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On the local time of a stopped random walk attaining a high level

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KEY WORDS: Stopped random walk, Brownian high jump, local time

MATHEMATICAL SUBJECT CLASSIFICATION: 60F17, 60G50

Abstract:

Let $X_1, X_2, \ldots$ be independent random variables with the same arithmetic distribution with the maximal span 1 and

$$\mathbf{E}X_1 = 0,\ \mathbf{E}X_1^2 := \sigma^2,\ 0 < \sigma^2 < +\infty. \tag{1}$$

Consider a random walk

$$S_0 = 0,\ S_i = \sum_{j=1}^{i} X_j,\ i \in \mathbf{N}.$$

Let T be the first hitting time of the semi-axis $(-\infty, 0]$ by the random walk $\{S_i\}$, i.e.

$$T = \min\{i > 0 : S_i \leq 0\}.$$

Set

$$\widetilde{S}_i = \begin{cases} S_i, 0 \leq i < T; \\ 0, i \geq T. \end{cases}$$

The sequence $\left\{\widetilde{S}_i, i \geq 0\right\}$ is called a *stopped random walk* (SRW). Let $\widetilde{\xi}(0) = 0$ and $\widetilde{\xi}(k)$ mean the number of visits of SRW to the state $k \in \mathbf{N}$, i.e.

$$\widetilde{\xi}(k) = \left|\left\{i \in \mathbf{N} : \widetilde{S}_i = k\right\}\right|.$$

The random variable $\widetilde{\xi}(k)$ is called *the local time* of SRW $\left\{\widetilde{S}_i, i \geq 0\right\}$ at the level k. Set for $x > 0$

$$T_x = \min\left\{i > 0 : \widetilde{S}_i > x\right\}.$$

We introduce a random process Z_n, given by the formula

$$Z_n(u) = \frac{\sigma^2 \widetilde{\xi}(\lfloor un \rfloor)}{n}, \; u \geq 0.$$

The main result is a theorem describing the limiting distribution of the process Z_n, considered under the condition that $T_n < +\infty$. Before we formulate this theorem, we define a random process that plays the role of a limiting one. Let $\{W(t), t \geq 0\}$ be a standard Brownian motion and

$$\tau_x = \inf\{t > 0 : W(t) = x\}.$$

We introduce the following two moments of attaining the state 0 by the Brownian motion: one of them $\tau_0^{(1)}$ precedes the time τ_1 and the other $\tau_0^{(2)}$ follows this time, i.e.

$$\tau_0^{(1)} = \sup\{t \in [0, \tau_1] : W(t) = 0\}, \; \tau_0^{(2)} = \inf\{t > \tau_1 : W(t) = 0\}.$$

The random process

$$W_0^{\uparrow}(t) = W\left(\tau_0^{(1)} + t\right), \; t \in \left[0, T_0^{\uparrow}\right],$$

where $T_0^{\uparrow} = \tau_0^{(2)} - \tau_0^{(1)}$, is called a *Brownian high jump.* We assume that $W_0^{\uparrow}(t) = 0$ for $t \geq T_0^{\uparrow}$. Let $l_0^{\uparrow}(u)$ be the local time of the process $\left\{W_0^{\uparrow}(s), s \in [0, t]\right\}$ at the level $u > 0$, i.e.

$$l_0^{\uparrow}(u) = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \int_0^{+\infty} I_{[u, u+\varepsilon]}\left(W_0^{\uparrow}(s)\right) ds.$$

(here we mean convergence almost surely).

Theorem 1. *If conditions* (1) *are satisfied, then, as* $n \to \infty$,

$$\{Z_n | T_n < +\infty\} \to l_0^{\uparrow}, \tag{2}$$

where the symbol $\to$ *means convergence in distribution in the space* $D[0, +\infty)$ *with the Skorokhod topology.*

Now consider a critical Galton-Watson branching process $\{\zeta_n, n \geq 0\}$, starting with a single particle and satisfying the condition

$$\mathbf{D}\zeta_1 := 2\beta \in (0, +\infty).$$

It turns out that, as $n \to \infty$,

$$\left\{ \frac{2\zeta_{\lfloor nt \rfloor}}{\beta n}, t \geq 0 \middle| \zeta_n > 0 \right\} \to l_0^{\uparrow}. \tag{3}$$

The right-hand sides of relations (2) and (3) coincide. This allows us to establish conditional limit theorems for various functionals from a stopped random walk, using the corresponding statements for the Galton-Watson branching process.

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Generalization of Lévy's problem

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KEY WORDS: Infinity divisible distributions, operator-stable laws, limit theorems, stable distributions

MATHEMATICAL SUBJECT CLASSIFICATION: 60F05

Abstract: Back in the 30s of the last century, P. Lévy proved that α-stable random variables and only they are limits for the sums of i.i.d. random variables with positive normalization and some centering. Later, Feldheim generalized this result to the case of random vectors. Namely, he proved that α-stable random vectors and only they are limits for sum i.i.d. random vectors with positive normalization and some vector centering. During this talk, a similar result will be obtained for the sum of i.i.d. complex-valued random variables and vectors with complex normalization and centering.

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Large Deviations of Random Walk in Random Environment

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Key words: Large Deviations, Random Walks, Random Environment, Regenerative Sequences

Mathematical Subject Classification: 60J80

Abstract: We consider a random walk $\{S_n, n \geq 0\}$ in random environment $\vec{p}$, where a sequence of independent identically distributed random variables taking values in $(0,1)$. We suppose that $\rho := \mathbf{E} \ln((1-p_1)/p_1)$ is less or equal than zero. Denote $T_0 := 0,\ T_n := \min\{k \geq 1 : S_k = n\},\ n \in \mathbf{N}$.

Solomon proved in [1] that if $\rho \leq 0$ then random variables $T_n, n \in \mathbf{N}$ are finite almost surely. Limit theorems for T_n were obtained by Kozlov, Kesten and Spitzer in [2]. Large deviation principle for $T_n, n \in \mathbf{N}$, was proved in [3]. Further development for the case of non-independent environment was considered in [4].

We obtain the exact asymptotics of probabilities

$$\mathbf{P}(T_n = k) = (1 + o(1))n^{-1/2}F(k/n)\exp(-L(k/n)n).$$

The relation holds uniformly in $k/n = k(n)/n$ from any compact subset $K \subset B'$. Here functions F, L and the set B do not depend on n. The proof is based on the theory of large deviations for generalized renewal processes developed in [5]. We also use the results of [6] to describe the functions F, L and the set B'.

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Long edges in birth-death trees

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Key words: Galton-Watson processes, birth-death processes

Mathematical Subject Classification: 60J80

Abstract: In this talk we consider constant rate birth-death processes, which are often used in Biology to model speciation and extinction. We shall establish a number of results concerning limiting behaviour of particles (species) with extreme life lengths.

Acknowledgement The speaker is supported by the NSFC grant no. 11731012.

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On the hitting time of a growing level by catalytic branching walk

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KEY WORDS: Catalytic branching random walk, hitting time, spread of population, supercritical regime, light tails.

MATHEMATICAL SUBJECT CLASSIFICATION: 60J80, 60F15.

Abstract: Branching random walks (BRWs) are probabilistic models allowing particles to move randomly (on a lattice or in the space) and occasionally produce offspring. We analyze catalytic branching random walk (CBRW) on an integer line **Z**. The main feature of the CBRW is that the particles may produce offspring at the presence of a finite collection of catalysts located arbitrarily at fixed integer points. For a supercritical BRW, an interesting problem is the study of asymptotic behavior of its maximum, that is the coordinate of the right-most particle at time t, as t tends to infinity. Such a problem for a CBRW with light tails of the walk jump is solved in [1] and [2]. Here we go further and, for the CBRW, establish the limit theorem describing almost sure behavior of the time of first hitting a linearly growing level. We consider constant growth rate for the increasing level to guarantee the non-trivial limit. The new problem is more complicated than the mentioned above since we have to take into account not only the population maximum at time t, but also its dynamics before t, as t grows unboundedly. However, the new result and the previous one in [1] turn out to be close and involve the same constant in asymptotic formula. The proof is based on a (rather intricate) system of non-linear integral equations, large deviations theory for random walks, renewal theory and other techniques.

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Local lower deviations of branching process in random environment with geometric number of descendants

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KEY WORDS: Branching processes, random environments, random walks, Cramer's condition, lower deviations, large deviations, local theorems

MATHEMATICAL SUBJECT CLASSIFICATION: 60J80

Abstract:
We consider local probabilities of lower deviations for branching process $Z_n = X_{n,1} + \cdots + X_{n,Z_{n-1}}$ in random environment $\boldsymbol{\eta}$. We assume that $\boldsymbol{\eta}$ is a sequence of independent identically distributed variables and for fixed $\boldsymbol{\eta}$ the distribution of variable $X_{i,j}$ is geometric. We suppose that the associated random walk $S_n = \xi_1 + \cdots + \xi_n$ has positive mean μ and satisfies left-hand Cramer's condition $\mathbf{E}\exp(h\xi_i) < \infty$ as $h^- < h < 0$ for some $h^- < -1$. Under these assumptions, we find an asymptotic representation for local probabilities $\mathbf{P}\left(Z_n = \lfloor \exp\left(\theta n\right) \rfloor\right)$ as $\theta \in (m^-; \mu)$ for some constant $m^- \geq 0$. Problem of large deviations for branching processes in random environment is well-studied: the asymptotics of $\mathbf{P}(Z_n > \exp(\theta n))$, where $\theta > \mu$, for branching processes in random environment with geometric number of descendants was studied by Kozlov ([1], [2]). Logarithmic asymptotics for probabilities of lower deviations $\mathbf{P}(1 \leq Z_n < \exp(\theta n))$, where $\theta < \mu$, was obtained in [3]. In this report the problem of lower deviations is considered in local form $\mathbf{P}(Z_n = k)$, where $k(n) = k \in \mathbb{N}$. We assume that $\theta(n) = \theta := \ln k/n$ lies in some interval $[\theta_1; \theta_2] \subset (m^-; \mu)$. Under these assumptions we define two deviation zones and obtain two different asymptotics for $\mathbf{P}(Z_n = k)$:

$$\mathbf{P}\left(Z_n = k\right) = \frac{1 + o(1)}{\sqrt{2\pi n}\sigma\left(h_\theta\right)} e^{-\Lambda(\theta)n - \theta n} \Gamma\left(1 + h_\theta\right) \mathbf{E}\widetilde{V}_\infty^{h_\theta - 1}$$

for $n \to \infty$ uniformly in the first zone $\theta \in [\theta_1; \theta_2] \subset (m(-1); \mu)$,

$$\mathbf{P}\left(Z_n = k\right) = \left(1 + o(1)\right) R^n(-1) \mathbf{E}\widehat{V}_\infty^{-2}$$

for $n \to \infty$ uniformly in the second zone $\theta \in [\theta_1; \theta_2] \subset (m^-; m(-1))$, where $m(-1)$, $\widetilde{V}_\infty$, h_θ and $\Lambda(\theta)$ are some parameters.

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The initial stage of the evolution for intermediately subcritical branching processes in random environment

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Key words: Branching process, random environment, random walk

Mathematical Subject Classification: 60J80

Abstract: We consider a Galton-Watson branching process $Z = (Z_0, Z_1, \ldots)$ evolving in i.i.d. random environment $\{f_0, f_1, ...\}$, where $f_n = f_n(s)$ is the generating function of the reproduction law of particles of the n-th generation. Let $X_n = \log f_n'(1)$. We assume that the process Z is intermediately subcritical, i.e.

$$\mathbf{E}X_0 = 0, \ \mathbf{E}[X_0 e^{X_0}] = 0. \tag{1}$$

Let $\mathbf{N} = \{1, 2, ...\}$. Introduce the so-called associated random walk $S = \{S_n\}_{n \geq 0}$

$$S_n = X_0 + ... + X_n, \ n > 0, \ S_0 = 0.$$

Let

$$\tau_n = \min\{k \leq n \mid S_k \leq S_0, S_1, \ldots, S_n\}$$

be the moment, when S takes its minimum for the first time on the interval $[0, n]$. Let $r_n \in \mathbf{N}, n > 0$, and $r_n \to \infty, n \to \infty$. For brevity we will use the notation $r = r_n, \ \tau = \tau_r$. Let the symbol $\Rightarrow$ denotes weak convergence.

We show that if (1) is valid and $r = r_n = o(n)$ as $n \to \infty$, then under some mild technical conditions

1) there is a random variable ξ with values in $\mathbf{N}$ such that as $n \to \infty$

$$\left(Z_{\tau_r} \mid Z_n > 0\right) \Rightarrow \xi; \tag{2}$$

2) there is a positive random variable η such that as $n \to \infty$

$$\left(\frac{Z_r}{e^{S_r - S_{\tau_r}}} \mid Z_n > 0\right) \Rightarrow \eta. \tag{3}$$

Note also that the distribution of the number of particles at the initial period of the evolution for critical and weakly subcritical BPRE given their survival up to a distant moment were investigated in [2] and [3].

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Quenched invariance principles for random walks in random environment conditioned to stay positive

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Key words: Random environment, random walk

Mathematical Subject Classification: 60G50, 60G57

Abstract

We consider a random walk $\{S_n\}_{n\in\mathbb{N}}$ in random environment (in time) ξ. For almost each realization of ξ, we prove a quenched invariance principles for the random walk conditioned to stay positive (which specified by the Doob h-transform of the original one). To this end, a key step is to formulate a (quenched) harmonic function. Although the traditional approach Wiener-Hopf factorisation dose not work in this case, we prove the existence of the (quenched) harmonic function under the annealed $2+\epsilon$ (for some $\epsilon > 0$) moment condition on the increments. This is a joint work with Shengli Liang.

Capacity of the range of a critical branching random walk

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Key words: Branching random walk, capacity, range.

Mathematical Subject Classification: 60J80.

Abstract: Let R_n be the range of a critical branching random walk with n particles on Z^d, which is the set of sites visited by a random walk indexed by a critical Galton–Watson tree conditioned on having exactly n vertices. For $d \in \{3, 4, 5\}$, we prove that $n^{-\frac{d-2}{4}} \mathcal{C}ap(R_n)$, the renormalized capacity of R_n, converges in law to the capacity of the support of the integrated super-Brownian excursion. The proof relies on a study of the intersection probabilities between the critical branching random walk and an independent simple random walk on Z^d.

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Remarks on the Kolmogorov constant in the theory of Galton-Watson branching processes

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Key words: Branching processes; transition probabilities; generating function; Kolmogorov's constant.

Mathematical Subject Classification: Primary 60J80; Secondary 60J85

Abstract: Consider a Galton-Watson branching process. In the subcritical case, the mean of the particles population on the positive trajectories of the process stabilizes and it approaches a constant $1/\mathcal{K}$, where $\mathcal{K}$ is called the Kolmogorov's constant. The report is devoted to the calculation of this constant in the Kolmogorov's moment conditions.

Let $\mathbf{N}_0 = \{0\} \cup \mathbf{N}$ and $\mathbf{N} = \{1, 2, \ldots\}$. We consider the Galton-Watson Branching (GWB) process as a reducible homogeneous-discrete time Markov chain with a state space $\mathcal{S}_0 = \{0\} \cup \mathcal{S}$, where $\{0\}$ is absorbing state and $\mathcal{S} \subset \mathbf{N}$ is a class of essential communicating states. Let $Z(n)$ be a population size at the time $n \in \mathbf{N}_0$ in the GWB process with offspring rates $\{p_k, k \in \mathcal{S}_0\}$. Define an appropriate probability generating function (GF) $f(s) := \sum_{j\in\mathcal{S}_0} p_j s^j$ for $s \in [0, 1)$. Then n-step transition probabilities $P_{ij}(n) := \mathsf{P}\left\{Z(n+k) = j \;\middle|\; Z(k) = i\right\}$, for any $k \in \mathbf{N}_0$, are

$$P_{ij}(n) = \text{coefficient of } \; s^j \; \text{ in } \; \big(f_n(s)\big)^i \quad \text{for any} \quad i, j \in \mathcal{S}_0,$$

where $f_n(s)$ is the n-fold iteration of $f(s)$; see [1, pp. 11–14].

In this work, we consider the non-critical case only, i.e. $m := \sum_{j\in\mathcal{S}} jp_j = f'(1-) \neq 1$.

Let $R_n(s) := q - f_n(s)$, where q is an extinction probability of the process starting with a single particle. In 1938, A.N.Kolmogorov [2] established that if $m < 1$, the survival probability $Q(n) := \mathsf{P}\big\{Z(n) > 0\big\} =$

$R_n(0)$ of the GWB process admits an asymptotic representation

$$Q(n) = \mathcal{K} m^n \big(1 + o(1)\big) \quad as \quad n \to \infty, \tag{1}$$

if and only if $f''(1-) < \infty$, where $\mathcal{K}$ is an absolute constant. Later, A.V.Nagaev and I.S.Badalbaev [3] refined Kolmogorov's result by proving the validity of the asymptotic representation (1) under the $x \log x$ condition.

In this report we find an explicit form of the constant $\mathcal{K}$ under the Kolmogorov theorem condition [2].

Theorem. *Let* $m \neq 1$, $\beta := f'(q)$, $2b_q := f''(q) < \infty$ *and* $\gamma := b_q \big/ (\beta - \beta^2)$. *Then*

$$R_n(s) = \mathcal{A}_\gamma(s) \cdot \beta^n \big(1 + o(1)\big) \quad as \quad n \to \infty,$$

where $\mathcal{A}_\gamma(s) = (q - s) \big/ \big(1 + \gamma(q - s)\big)$.

Corollary. *Let* $m < 1$, $2b := f''(1-) < \infty$ *and* $\gamma := b \big/ (m - m^2)$. *Then*

$$\mathcal{K} = \frac{1}{1 + \gamma}.$$

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Asymptotic behaviour of the survival probability of almost critical branching processes in a random environment with geometric distribution

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KEY WORDS: Random walks, branching processes, random environments

MATHEMATICAL SUBJECT CLASSIFICATION: 60J80

Abstract:

We consider the branching process in random environment, given by the sequence of independent probability generating functions

$$f_{i-1,n}(s) := \frac{1-p_{i,n}}{1-p_{i,n}s}, \quad p_{i,n} := \frac{1}{1+e^{-X_i - b_{i,n}}},\ i \in \{1,\dots,n\},$$

where X_i – independent identically distributed random variables with $\mathsf{E}X_1 = 0$, $\mathsf{D}X_1 \in (0,\infty)$, $b_{i,n}$ is some sequence of real numbers. Let $Z_{k,n}$ be the population size at moment k, $Z_{0,n} = 1$. Set

$$\widehat{X}_{i,n} := \ln f'_{i-1,n}(1) = X_i + b_{i,n}, \quad \widehat{S}_{0,n} := 0, \quad \widehat{S}_{k,n} := \widehat{X}_{1,n} + \ldots + \widehat{X}_{k,n}.$$

We will call the sequence $\widehat{S}_{k,n}$, $k \geq 0$, *the associated random walk* for $Z_{k,n}$. In the case $b_{i,n} \equiv 0$, the associated random walk is random walk with finite variance and zero drift. In this case we denote the population size at moment k by Z_k^0.

Our main result is the following theorem.

Theorem 1 *Assume that there exists $\delta \in (0, 1/2)$ such that,*

$$\max_{k \leq n} k^{\delta - 1/2} \left| \sum_{i=1}^{k} b_{i,n} \right| \to 0,\ n \to \infty.$$

Then

$$\mathsf{P}(Z_{n,n} > 0) \sim \mathsf{P}(Z_n^0 > 0),\ n \to \infty.$$

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On sizes of trees in a Galton-Watson forest with power-law distribution

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KEY WORDS: Galton – Watson forest, limit distribution, maximum tree size, number of trees of a given size

MATHEMATICAL SUBJECT CLASSIFICATION: 05C80

Abstract: Let G_N be a critical Galton – Watson branching process with N initial particles and let the number of offspring of each particle be a random variable ξ following the distribution

$$p_k = \mathbf{P}\{\xi = k\} = \frac{1}{(k+1)^\tau} - \frac{1}{(k+2)^\tau},\ k = 0, 1, 2, \ldots \tag{4}$$

The process G_N induces a conditional probability distribution on the subset $F_{N,n}$ of its trajectories with $N+n$ vertices provided that the number of vertices is equal to $N+n$. We denote by $\mathcal{F}_{N,n}$ the thus constructed Galton – Watson forest with N trees and n non-rooted vertices. It is easy to show that $\mathbf{E}\xi = \zeta(\tau, 2)$, where $\zeta(s, v) = \sum_{k=0}^{\infty}(k+v)^{-s}$ is the generalized zeta-function. Since the branching process G_N is critical, the equality $\zeta(\tau, 2) = 1$ holds and therefore $\tau \approx 1.728$. For such a parameter value only the first moment of the distribution (4) is finite.

Let $\eta(\mathcal{F})$ be a random variable equal to the maximum tree size and $\mu_r(\mathcal{F})$ be a random variable equal to the number of trees of size r in the forest $\mathcal{F}_{N,n}$. Limit distributions of $\eta(\mathcal{F})$ and $\mu_r(\mathcal{F})$ are obtained as $N, n \to \infty$, $n/N^\tau \geq C > 0$.

We denote by $g(x)$ a stable distribution density with a parameter τ and a characteristic function

$$f(t) = \exp\left\{-\Gamma(1-\tau)|t|^\tau e^{-i\pi\tau t/2|t|}\right\},$$

and let $p(x)$ be a stable distribution density with a parameter $1/\tau$ and a characteristic function

$$h(t) = \exp\left\{-\left(-\Gamma(1-\tau)\right)^{-1/\tau}|t|^{1/\tau} e^{-i\pi t/2\tau|t|}\right\}.$$

In particular the following statements hold.

Theorem 1. *Let $N, n \to \infty$ in such a way that $n/N^\tau \to \gamma$, where γ is a positive constant. Then for any positive z*

$$\mathbf{P}\left\{\frac{\eta(\mathcal{F})}{n} \le z\right\} \to \frac{1}{2\pi p(\gamma)} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} I_k(\gamma z, \gamma),$$

where

$$I_0(u,v) = p(v), \;\; I_k(u,v) = \int\limits_{x_k(u,v)} \frac{p(v - x_1 - \ldots - x_k)\, dx_1 \ldots dx_k}{(2\pi C(\tau))^k (x_1 \ldots x_k)^{(\tau+1)/\tau}},$$

$$x_k(u,v) = \{x_i \ge u, \;\; i = 1, \ldots, k, \;\; x_1 + \ldots + x_k \le v\}, \quad k = 1, 2, \ldots,$$

$$C(\tau) = 1/\tau\Gamma(1 - 1/\tau)\,(-\Gamma(1-\tau))^{1/\tau}.$$

Theorem 2. *Let $N, n \to \infty$ in such a way that $n/N^\tau \to \infty$. Then for any fixed positive z*

$$\mathbf{P}\left\{\frac{n - \eta(\mathcal{F})}{N^\tau} < z^{-\tau}\right\} \to \tau \int\limits_{-\infty}^{-z} g(y)dy.$$

Branching processes in random environment with cooling

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Key words: Branching processes, random walks, random environment

Mathematical Subject Classification: 60J80

Abstract: It is well known that a branching process in random environment can be described by the associated random walk

$$S_n = \xi_1 + \ldots + \xi_n,$$

where $\xi_k = \ln \varphi'_{\eta_k}(1)$, $\varphi_x(t)$ and η_k are the generating functions of the number of descendants and the random environment respectively. The talk will address the issue of degeneration of a branching process in random environment with cooling with $\mathbf{E}\xi_1 > 0$ which differs from the classic BPRE in that each environment lasts for several generations. It turns out that this varianant of BPRE is also closely related to random walk

$$S_n = \tau_1\xi_1 + \ldots + \tau_n\xi_n,$$

where $\xi_k = \ln \varphi'_{\eta_k}(1)$ and $\varphi_x(t)$ and η_k are generating functions of the number of descendants and the random environment ewspectively and τ_k is a duration of the k-th cooling.

In this talk we will show that if for any $\varepsilon > 0$

$$\sum_{n=1}^{\infty} \mathbf{P}\left(\varepsilon\xi_1 < -\frac{\tau_1 + \ldots + \tau_n}{\tau_n}\right)$$

is divergent then the process degenerates with probability 1. Also we will show that if $0 < \mathbf{D}\xi_1 < \infty$ and

$$\sum_{n=1}^{\infty} \frac{\tau_n^2}{(\tau_1 + \ldots + \tau_n)^2} < \infty$$

then the process degenerates with probability less than 1.

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Branching random walks with the generation of particles determined by Gumbel-type random potential. Simulation.

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Key words: Branching random walks, simulation, multidimensional lattices, evolutionary operators

Mathematical Subject Classification: 60J27; 60J80; 05C81; 60J85

Abstract:

We consider continuous-time branching random walks (BRWs) on a multidimensional lattice in a random branching medium. The branching medium may contain a finite or non-finite number of particle generation sources. The underlying walk of particles is symmetric, homogeneous by space, and irreducible. In such BRWs, at large times, rare fluctuations of the medium may lead to "intermittency" which is an anomalous property of the limiting distribution of the random field that occurs in a random media. An intermittent field cannot be described correctly with its moments. In the case of BRW in random media, the field of quenched moments of particles turns out to be intermittent under specific conditions [1,2].

The study of BRWs at finite time intervals seems to be a difficult task that has not yet been solved satisfactorily enough. In the work [3] devoted to the comparison of BRW in random and non-random media, we have shown that for BRW in random media with potential with Weibull-type tails it is possible to obtain qualitative intermittency predicted by the theory already at finite times. In addition, we suggested a measure that allows numerical estimation of the intermittency of the field of quenched moments. The purpose of this work was to study whether it is possible to obtain similar results for a potential with Gumbel-type tails. In particular, to evaluate whether it is possible to use the same measure of intermittency as for potential with Weibull-type tails. Based on the

simulation results, we showed that intermittency can be observed and numerically estimated for a potential with Gumbel-type tails.

Acknowledgement. The research was supported by the Russian Foundation for the Basic Research (RFBR), project No. 20-01-00487.

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A persistense result for a critical multitype branching system

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KEY WORDS: Multitype branching process, persistence, Markov renewal process, renewal equations.

MATHEMATICAL SUBJECT CLASSIFICATION: 60J80, 60K15

Abstract: We consider a critical branching system of particles living in R^d with a finite number of types, in which an individual of type i lives a random lifetime with distribution functions Γ_i, during which it moves according to a symmetric α_i-stable motion. We consider the case when the lifetime distribution Γ_1 of particles of type 1 has a power tail $t^{-\gamma}, \gamma \in (0, 1]$, while the lifetimes of the other particle types have finite means. Under the usual independence assumptions in branching systems, we obtain a sufficient condition for the persistence of the system which is valid for a class of branching laws. Our result compelements the extinction result obtained by Kevei and Lopez-Mimbela [1].

Acknowledgement This research was supported in part by CONACyT Grant No. 652255 C.F. 2019.

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Exponential ergodicity of branching processes with immigration and competition

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KEY WORDS: Continuous-state branching process; immigration; competition; exponential ergodicity; stochastic equation; Markov coupling; Lyapunov function; control function

MATHEMATICAL SUBJECT CLASSIFICATION: 60J80, 60J25, 60G51, 60G52

Abstract: We study the ergodic property of a continuous-state branching process with immigration and competition, which is an extension of the models studied by Pardoux (2016, Springer) and Berestycki et al. (Probab. Theory Related Fields, 2018) with an additional immigration structure. The exponential ergodicity in a weighted total variation distance is proved under natural assumptions. The result applies to general branching mechanism including all stable types. The proof is based on a Markov coupling process and a nonsymmetric control function for the distance, which are designed to identify and to take the advantage of the dominating factor among the branching, immigration and competition mechanisms in different parts of the state space. The approach provides a way of finding explicitly the ergodicity rate.

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L^p convergence and large deviations for supercritical multi-type branching processes in random environments

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Key words: multi-type Branching processes, random environment, L^p convergence, large deviations, products of random matrices, martingales

Mathematical Subject Classification: 60J80, 60K37, 60J85

Abstract: Consider a d-type supercritical branching process $Z_n^i = (Z_n^i(1), \cdots, Z_n^i(d))$, $n \geq 0$, in an independent and identically distributed random environment $\xi = (\xi_0, \xi_1, \ldots)$, starting with one initial particle of type i, whose offspring distributions of generation n depend on the environment ξ_n at time n. In [1] we have established a Kesten-Stigum type theorem for Z_n^i, which implies that for any $1 \leq i, j \leq d$, $Z_n^i(j)/E_\xi Z_n^i(j) \to W^i$ in probability as $n \to +\infty$, where E_ξ denotes the conditional expectation given the environment ξ, and W^i is a non-negative and finite random variable for which a criterion for non-degeneracy is obtained. Here we present the following results established in [2]: a necessary and sufficient condition for the convergence in L^p of the normalized population size $Z_n^i(j)/E_\xi Z_n^i(j)$, a theorem giving its exponential convergence rate, and similar results for the associated fundamental martingale (W_n^i). We also present a result on the precise large deviations for the total population size $\|Z_n\|_1 := \sum_{j=1}^n Z_n(j)$ of generation n recently established in [3], whose proof uses the L^p convergence and a similar large deviation result on products of random matrices proved in [4].

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Fluctuation limit theorem for the occupation time of a branching systems with long individual lifetimes

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Key words: Branching particle system, rescaled occupation time, functional limit theorem.

Mathematical Subject Classification: 60G15, 60G22, 60F17, 60G20

Abstract: We give a functional limit theorem for the fluctuations of the rescaled occupation time process of a critical branching particle system $\{N_t,\ t \geq 0\}$ in R^d with symmetric α-stable motion. The branching law is binary critical. We consider the case where the distribution function F of the particle lifetimes satisfies $F(0) = 0$, $F(x) < 1$ for all $x \in [0, \infty)$, and

$$1 - F(u) \sim \frac{u^{-\gamma}}{\Gamma(1-\gamma)} \quad \text{when} \quad u \to \infty \tag{5}$$

for some $\gamma \in (0, 1)$, where $\Gamma(\cdot)$ denotes the Gamma function. We assume that N_0 is a Poisson random field with Lebesgue intensity measure. Let us write $< \mu, f > := \int f d\mu$, where μ is a measure and f is a measurable function. For $T > 0$, let $(L_T(t))_{t\geq 0}$ be the rescaled occupation time process of $\{N_t,\ t \geq 0\}$, which is defined by

$$< L_T(t), \phi > = \int_0^{Tt} < N_s, \phi > ds = T \int_0^t < N_{Ts}, \phi > ds, \quad \phi \in \mathcal{S}(R^d), \tag{6}$$

where $S(R^d)$ is the space of rapidly decreasing functions. Let $(X_T(t))_{t\geq 0}$ be the occupation time fluctuations process, that is,

$$< X_T(t), \phi > := \frac{1}{F_T}\Big(< L_T(t), \phi > - E(< L_T(t), \phi >)\Big),$$

where F_T is a normalizing constant. Our objective is to find a suitable F_T such that X_T converges in distribution on $C([0, \tau], \mathcal{S}'(R^d))$

as $T \to \infty$ for any $\tau > 0$. We will show that under the assumption $\alpha\gamma < d < \alpha(1+\gamma)$, weak convergence of X_T on $C([0,\tau], \mathcal{S}'(R^d))$ as $T \to \infty$ holds for any $\tau > 0$.

Inequalities for the mean time to reach the level by a random walk with delay at zero

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KEY WORDS: Random walk, first exit time, probabilistic inequalities, change point problem

MATHEMATICAL SUBJECT CLASSIFICATION: 60G50

Abstract:
Let $X_1, X_2, \ldots$ be a sequence of i.i.d. random variables, and

$$W_{n+1} = \max\{0, W_n + X_{n+1}\}, \qquad W_0 = 0.$$

We introduce stopping time

$$T = \inf\{n \geq 1 : W_n \geq b\}, \qquad b > 0.$$

The goal is to obtain two-sided inequalities for ET under conditions $EX_1 > 0$ and $EX_1 < 0$. These bounds are then used to characterize the quality of the sequential procedure of cumulative sums (CUSUM procedure) for the early detection of change in distribution.

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On the probabilistic representation of the resolvent of the two-dimensional Laplacian

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Key words: Stochastic processes, local time, two-dimensional Wiener process.

Mathematical Subject Classification: 60J55

Abstract:

Let $w(\tau) = (w_1(\tau), w_2(\tau))$, $\tau \geq 0$, $w(0) = (0,0)$ be a two-dimensional Wiener process. Consider a family of random linear operators

$$\mathcal{A}_\lambda^t f(x) = \int_0^t e^{\lambda\tau} f(x - w(\tau))\, d\tau, \tag{7}$$

defined on the functions $f(x) \in L_\infty \cap C(\mathbb{R}^2)$ for all $t > 0$ and $\lambda \in \mathbb{C}$, $\operatorname{Re}\lambda < 0$.

Such an operator family arises in the construction of a probabilistic representation of the resolvent of the two-dimensional Laplacian.

Namely, the following relation holds

$$(-\frac{1}{2}\,\Delta - \lambda\, I)^{-1} f(x) = \int_0^\infty e^{\lambda\,\tau}\, \mathbf{E}\, f(x - w(\tau))\, d\tau = (u) \lim_{t\to\infty} \mathbf{E}\,[\mathcal{A}_\lambda^t f(x)] \tag{8}$$

for all functions $f(x) \in L_\infty \cap C(\mathbb{R}^2)$.

Note that the operator $\mathcal{A}_\lambda^t$ cannot be extended to an integral operator on the entire space $L_2(\mathbb{R}^2)$. In particular, from a probabilistic point of view, this means that the process $w(\tau)$ does not have local time at an arbitrary point $x \in \mathbb{R}^2$ by time $t > 0$.

We will construct a family of random integral operators $\mathcal{R}_\lambda^t$ defined on the entire space $L_2(\mathbb{R}^2)$ and satisfying the relation

$$(-\frac{1}{2}\,\Delta - \lambda\, I)^{-1} f(x) = (L_2) \lim_{t\to\infty} \mathbf{E}\,[\mathcal{R}_\lambda^t f(x)] \tag{9}$$

for all $\lambda \in \mathbb{C}$, $\operatorname{Re}\lambda \leq 0$.

It will be shown that the kernels $r_\lambda(t,\cdot)$ of the corresponding operators belong with probability 1 to the Sobolev class $W_2^\alpha(\mathbb{R}^2)$, $0 \le \alpha < 1/2$. Also, for the function $r_\lambda(t,\cdot)$, an explicit formula will be obtained in the form of a trajectory functional of the two-dimensional Wiener process $w(\tau)$.

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New results on random forests

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KEY WORDS: random forest, configuration graph, tree size, limit distribution

MATHEMATICAL SUBJECT CLASSIFICATION: 05C80

Abstract: We consider the set of Galton-Watson forests consisting of N rooted trees and n nonroot vertices. Let ξ denote the number of offspring of each particle in the critical forest-generating branching process. Assume that

$$\mathbf{P}\{\xi = k\} = \frac{h(k+1)}{(k+1)^\tau}, \quad k = 1, 2, \ldots, \quad \tau \in (2, 3), \tag{10}$$

where the slowly varying function $h(x)$ for $x \geq 1$ takes only positive values. Such branching processes are used successfully to study random graphs intended for modeling complex communication networks in particular the Internet. The papers [1, 2] were the first to propose using the results on random forests in order to study the asymptotics of the structure of configuration graphs. The known results of Galton-Watson forests were obtained under the condition that the offspring distribution of the branching process has a finite variance. We can see that the distribution (10) has an infinite variance. This means that the present theory should be developed further. Now we have proved theorems on the limit distributions of the maximum tree size and of the number of trees of a given size for various relations between N and n as they tend to infinity.

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Probabilistic approximation of evolution operators related to higher order Schrödinger equations

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KEY WORDS: Evolution equations, Poisson random measures, Schrödinger equation

MATHEMATICAL SUBJECT CLASSIFICATION: 28C20, 60H05, 60G57

Abstract: We consider the Cauchy problem for the higher order Schrödinger equation

$$i\,\frac{\partial u}{\partial t} = \frac{(-1)^m}{(2m)!}\frac{\partial^{2m} u}{\partial x^{2m}} + V(x)u,\ u(0,x) = \varphi(x),\ m \in \mathbf{N}.$$

Probabilistic approximations of the Cauchy problem solution $u(t,x)$ for the Schrödinger equation ($m = 1$) by expectations of functionals of stochastic processes were constructed in [1]. The case when $V = 0$ and $m \geq 2$ was considered in [2]. Now we extend our results to the case when $m \geq 2$. As before the approximating operators take the form of expectations of functionals of a certain random point field.

Acknowledgement This work was supported by the Russian Science Foundation (grant 22-21-00016).

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Functional Limit Theorems for Continuous-Time Critical Recurrent Branching Random Walks

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Key words: Branching random walk, Multidimensional lattices, Limit distributions, Recurrent stochastic walk

Mathematical Subject Classification: Probability theory and stochastic processes

Abstract: We consider continuous-time critical symmetric branching random walks on a multidimensional lattice $\mathbb{Z}^d$, $d \geq 1$, with the source of particle generation at the origin. We assume that the underlying random walk is symmetric, spatially homogeneous, and irreducible, and that the birth and death of particles at the source is described by a Markov branching process. One of the main problems is to study the exact form of the limiting distribution of the particle population at the source. This problem has been solved so far only for some relations between the parameters specifying walking and branching of particles. Based on limit theorems about the distribution of the sojourn time of the underlying recurrent stochastic random walk at the origin (see Aparin, Popov, and Yarovaya, 2021), we obtain limit theorems for the distribution of the particle population at the source with finite variance of the jumps of the random walk. Currently, stochastic walks with infinite variance of jumps have been much less studied than those with finite variance. In this context, the theorems for such stochastic walks deserve special attention. For $d = 1$, the limiting distribution of the particle population at the source under normalization on the Green's function of the transition probabilities depends on the parameters of the system and may take the form of the Mittag-Leffler or the exponential distribution for a recurrent random walk.

Acknowledgement The research was supported by the Russian Foundation for the Basic Research (RFBR), project N 20-01-00487.

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Some functional limit theorems for branching processes with dependent immigration

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KEY WORDS: Branching process, immigration, regularly varying functions, $m-$dependence, $\psi-$mixing, a fluctuation limit theorem

MATHEMATICAL SUBJECT CLASSIFICATION: 60J80, 62F12

Let for each $n \geq 1$, $\left\{\xi_{k,j}^{(n)},\, k,j \geq 1\right\}$ and $\left\{\varepsilon_k^{(n)},\, k \geq 1\right\}$ be two independent families of independent identically distributed random variables with nonnegative integer values which are defined on a fixed probability space $(\Omega, \mathcal{F}, \mathbf{P})$. The sequence of branching processes with immigration $\left\{X_k^{(n)},\, k \geq 0\right\}$, $n \geq 1$ is defined by recursion:

$$X_0^{(n)} = 0, \quad X_k^{(n)} = \sum_{j=1}^{X_{k-1}^{(n)}} \xi_{k,j}^{(n)} + \varepsilon_k^{(n)}, \quad k,n \geq 1. \qquad (11)$$

We discuss conditions on validity of weak convergence of properly normalized process (1) to the deterministic function under assumption that immigration is a rowwise $\psi-$mixing and the offspring mean tends to its critical value 1, moreover, immigration mean and variance controlled by regularly varying functions. Furthermore, we obtain a fluctuation limit theorem for branching process with immigration when immigration is $m-$dependent where m may tend to infinity with the row index at a certain rate. In this case the limiting process is a time-changed Wiener process. Our results extend and improve the results in [1] and [2].

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Large Deviations of Subcritical Branching Processes in Random Environment with and without Immigration.

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KEY WORDS: Large Deviations, Branching Processes, Random Environment, Immigration, Cramer Condition

MATHEMATICAL SUBJECT CLASSIFICATION: 60J80.

Abstract: We consider a strongly subcritical branching process $\{Z_n, n > 0\}$ in a random environment (BPRE). We assume that $\mathbf{E}Z_1^h < +\infty$ for some $h > 1$ and consider large deviations probabilities in integral $\mathbf{P}(\ln Z_n \geq x)$ and integro-local $\mathbf{P}(\ln Z_n \in [x, x + \Delta))$ form, $x/n \in (0, \gamma)$, where γ is some constant. D. Buraczewski and P. Dyszewski ([1]), A. Shklyaev ([2]) considered the supercritical BPRE for $x/n \in (\mu, m^+)$, critical, weakly and intermediately subcritical BPREs for $x/n \in (0, m^+)$ and the strongly subcritical BPRE for $x/n \in (\gamma, m^+)$, where m^+ is some positive constant. E. Prokopenko, M. Struleva ([3]) considered large deviations for the supercritical case. It's known that in the strongly subcritical case for $x/n \in (0, \gamma)$ the asymptotical behaviour of $\mathbf{P}(\ln Z_n \geq x)$ has another form. It was proved by Kozlov ([4]) in the case of geometric conditional distribution and in LDP form by C. Bounghoff and G. Kersting ([5]). We'll discuss the results of A. Shklyaev ([6]) about precise asymptotics of large deviation probabilities in that case.

After that we consider branching process Z_n^* with immigration in random environment (BPIRE). We assume that $\mathbf{E}Z_1^h < +\infty$ for some $h > 1$ (including the immigration). Large deviations for BPIRE were considered by D. Dmitrusenkov and A. Shklyaev ([7]) in the geometric case and A. Shklyaev ([2]) for the general case. Both works deal with the supercritical and critical case for $x/n \in (\mu, m^+)$ and for subcritical case for $x/n \in (\gamma^*, m^+)$, where γ^* is some constant. The situation of $x/n \in (0, \gamma^*)$ was never studied, but there was a hypothesis that the behaviour of the process is close to those of strongly subcritical BPRE. We obtain the precise asymptotics of large deviation probabilities in that case. We'll discuss the difference between large deviations of BPRE and BPIRE.

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Parasite infection in a cell population with deaths

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Key words: Continuous-time and space branching Markov processes, Structured population, Long time behaviour, Birth and Death Processes

Mathematical Subject Classification: 60J80, 60J85, 60H10

Abstract: We introduce a general class of branching Markov processes for the modelling of a parasite infection in a cell population. Each cell contains a quantity of parasites which evolves as a diffusion with positive jumps. The drift, diffusive function and positive jump rate of this quantity of parasites depend on its current value. The division rate of the cells also depends on the quantity of parasites they contain. At division, a cell gives birth to two daughter cells and shares its parasites between them. Cells may also die, at a rate which may depend on the quantity of parasites they contain. We study the long time behaviour of the parasite infection.

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On a family of random operators

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Key words: Resolvent, local time, random operator

Mathematical Subject Classification: 60G17

Abstract: We study random operators arising when one constructs a probabilistic representation of the resolvent of an operator $\mathcal{A} = -\frac{1}{2}\frac{d}{dx}\big(b^2(x)\frac{d}{dx}\big)$. Namely, consider the family of operators $\mathcal{R}_\lambda^t$, $\operatorname{Re}\lambda \le 0$ defined by

$$\mathcal{R}_\lambda^t f(x) = \int_0^t e^{\lambda\tau} f(\xi_x(\tau))\,d\tau, \tag{12}$$

where $\xi_x(t)$ is a solution of the stochastic differential equation

$$d\xi_x(t) = b(\xi_x(t))b'(\xi_x(t))\,dt + b(\xi_x(t))\,dw(t), \qquad \xi_x(0) = x. \tag{13}$$

We show that under some conditions on the function $b(x)$ with probability one the operator $\mathcal{R}_\lambda$ is an integral operator in L_2 and study some properties of its kernel. We also construct a similar family of random operators for the case $\operatorname{Re}\lambda \ge 0$. Namely, we construct a family of random integral operators

$$\mathcal{R}_\lambda^t f(x) = \int_{\mathbf{R}} r_\lambda(t,x,y)f(y)\,dy,$$

where $\lambda \in \mathbf{C}$, $t \in [0,\infty]$ if $\operatorname{Re}\lambda < 0$ and $t \in [0,\infty)$ if $\operatorname{Re}\lambda \ge 0$ having the following properties.

1. For every $\lambda \in \mathbf{C}$, $t \in [0,\infty)$ with probability one the operator $\mathcal{R}_\lambda^t$ is a bounded operator in $L_2(\mathbf{R})$.
2. If $\operatorname{Re}\lambda \le 0$ then (12) holds, and under the condition $\operatorname{Re}\lambda < 0$ the equality (12) holds for $t = \infty$.
3. For every λ, t, x with probability one the function $r_\lambda(t,x,\cdot)$ belongs to the Sobolev space W_2^α for every $\alpha \in [0, \frac{1}{2})$.
4. At $\lambda = 0$ the function $r_\lambda(t,x,y)$ coincides with the local time of the process $\xi_x(\cdot)$ at point y up to the time t (see [1]).
5. If $\operatorname{Re}\lambda < 0$ then for every $f \in L_2(\mathbf{R})$ we have

$$\mathbf{E}\int_{\mathbf{R}} r_\lambda(\infty,\cdot,y)f(y)\,dy = (\mathcal{A} - \lambda I)^{-1}f. \tag{14}$$

6. If $\operatorname{Re}\lambda \le 0$ and $\lambda \notin \sigma(\mathcal{A})$ (by $\sigma(\mathcal{A})$ we denote the spectrum of the operator $\mathcal{A}$), then for every $f \in L_2(\mathbf{R})$ we have

$$\lim_{t\to\infty} \mathbf{E} \int_{\mathbf{R}} r_\lambda(t, \cdot, y) f(y)\, dy = (\mathcal{A} - \lambda I)^{-1} f. \tag{15}$$

7. If $\lambda \in \sigma(\mathcal{A})$ then (15) holds for every $f \in \mathcal{D}(\mathcal{A} - \lambda I)^{-1}$.

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The probability of reaching a receding boundary by a random walk on branching process with fading branching and heavy-tailed jump distribution

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Key words: Random walk, branching process, heavy-tailed distribution

Mathematical Subject Classification: 60J80

Let Z_n be a branching process in varying environment with the set of offspring distributions $\widehat{\mathcal{P}} = (\mathcal{P}_0, \mathcal{P}_1, \ldots)$ and let $\mathcal{T}$ be a genealogical tree for Z_n. Define a *random walk on* $\mathcal{T}$ as follows:

$$S(\pi) = \sum_{e \in \pi} \xi_{n(e),j(e)},$$

where π is an arbitrary path in $\mathcal{T}$ starting in root, $n(e)$ is the number of generation in which e ends, $j(e)$ is the number of particle in generation $n(e)$ in which e ends and $\{\xi_{n,j}\}_{n,j \geq 1}$ is the sequence of independent and identically distributed random variables that does not depend on genealogical tree $\mathcal{T}$.

We are interested in studying tail asymptotics for the

$$R^g_\mu = \sup_{\pi : |\pi| \leq \mu} \left(S(\pi) - g(|\pi|) \right),$$

that is the rightmost point of g-shifted random walk on $\mathcal{T}$, where $\mu \leq \infty$ is an arbitrary counting random variable and g is an arbitrary function on $\{0, 1, 2, \ldots\}$).

We obtain conditions under which

$$\mathbb{P}\left(R^g_\mu > x\right) = (1 + o(1)) H^g_\mu(x; \widehat{\mathcal{P}}) \qquad \text{as } x \to \infty,$$

uniformly over all suitable classes of time moments μ and functions g, where

$$H^g_\mu(x; \widehat{\mathcal{P}}) = \sum_{n=1}^{\infty} \mathbb{E}\left[Z_n \mathbb{I}(\mu \geq n)\right] \overline{F}(x + g(n)).$$

This talk is based on the joint work with Sergey Foss.

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Properties of ongoing critical branching processes with countable particle types

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Key words: Galton-Watson branching processes with a countable number of particle types, critical branching processes, limit theorems, reduced genealogical trees

Mathematical Subject Classification: 60J80

Abstract: Let $\mathbf{X}^{(l)}(n) = \big(X_j^{(l)}(n)\big)_{j\in\mathcal{Z}}$, $n \in \mathcal{N}_0 := \mathcal{N} \cup 0$, be the Galton-Watson branching process starting from one particle of type $l \in \mathcal{Z}$, where $Z_j^{(l)}(n)$ is the number of particles of type j at time n. Let's put $\mathbf{X}(n) := \mathbf{X}^{(0)}(n) =: \big(X_j(n)\big)_{j\in\mathcal{Z}}$. Define the generating function $f(\mathbf{s})$ for the random vector $\xi := (\xi_i)_{i\in\mathcal{Z}} \in \mathcal{N}_0^{\mathcal{Z}}$, where ξ_i is the type i offspring number for a particle of type 0, with its own distribution $p_{\mathbf{j}} := \mathcal{P}\big(\xi = \mathbf{j}\big)$

$$f(\mathbf{s}) := \mathbf{E}\mathbf{s}^{\xi} = \sum\nolimits_{\mathbf{j}\in\mathcal{N}_0^{\mathcal{Z}}} p_{\mathbf{j}}\mathbf{s}^{\mathbf{j}}, \quad \mathbf{j} = \big(j_i\big)_{i\in\mathcal{N}} \in \mathcal{N}_0^{\mathcal{Z}},$$

where $\mathbf{s} = \big(s_i\big)_{i\in\mathcal{Z}} \in [0,1]^{\mathcal{Z}}$, and $\mathbf{s}^{\mathbf{j}} := \prod_{i\in\mathcal{Z}} s_i^{j_i}$. An analogous generating function for a particle of type $m \in \mathcal{Z}$ has the form $f_m(\mathbf{s}) = f\big(\mathbf{s}^{(m)}\big)$, where $\mathbf{s}^{(m)} := \big(s_{i+m}\big)_{i\in\mathcal{Z}} \in [0,1]^{\mathcal{Z}}$, $\mathbf{s} = \mathbf{s}^{(0)}$.

Process $X(n) := \big|\mathbf{X}(n)\big| = \sum_{j=-\infty}^{\infty} X_j(n)$ with generating function $p(s) := \mathbf{E}s^{|\xi|} = f(\mathbf{1}s) =: \sum_{i=0}^{\infty} p_i s^i$ will be called the accompanying one. In terms of $X(n)$, we study only the critical case with a finite variance for the number of offspring.

We fix n and from the processes $X(k)$ and $\mathbf{X}(k)$, $k = 0, 1, \cdots, n$, we exclude all particles that have no offspring at time n. The resulting processes are called reduced and are denoted by $X(k,n)$ and $\mathbf{X}(k,n) = \big(X_j(k,n)\big)_{j\in\mathcal{Z}}$, $k = 0, 1, \cdots, n$. Set $\eta := \sum_{j=-\infty}^{\infty} jX_j(1)$ and $V(k,n) := \sum_{j=-\infty}^{\infty} jX_j(k,n)$. It is obvious that $X(n,n) = \{X(n)|X(n) > 0\}$ and $\mathbf{X}(n,n) = \{\mathbf{X}(n)|X(n) > 0\}$.

It is well known (see [1], Ch. I, §10) that in the critical case for $\mathbf{D}\xi = \sigma^2$ and finite third moment $p'''(1) < \infty$ for $Q_n := \mathbf{P}(X(n) > 0)$

$$Q_n^{-1} = 0.5\sigma^2 n + O\big(\ln n\big),$$

while Yaglom's theorem asserts the convergence $\lim_{n\to\infty} \mathbf{P}\big\{Q_n X(n) > x \big| X(n) > 0\big\} \to e^{-x}$.

In [2] a generalization of Yaglom's theorem for processes with a countable number of particle types is proved. The history of the problem is also described in some detail there. The essence of this generalization was that if in the limit, particles with small numbers of types are mainly preserved.

Suppose that $\mathbf{E}\xi = 1$, $\mathbf{D}\xi = \sigma^2$, $\mathbf{E}\eta = a_1 \neq 0$ and among the $p_{\mathbf{j}}$, only a finite number are nonzero. Than

$$\mathbf{E}X(k,n) = \frac{Q_{n-k}}{Q_n};\ \mathbf{D}X(k,n) = \frac{kQ_{n-k}^2}{nQ_n^2}\big(1 + o_n\big(1\big)\big);\ \mathbf{E}V(k,n) = ka_1\frac{Q_{n-k}}{Q_n};$$

$$\mathbf{D}V(k,n) = \big(a_1 k + a_1^2(k-1)\big)\big(1 + o_n\big(1\big)\big), \quad k = O(1);$$

$$\mathbf{D}V(k,n) = (a_1 + a_1^2)k(1 - 0.5kn^{-1})\frac{Q_{n-k}^2}{Q_n^2}\big(1 + o_n\big(1\big)\big), \quad k \to \infty.$$

$$\lim_{M\to+\infty}\lim_{n\to\infty} \mathbf{P}\left(\frac{\big|Q_{n-k}^{-1}V(k,n) - 0.5a_1\sigma^{-2}n^2\big|}{n\sqrt{n}} > M\right) = 0, \text{ for } n - k = o(n).$$

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Branching processes in non-favorable random environment

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Key words: Branching processes, random environment, survival probability

Mathematical Subject Classification: 60J80

Abstract:
Let $\mathcal{Z} = \{Z_n,\ n = 0, 1, 2, ...\}$ be a critical branching process evolving in a random environment generated by a sequence $\{F_n(s), \quad s \in [0,1], n = 1, 2, ...\}$ of i.i.d. probability generating functions. Denote $X_i = \log F_i'(1), i = 1, 2, ...$ and introduce a random walk

$$S_0 = 0, \quad S_n = X_1 + ... + X_n,\ n \geq 1.$$

We impose the following restrictions on the characteristics of the process.

Assumption B1. The random variables $X_n, n = 1, 2, ...$ are independent and identically distributed with

$$\mathbf{E}X_1 = 0, \quad \sigma^2 = \mathbf{D}X_1 \in (0, \infty).$$

Besides, the distribution of X_1 is non-lattice.

Assumption B2. There is an $\varepsilon > 0$ such that

$$\mathbf{E}\left(\log^+ \frac{F_1''(1)}{(F_1'(1))^2}\right)^{2+\varepsilon} < \infty.$$

Theorem 2 *Let Assumptions B1-B2 be valid. If $\varphi(n), n = 1, 2, ...$ is a sequence of positive numbers such that $\varphi(n) \to \infty$ as $n \to \infty$ and $\varphi(n) = o(\sqrt{n})$, then there is a constant $\Theta \in (0, \infty)$ such that*

$$\mathbf{P}\left(Z_n > 0; S_n \leq \varphi(n)\right) \sim \frac{\Theta\varphi^2(n)}{n^{3/2}}, \quad n \to \infty.$$

Theorem 2 compliments Theorem 1.1 in [1] where it was shown that there is a constant $C \in (0, \infty)$ such that $\mathbf{P}\left(Z_n > 0\right) \sim C\sqrt{n}$ as $n \to \infty$.

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On structural equivalence of S-tuples in Markov chains

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Key words: Markov chain, s-tuple

Mathematical Subject Classification: Theory of probabilities

Abstract: The paper presents the main results of [3]. Let H be the permutation group on the set $\{1, \dots, N\}$. Typles $(a_1, \dots, a_s)$, $(b_1, \dots, b_s)$ of elements sets $\{1, \dots, N\}$ are called *H-equivalent* if there is a permutation $h \in H$ such that $b = h(a)$, i.e.

$$b_i = h(a_i), \quad i = 1, \dots, s.$$

For H-equivalent tuples $a = (a_1, \dots, a_s)$ and $b = (b_1, \dots, b_s)$ we will use the notation aHb. If the tuples a and b are not H-equivalent, then we use the notation $\overline{H}$.

Let $x_1, x_2, \dots$ be the sequence elements of the set $\{1, \dots, N\}$. We will say that the tuple $z = (x_j, \dots, x_{j+s-1})$ is the H-repetition of the tuple $y = (x_i, \dots, x_{i+s-1})$, $j > i$, if yHz.

Further as a sequence $x_1, x_2, \dots$ consider a nonperiodic homogeneous Markov chain $\mathbf{X} = \{X_0, X_1, \dots, X_n, \dots\}$ with outcomes $1, \dots, N$, indecomposable matrix transition probabilities $\mathbb{P} = \|p_{k,l}\|$ and arbitrary initial distribution. Denote $\pi = (\pi_1, \dots, \pi_N)$, where $\pi_k > 0,\ k = 1, ..., N$, stationary distribution of the chain $\mathbf{X}$.

We are interested in events $\left\{Y_{i_1-1}\overline{H}Y_{i_2-1},\ Y_{i_1}(s)HY_{i_2}(s)\right\}$, consisting in the fact that at the moments i_1 and i_2 the series begins H-repetitions of s-tuples. We study the asymptotic behavior of the distribution of the number of series of H-repetitions s-tuples starting up to the moment n:

$$\widetilde{\xi}_2(n, s, H) = \sum_{1 \le i_1 < i_2 \le n} I\Big\{Y_{i_1-1}\overline{H}Y_{i_2-1},\ Y_{i_1}(s)HY_{i_2}(s)\Big\}.$$

The problem of the number of equivalent tuples in random discrete sequences was first considered in [1]. In this paper, sufficient

conditions for the Poisson approximation were obtained for the number of pairs of equivalent tuples in a sequence independent random variables distributed uniformly on set $\{1,\dots,N\}$. Further development of this direction is reflected in the review paper [2], which describes the results of works that appeared before 2003 year, and also announced a number of results published a little later.

Theorem 1. *Let the matrix $\mathbb{P}$ be indecomposable, $p^2 < \rho$, $n \to \infty$, and $s = s(n) \to \infty$ so that the condition holds $n^2\rho^s = O(1)$. Then*

$$\mathbf{P}\Big\{\tilde{\xi}_2(n,s,H) = \tilde{\xi}_2(n,s,H_{\mathbb{P}})\Big\} \to 1.$$

Let us introduce the notation $R^2_{H_{\mathbb{P}}} = \rho^{s-2}(1-\rho)|H_{\mathbb{P}}| \sum\limits_{a,b\in\{1,\dots,N\}} \pi_a^2 p_{a,b}^2$.

Theorem 2. *Let the matrix $\mathbb{P}$ be indecomposable, $p^2 < \rho$, $n \to \infty$, and $s = s(n)$ changes so that $s^2/n \to 0$ and $n^2R^2_{H_{\mathbb{P}}}/2 \to \lambda \in (0,\infty)$. Then the distribution of the random variable $\tilde{\xi}_2(n,s,H)$ converges to Poisson distribution with parameter λ.*

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Processes with generation and transport of particles

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Key words: Multitype branching random walks, multidimensional lattices, evolutionary operators, limit theorems

Mathematical Subject Classification: 60J27; 60J80; 05C81; 60J85

Abstract:

The talk is devoted to continuous-time stochastic processes, which can be described in terms of birth, death and transport of particles. Such processes on multidimensional lattices are called branching random walks, and the points of the lattice at which the birth and death of particles can occur are called branching sources. Particular attention is paid to the analysis of the asymptotic behavior of particle numbers and their moments for symmetric branching random walks with a finite set of branching sources and a finite or infinite number of initial particles under various assumptions on the variance of random walk jumps. The behavior of moments is mainly determined by the structure of the spectrum of the evolutionary operator of average particle numbers and requires the use of the spectral theory of operators in a Banach space. The proof of some limit theorems on branching random walks with a finite number of sources and pseudo-sources, in which random walk symmetry breaking is based on checking the conditions that guarantee the uniqueness of the definition of the limit probability distribution of particle numbers by their moments. For branching random walks with branching sources at each point of the lattice, in which the rates of birth and death of particles are equal and the underlying random walk is recurrent, limit theorems on the behavior of populations and subpopulations of particles are given. One of the new directions in the theory of branching random walks is the study of multitype branching random walks both in a non-random and in a random "branching" environment. A series of results of numerical simulation of branching random walks are presented and the

possibility of applying such processes in medicine and genetics are discussed. The talk is partly based on papers [1-4].

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Branching processes on finite sets and iterations of random mappings

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KEY WORDS: branching processes, random allocation of particles, nearest common ancestor

MATHEMATICAL SUBJECT CLASSIFICATION: 60J80, 60C05

Abstract:

I plan to review several problems and results connected with branching processes and processes of random allocations of particles into cells.

1. First we consider a model of populations evolving in a sequence of layers consisting of finite sets of cells.

Let $S = \{1, \ldots, N\}$ be a finite set, $\nu_{t,i}$ $(t, i = 1, 2, \ldots)$, be independent random variables with generating function $f(s) = \sum_{k=0}^{\infty} p_k s^k$.

The process $\{\xi_t\}_{n \geq 0}$ begins with $\xi_0 = s \leq N$ particles in 0-th layer. For any $t = 0, 1, \ldots$ let $\eta_{t+1} = \nu_{t,1} + \ldots + \nu_{t,\xi_t}$ be the number of particles born by ξ_t particles in the t-th layer.

This η_{t+1} particles are allocated over the cells of $(t+1)$-th layer independently and equiprobably. The value ξ_{t+1} equals the number of non-empty cells after this allocation. In other words, particles allocated in the same cell are glued together.

For the case $s_0 = N$ some estimates of the probability of extinction at least at t-th layer are obtained.

2. Second problem relates to the case $f(s) = s$, $s \in [0, 1]$, where there are no branching. Here the limit distribution of $\tau_N = \min\{t \colon \xi_t = 1\}$ as $N \to \infty$ is found.

3. We discuss also limit theorems for the distributions of distance to the nearest common ancestor of all particles existing in a branching process at the moment t under condition that the process does not extinct at the moment $t \to \infty$.

4. Finally we consider theorems on the value ξ_t under the condition that $\xi_0 = s = o(N)$, $N, s \to \infty$.