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Abstracts

- *Complexity of structures*
  Pavel Alaev (Sobolev Institute of Mathematics and Novosibirsk State University)
  Victor Selivanov (Ershov Istitute of Informatics Systems SB RAS)

  We consider notions and techniques needed for the development of the computable structure theory with bounded resourses. We are mainly interested in polynomial-time computable and primitive recursive structures. We analyze presentation complexity of some important concrete structures, in particular, number fields and finitely generated structures. We show that some problems in this area turns out to depend on complexity-theoretic conjectures like P=NP.

- *Группы с n-кручением, их расширения и эндоморфизмы*
  Варужан Атабекян (Ереванский Государственный Университет, Армения)

  Группа G с множеством порождающих X называется группой с n-кручением, если она имеет систему определяющих соотношений вида $R^n = 1$, где R пробегает множество всех слов в алфавите X, которые имеют конечный порядок в G. При нечетных $n \geq 665$ для каждой n-крученой группы можно построить теорию, аналогично теории построенной в известной монографии С.И.Адяна, что позволяет n-крученые группы исследовать развитыми в ней методами. Получено, что n-периодическое произведение любого семейства n-крученых групп является n-крученой группой, любая n-крученая группа может быть задана с помощью некоторой независимой системы определяющих соотношений вида B^n, любая m-порожденная абелева группа D может быть вложена в качестве центра в некоторую группу A так, что фактор группа A/D изоморфна заданной n-крученой группе с не менее чем m независимыми определяющими соотношениями. Далее, центр любой n-крученой группы тривиален, группа автоморфизмов $Aut(End(F))$ канонически вложена в группу $Aut(Aut(F))$ для любой относительно свободной n-крученой группы F и т.д.

- *Combinatorial approach for Burnside groups of relatively small odd exponents*
  Agata Atkarskaya (Bar-Ilan University, Israel)

  We consider a finitely generated group of a given exponent n. The bounded Burnside problem, which was stated in 1902, asks whether there exists an infinite such group. The first solution of this problem was given by P. Novikov and S. Adian in their famous work in 1968. They proved that there exists such group for odd exponents $n \geqslant 4381$. After that there was a series of works that decreases a lower bound of the exponent (including works of S. Adian), and a series of works that gives a solution for even n (S. Ivanov, I. Lysenok). Following the spirit of the proof of P. Novikov and S. Adian, in our work we develop a new combinatorial approach for the bounded Burnside problem, which is based on the Rips's idea of the canonical form. We prove that there exist infinite finitely generated groups of odd exponents $n \geqslant 297$.

  Joint work with Professor Eliyahu Rips and Professor Katrin Tent.

- *Primitive recursive and automatic structures*
  Nikolay Bazhenov (Sobolev Institute of Mathematics),
  Iskander Kalimullin (Kazan Federal University)

  We will discuss the recent progress in the studies of sub-recursive presentations for algebraic structures. The key notion in these developments is the notion of a punctual structure, introduced in [1]. A countably infinite structure S is punctual if the domain of S is the set of natural numbers, and the signature functions and relations of S are uniformly primitive recursive.

  The theory of punctual structures blends feasible computations with the methods of constructive model theory. The developed techniques can be used to derive interesting unexpected results. In particular, in a joint work with Harrison-Trainor, Melnikov, and Ng [2], we prove that the class of automata-presentable structures (in the sense of Khoussainov and Nerode) does not admit a simple syntactic characterization. A similar result holds for the structures with a polynomial-time computable presentation. We will discuss in detail these and other results.

  [1] I. Kalimullin, A. Melnikov, and K. M. Ng. Algebraic structures computable without delay. *Theor. Comput. Sci.*, 674:73–98, 2017.

  [2] N. Bazhenov, M. Harrison-Trainor, I. Kalimullin, A. Melnikov, and K. M. Ng. Automatic and polynomial-time algebraic structures. *J. Symb. Log.*, 84(4):1630–1669, 2019.

- *On the homology of Torelli groups*
  Alexander Gaifullin (Steklov Mathematical Institute, Moscow)

  Let S be an oriented closed surface of genus g. The mapping class group of S is the group of orientation preserving homeomorphisms of S onto itself considered up to isotopy. Theory of mapping class groups is closely related to geometry and topology of moduli spaces, topology of three-dimensional manifolds, braid groups, and automorphisms of free groups. The action of the mapping class group on the first homology group of the surface S yields a surjective homomorphism of it to the arithmetic group $\mathrm{Sp}(2g, \mathbb{Z})$. The kernel of this homomorphism is called the Torelli group and denoted by $\mathcal{I}_g$; it is the most mysterious part of the mapping class group. It is well known that the group $\mathcal{I}_1$ is trivial. Mess (1992) proved that $\mathcal{I}_2$ is an infinitely generated free group. On the other hand, Johnson (1983) showed that $\mathcal{I}_g$ is finitely generated, provided that $g > 2$. One of the most interesting questions concerning Torelli groups is whether the groups $\mathcal{I}_g$ are finitely presented or not for $g > 2$. This question is closely related to the problem of computing the homology of the Torelli groups. In the talk I will give a survey of recent results on the homology of Torelli groups, focusing on a possible approach to proving that $\mathcal{I}_3$ is not finitely presented. The main tool is the study of the action of I_g on a contractible CW complex constructed by Bestvina, Bux, and Margalit (2007) and called the complex of cycles.

- *Minimal degree and setwise stabilizers in profinite permutation groups*
  Laszlo Babai (University of Chicago, USA)

- *On a tropical version of the Jacobian conjecture*
  Dima Grigoriev (Institut des Mathématiques de Lille, France)

  We prove that, for a tropical rational map if for any point the convex hull of Jacobian matrices at smooth points in a neighborhood of the point does not contain singular matrices then the map is an isomorphism. We also show that a tropical polynomial map on the plane is an isomorphism if all the Jacobians have the same sign (positive or negative). In addition, for a tropical rational map we prove that if the Jacobians have the same sign and if its preimage is a singleton at least at one regular point then the map is an isomorphism.

  Joint work with Danylo Radchenko.

- *Determinicity properties, Mozes-Goodman-Strauss theorems and the construction of algebraic objects.*
  Ilya Ivanov-Pogodaev (Moscow Institute of Physics and Technology)

  The theorems of Moses and Goodman-Strauss are widely known in the theory of tilings. They link the language of local rules and the language of substitution systems. Let a substitution rule be given, according to which a tile of the next level can be assambled from polygon tiles of several types. Then the sides of the polygons can be decorated with a finite number of colors so that all tilings that follow the local rules (tiles are applied to each other only by sides of the same colors) will be generated by this substitution system. Thus, substitution systems can be specified using local rules. Determinicity property is also very interesting. For example, consider square Wang tiles, the sides of which are painted in a finite set of colors and only tiles with sides of the same color can be placed side by side. There are aperiodic sets of tiles that can be used to tile a plane only in an aperiodic manner. As Kari and Papasoglu showed, there are also aperiodic and deterministic sets, where the colors of two adjacent sides uniquely determine the colors of the other two sides (there is at most one tile with adjacent sides with a given pair of colors).

  It is interesting that determinicity can also be achieved in more general situations. The talk deals with substitution systems of flat complexes, with a substitution rule, according to which a tile (4-cycle) of level k is subdivided into tiles of level k-1. The substitution system specifies a sequence of complexes of all possible levels.

  Let the path consist of two adjacent edges of some minimal tile T and passes through three of its four vertices. Let X be the fourth vertex of T through which the path does not pass. If X lies on the boundary of a tile of strictly higher level than the other three vertices of T, we will call the path irregular. In other cases, we will call the path regular. Let the vertices and edges of the sequence of complexes obtained by some substitution be colored in a finite number of colors. We will assume that the coloring of a sequence of complexes has weak determinicity if known colors of the edges and vertices of the path P 'are uniquely determine the colors of the edges and vertices of the path P', with the same beginning and end as P, but passing along the other two adjacent sides of the same tile.

  The main result is that, for a given substitution system, a coloring in a limited number of colors with weak determinicity is always possible.

  The specification of substitution tilings by local rules and determinicity (including weak determinicity) can be useful in constructing algebraic objects with a finite number of defining relations. Words from a semigroup or ring are considered as paths on a specially constructed sequence of complexes glued together from 4-cycles (tiles). Let a coloring with a globally bounded number of colors be introduced on the vertices and edges of a sequence of complexes.

The colors of the edges and vertices correspond to the letters of the finite alphabet, the vertices and edges traversed along the path correspond to a word. The defining relations correspond to pairs of equivalent paths of length 2, consisting of two adjacent sides of the square. For rings and semigroups, monomial relations are also introduced for some types of forbidden paths. In this case, the geometric properties of the complex correspond to some useful properties in the resulting object. This method can be useful for constructing finitely presented objects of Burnside type and has been applied in constructing a finitely presented nilsemigroup.

1. S. Mozes, Tilings, substitution systems and dynamical systems generated by them, J. Analyse Math 53 (1989), no. 1, 139–186.

2. C. Goodman-Strauss, Matching rules and substitution tilings, Ann. of Math. (2) 147 (1998), no. 1, 181–223.

3. J. Kari J, P. Papasoglu. Deterministic aperiodic tile sets, GAFA, Geom. funct. anal. Vol. 9 (1999) 353–369

- *Paradoxical partitions of the reals by Robert Solovay*
  Vladimir Kanovei (Institute of Information Transmission Problems, Moscow)

  It holds in some generic extensions of the constructible universe L (those by a single Sacks, Silver, Miller, Laver real) that there exists an OD (ordinal-definable) partition of the reals into two non-OD parts. The result for the Sacks extensions was announced by Solovay in 2002 but never published. The proofs combine forcing and descriptive set theoretic methods.

- *Countable elementary free groups*
  Olga Kharlampovich (City University of New York, USA)

  We modify the notion of a Fraisse class and show that various interesting classes of groups, notably the class of nonabelian limit groups and the class of finitely generated elementary free groups, admit Fraisse limits. We rediscover Lyndon's $\mathbb{Z}[t]$-exponential completions of countable torsion-free CSA groups, as Fraisse limits with respect to extensions of centralizers.

- *Large scale geometries of infinite strings*
  Bakhadyr Khoussainov
  (The UESTC, Chengdu, China, and The University of Auckland, New Zealand)

  We aim to shed light on our understanding of large-scale properties of infinite strings. Say that an infinite string X has weaker large-scale geometry than that of Y if there is color preserving bi-Lipschitz map from X to Y with small distortion. This defines a partially ordered set of large-scale geometries on infinite strings. This partial order presents an algebraic tool for classification of global patterns. We prove that this partial order has a greatest element and has infinite chains and anti-chains. We study the sets of large-scale geometries of strings accepted by finite state machines. We provide an algorithm that describes large scale geometries of strings accepted by ω- automata. This connects the work with the complexity theory. We prove that the quasi-isometry problem is Σ_2^0-complete, thus providing a bridge with computability theory. We build algebraic structures that are invariants of large-scale geometries. We invoke asymptotic cones, a key concept in geometric group theory, defined via model-theoretic notion of ultra-product. We study asymptotic cones of algorithmically random strings, thus connecting the topic with algorithmic randomness.

- *Generalizing a question of Gromov*
  Julia Knight (University of Notre Dame, USA)

  Gromov asked, "What does a typical group look like? He suggested a way of describing typical behavior in terms of limiting density. Based on a remark of Fine, I conjectured that for groups on $n \geq 2$ generators, and presentations with a single relator, an elementary first order sentence has limiting density 1 iff it is true in the non-Abelian free groups. There are partial positive results due to Coulon, Ho, and Logan, and to Karlampovich and Sklinos, but the full conjecture remains open. I will describe joint work with Johanna Franklin and Meng-Che (Turbo) Ho, in which we generalize Gromov's question to other algebraic varieties (in the sense of universal algebra). We have results and examples illustrating different possible behaviors.

- *First order rigidity of high-rank arithmetic groups*
  Alex Lubotzky (Hebrew University, Israel)

  The family of high rank arithmetic groups is a class of groups playing an important role in various areas of mathematics. It includes $\mathrm{SL}(n, \mathbb{Z})$, for $n > 2$, $\mathrm{SL}(n, \mathbb{Z}[1/p])$ for $n > 1$, their finite index subgroups and many more. A number of remarkable results about them have been proven including; Weil local rigidity, Mostow strong rigidity, Margulis Super rigidity and the Schwartz-Eskin-Farb Quasi-isometric rigidity.

  We will add a new type of rigidity : "first order rigidity". Namely if D is such a non-uniform characteristic zero arithmetic group and L a finitely generated group which is elementary equivalent to D then L is isomorphic to D.

  This stands in contrast with Zlil Sela's remarkable work which implies that the free groups, surface groups and hyperbolic groups (many of which are low-rank arithmetic groups) have many non isomorphic finitely generated groups which are elementary equivalent to them.

  Based on a joint paper with Nir Avni and Chen Meiri (Invent. Math. 217(2019) 219-240).

- *Rich structures and weak second order logic*
  Alexei Miasnikov (Stevens Institute of Technology, USA)

  "What can one describe by first-order formulas in a given structure A?" - is an old and interesting question. Of course, this depends on the structure A. For example, in a free group only cyclic subgroups (and the group itself) are definable in the first-order logic, but in a free monoid of finite rank any finitely generated submonoid is definable. A structure A is called rich if the first-order logic in A is equivalent to the weak second order logic. Surprisingly, there are a lot of interesting groups, rings, semigroups, etc., which are rich. I will talk about some of them and then describe various algebraic, geometric, and algorithmic properties that are first-order definable in rich structures. Weak second order logic can be introduced into algebraic structures in different ways: via HF-logic, or list superstructures over A, or computably enumerable infinite disjunctions and conjunctions, or via finite binary predicates, etc. I will describe a particular form of this logic which is especially convenient to use in algebra and show how to effectively translate such weak second order formulas into the equivalent first-order ones in the case of a rich structure A.

- *New examples of torsion groups*
  Volodymyr Nekrashevych (Texas A&M University, USA)

  We will talk about amenable infinite finitely generated torsion groups. The classical methods of constructing groups of Burnside type (by adding defining relations to the free group) typically produce non-amenable groups. A method inspired by topological dynamics produces amenable finitely generated infinite torsion groups starting from locally finite groups. This includes simple groups and simple groups of subexponential growth. The method also includes some old groups (e.g., the Grigorchuk groups) but it provides a perspective on these old examples.

- *С.И.Адян и алгоритмические проблемы топологии*
  Sergei Novikov (Steklov Mathematical Institute, Moscow)

- *Bounded generation for linear and Kac-Moody groups*
  Eugeny Plotkin (Bar-Ilan University, Israel)

  In our talk we will go over the development of bounded generation results, starting from Carter-Keller (1983) and Adian-Mennicke (1991) papers till the very recent achievements. We will also dwell on some useful applications of this notion.

- *Small cancellation rings*
  Eliyahu Rips (Hebrew University, Israel)

  All the rings we deal with are algebras over a field with a basis of invertible elements. The small cancellation theory for groups (along with its generalizations) is a well-known and quite useful part of the combinatorial theory of groups. When a group is given by generators and defining relations, the small cancellation condition means that in a certain sense there is a weak interaction between the defining relations. In order to define an analogous notion for rings, we formulate an appropriate Small Cancellation axiom. However, we need to add what we call the Isolation axiom. Then we develop the structure theory for rings satisfying these axioms. We construct a filtration on a small cancellation ring and find a linear basis of it. In particular, we show that the small cancellation ring is non-trivial. We anticipate that many of the properties of small cancellation groups or of hyperbolic groups have appropriate analogs in the case of small cancellation rings. On the other hand, one can consider rings with small cancellation as the first step towards a more general theory of rings with iterated small cancellation yet to be constructed, that could yield examples of rings with exotic properties.

- *On maximal subgroups in infinite finitely generated groups*
  Tatiana Smirnova-Nagnibeda (University of Geneva and St.Petersburg State University)

  Margulis and Soifer proved that a finitely generated linear group has all maximal subgroups of finite index (has the MF property) if and only if it is virtually solvable. Otherwise it has uncountably many maximal subgroups of infinite index. Pervova later proved that outside the world of linear groups Grigorchuk's group also has the MF property. However it is known to have a huge variety of weakly maximal subgroups. I will discuss some questions stemming from the aforementioned results about the richness versus rigidity of maximal and weakly maximal subgroups in various classes of groups, in particular in branch groups and in Thompson groups.

- *Growth rates of Coxeter groups and Perron numbers*
  Alexey Talambutsa (Steklov Mathematical Institute, Moscow)

  A classical formula obtained by Steinberg in 1968 shows that the growth series of a Coxeter group (with respect to its standard generating set consisting of involutions) is a rational function, hence the growth rates of these groups are algebraic numbers. In 1980's Cannon discovered a remarkable connection between Salem polynomials and growth functions of surface groups and some cocompact Coxeter groups. Later Floyd and Parry obtained remarkable results that in many cases the growth rates of cocompact and cofinite Coxeter groups are either Salem or Pisot numbers, which belong to a wider class of Perron numbers.

  According to the conjecture of Kellerhals–Perren, all standard growth rates of all cofinite Coxeter groups are Perron numbers. For many classes of such groups the conjecture has been confirmed in two recent decades by different authors, however all these results were obtained using the recursive Steinberg's formula, so the groups with big number of generators (more than 15) could not be treated.

  We define a large class of abstract Coxeter groups, that we call ∞–spanned, and for which we prove that the word growth rate and the geodesic growth rate are Perron numbers. This class contains a fair amount of Coxeter groups acting on hyperbolic spaces, including a notable example of Vinberg and Kaplinskaya of a cofinite 50-generated group which acts on the 19-dimensional hyperbolic space. In order to overcome the computational difficulties, instead of Steinberg formula, we use finite automata constructed by Brink and Howlett and Perron-Frobenius theory.

  Joint work with Alexander Kolpakov.

- *Finite and locally finite groups generated with involutions*
  Anatoly Vershik (PDMI RAS, Saint Petersburg)

  We define the class of countable groups of the symmetries of the spaces of paths of the graded N-graphs which are Hasse diagrams of the distributed lattices. From other side the is the group of symmetries of the monotonic numerations of a poset and are natural generalizations of the Coxeters groups. The countable groups of that type are the inductive limit of countable symmetric groups. We are interested in the actions with invariant measures and with representations of these groups.

- *Identities in twisted Brauer monoids*
  M.V.Volkov (Ural Federal University, Ekaterinburg)

  The twisted Brauer monoid $\mathcal{W}_n$ is the monoid generated by $s_1, \ldots, s_{n-1}, h_1, \ldots, h_{n-1}, c$, subject to the following relations that hold for all $i, j = 1, \ldots, n-1$:

  $$\begin{array}{ll} h_i h_j = h_j h_i, \quad s_i s_j = s_j s_i, \quad h_i s_j = s_j h_i & \text{if } |i-j| \geq 2; \\ h_i h_j h_i = h_i, \quad s_i s_j s_i = s_j s_i s_j, \quad s_i s_j h_i = h_j s_i s_j & \text{if } |i-j| = 1; \\ h_i^2 = c h_i = h_i c, \quad s_i^2 = 1, \quad c s_i = s_i c, \quad s_i h_i = h_i s_i = h_i. & \end{array}$$

  The submonoid $\mathcal{K}_n$ of $\mathcal{W}_n$ generated by $h_1, \ldots, h_{n-1}, c$ is called the Kauffman monoid. Identities in Kauffman monoids were studied in [1,2]. It has been shown that the identity checking problem for the monoids $\mathcal{K}_3$ and $\mathcal{K}_4$ is decidable in polynomial time.

  Theorem. The identity checking problem for the monoid $\mathcal{W}_n$ with $n \geq 4$ is coNP-complete.

The complexity of identity checking problem for the monoid $\mathcal{W}_3$ still remains unknown.

[1] Chen Yuzhu, Hu Xun, Kitov N. V., Luo Yanfeng, Volkov M. V. Identities of the Kauffman monoid $\mathcal{K}_3$. Comm. Algebra 48:5 (2020), 1956–1968.

[2] Kitov N. V., Volkov M. V. Identities of the Kauffman monoid $\mathcal{K}_4$ and of the Jones monoid $\mathcal{J}_4$. In: Fields of Logic and Computation III Lect. Notes Comp. Sci., volume 12180, Springer, Cham, 2020, 156–178.

- *Logic and Mathematics. An interaction via Model Theory.*
  Boris Zilber (Oxford University, United Kingdom)

  The study of fundamental logical notions such as a formal language, theory, completeness, categoricity and other lead to an interaction with core mathematical theories at a very deep level.