Multidimensional Residues and Tropical Geometry

June 14 – 18, 2021, Sochi

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- Sirius Mathematics Center
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Location

- Sirius Mathematics Center, Sochi, Olimpiyskiy av., 3

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List of speakers

- **Alexander Aleksandrov**, *Institute of Control Sciences, Russian Academy of Sciences*, Moscow, Russia
- **Mats Andersson**, *Chalmers University of Technology and the University of Gothenburg*, Göteborg, Sweden
- **Irina Antipova**, *Siberian Federal University*, Krasnoyarsk, Russia
- **Victor Batyrev**, *Universität Tübingen*, Tübingen, Germany
- **Valery Beloshapka**, *Lomonosov Moscow State University*, Moscow, Russia
- **Matvey Durakov**, *Siberian Federal University*, Krasnoyarsk, Russia
- **Yury Eliyashev**, *HSE University*, Moscow, Russia
- **Alexander Esterov**, *HSE University*, Moscow, Russia
- **Sergey Feklistov**, *Siberian Federal University*, Krasnoyarsk, Russia
- **Alexey Golota**, *HSE University*, Moscow, Russia
- **Sergey Gorchinskiy**, *Steklov Mathematical Institute of Russian Academy of Sciences*, and *HSE University*, Moscow, Russia
- **Sabir Gusein-Zade**, *Lomonosov Moscow State University*, *Moscow Center for Fundamental and Applied Mathematics* and *HSE University*, Moscow, Russia
- **Ilia Itenberg**, *Sorbonne Université*, Paris, France
- **Burglind Jöricke**, *Institut des hautes études scientifiques*, Bures-sur-Yvette, France
- **Dmitry Kaledin**, *Steklov Mathematical Institute of Russian Academy of Sciences*, Moscow, and *HSE University*, Moscow, Russia
- **Nikita Kalinin**, *Saint Petersburg State University* and *HSE University*, St. Petersburg, Russia
- **Sergei Kalmykov**, *Shanghai Jiao Tong University*, Shanghai, China and *Institute for Applied Mathematics, Far Eastern Branch of the Russian Academy of Sciences*, Vladivostok, Russia
- **Askold Khovanskii**, *University of Toronto*, Toronto, Canada and *Moscow Independent University*, Moscow, Russia
- **Gulmiza Khudayberganov**, *National University of Uzbekistan*, Tashkent, Uzbekistan
- **Ilya Kossovskiy**, *Masaryk University*, Brno, Czech Republic

- **Igor Krichever**, *Columbia University*, New York, NY, USA, *Skoltech*, Moscow, and *Kharkevich Institute for Information Transmission Problems*, Moscow, Russia

- **Nikolai Kruzhilin**, *Steklov Mathematical Institute of Russian Academy of Sciences*, Moscow, Russia

- **Victor Kulikov**, *Steklov Mathematical Institute of Russian Academy of Sciences*, Moscow, Russia

- **Richard Lärkäng**, *Chalmers University of Technology and the University of Gothenburg*, Göteborg, Sweden

- **Alexander Loboda**, *Voronezh State University*, and *Voronezh State Technical University*, Voronezh, Russia

- **Konstantin Loginov**, *Steklov Mathematical Institute of Russian Academy of Sciences*, *HSE University*, and *Moscow Institute of Physics and Technology*, Moscow, Russia

- **Alexander Mednykh**, *Sobolev Institute of Mathematics*, and *Novosibirsk State University*, Novosibirsk, Russia

- **Evgeny Mikhalkin**, *Siberian Federal University*, Krasnoyarsk, Russia

- **Ildar Musin**, *Institute of Mathematics with Computer Centre of the Ufa Scientific Centre of the Russian Academy of Sciences*, Ufa, Russia

- **Simona Myslivets**, *Siberian Federal University*, Krasnoyarsk, Russia

- **Mounir Nisse**, *Xiamen University Malaysia*, Jalan Sunsuria, Bandar Sunsuria, 43900 Sepang, Selangor, Malaysia

- **Denis Osipov**, *Steklov Mathematical Institute of Russian Academy of Sciences*, *HSE University*, and *Moscow Institute of Steel and Alloys State Technological University*, Moscow, Russia

- **Dmitry Pochekutov**, *Siberian Federal University*, Krasnoyarsk, Russia

- **Vladimir Popov**, *Steklov Mathematical Institute of Russian Academy of Sciences*, Moscow, Russia

- **Yuri Prokhorov**, *Steklov Mathematical Institute of Russian Academy of Sciences*, Moscow, Russia

- **Victor Przyjalkowski**, *Steklov Mathematical Institute of Russian Academy of Sciences*, and *HSE University*, Moscow, Russia

- **Alexander Rashkovskii**, *University of Stavanger*, Stavanger, Norway

- **Azimbay Sadullaev**, *National University of Uzbekistan*, Tashkent, Uzbekistan

- **Timur Sadykov**, *Plekhanov Russian University of Economics*, Moscow, Russia

- **Håkan Samuelsson Kalm**, *Chalmers University of Technology and the University of Gothenburg*, Göteborg, Sweden
- **Gerd Schmalz**, *University of New England*, Armidale, Australia
- **Armen Sergeev**, *Steklov Mathematical Institute of Russian Academy of Sciences*, Moscow, Russia
- **Rasul Shafikov**, *University of Western Ontario*, London, Ontario, Canada
- **Nikolay Shcherbina**, *University of Wuppertal*, Wuppertal, Germany
- **Alexander Shlapunov**, *Siberian Federal University*, Krasnoyarsk, and *Sirius University of Science and Technology*, Sochi, Russia
- **Constantin Shramov**, *Steklov Mathematical Institute of Russian Academy of Sciences*,and *HSE University*, Moscow, Russia
- **Frank Sottile**, *Texas A&M University*, College Station, USA
- **Vitaly Stepanenko**, *Siberian Federal University*, Krasnoyarsk, Russia
- **Maria Stepanova**, *Lomonosov Moscow State University*, Moscow, Russia
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- **Alexander Tumanov**, *University of Illinois at Urbana-Champaign*, Urbana, USA
- **Dimitrii Tyurin**, *HSE University*, Moscow, Russia
- **Roman Ulvert**, *Reshetnev University of Science and Technology*, Krasnoyarsk, Russia
- **Elizabeth Wulcan**, *Chalmers University of Technology and the University of Gothenburg*, Göteborg, Sweden
- **Alain Yger**, *Université Bordeaux 1*, Talence, France

Schedule

Click on the personality to proceed to the corresponding abstract

June 14, Monday

9:15-09:30	*Opening ceremony*, conference hall *Torino*	
Chairman	**August Tsikh**, conference hall *Torino*	
09:30-10:30	Dmitry Kaledin	
10:30-11:00	*Coffee-break*	
11:00-12:00	Burglind Jöricke	
12:00-13:00	Igor Krichever	
13:00-14:30	*Lunch*	
Chairman	**Victor Przyjalkowski**, conference hall *Torino*	**Ilya Kossovskiy**, conference hall *Chamonix*
14:30-15:30	Sergey Gorchinskiy	Gerd Schmalz
15:30-16:30	Denis Osipov	Nikolai Kruzhilin
16:30-17:00	*Coffee-break*	
Chairman	**Dmitry Kaledin**, conference hall *Torino*	**Valery Beloshapka**, conference hall *Chamonix*
17:00-18:00	Irina Antipova	Alexandre Sukhov
18:00-18:30	Dimitrii Tyurin	Ilya Kossovskiy
18:30-19:00	Sergey Feklistov	
19:00	*Welcome party*	

June 15, Tuesday

Chairman	**Igor Krichever**, conference hall *Torino*	
09:30-10:30	Armen Sergeev	
10:30-11:00	*Coffee-break*	
11:00-12:00	Mats Andersson	
12:00-13:00	Yuri Prokhorov	
13:00-14:30	*Lunch*	
Chairman	**Irina Antipova**, conference hall *Torino*	**Timur Sadykov**, conference hall *Chamonix*
14:30-15:30	Elizabeth Wulcan	Azimbay Sadullaev
15:30-16:30	Håkan Samuelsson Kalm	Alexander Rashkovskii
16:30-17:00	*Coffee-break*	
Chairman	**Azimbay Sadullaev**, conference hall *Torino*	**Nikolai Kruzhilin**, conference hall *Chamonix*
17:00-17:30	Richard Lärkäng	Alexander Tumanov
17:30-18:00	Alexey Golota	
18:00-18:30	Sergei Kalmykov	Rasul Shafikov
18:30-19:00	Matvey Durakov	

June 16, Wednesday

Chairman	**Victor Kulikov**, conference hall *Torino*
09:30-10:30	Vladimir Popov
10:30-11:00	*Coffee-break*

11:00-12:00	Alain Yger
12:00-13:00	Victor Batyrev
13:00-14:00	*Lunch*
14:00-19:00	*Excursion*
19:00	*Banquet*

June 17, Thursday

Chairman	**Vladimir Popov**, conference hall *Torino*	**Armen Sergeev**, conference hall *Chamonix*
09:30-10:30	Victor Przyjalkowski	Timur Sadykov
10:30-11:00	*Coffee-break*	
11:00-12:00	Victor Kulikov	Nikolay Shcherbina
12:00-12:30	Constantin Shramov	Vitaly Stepanenko
12:30-13:00		Dmitry Pochekutov
13:00-14:30	*Lunch*	
Chairman	**Constantin Shramov**, conference hall *Torino*	**Nikolai Kruzhilin**, conference hall *Chamonix*
14:30-15:00	Andrey Trepalin	Alexander Loboda
15:00-15:30	Konstantin Loginov	
15:30-16:30	Sabir Gusein-Zade	Valery Beloshapka
16:30-17:00	*Coffee-break*	
Chairman	**Sergey Gorchinskiy**, conference hall *Torino*	
17:00-18:00	Askold Khovanskii	
18:00-19:00	Frank Sottile	

June 18, Friday

Chairman	**Yuri Prokhorov**, conference hall *Torino*	**Sabir Gusein-Zade**, conference hall *Chamonix*
09:30-10:30	Mounir Nisse	Alexander Aleksandrov
10:30-11:00	*Coffee-break*	
11:00-12:00	Ilia Itenberg	Alexander Mednykh
12:00-13:00	Alexander Esterov	Alexander Shlapunov
13:00-14:30	*Lunch*	
Chairman	**Denis Osipov**, conference hall *Torino*	**Alexander Shlapunov**, conference hall *Chamonix*
14:30-15:00	Evgeny Mikhalkin	Simona Myslivets
15:00-15:30	Yury Eliyashev	Roman Ulvert
15:30-16:00	Nikita Kalinin	Maria Stepanova
16:00-17:00	Ildar Musin	Gulmiza Khudayberganov
17:00-17:15	*Closing ceremony*, conference hall *Torino*	
17:15	*Dinner*	

Abstracts

Logarithmic differential forms on singular varieties

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The concept of logarithmic differential forms with poles along a *reduced* (i.e., without multiple components) divisor D defined on a complex smooth manifold M, appeared in the early 1960s in relation with the study of Hodge structures and the Gauss-Manin connection in the cohomology of singular varieties. More precisely, a meromorphic differential form ω with poles along D is called *logarithmic* if ω and the total differential $d\omega$ have at worst simple poles only along the divisor D. The corresponding sheaves are usually denoted by $\Omega^p_M(\log D)$, $p \geq 0$. First, P. Deligne, N. Katz, Ph. Griffiths and others considered this notion for a union of smooth subvarieties with *normal crossings*, then K. Saito and his successors analyzed the case of divisors with other types of singularities, and so on.

We are developing a different approach to the study of logarithmic differential forms; it is based on an original interpretation of the classical de Rham lemma, adapted to the study of differential forms defined on hypersurfaces with arbitrary singularities. This approach allows us to extend the concept of logarithmic differential forms to the case of effective Cartier divisors defined on singular varieties. Some useful applications and further generalizations, as well as connections with the theory of multi-logarithmic differential forms and their residues, will be also discussed. ⇑

Pseudomeromorphic currents

Mats Andersson
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To begin with I will try to give an overview of some aspects of the development of multivariable residue theory, starting with the groundbreaking results by Coleff, Herrera, and Lieberman in the 70'. I will then motivate the introduction (A-Wulcan, 2010) of the sheaf of pseudomeromorphic currents, discuss its basic properties and sketch some recent works where it plays a role. In particular I will mention a Serre duality theorem on a non-reduced analytic space (A-Lennartsson-Lärkäng-Samuelsson Kalm 2021). ⇑

Multidimensional Mellin Transforms: Fundamental Correspondence

Irina Antipova
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There is a precise correspondence between individual terms in the asymptotic expansion of an original function and singularities of its Mellin transform. This general phenomenon is called the fundamental correspondence which determines the scope of application for Mellin fransforms. A pair of convex domains $\Theta, U \subset \mathbb{R}^n$ encodes isomorphic functional spaces M^U_Θ, W^Θ_U which are transformed to each other by the direct and inverse Mellin transforms. The domains Θ and U predetermine the asymptotic behaviour of functions in classes M^U_Θ and W^Θ_U respectively.

Mellin transforms figure prominently in the complex analysis due to being the most appropriate for using the residue theory techniques. In particular, the direct Mellin transform is used for computing the Bochner-Martinelli residue currents, and the inverse transform (the Mellin-Barnes integral representation) serves as a tool for analytic continuation of algebraic functions.

In my talk, I will focus on the properties of the multidimensional Mellin transform for rational functions with quasi-elliptic or hyperelliptic denominators. ⇑

Variations on the theme of classical discriminant

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The classical discriminant $\Delta_n(f)$ of a degree n polynomial $f(x)$ is an irreducible homogeneous polynomial of degree $2n-2$ on the coefficients $a_0, \ldots, a_n$ of f that vanishes if and only if f has a multiple zero. I will explain a tropical proof of the theorem of Gelfand, Kapranov and Zelevinsky (1990) that identifies the Newton polytope P_n of Δ_n with an $(n-1)$-dimensional combinatorial cube obtained from the classical root system of type A_{n-1}. Recently Mikhalkin and Tsikh (2017) discovered a nice factorization property for truncations of Δ_n with respect to facets Γ_i of P_n containing the vertex $v_0 \in P_n$ corresponding to the monomial $a_1^2 \cdots a_{n-1}^2 \in \Delta_n$. I will give a GKZ-proof of this property and show its connection to the boundary stata in the $(n-1)$-dimensional toric Losev-Manin moduli space $\overline{L_n}$. Some variations on the above statements will be discussed in connection to the toric moduli space associated with the root system of type B_n and to the mirror symmetry for 3-dimensional cyclic quotient singularities $\mathbb{C}^3/\mu_{2n+1}$. ⇑

The model surface method: a new step

Valery Beloshapka
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There were several steps of construction of the model surface method. At the beginning, H. Poincare used as a model surface the 3-dimensional sphere, then J. Moser used hyperquadrics. Further appeared quadrics of higher codimension, then totally nondegenerate model surfaces were introduced. In 2020, the theory made the next step: model surfaces of an arbitrary Bloom-Graham type were considered. This was crucial for the general appearance of the theory and stated many new questions. ⇑

About the Blaschke products in polydiscs

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We consider multidimensional analogs of Blaschke's products. The motivation for this consideration was the articles by D. Alpay and A. Iger in connection with the interpolation theory in some functional spaces (Hilbert, Hardy, etc.). We construct the Blaschke multipliers using the Rudin characterization [4] of interior functions in polydisc and the Lee-Yang polynomials from the theory of phase transitions in statistical mechanics [5], [1]. As shown by M. Passare and A. Tsikh in [2], the amoeba of the Lee-Yang polynomial adjoins the log-image of a unit polydisc only on the skeleton of the polydisc. The main result of my talk is a theorem about the construction of a multidimensional Blaschke multiplier in odd- dimensional spaces. In such spaces, the Lee-Yang polynomial naturally decomposes into the sum of two polynomials that make up the inner rational function in the polydisc. The description of the permissible denominators of inner rational functions is made by the language of the polar [3] of the real cube $[-1, 1]^n$. ⇑

[1] T. D. Lee and C. N. Yang. Statistical theory of equations of state and phase transitions. II. Lattice gas and Ising model. *Phys. Rev. (2)*, 87:410–419, 1952.

[2] Mikael Passare and August Tsikh. Amoebas: their spines and their contours. In *Idempotent mathematics and mathematical physics*, volume 377 of *Contemp. Math.*, pages 275–288. Amer. Math. Soc., Providence, RI, 2005.

[3] R. Tyrrell Rockafellar. *Convex analysis.* Princeton Mathematical Series, No. 28. Princeton University Press, Princeton, N.J., 1970.

[4] Walter Rudin. *Function theory in polydiscs.* W. A. Benjamin, Inc., New York-Amsterdam, 1969.

[5] C. N. Yang and T. D. Lee. Statistical theory of equations of state and phase transitions. I. Theory of condensation. *Phys. Rev. (2)*, 87:404–409, 1952.

Tropical Chern classes and Chern-Weil theory

Yury Eliyashev
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There is a natural analog of a vector bundle in the tropical setting. The Chern classes of a tropical vector bundle was introduced by L. Allermann in terms of intersection theory. In our talk we will construct the Chern classes of a tropical vector bundle in terms of differential geometry. The classical Chern-Weil theory constructs the Chern classes of a complex vector bundle in terms of differential geometry, in particular using curvature and connection of a vector bundle. We will describe the tropical analog of this construction. Also we will discuss a tropical analog of the exponential long exact sequence of a line bundle. ⇑

Tropical characteristic classes

Alexander Esterov
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To every affine algebraic variety one can assign its tropical fan, which remembers a lot about the intersection theory of the variety. Moreover, to every k-dimensional variety one can associate its tropical characteristic classes (a tuple of tropical fans of dimensions from 0 to k), which remember much more. I will introduce tropical characteristic classes, discuss how to compute them, and review some of their applications. ⇑

The Hartogs extension phenomenon in toric varieties

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We study the Hartogs extension phenomenon in non-compact toric varieties and its relation to the first cohomology group with compact support. We show that a toric variety admits this phenomenon if at least one connected component of the fan complement is concave, proving by this an earlier conjecture M. Marciniak. ⇑

Positivity for codimension one holomorphic foliations

Alexey Golota
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Let $\mathscr{F}$ be a codimension one holomorphic foliation on a smooth projective manifold X. We can associate to $\mathscr{F}$ its canonical line bundle (analogue of the canonical line bundle of a manifold) and its conormal line bundle. I will discuss positivity properties of these line bundles; also I will use positivity to prove some classification results for foliations. ⇑

Higher-dimensional Contou-Carrère symbol, I

Sergey Gorchinskiy
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The talk is based on a common work with Denis Osipov. Contou-Carrére symbol in dimension n is a way to construct an invertible element of an arbitrary commutative ring A from $n+1$ Laurent series in n variables over A. This symbol arises when considering families of n-dimensional varieties and chains of irreducible subvarieties on them. The higher-dimensional Contou-Carrere symbol satisfies many fundamental properties, among them, a higher-dimensional reciprocity law, which implies basically all known reciprocity laws. In our survey talk, we will discuss all these phenomena starating from the Weil reciprocity law on a curve. ⇑

Indices of 1-forms and quadratic forms

Sabir Gusein-Zade
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The index of an isolated singular point of a vector field or of a 1-form on a smooth manifold can be expressed as the dimension of a certain algebra in the complex analytic setting and as the signature of a quadratic form on its real analogues (part) in the real analytic setting. This quadratic form has a natural definition in terms of residues. We shall discuss indices of singular points of 1-forms on isolated complete intersection singularities and quadratic forms related to them. ⇑

Refined invariants for real elliptic curves

Ilia Itenberg
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The talk is devoted to several real and tropical enumerative problems. We suggest new invariants of the projective plane (and, more generally, of toric surfaces) that arise as results of an appropriate enumeration of real elliptic curves. These invariants admit a refinement similar to the one introduced by Grigory Mikhalkin in the rational case. We discuss tropical counterparts of the elliptic invariants under consideration and prove a recursive formula allowing one to compute them.

This is a joint work with Eugenii Shustin. ⇑

Mappings from open Riemann surfaces to the twice punctured complex plane and the restricted validity of Gromov's Oka principle

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According to Gromov the Oka principle holds for holomorphic mappings from a complex manifold X to a complex manifold Y if each continuous mapping $X \to Y$ is homotopic to a holomorphic mapping. Giving sufficient conditions on the target Y for the validity of the Oka principle for holomorphic mappings from any Stein manifold to Y, he initiated a line of interesting and fruitful research. On the other hand he mentions mappings from annuli to the twice punctured complex plane as simplest example for which this Oka principle fails and draws attention to the fact that mappings from annuli play a crucial role for understanding the "rigidity" of the target Y in case the Oka principle fails for mappings from some Stein manifolds to Y.

We will say that a continuous mapping f from a finite open Riemann surface X to the twice punctured complex plane has the Gromov-Oka property if for each orientation preserving homeomorphism $\omega : X \to \omega(X)$ onto a Riemann surface $\omega(X)$ with only thick ends the mapping $f \circ \omega^{-1}$ is homotopic to a holomorphic mapping. For finite open Riemann surfaces we show the existence of finitely many embedded annuli in X, such that f has the Gromov-Oka property iff its restriction to each of the annuli has this property, and describe all mappings with the Gromov-Oka property. On the other hand we show that for X_ε being the ε-neighbourhood of a skeleton of a torus with a hole, the number of irreducible holomorphic mappings $X_\varepsilon \to \mathbb{C} \setminus \{-1, 1\}$ up to homotopy grows exponentially in $\frac{1}{\varepsilon}$. ⇑

Witt vectors ring and the residue map

Dmitry Kaledin
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I will show how the residue map provides a simple and natural construction of the multiplication in the ring of Witt vectors (and I will remind the audience what is this ring). If time permits, I will also describe a non-commutative generalization of this construction. ⇑

Tropical singular points

Nikita Kalinin
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We will consider tropicalisations of m-fold points of algebraic curves. Certain estimates on the area of the corresponding dual part of the Newton polygon will be provided. Also we will speak about possible generalisations of this approach for higher dimensions. ⇑

On interpolation with finite Blaschke products

Sergei Kalmykov
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In this talk, we discuss an alternative proof of the Jones-Ruscheweyh theorem concerning boundary interpolation with finite Blaschke products. The approach is based on Positivstellensatz by Prestel and Delzell and a representation of positive polynomials in a special form due to Berr and Wörmann together with a particular structure of equations.

This is based on joint work with Béla Nagy. ⇑

Balance and rigidity conditions for subvarieties in $(\mathbb{C}^*)^n$

Askold Khovanskii
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An enriched tropical fan of an algebraic variety satisfies well known additive balanced conditions. Beside these conditions it satisfies multiplicative balance conditions and rigidity conditions which I will discuss in the talk. For hypersurfaces these conditions are not only necessary but also sufficient. ⇑

Laurent series with respect to a matrix ball from the space $\mathbb{C}^n\,[m \times m]$

Gulmiza Khudayberganov
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In this work we suggest analogues of the Laurent series with respect to a matrix ball from the space $\mathbb{C}^n\,[m \times m]$. For this purpose we introduce first the notion of the layer of a matrix ball in $\mathbb{C}^n\,[m \times m]$, and then we use the properties of the Bochner-Luogeng type intergrals in this layer to obtain the analogues of the Laurent series. ⇑

Applications of the Multisummability theory in Complex Analysis

Ilya Kossovskiy
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The theory of multisummability for solutions of complex differential equations with an isolated singularity is an important achievements in Dynamics of the 20's theory. In a certain sense, it is a theory of how to describe solutions of such differential equations in a "large" sector with the vertex at the singularity, and how then to uniquely recognize a solution by its Taylor series (at the singularity). In this talk, we show unexpected applications of the multisummability theory for classification problems for real submanifolds in complex space. ⇑

Generalized amoebas of the second kind

Igor Krichever
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The notion of a meromorphic real normalized differential is equivalent to the notion of a harmonic function on an algebraic curve with "algebraic type" singularities at punctures. A pair of such differentials determines the harmonic map of the complement on the curve to the punctures on (in) the two-dimensional real plane, which can be regarded as a generalization of the amoeba map of a plane algebraic curve. The moduli spaces of algebraic curves with a pair of real normalized meromorphic differentials are fundamental in the theory of integrable systems and their perturbations, in the Seiberg-Witten solution to $N = 2$ SUSY gauge models. Recently they found applications to the study of geometry of the moduli spaces of curves. In the talk I will present the basics of the theory of real normalized differentials and their applications and some open problems. ⇑

Fundamental theorem of projective geometry in multidimensional complex analysis

Nikolai Kruzhilin
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A local holomorphic version of the fundamental theorem of projective geometry is discussed along with some applications to problems in CR-geometry. ⇑

Germs of finite covers of the plane branched in germs of curves with ADE singularities and Belyi pairs

Victor Kulikov
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In the talk, a mapping to the set of equivalence classes of Belyi pairs from the set of equivalence classes of rigid germs of finite covers of the plane, branched in the germs curves with ADE-type singularities, will be constructed. ⇑

Chern Currents of Coherent Sheaves

Richard Lärkäng
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I will present some joint work with Elizabeth Wulcan. Given a locally free resolution of a coherent analytic sheaf F, equipped with Hermitian metrics and connections, we construct an explicit current, obtained as the limit of smooth Chern forms of F, that represent the Chern class of F and has support on the support of F. If the connections are $(1, 0)$-connections and F has pure dimension, then the non-trivial part of lowest degree of this Chern current coincides with (a constant times) the fundamental cycle of F. The proof of this result utilizes the theory of residue currents, and goes through a generalized Poincaré-Lelong formula, previously obtained by the authors, and a result that relates the Chern current to the residue current associated with the locally free resolution. ⇑

On holomorphically homogeneous hypersurfaces in $\mathbb{C}^4$

Alexander Loboda
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In connection with the description problem of (locally) holomorphically homogeneous real hypersurfaces of multidimensional complex spaces the case of the space $\mathbb{C}^4$ is discussed (the 2-dimensional case was described by E. Cartan, the study of the 3-dimensional one was completed last year by the author of this report).

Three approaches to the problem are considered:

1) representations of abstract Lie algebras in the form of holomorphic vector fields algebras on the homogeneous hypersurfaces under consideration;

2) a coefficient approach using the properties of normal (canonical) equations of homogeneous manifolds;

3) computer calculations associated with the homogeneity property.

Within the framework of the first approach, in the space $\mathbb{C}^4$, all the Levi nondegenerate orbits of 7-dimensional nilpotent Lie algebras are studied. It is proven that for the entire collection of 180 types of such algebras there are only 4 (up to holomorphic equivalence) nondegenerate surfaces: two quadrics and two generalizations of the known Winkelmann surface from $\mathbb{C}^3$.

For analytic strictly pseudoconvex hypersurfaces of the space $\mathbb{C}^4$, the properties of fourth degree polynomials from the normal Moser equations are studied. It is expected that, by analogy with the 3-dimensional case, the using of these properties will provide a description of large families homogeneous surfaces with rich symmetry algebras.

On the base of algorithms of symbolic mathematics, a large number of examples of affine homogeneous hypersurfaces of the space $\mathbb{C}^4$ (both nondegenerate and degenerate) are constructed.

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Boundedness of divisors on Fano fibrations

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Fano fibrations are natural objects that appear in the Mori program, which aims to classify higher-dimensional algebraic varieties. Unlike Fano varieties, Fano fibrations of a given dimension and with restricted singularities are not bounded. Nevertheless, we show that under some conditions, divisors on such fibrations are bounded. This result has applications for bounding the irrationality of fibers in del Pezzo fibrations (a del Pezzo surface is a Fano variety of dimension 2). Also it implies boundedness for divisors on resolutions of singularities on threefolds under some conditions.

The talk is based on a joint work with C. Birkar. ⇑

On Kirchhoff index and the number of spanning trees and rooted spanning forests in circulant graphs

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The aim of this report is to find analytical formula for the Kirchhoff index and the number of spanning trees and rooted spanning forests in circulant graphs $C_n(s_1, s_2, ..., s_k)$ on n vertices. Asymptotic behavior of the above mentioned quantities is investigated as n tends to the infinity. We proof that Kirchhoff index of a circulant graph can be expressed as a sum of a cubic polynomial of n and an exponentially small remainder. ⇑

Blow-ups for the Horn-Kapranov parametrization of the classical discriminant

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It is useful to consider a mermorphic map $f : X \to Y$ of complex analytic sets (spaces) as an analytic subset in $X \times Y$, which is the closure $\overline{G}_f$ of the graph of f. A fiber $\overline{G}_f$ over an indeterminacy point is interpreted through 'blow up' or 'blow down'. The most transparent scheme for these procedures is realized for mappings that are inverses of the logarithmic Gauss mappings for hypersurfaces. In the theory of hypergeometric functions these inverses are called *the Horn-Kapranov parametrizations.*

In the paper, on which this talk is based, we study the Horn-Kapranov parametrizations for the classical discriminant to prove factorization identities for its 'cut-offs'. More precisely, consider a polynomial in one variable

$$f(y) = a_0 + a_1 y + \ldots + a_n y^n.$$

Its discriminant is the irreducible polynomial $\Delta_n = \Delta_n(a_0, a_1, \ldots, a_n)$ with integer coefficients that vanishes if and only if f has multiple roots. In [4] it is proved that the Newton polytope $\mathcal{N}(\Delta_n) \subset \mathbb{R}^{n+1}$ of this discriminant is combinatorially equivalent to an $(n-1)$-dimensional cube. Note that another proof of this fact was given by V. Batyrev, see [2].

We are interested in 'cut-offs' of the discriminant Δ_n to faces of its polytope $\mathcal{N}(\Delta_n)$ formed by intersections of p facets. Recall that a *'cut-off'* (restriction) of a polynomial Δ to a face h of its polytope $\mathcal{N}(\Delta)$ is the sum of monomials of Δ whose exponents belong to h. In the talk we present explicit factorization formulas for 'cut-offs' of Δ_n to faces of $\mathcal{N}(\Delta_n)$. The study of such formulas is motivated by investigations of the structure of the universal algebraic function, see [1], [3]. The main point of these formulas is that these 'cut-offs' are products of discriminants of polynomials of degrees less than n.

These results are obtained together with V.A. Stepanenko and A.K. Tsikh. ⇑

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Analytical realization of the strong dual of a space of holomorphic functions with boundary smoothness. Applications

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We consider the space of functions holomorphic in a bounded convex domain of a multidimensional complex space and infinitely differentiable up to its boundary, with given estimates of all derivatives. For this space a Polya-Martineau-Ehrenpreis type theorem is established. This result allows to study the classical problems of the theory of linear differential operators with constant coefficients in the space under consideration, in particular, to obtain analogs of Theorems 7.6.13 and 7.6.14 from the book by L. Hormander (L. Hormander. An Introduction to Complex Analysis in Several Variables, Van Nostrand, London, 1966). ⇑

The holomorphic extension of functions with the boundary Morera properties in domains with piecewise-smooth boundary

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The problem of holomorphic extension of functions defined on the boundary of a domain into this domain is actual in multidimensional complex analysis. It has a long history, starting with the proceedings of Poincaré and Hartogs. We consider continuous functions defined on the boundary of a bounded domain D in $\mathbb{C}^n$, $n > 1$ with piecewise-smooth boundary, and possessing the generalized boundary Morera property along the family of complex lines that intersect the boundary of a domain. Morera property is that the integral of a given function is equal to zero over the intersection of the boundary of the domain with the complex line. It is shown that such functions extend holomorphically to the domain D. For functions of one complex variable, the Morera property obviously does not imply a holomorphic extension. Therefore, this problem should be considered only in the multidimensional case ($n > 1$). The main method for studying such functions is the method of multidimensional integral representations, in particular, the Bochner-Martinelli integral representation. ⇑

Phase tropical varieties those are degenerations of complete intersections are topological manifolds

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In the first part of this talk we show that a smooth complex projective complete intersection variety of arbitrary codimension can be decomposed into pairs-of-pants, where a k-dimensional pair-of-pants is diffeomorphic to the complement of $k + 2$ generic hyperplanes in $\mathbb{CP}^k$. This generalizes an earlier theorem of Mikhalkin. Moreover, we prove that a phase tropical variety which is a degeneration of a smooth complete intersection varieties is a topological manifold. This gives a positive answer to Viro's conjecture in the case of complete intersections. ⇑

Higher-dimensional Contou-Carrère symbol, II

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The talk is based on a joint work with Sergey Gorchinskiy. The n-dimensional Contou-Carrère symbol is a universal deformation of the n-dimensional tame symbol such that it satisfies the Steinberg property from algebraic K-theory and it is possible to obtain the n-dimensional residue from this symbol. I will give various equivalent definitions of the n-dimensional Contou-Carrère symbol: 1) by an explicit "analytic" formula over $\mathbb{Q}$-algebras, 2) by means of the action of the group of continuous automorphisms of iterated Laurent series over a ring, 3) by means of algebraic K-theory. I will explain also the universal property for the n-dimensional Contou-Carrère symbol, the proof of which is based on the statement that the tangent map to the map given by the n-dimensional Contou-Carrère symbol is the n-dimensional residue. ⇑

Analytic Continuation of Diagonals of Laurent Series for Rational Functions

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We discuss branch points of complete q-diagonals of Laurent series for rational fuctions in several complex variables in terms of logarithmic Gauss mapping. In particular, we are interested in conditions of non-algebraicity of such diagonals. ⇑

Underlying algebraic varieties of algebraic groups

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The talk is aimed to discuss to what extent the geometric properties of the underlying algebraic variety of an algebraic group determine its group properties, and the related problems about such varieties. ⇑

Automorphism groups of compact complex varieties

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The purpose of the talk is to discuss recent results on finite subgroups of automorphism groups of compact complex varieties. We concentrate on boundedness properties and some classification theorems in low dimension. The talk is based on joint works with Constantin Shramov. ⇑

Landau–Ginzburg models, Calabi–Yau compactifications, and KKP and P=W conjectures

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Mirror Symmetry predicts a correspondence between Fano varieties and Landau–Ginzburg models, that is families of varieties (parameterized by complex numbers) with non-degenerate everywhere defined top holomorphic forms. By geometric reasons, as well as because of recent Katzarkov–Kontsevich–Pantev conjectures, P=W conjecture, and the conjecture about the dimension of anticanonical linear system, the challenging problem is to construct log Calabi–Yau compactifications of Landau–Ginzburg models. The canonical class of the compactification has first order poles in components of the fiber over infinity of the compactification. We discuss the notions, constructions, and conjectures mentioned above. ⇑

Set interpolation by plurisubharmonic geodesics

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Given a pair of compact, non-pluripolar, polynomially convex subsets K_0, K_1 of a bounded hyperconvex domain $\Omega \subset \mathbb{C}^n$, we consider a plurisubharmonic geodesic $u_t(z)$, $0 < t < 1$, between the functions $c_j\,\omega_j(z)$, $j = 0, 1$, where c_j are positive constants and ω_j are the extremal functions of the sets K_j relative to Ω: $\omega_j(z) = \sup\{u(z) : u \in PSH(\Omega),\ u < 0, u|_{K_j} \le -1\}$.

The sets $K_t = \{z \in \Omega : u_t(z) = \min_\Omega u_t\}$ interpolate K_0 and K_1. For a good choice of the constants c_j, the relative capacities $Cap\,(K_t, \Omega)$ are proved to satisfy a stronger version of Brunn-Minkowski type inequality. This is achieved by using linearity of the Monge-Ampère energy functional $\int_\Omega u_t(dd^c u_t)^n$.

When the sets K_j are Reinhardt subsets of the unit polydisk, K_t do not depend on the choice of the constants c_j and are the geometric means of K_0 and K_1: $K_t = K_0^{1-t} K_1^t$, and their capacities are $n!$ times the covolumes of certain unbounded convex subsets of $\mathbb{R}^n_+$. ⇑

Holomorphic continuation of a formal series along analytic curves

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This talk is devoted to a curvilinear analogue of the well-known Forelli theorem [3]: if a function f is infinitely smooth in a neighborhood of the origin $0 \in \mathbb{C}^n$, $f \in C^\infty\{0\}$, and for every complex line l passing through the origin the restriction $f|_l$ continues holomorphically into the unit disk $l\bigcap B(0,1)$, then f continues holomorphically into the unit ball $B(0,1)\subset\mathbb{C}^n$.

An example of a function

$$f(z_1,z_2)=\frac{z_1^{k+1}\bar{z}_2}{z_1\bar{z}_1+z_2\bar{z}_2}\in C^k\left(\mathbb{C}^2\right)$$

shows that the condition of infinite smoothness in Forelli's Theorem is essential. The restrictions $f|_l$ to complex lines $l\ni 0$ are polynomials, but $f(z_1,z_2)$ is not holomorphic.

The following takes place

Theorem 1. Let the unit ball $B(0,1)\subset\mathbb{C}^n$ be fibered by a smooth family of analytic curves $A_\lambda=\{z=p_\lambda(\xi)\},\lambda\in\mathbb{P}^{n-1}$, at the point 0, where $p_\lambda(\xi)=\left(p_\lambda^1(\xi),p_\lambda^2(\xi),...,p_\lambda^n(\xi)\right)$ is a holomorphic vector function in the unit disk $U=\{|\xi|<1\}:p_\lambda(\xi)=a_1(\lambda)\xi+a_2(\lambda)\xi^2+...,a_k(\lambda)\in C^1(\mathbb{C}^n),\ k=1,2,...,\ B(0,1)=\bigcap_\lambda A_\lambda$. If a function $f\in C^\infty\{0\}$ has the property that each restriction $f|_{A_\lambda},\ \lambda\in\mathbb{P}^{n-1}$, that is defined in the neighborhood of 0, holomorphically continues to the whole A_λ, then f continues holomorphically to $B(0,1)$.

Theorem 1 in the following version is also true under a weaker requirements.

Theorem 2. Under the conditions of Theorem 1, if each restriction $f|_{A_\lambda},\ \lambda\in W\subset\mathbb{P}^{n-1}$, holomorphically continues to the whole A_λ, then f continues holomorphically to the domain $\hat{O}=\left\{z\in\mathbb{C}^n:\ |z|\exp V^*\left(\frac{z}{|z|},O\right)<1\right\}$. Here $W\neq\emptyset$ is an open subset of $\mathbb{P}^{n-1}$, $O=\bigcap_{\lambda\in W}A_\lambda$, $V^*(\omega,O)$ is the Green's function in $\mathbb{C}^n$.

In the work [1] Chirka showed the validity of the curvilinear analogue of Forelli's theorem for $n=2$. Further advances on variations of the Forelli's theorem, were obtained in the works Kim et al., [5], [4], [2].

Keywords: Formal series, Forelli's theorem, Green function, plurisubharmonic functon.
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Multivariate hypergeometric polynomials and their amoebas

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Amoebas of complex algebraic varieties have attracted substantial attention in the recent years after their inception in the work of Gelfand, Kapranov and Zelevinsky. Being a semi-analytic subset of the real space, the amoeba carries a lot of geometric, algebraic, topological, and combinatorial information on the corresponding (tropical) algebraic variety.

Alongside with the definition of unbounded affine amoeba of an algebraic hypersurface, an alternative definition of compactified amoeba has been introduced. While the affine amoeba of a hypersurface is defined to be its Reinhardt diagram in the logarithmic scale, the compactified amoeba is the image of the hypersurface under the moment map providing a homeomorphism between the Newton polytope of the defining polynomial of that hypersurface and the positive orthant of the real vector space. Being topologically equivalent to the standard affine amoeba, its compactified counterpart often has the substantial disadvantage of exhibiting complement components of very different relative size. This makes it difficult to work with compactified amoebas in a computationally reliable way and probably explains the focus of research on affine amoebas.

In the talk (mainly based on a joint work with Dmitry Bogdanov) I will discuss the definition of an amoeba-shaped polyhedral complex of an algebraic hypersurface. Like the compactified amoeba, this polyhedral complex is a subset of the Newton polytope of the defining polynomial of the hypersurface. Besides, it is a deformation retract of the compactified amoeba and provides the straightforward solution to the membership problem: the order of a connected component in its complement is itself a point in this component. An explicit formula for this polyhedral complex will be given in the case when the hypersurface is optimal. Furthermore, the tropical-geometric obstructions for the amoeba of a multivariate hypergeometric polynomial to be optimal will be introduced and discussed. ⇑

A pullback on a class of currents

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In joint works with Andersson, Eriksson, Wulcan, and Yger we introduced a currential approach to non-proper intersection theory. In this talk I will explain how ideas from these works can be used to obtain a reasonable pullback of certain currents under any holomorphic mapping. ⇑

Rigid spheres and homogeneous Sasakian manifolds

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Sasakian manifolds can be defined as CR manifolds with a fixed translational symmetry transversal to the CR distribution. Locally, a Sasakian manifold of dimension $2n+1$ can be embedded into $\mathbb{C}^{n+1}$ as a real hypersurface with defining equation $\operatorname{Im} w = f(z)$, which does not depend on $\operatorname{Re} w$. Such hypersurfaces have been coined "rigid". N. Stanton has developed a version of the Chern-Moser normal form that takes into account rigidity. Rigidity can also be considered as a weaker structure than a Sasakian structure, by fixing a translational symmetry only up to scale.

Stanton's rigid normal form is very useful in the study of Sasakian and rigid structures. We demonstrate this in relation to homogeneous 3-dimensional Sasakian and rigid manifolds.

This is joint work with V. Ezhov and D. Sykes. ⇑

Topological insulators invariant under time reversal: an overview

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The talk is devoted to the theory of topological insulators which is an actively developing direction in the solid state physics. The search for new topological objects is reduced to the search of appropriate topological invariants and systems having non-trivial invariants. Such systems are characterized by wide energetic gaps stable with respect to small deformations. The quantum spin Hall insulator may be considered as a non-trivial example of such systems. It is a two-dimensional insulator invariant under time reversal having a non-zero topological $\mathbb{Z}_2$-invariant introduced by Kane and Mele.

Our talk is devoted to the topological insulators invariant under time reversal transform. In the first part we consider the physical basics of the theory of topological insulators while in the second part we deal with its mathematical aspects. ⇑

Polynomially convex embeddings and approximation of functions

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I will discuss the problem of polynomially convex embeddings of closed real manifolds into complex Euclidean spaces and its applications to the approximation of continuous functions. ⇑

On compacts possessing strictly plurisubharmonic functions

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We give a geometric condition on a compact subset of a complex manifold which is necessary and sufficient for the existence of a smooth strictly plurisubharmonic function defined in a neighbourhood of this set. ⇑

Approximation of solutions of the heat equation of Lebesgue class L^2 by more regular solutions

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Let $s \in \mathbb{N}$, $T_1, T_2 \in \mathbb{R}$, $T_1 < T_2$, and let Ω, ω be bounded domains with smooth boundaries in $\mathbb{R}^n$, $n \geq 1$ such that $\omega \subset \Omega$. We prove that the space $H^{2s,s}_{\mathcal{H}}(\Omega \times (T_1, T_2))$ of solutions of the heat operator $\mathcal{H} = \frac{\partial}{\partial t} - \sum_{j=1}^{n} \frac{\partial^2}{\partial x_j^2}$ in the cylinder domain $\Omega \times (T_1, T_2)$ belonging to anisotropic Sobolev space $H^{2s,s}(\Omega \times (T_1, T_2))$ is everywhere dense in the space $L^2_{\mathcal{H}}(\omega \times (T_1, T_2))$, consisting of solutions in the domain $\omega \times (T_1, T_2)$ of the Lebesgue class $L^2(\omega \times (T_1, T_2))$, if and only if the complement $\Omega \setminus \omega$ has no compact components in Ω. As an important corollary we obtain the theorem on the existence of a basis with the double orthogonality property for the pair of the Hilbert spaces $H^{2s,s}_{\mathcal{H}}(\Omega \times (T_1, T_2))$ and $L^2_{\mathcal{H}}(\omega \times (T_1, T_2))$. ⇑

Finite groups acting on algebraic varieties in positive characteristic

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I will give a survey of the results concerning finite subgroups in automorphism and birational automorphism groups of algebraic varieties over fields of positive characteristic. Similarly to the case of algebraic varieties over fields of characteristic zero and compact complex manifolds, we will focus on an appropriate analog of Jordan property for the relevant groups. ⇑

To be announced

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Integral representation of the group operation of the local Lie group. The reduced form of the Campbell-Hausdorff series

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We obtain the parametric solution of the system of Lie differential equations in the first kind canonical coordinates using the given structural constants. After complexification via Yuzhakov formulas we obtain the integral representation of the group operation of the local Lie group and the reduced form of the Campbell-Hausdorff series. We consider the example.

Keywords: multiple logarithmic residue, implicit function theorem in several complex variables, canonical coordinates of the first kind, Dynkin's form of the Campbell-Hausdorff series. ⇑

CR manifolds of infinite Bloom-Graham type

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We will provide a criterion for finite dimensionality of Lie algebra of infinitesimal holomorphic automorphisms in the case of real analytic manifolds of uniformly infinite Bloom-Graham type (i.e., infinite everywhere). This criterion was obtained with the help of an analogue of the Bloom-Graham theorem for germs of real analytic manifolds of infinite type and the description of a standard form (reduced form), in which it is possible to transform such germs. This form also enables us to construct a system of new biholomorphic invariants. ⇑

Local polynomially convex hulls of Levi-flat singularities

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We study the local polynomial convexity near a singular point of a real analytic Levi-flat hypersurface. It turns out that the answer depends on the geometry of the Levi foliation of a hypersurface.

This is a joint work with R.Shafikov, Univ. W. Ontario, Canada. ⇑

Automorphisms groups of pointless del Pezzo surfaces

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In the talk we discuss some properties of automorphisms groups of pointless del Pezzo surfaces of higher degree, in particular Severi—Brauer surfaces. We consider structure of finite subgroups in automorphisms groups and rationality of the quotient surfaces. ⇑

Finite jet determination for CR mappings

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A CR mapping is a diffeomorphism between two real manifolds in complex space that satisfies tangential Cauchy-Riemann equations. We are concerned with the problem whether a CR mapping is uniquely determined by its finite jet at a point. This problem has been popular since 1970-s and the number of publications on the matter is enormous. Nevertheles, natural fundamental questions have been open. I will present a solution to a version of the problem and discuss old and recent results. ⇑

Relative Milnor K-groups and Kahler differentials

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I will tell about the connection between relative Milnor K-groups $K_n^M(R, I)$ of a split nilpotent extension (R, I) and it's modules of Kahler differential forms. Namely, let R be a ring and $I \subset R$

be its nilpotent ideal such that R/I splits and $I^N = 0$ for some natural N such that $N!$ is invertible in R. I will show that in this case there exists functorial homomorphism called Bloch map (since it was originally developed by S.Bloch) from relative Milnor K–group $K^M_{n+1}(R, I)$ to the quotient group $\Omega^n_{R,I}/d\,\Omega^{n-1}_{R,I}$ wich can be considered as a canonical integral for the map $d\log$. Moreover, under the assumption that R is weakly 5-fold stable this map is an isomorphism.

If time permits I will also tell about a particulary interesting geleralization of the Bloch map B to the case of p-adically complete ring R with a δ-structure. ⇑

The Generalized Long Mayer–Vietoris Sequence and Separating Cycles

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The notion of a cycle separating a collection of hypersurfaces in a complex-analytic manifold appears in the theory of multidimensional residue in connection with the Grothendieck residue. Namely, the residue $\mathrm{res}_a\,\omega$ of the meromorphic n-form ω on an n-dimensional manifold is represented by the integral in which n-cycle of integration (the local cycle at the point a) in a certain sense separates the set of polar hypersurfaces of the form ω. The most complete results on separating cycles were obtained by Tsikh and Yuzhakov (1975–1988) under the condition of Steinnes of the manifold. The use of the generalized long Mayer –Vietoris sequence allows us to replace this condition with more flexible conditions, described in the homology of intersections of elements of the open cover of the manifold associated with the family of hypersurfaces. As a result, we obtain sufficient conditions of representation of the integral of a meromorphic form in terms of Grothendieck residues.

Another application of the generalized long Mayer –Vietoris sequence is related to the calculation of the combinatorial coefficients involved in the Gelfond –Khovanskii formula for the global residue in an algebraic torus (2002). In this case, we consider 0-dimensional cycles separating some connection of tropical hypersurfaces in $\mathbb{R}^n$. ⇑

An extended Monge-Ampère operator

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I will discuss a joint work in progress with Mats Andersson and David Witt Nyström. We introduce a class of plurisubharmonic functions $\mathcal{G}$, for which there is a natural Monge-Ampère operator with nice continuity properties. The class $\mathcal{G}$ includes plurisubharmonic functions with analytic singularities and has certain convexity properties, and thus it has a quite rich structure. ⇑

Multidimensional residue calculus over $\mathbb{Q}$ or $\overline{\mathbb{Q}}$

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Toric geometry (more specifically within the context of products of projective spaces through the concept of Chow or multi-Chow forms or cycles) plays an important role with respect to upper estimates for the arithmetic complexity of geometric cycles (in terms of logarithmic heights such as Ronkin functions evaluated at the origin) in the affine space $\mathbb{A}^n_{\mathbb{C}}$ or the projective space $\mathbb{P}^n_{\mathbb{C}}$ which are defined over $\mathbb{Q}$ or over the algebraically closed field of algebraic numbers $\overline{\mathbb{Q}}$ (W. Stoll, Y. Nesterenko, D. W. Brownawell, P. Philippon, V. Maillot, J. Kollár, M. Sombra, etc.). An important formula based on simple linear algebra arguments (basically relying on comparison of dimensions) which arose from Oskar Perron's pioneer book *Algebra* (1927) [Satz 57, S. 129] appears to be today, since the works of A. Płoski (2005), Z. Jelonek (2005), C. d'Andrea - T. Krick - M. Sombra (2013), a major base stone towards the realization of nearly sharp versions of Hilbert's arithmetic nullstellensatz (or membership problem within the complete intersection setting). Transformation Laws inherent to multivariate residue calculus materialize the bridge between Oskar Perron's result and estimates (which sharpness still remains in general conjectural) for the arithmetic complexity of fully developped trace formulae of the Taylor type (such as Bergman-Weil expansions). I will discuss such questions in this lecture and in particular focus on still un-answered ones. Among such open problems, I will insist on the following : could one expect an effective dynamical approach (based on perturbation arguments which arise from the pioneer work of Krasnoyarsk's mathematical school) be a substitute for the somehow un-effective (but nevertheless extremely efficient!) Oskar Perron's theorem (once combined with Transformation Laws)? A large part of this lecture is inspired by my recent joint work (2021) with Martin Sombra (Barcelona) and the finalization of our book project with Alekos Vidras (Nicosia). ⇑