of probability measures is defined by choosing another topological vector space of function for convolution operators. This approach gives the opportunity to obtain the limit theorem for the distribution of the sums of independent random vector values variables and for the distribution of the compositions of independent random mappings. The limit theorem for the composition of independent random orthogonal operators in finite dimensional Euclidian space is obtained.

Miguel A.F. Sanjuan

Rey Juan Carlos University

Binary Black Hole Shadows: Chaos in General Relativity

General relativity -- itself a nonlinear field theory -- naturally leads to deterministic chaos. For example, the fate of a photon approaching a pair of black holes can be essentially indeterminate, even though it is governed by a deterministic set of equations. Here we explore a topic of interest to astronomers, relativists, and nonlinear dynamicists alike: the intricate structure of the shadow cast by the event horizons of a pair of black holes (BHs). An exciting era for gravitational astronomy is underway. In 2015, the first direct observation of gravitational waves (GWs), by the LIGO/Virgo collaboration, confirmed that binary black holes exist in Nature. The Event Horizon Telescope (EHT) has begun observing nearby galactic centres, such as Sagittarius A* and M87. On April 10, 2019, the first picture of a BH shadow at the center of the M87 galaxy was shown. A BH shadow is associated with the set of all photons which, when traced backwards in time f rom the observer, asymptote towards the event horizon of the BH. In the language of nonlinear dynamics, a BH shadow is an exit basin in an open Hamiltonian dynamical system. Motivated by the GW detections from merging binary BHs, and the future prospects of the EHT, a strand of recent work has focused on what the shadow of a pair of BHs would look like. The null geodesic equations, which describe the propagation of photons, are non-integrable, and chaotic scattering of photons emerges naturally. One of the hallmarks of chaos is the presence of fractal structures in phase space. In a binary BH system, a photon meets one of three possible fates: it falls into the first BH, the second BH, or it escapes to infinity. Thus, it is natural to define three exit basins. As we shall show, across the phase space the shadow may exhibit both a regular and a fractal structure. Furthermore, in certain parts of the phase space, the three basins have the more restrictive property of Wada, with all th ree basins sharing a common fractal boundary. For the binary BH system, this means that a photon which starts close to a Wada boundary in phase space is uncertain and could end up in one of three final states: the photon could fall into either of the black holes, or escape to spatial infinity. Here we apply a recently-developed numerical method, the merging method to test for the Wada property to study the fractal structures that arise in a binary BH model in general relativity. We have verified the Wada property in both (i) the exit basins in phase space; and (ii) exit basins on an image plane which define the shadow cast by the BHs. We have demonstrated that the BH shadow can exhibit either the partial Wada or the full Wada property, depending on the value of the BH separation. To our knowledge, this work represents the first demonstration of the Wada property for a general-relativistic system. As well as demonstrating that tools from the field of chaos theory can be used to under stand the rich dynamics of scattering processes in general relativity, this work highlights that there exist novel dynamical systems in gravitational physics which can be fruitfully explored by nonlinear dynamicists. This is a joint work with A. Daza (Spain), J. Shipley, and S. Dolan (UK).

Armen Sergeev

Steklov Mathematical Institute of RAS

Topological insulators invariant under time reversal

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The talk is devoted to the theory of topological insulators which is an actively developing direction in the solid state physics. The search for new topological objects is reduced to the search of appropriate topological invariants and systems having non-trivial invariants. Such systems are characterized by wide energetic gaps stable with respect to small deformations. The quantum spin Hall insulator may be considered as a non-trivial example of such systems. It is a two-dimensional insulator invariant under time reversal. It has a non-zero topological $\mathbb Z_2$-invariant introduced by Kane and Mele. Our talk is devoted to the topological insulators invariant under time reversal transform. In the first part we consider the physical basics of the theory of topological insulators while in the second part we deal with its mathematical aspects.

Andrei Shafarevich

Moscow State University

Localized asymptotic solutions of hyperbolic systems, homogeneous Lagrangian manifolds and modifications of Maslov canonic operator

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Evgeniy Shavgulidze

Moscow State University

Polar Decompozition of the Wiener Measure and the Schwarzian Theory

We construct the series of functional integrals in the Schwarzian theory as the integrals on the group of diffeomorphisms. The Schwarzian theory is behind various physical models including the SYK model and the two-dimensional dilaton gravity. A polar decomposition of the Wiener measure based on its quasi-invariance under the group of diffeomorphisms is proposed. As a result, the functional integrals in the Schwarzian theory can be written as Fourier transform of the integrals in a model with the Calogero potential.

Maksim Shirokov

Steklov Mathematical Institute of RAS

Optimal form of the Kretschmann-Schlingemann-Werner theorem for energy-constrained quantum channels and operations

It is shown that the energy-constrained Bures distance between arbitrary infinite-dimensional quantum channels is equal to the operator E-norm distance from any given Stinespring isometry of one channel to the set of all Stinespring isometries of another channel with the same environment. The same result is proved for arbitrary quantum operations.

Nikita Slavnov

Steklov Mathematical Institute of RAS

Quantum Inverse Scattering Method and scalar products

I will talk about a new method for calculating scalar products in the framework of the Quantum Inverse Scattering Method. The new method is based on the formula of the transfer matrix action on Bethe vectors. This formula allows us to obtain a system of linear equations, the solutions of which are scalar products of Bethe vectors.

Oleg Smolyanov

Moscow State University

Quantum anomalies and differential properties of generalized Lebesgue-Feynman measures

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Abdessatar Souissi

Qassim University, Saudi Arabia

Diagonalizability of quantum Markov States on trees

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We introduce quantum Markov states (QMS) in a general tree graph $G= (V, E)$, extending the Cayley trees case. We investigate the Markov property w.r.t. the finer structure of the considered tree. The main result of this paper concerns the diagonalizability of a locally faithful QMS φ on a UHF-algebra $\mathcal A_V$ over the considered tree by means of a suitable conditional expectation into a maximal abelian subalgebra. Namely, we prove the existence of a Umegaki conditional expectation $\mathfrak E : \mathcal A_V \to \mathcal D_V$ such that $$\varphi = \varphi_{\lceil \mathcal D_V}\circ \mathfrak E.$$ Moreover, we clarify the Markovian structure of the associated classical measure on the spectrum of the diagonal algebra $\mathcal D_V$.

Alexander Teretenkov

Steklov Mathematical Institute of RAS

Reduced quantum dynamics in all orders of perturbation theory with Bogolubov-van Hove scaling

We consider the dynamics of the reduced density matrix for the spin-boson model in the rotating wave approximation with the reservoir at zero temperature. We show that if one considers the perturbation theory with Bogolubov-van Hove scaling, then the dynamics of the perturbative part of the reduced density matrix is described by the Gorini – Kossakowski – Sudarshan – Lindblad equation with constant coefficients. So, it is simpler than the usual time-convolutionless master equation derived by usual perturbation methods (without time scaling). We also show that the initial conditions for the exact reduced density matrix and for its perturbative part generally do not coincide. Moreover, under certain resonance conditions the initial condition for the asymptotic equation should be not a density matrix to provide the given asymptotic precision outside the reservoir correlation time. This work is supported by the Russian Science Foundation under grant 17-71-20154.

Dmitry Treschev

Steklov Mathematical Institute of RAS

Quantum heavy particle in a periodic potential

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Anton Trushechkin

Steklov Mathematical Institute of RAS

Mathematical methods of quantum cryptography

Quantum cryptography is a modern branch of science where methods of secure communication based on principles of quantum mechanics are studied. The rigorous proof of the security of quantum cryptography (more precisely, quantum key distribution) gave rise to a complex and beautiful mathematical theory, which is based on methods of quantum information theory, in particular, quantum entropic measures and entropic uncertainty relations. In the talk, well-known results in the proof of security of quantum key distribution will be reviewed. Currently, an important task is to prove the security of quantum key distribution with imperfect devices. This is important for practical implementations. In papers [1, 2], theorems on the security of the famous BB84 quantum key distribution protocol (the first and most widely used protocol) with detection-efficiency mismatch were rigorously proved for the first time. The differences in the detector efficiencies (the probabilities of detecting a photon) destroys certain symmetries in the problem, which makes the earlier methods of security proofs insufficient. In [1, 2], new methods for analytic solution of a minimization problem of a certain convex functional (quantum relative entropy) and methods for bounding the dimensionality of the search space under were developed. This was one of the key problems for solving this problem. In [3], a number of statements were proved that explain the operational meaning of the security parameter widely used in quantum key distribution.

Anton Trushechkin

Steklov Mathematical Institute of RAS

Redfield, local and global quantum master equations from the viewpoint of quantum stochastic limit

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Quantum master equations are at the heart of theory of open quantum systems. Many present-day works are devoted to the correct form of quantum master equation for a system weakly interacting with the bath. A known rigorous mathematic derivation leads to an equation (often referred to as the "global" mater equation) in the form of Gorini–Kossakowski–Lindblad–Sudarshan (GKLS). It has good properties (conservation of positivity, agreement with the second law of thermodynamics), but is too restrictive to cover all possible physical situations. Namely, all Bohr frequencies are assumed to be well separated from each other. If this is not the case, the Redfield equation with the so called non-secular terms and the so called local mater equation are discussed. However, the Redfield equation does not conserve positivity and the local master equation violates the second law of thermodynamics. In the talk, we approach the problem of the correct form of quantum mater equation from the viewpoint of quantum stochastic limit [1]. In contrast to other approaches, which describe the dynamics of the reduced density operator of the system, an approximate unitary dynamics of the system and bath together is rigorously derived in this framework. Unitarity automatically guarantees the conservation of positivity. A unified approach to a rigorous derivation of the master equation in the GKLS form which includes secular, non-secular, and local terms is presented. [1] L. Accardi, Y.G. Lu, I. Volovich, Quantum Theory and Its Stochastic Limit (Springer, Berlin, 2002).

Bassano Vacchini

University of Milan

Role of local and non-local master equations in the description of non-Markovian open quantum system dynamics

The study of open quantum systems has led to identify notions of quantum non-Markovian dynamics, which have recently been the object of quite intense research. Such dynamics should naturally provide a description of memory effects in the quantum domain. Master equations are a useful tool to describe such dynamics. In order to characterize the mathematical features and the physical origin of the dynamics, it is often useful to consider different kinds of master equations for the same system. Here we explore this connection and derive an exact connection between the time-local and the integro-differential descriptions.

Mikhail Vasiliev

Lebedev institute of physics RAS

Spin-Locality and Star-Product Functions in Higher-Spin Theory

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Sergey Vernov

Moscow State University

The effective potential method for cosmological models with the Gauss-Bonnet term

We investigate cosmological dynamics in the Einstein-Gauss-Bonnet gravity using the effective potential. The model under consideration includes a scalar field, that is non-minimally coupled to both with the Ricci scalar and the Gauss-Bonnet curvature invariant. We derive conditions for stable de Sitter solutions via the effective potential. Specializing in specific couplings, we explore the possibility of realizing the stable de Sitter configurations which may have implications for both the early Universe and late-time evolution. In particular, we express the observable inflationary parameters via derivatives of the effective potential. The talk is based on the following papers: E.O. Pozdeeva, M. Sami, A.V. Toporensky, S.Yu. Vernov, Phys. Rev. D 100 (2019) 083527 and E.O. Pozdeeva, M.R. Gangopadhyay, M. Sami, A.V. Toporensky, S.Yu. Vernov, Phys. Rev. D 102 (2020) 043525.

Antonio Vidiella-Barranco

Gleb Wataghin Institute of Physics University of Campinas, Brazil

How faithfully the evolution of composite open quantum systems can be modeled?

I will discuss the dynamics of two coupled qubits, having one of them coupled to an external environment. Several aspects of the evolution of such system will be analyzed, by using different approaches. I will compare the results obtained from i) local and ii) global models, and also discuss the dynamics of a simplified (related) toy model.

Boris Volkov

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Moscow Institute of Physics and Technology

Levy Laplacians, holonomy group and instantons on 4-manifolds

The relationship between the modified Levy Laplacians parameterized by the choice of a curve in the group $SO(4)$ and the gauge fields over 4-dimensional orientable compact Riemannian manifold is studied. It is shown that if the holononomy group of the bundle of self-dual 2-forms is nontrivial then the anti-selfduality Yang--Mills equations are equal to the Laplace equation for some modified Levy Laplacian.

Igor Volovich

Steklov Mathematical Institute of RAS

Integrability of quantum dynamical systems and categories

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Noboru Watanabe

Tokyo University of Science

Note on complexity for the quantum compound systems

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Evgeny Zelenov

Steklov Mathematical Institute of RAS

p-Adic quantized calculus and ideals of compact operators

A quantum Connes calculus is constructed on the spaces of complex-valued functions of the p-adic argument. Preimages of ideals of compact operators (Macaev, Schatten-von Neumann) under the quantization map investigated.

Victor Zharinov

Steklov Mathematical Institute of RAS

Binary relations and fuzzy logic

A mathematical apparatus based on binary relations is proposed expanding the feasibility of traditional analysis in applications to problems of mathematical and theoretical physics. General constructions are illustrated by the algebraic approach to Backlund transformations of nonlinear systems of PDE and by the dynamics of traveling wave packets.

Alexander Zubarev

Samara University

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