International Seminar for Young Researchers

"Algebraic, Combinatorial and Toric Topology"

December 17 – 18, 2020 (online)

Organizers

Steklov Mathematical Institute of Russian Academy of Sciences, Moscow

Steklov International Mathematical Center, Moscow

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ABSTRACTS

Thursday, December 17

Morning session

Holomorphic foliations and transversal real sub manifolds in Kahler manifolds

Hiroaki Ishida

This talk is based on the ongoing joint work with Hisashi Kasuya (Osaka University). In this talk I will explain a new construction of a pair of a real submanifold and a holomorphic foliation in a Kahler manifold equipped with a torus action. This construction provides non-invariant complex structures on compact Lie groups and a generalization of LVM theory.

Topics on Hessenberg varieties

Tatsuya Horiguchi

Hessenberg varieties are subvarieties of flag varieties. Recently, it turned out that their topology makes connections with other research areas, such as logarithmic derivation modules in hyperplane arrangements and chromatic symmetric functions in graph theory. In this talk, I will explain their connections. Also, if time permits, then I would like to talk about a recent result of a connection between Hessenberg varieties and some toric orbifolds. This talk is based on three joint works with H.Abe-M.Harada-M.Masuda, T.Abe-M.Masuda-S.Murai-T.Sato, M.Masuda-J.Shareshian-J.Song.

On homology of the MSU spectrum

Semyon Abramyan

The theory of bordism and cobordism was actively developed in the 1950–1960s. Most leading topologists of the time have contributed to this development. The idea of bordism was first explicitly formulated by Pontryagin who related the theory of framed bordism to the stable homotopy groups of spheres using the concept of transversality. Key results of bordism theory were obtained in the works of Rokhlin, Thom, Novikov, Wall, Averbuch, Milnor, Atiyah.

The paper of Adams gave an opportunity to enter the new stage of developing bordism theory. It culminated in the calculation of the complex (or unitary) bordism ring Ω^U in the works of Milnor and Novikov. The ring Ω^U was shown to be isomorphic to a graded integral polynomial ring $\mathbb{Z}[a_i\colon i \geq 1]$ on infinitely many generators, with one generator in every even degree, $\deg a_i = 2i$. This result has since found numerous applications in algebraic topology and beyond.

In Novikov's 1967 work a brand new approach to cobordism and stable homotopy theory was proposed, based on application of the Adams–Novikov spectral sequence and formal group laws techniques. This approach was further developed in the context of bordism of manifolds with singularities in the works of Mironov, Botvinnik and Vershinin. The Adams-Novikov spectral sequence has also become the main computational tool for stable homotopy groups of spheres.

As an illustration of his approach, Novikov outlined a complete description of the additive torsion and the multiplicative structure of the SU-bordism ring Ω^{SU}, which provided a systematic view on earlier geometric calculations with this ring. A modernised exposition of this description is given in the survey paper of Chernykh, Limonchenko and Panov; it includes the geometric results by Wall, Conner-Floyd and Stong, the calculations with the Adams-Novikov spectral sequence, and the details of the arguments missing in Novikov's work. A full description of the SU-bordism ring Ω^{SU} relies substantially on the calculation of Ω^{SU} with 2 invereted, namely on proving the ring isomorphism

$$\Omega^{SU} \cong \mathbb{Z}[\tfrac{1}{2}][y_2, y_3, \ldots], \quad \deg y_i = 2i.$$

This result first appeared in Novikov's work with only a sketch of the proof, stating that it can be proved using Adams' spectral sequence in a way similar to Novikov's calculation of the complex bordism ring Ω^U. Although the result has been considered as known since the 1960s, its full proof has been missing in the literature, and also not included in the survey.

The main goal of the talk is to give a complete proof of the isomorphism above using the original methods of the Adams spectral sequence. While filling in details in Novikov's sketch we faced technical problems that seemed to be unknown before. For example, the comodule structure of $H(MSU; \mathbb{F}_p)$ over the dual Steenrod algebra $\mathcal{A}_p^*$ with odd prime p has not been sufficiently detailed in the literature. The latter computation is one of the main results of the talk. We also discuss results on the structure of the Hurewicz map and the divisibility of characteristic numbers of SU-manifolds.

Distinguishing 4-dimensional geometries via profinite completions

Zixi Wang

Abstractly saying, it is well-known that there are 19 classes of geometries for 4-dimensional manifolds in the sense of Thurston. We could ask that to what extent the geometric information is revealed by the profinite completion of the fundamental group of a closed smooth geometric 4-manifold. In this paper, we show that the geometry of a closed orientable 4-manifold in the sense of Thurston could be detected by the profinite completion of its fundamental group except when the geometry is $\mathbb{H}^4$, $\mathbb{H}^2_{\mathbb{C}}$ or $\mathbb{H}^2 \times \mathbb{H}^2$. Moreover, despite the fact that not every smooth 4-manifold could admit one geometry in the sense of Thurston, some 4-dimensional manifolds with Seifert fibred structures are indeed geometric. For a closed orientable Seifert fibred 4-manifold M, we show that whether M is geometric could be detected by the profinite completion of its fundamental group.

A combinatorial formula for the first Pontryagin class in terms of combinatorial curvature redistribution

Denis Gorodkov

The problem of finding explicit formulae for the rational Pontryagin classes of a combinatorial manifold is classical in algebraic topology. In the case of the first Pontryagin class Prof. Alexander Gaifullin devised a computable explicit algorithm in 2004. More precisely, he described the set of all local formulae for the first Pontryagin class of a combinatorial manifold, and, therefore, it is natural to look for a canonical precise formula from this set. We will demonstrate such a choice based on the redistribution of combinatorial curvature under bistellar flips of two-dimensional spheres. The talk is based on a joint work with Prof. Alexander Gaifullin.

Evening session

On the structure of the top homology group of the Johnson kernel

Igor Spiridonov

The Johnson kernel of a genus g oriented surface Σ_g is a subgroup $\mathcal{K}(\Sigma_g)$ of the mapping class group $\mathrm{Mod}(\Sigma_g)$ generated by all Dehn twists along separating curves. Given a family of $2g-3$ pairwise disjoint separating curves on Σ_g one can construct the corresponding abelian cycle in the top homology group $H_{2g-3}(\mathcal{K}(\Sigma_g), \mathbb{Z})$; such abelian cycles we call primitive. We will discuss the structure of the subgroup of $H_{2g-3}(\mathcal{K}(\Sigma_g), \mathbb{Z})$ generated by all primitive abelian cycles. In particular, we will describe the relations between them.

A K-theory criterion for p-hyperbolicity

Guy Boyde

For a (nice enough) finite CW-complex, consider the sequence of non-negative integers whose k-th term is the number of $\mathbb{Z}$-summands appearing in the direct sum of the first k homotopy groups. A famous dichotomy in rational homotopy theory says that either this sequence is bounded (hence eventually constant) or it grows exponentially. For example, this means that no finite CW-complex whose rational homotopy grows polynomially exists. Huang and Wu (arXiv 2017) introduced the definitions of p- and $\mathbb{Z}/p^r$-hyperbolicity in order to study the growth of the number of torsion summands at a given prime p. I will give an overview, focussing on a condition on K-theory which implies p-hyperbolicity, and deduce some examples of p-hyperbolic suspensions. This condition is based on work of Selick on Moore's conjecture for torsion-free suspensions.

H-maps of p-regular Lie groups

Holly Paveling

Consider the set of homotopy classes of maps between Lie groups G and L; and in particular the subset $H[G, L]$ of homotopy classes of H-maps. We will give an overview of how to describe this set, under various conditions on G and L.

Homotopy types of gauge groups of PU(p)-bundles over spheres

Simon Rea

The gauge group $\mathcal{G}(P)$ of a principal G-bundle $P \to X$ is the group of G-equivariant homeomorphisms of P that cover the identity on X. Under certain conditions on G and X, the number of possible homotopy types of $\mathcal{G}(P)$ is finite. This number has been determined only in a few special cases. In this talk I will introduce the methods to determine this number and discuss how, for bundles over even dimensional spheres, the $PU(p)$-gauge groups are related to $SU(p)$-gauge groups.

The minimal Hirsch-Brown model for moment-angle complexes

Steven Amelotte

In this talk we will describe certain higher cohomology operations induced by the torus action on a moment-angle complex. Focusing on examples, we explain how these operations assemble into an explicit Hirsch-Brown model of the torus action and can be used to give a combinatorial description of the minimal free resolution of Stanley-Reisner rings. Time permitting, we will indicate the relevance of these operations to cohomological rigidity and outline a proof that B-rigid simple polytopes are closed under products. This talk is based on joint work with Benjamin Briggs.

Friday, December 18

Morning session

Toric varieties associated with partitioned weight polytopes

Jongbaek Song

A weight polytope is a convex polytope obtained by taking the convex hull of the Weyl group orbit of a point in a weight space. For the Lie type A, a weight polytope is called a permutohedron. In this talk, we consider the toric variety X associated with a weight polytope together with a natural Weyl group action on it. In particular, we study the invariant part of the cohomology ring of X for the action of a parabolic subgroup of the given Weyl group, which is indeed isomorphic to the cohomology ring of a toric orbifold associated with a 'partitioned weight polytope'. This result provides explicit cohomology presentations of a certain class of regular Hessenberg varieties. It is a joint work with T. Horiguchi, M. Masuda and J. Shareshian.

The alpha invariant of twisted Milnor hypersurfaces

Jingfang Lian

In this talk, we give a computable formula about alpha invariant of twisted Milnor hypersurfaces with spin structure and dimension no less than 5. Using the result, we can also give the formula about $\hat{A}$-genus, and we do not restrict conditions on the spin structure and the dimension in this formula. As an important application, we show that Milnor hypersurfaces always admit PSC and give the sufficient and necessary conditions for twisted Milnor hypersurfaces with dimension restrictions non-existing PSC. During the calculation, we mentioned two methods, and one is concerned with important binomial numbers such as Bernoulli numbers, and the other can be applied in more general cases.

The integral homology of Coxeter cellular complexes

Lisu Wu

We define a special q-CW complex, named Coxeter cellular complex, and give an explicit boundary map which can reflect the information of local groups. The orbifold homology groups we defined have some similar properties as usual cellular homology theory, such as Hurewize Theorem. Moreover, I will compute the homology groups of Coxeter orbifolds and some (non-Coxeter) orbifolds in this talk. This is a joint work with Professor Zhi Lü and Li Yu.

Braid varieties, torus actions and stratifications

Mikhail Gorsky

I will discuss braid varieties, a class of affine algebraic varieties associated to positive braids and related to augmentation varieties and open Bott-Samelson varieties. First, I will explain the geometric properties of braid varieties, including the construction of torus actions and holomorphic symplectic structures on their quotients. Then, we will discuss correspondences between these braid varieties constructed by using moduli spaces associated to certain labeled planar diagrams. I will explain how these geometric correspondences induce stratifications for braid varieties and their quotients, unifying known constructions of A. Mellit, in the case of character varieties, and M. Henry and D. Rutherford, in the case of augmentation varieties. If time permits, I will briefly explain the relation between open toric charts appearing in these stratifications and cluster algebras. (Based on joint work with R. Casal, E. Gorsky, and J. Simental.)

SU-linear operations in complex cobordism and c_1-spherical bordism theory

George Chernykh

In the first part of the talk I will give a description of the set of SU-linear cohomological operations in complex cobordism and will show that they are generated by known geometrical operations ∂_i. Then I will talk about some properties of c_1-spherical bordism theory W, in particular, I will discuss SU-bilinear multiplications on W and projections from complex cobordism onto W. At the end I will talk about complex orientations of W and some particular results on the coefficients of the corresponding formal group laws.

Evening session

One-relator loop homology algebras of moment-angle complexes

George Simmons

For a flag complex K, we give a necessary and sufficient combinatorial condition for the loop homology algebra of the moment-angle complex $\mathcal{Z}_K$ to be a one-relator algebra. The condition we specify also characterises when the commutator subgroup of the right-angled Coxeter group corresponding to K is a one-relator group, providing a combinatorial link between distinct concepts of geometric group theory and homotopy theory. Returning to moment-angle complexes, we give other equivalent algebraic and homotopy-theoretic formulations of our condition. In the non-flag case, many of these equivalences no longer hold, and we will present some of the key differences.

Duality in Toric Topology

Matthew Staniforth

There are numerous notions of duality across mathematics. In this talk we explore the interplay between dualities in combinatorics, topology, and algebra, via a study of cup and cap products in moment-angle complexes and related polyhedral product spaces. We obtain both a description of the cap product and a characterisation of Poincaré duality in $\mathcal{Z}_K$, in terms of the combinatorics of the simplicial complex K. We then show how these results can be extended to a broader class of polyhedral products via simplicial substitution, and give a similar combinatorial characterisation of duality in such spaces.

New parametric families of graph-associahedra arising in toric topology

Ivan Limonchenko

In this talk, using the theory of direct families of polytopes developed by V.M.Buchstaber and the author, we introduce sequences of moment-angle manifolds over flag nestohedra $\{M_n\}_{n=1}^{\infty}$ such that M_{n-1} is a submanifold and a retract of M_n for any $n \geq 2$, and there exists a non-trivial (higher) Massey product $\langle \alpha_{n,1}, \ldots, \alpha_{n,k} \rangle$ in $H^*(M_n)$ with $\dim \alpha_{n,i} = 3, 1 \leq i \leq k$ for every $2 \leq k \leq n$.

Furthermore, we introduce new closed 2-parametric families of graph-associahedra, study their combinatorial properties, and find their applications in toric topology by means of V.M.Buchstaber's theory of the differential ring of simple polytopes.

The talk is based in part on a joint work with Victor Buchstaber.

Basic de Rham cohomology of manifolds with maximal torus action

Roman Krutowski

The family of complex manifolds with maximal torus action introduced by Ishida is a wide family of complex manifolds which includes complex moment-angle and LVMB-manifolds and it is the most general in the sense of admitting certain type of torus action. Any manifold M in this family possesses the canonical foliation $\mathcal{F}$ (like LVMB-manifolds and moment-angle manifolds) which is reconstructed from complex structure and torus action. In the talk, I will present the calculation of basic de Rham cohomology $H^*_{\mathcal{F}}(M)$ obtained in the joined paper with Hiroaki Ishida and Taras Panov. The key ingredients are transverse equivalence of pair $(M, \mathcal{F})$ with a canonical foliation on some moment-angle manifold and the actual calculation for moment-angle manifolds.