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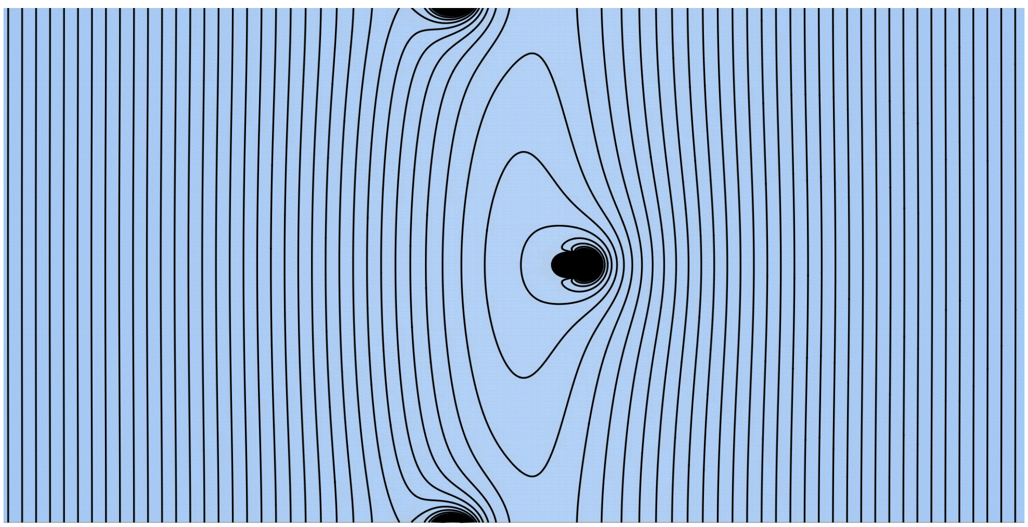
Abstracts

George Alekseev
(Steklov Mathematical Institute of RAS)

Charged black hole accelerated by spatially homogeneous electric field of Bertotti-Robinson ($AdS^2 \times \mathbb{S}^2$) space-time

The exact solution of Einstein - Maxwell field equations for charged non-rotating black hole accelerated by an external electric field is constructed as nonlinear superposition of electrovacuum fields using the monodromy transform approach to solution of integrable reductions of Einstein - Maxwell equations. The background space-time, described by the well known Bertotti-Robinson solution, contains a spatially homogeneous electric field and possess the topology $\mathrm{AdS}^2 \times \mathbb{S}^2$. The black hole mass m, its charge e and the value of the background electric field E are free parameters of the constructed solution. In the reference frame comoving a black hole, this solution is static. The value of acceleration of a black hole due to its interaction with the external electric field is determined by the condition of vanishing of conical singularities on the axis of symmetry. The dynamics of a charged black hole in the external electric field is compared with the behaviour of a charged test particle with the same charge to mass ratio and in the same background space-time.



Irina Aref'eva
(Steklov Mathematical Institute of RAS)

Wormholes/Boltzmann Fields Duality

It is known that the master fields for large N matrix fields satisfy the Boltzmann statistic. We consider holographic duals for correlators of Boltzmann fields at the boundary. It is shown that summation over wormhole configurations in the bulk corresponds to computation of correlation functions of fields with Boltzmann statistic on the boundary, i.e. wormhole configurations are dual to Boltzman correlators.

Gleb Arutyunov
(Hamburg University)

R-matrix structures for classical and quantum Ruijsenaars-Schneinder models

We use the reduction approach to derive a hamiltonian structure of the classical spin hyperbolic Ruijsenaars-Schneinder model. We show that the model enjoys the Poisson-Lie symmetry which explains its superintegrability. For the quantum model without spin we obtain the L-operator algebra and integrals of motion in the R-matrix formalism.

Alexander Belavin
(Landau Institute for Theoretical Physics of RAS)

Spectral Flow construction of $N = 2$ superconformal orbifolds

In this talk I present a new approach to constructing Calabi-Yau orbifolds required for Compactification in Superstring theory. We use the connection of CY orbifolds with the class of exactly solvable N=2 SCFT minimal models to explicitly construct a complete set of fields in these orbifold models using the twisting of the spectral flow and the requirement of mutual locality of the fields. The talk is based on a joint work with V. Belavin and S. Parkhomenko.

Samuel Belliard
(IDP université de Tours France)

Modified Algebraic Bethe Ansatz

I will discuss a way to calculate spectrum and states for models without U(1) symmetries such as the open XXZ spin chain. The modified algebraic Bethe ansatz will be described in that case, and off-shell action of the related transfer matrix will be established. It will allows to calculate Slavnov's formula for scalar product of the associated states.

Victor Buchstaber
(Steklov Mathematical Institute of RAS)

Quantum Novikov equations

In the first part of the talk, we will discuss the well-known Korteweg-de Vries hierarchy in the free associative algebra of an infinite number of variables. For each natural number N, we give an explicit description of the noncommutative version of the N-th Novikov equation and its first integrals in a free associative algebra of 2N variables. In the second part of the talk, we will introduce the N-th quantum Novikov equations and describe their first integrals. Using the examples, we will show how work the general method of quantization ideals, recently introduced by A.V. Mikhailov. In our case, we are talking about a two-sided ideal in the free associative algebra of 2N variables, which is invariant under the noncommutative N-th Novikov equation in this algebra. A factor by such ideal defines a dynamical system in an associative algebra AN of 2N variables with the additive Poincaré–Birkhoff–Witt basis. In the third part of the talk, we will introduce a polynomial invertible transformation of the algebra AN, which transforms the N-th quantum Novikov equation and the corresponding quantum hierarchy to the standard Heisenberg form. As result we will obtain the operator representation of explicitly given quantum Hamiltonians. The talk is based on the results obtained jointly with A.V. Mikhailov.

Dmitri Bykov
(Steklov Mathematical Institute of RAS)

Sigma models as Gross-Neveu models

I will show that there is a wide class of integrable sigma models that are exactly and explicitly equivalent to bosonic/fermionic Gross-Neveu models. In full generality these are models with quiver variety phase spaces, but the familiar CP^n, Grassmannian or flag manifold sigma models belong to this class as well. This approach leads to a new take on topics such as RG flow (potentially allowing an all-loop calculation), construction of integrable deformations and the inclusion of fermions. In particular, it provides a way of obtaining worldsheet SUSY theories from target space SUSY theories by means of a supersymplectic quotient. Generalizations to Riemann surface worldsheets will also be mentioned.

This is mostly a review talk based on arXiv:2006.14124, 2009.04608, 2101.11638, 2106.15598, 2202.12805 but will include latest developments as well.

Sergey Derkachov
(St. Petersburg Department of Steklov Mathematical Institute of RAS)

Conformal Triangles and Zig-Zag Diagrams

We consider the convenient operator representation for so called zig-zag diagrams in the 4-dimensional φ^4 quantum field theory. The explicit diagonalization of the graph-building operator allows to prove in a very simple way the Broadhurst-Kreimer conjecture about the values of zig-zag multi-loop diagrams which make a significant contribution to the renormalization group β-function.

The talk is based on our work with A. P. Isaev and L. Shumilov Phys.Lett.B 830 (2022) 137150 arxiv: 2201.12232

Sergey Dobrokhotov
(Ishlinsky Institute for Problems in Mechanics RAS)

Keplerian trajectories and simple global Airy-type asymptotic solution of the scattering problem for the Schroedinger equation with a repulsive Coulomb potential.

Joint work with S.B.Levin (St. Petersburg State University), A.A.Tolchennikov (Ishlinsky Institute for Problems in Mechanics RAS).

The exact solution of the scattering problem for the Schroedinger equation with a repulsive Coulomb potential in the form of a degenerate hypergeometric is well known (see the book S. P. Merkuryev, L.D. Faddeev, Quantum scattering theory for systems of several particles, Moscow: Nauka, 1998; Springer 1993).

With the help of recently developed approaches, we construct simple explicit semiclassical asymptotic formulas for solving this problem in the form of the Airy function of a complex argument. The answer relies on a suitable Lagrangian manifold woven from well-known Keplerian trajectories.

The work was supported by the Russian Science Foundation under grant 21-71-3001.

Dimitri Gurevich
(Valenciennes University, France)

q-cut-and-join operators related to Reflection Equation algebras

The classical cut-and-joint operators, playing a very important role in Hurwitz theory, differ from the Casimir operators, coming from the enveloping algebras U(gl(N)), by normal ordering. Construction of the cut-and-join operators is based on the fact that the product of two square matrices L=MD, where M has commutative entries and D is composed from the partial derivatives in these entries, generates the algebra U(gl(N)). I'll define analogs of all these objects related to the so-called Reflection Equation algebras and describe a way to perform spectral analysis of the corresponding operators. Also, I plan to exhibit other applications of the formula L=MD in the "q-setting".

Gerard Helminck
(KdV Institute University of Amsterdam)

TBA

Alexander Its
(Indiana University — Purdue University Indianapolis)

Operator Valued Riemann-Hilbert Problems. Then and Now.

In the context of integrable systems, the operator valued Riemann-Hilbert problems first appeared in the late 80s early 90s work of Nikita Slavnov and his collaborators (the speaker included). It turns out that the transition to the operator valued Riemann-Hilbert setting manefistates the transition from free fermion to non free fermion exactly solvable quantum models. Very recently, the operator valued Riemann-Hilbert problems started to show up in the problems related to the integrable probability; notably, in the description of the important solutions of the KPZ equations. These are the works of I. Corwin, P. Ghosal, T. Bothner, M. Cafasso, and S. Tarricone.

The principal goal of the talk is to remind the old, still unsolved important questions associated with the quantum operator valued Riemann-Hilbert problems and to highlight the new challenges emerged in the area due to its involvement in the probabilistic applications.

Maxim Kazarian
(Higher School of Economics)

Symplectic duality for topological recursion

Joint work with B. Bychkov, P. Dunun-Barkovski, and S. Shadrin, work in progress.

There are many enumerative problems whose answers are encoded in the Taylor coefficients of a sequence of the so-called m-point correlator functions. The topological recursion (due to Chekhov-Eunard-Orantin) is an inductive procedure for explicit computation of these functions in a closed form starting from a relative small amount of initial data. A small suspension of the problem leads to a collection of (m,n)-point correlator functions such that the original ones correspond to the case n=0. It proves out that the sequence of (0,n) functions also satisfies its own topological recursion with its own initial data. This fact was known before for the two-matrix model related to the problem of enumeration of (hyper)maps. The two recursions are related in this case by the x-y duality which is well studied in a general formalism of topological recursion. We generalize this fact to the case of enumeration of generalized Hurwitz numbers. The former x-y duality does not hold literally in this case; its analogue for the generalized Hurwitz numbers is exactly what we mean by the symplectic duality.

Sergey Khoroshkin
(Higher School of Economics)

Wave functions of hyperbolic Sutherland model

We present the wave functions of hyperbolic Sutherland model by Mellin-Barnes integrals and show that they solve the related bispectral problem.

Igor Krichever
(Columbia University, Skoltech)

Monodromy free linear equations and Abelian pole systems

Sergey Lando
(Higher School of Economics and Skoltech)

Skew characteristic polynomial of graphs and embedded graphs

We introduce a new one-variable polynomial invariant of graphs, which we call the skew characteristic polynomial. For an oriented simple graph, this is just the characteristic polynomial of its anti-symmetric adjacency matrix. For nonoriented simple graphs the definition is different, but for a certain class of graphs (namely, for intersection graphs of chord diagrams), it gives the same answer if we endow such a graph with an orientation induced by the chord diagram.

We prove that this invariant satisfies Vassiliev's 4-term relations and determines therefore a finite type knot invariant. We investigate the behaviour of the polynomial with respect to the Hopf algebra structure on the space of graphs and show that it takes a constant value on any primitive element in this Hopf algebra. We also provide a two-variable extension of the skew characteristic polynomial to embedded graphs and delta-matroids. The 4-term relations for the extended polynomial prove that it determines a finite type invariant of multicomponent links.

The talk is based on a joint work with R.Dogra.

Andrei Marshakov
(Skoltech)

Cluster reductions and Painleve equations

Alexander Mikhailov
(University of Leeds)

Quantisation of free associative dynamical systems

Traditional quantisation theories start with classical Hamiltonian systems with variables taking values in commutative algebras and then study their non-commutative deformations, such that the commutators of observables tend to the corresponding Poisson brackets as the (Planck) constant of deformation goes to zero. I am proposing to depart from dynamical systems defined on a free associative algebra $\mathfrak{A}$. In this approach the quantisation problem is reduced to the problem of finding of a two-sided ideal $\mathfrak{J} \subset \mathfrak{A}$ satisfying two conditions: the ideal $\mathfrak{J}$ has to be invariant with respect to the dynamics of the system and to define a complete set of commutation relations in the quotient algebras $\mathfrak{A}_{\mathfrak{J}} = \mathfrak{A}/\mathfrak{J}$ [1].

To illustrate this approach I'll consider the quantisation problem for the Volterra family of integrable systems. In particular, I will show that odd degree symmetries of the Volterra chain admit two quantisations, one of them is a standard deformation quantisation of the Volterra chain, and another one is new and not a deformation quantisation. The periodic Volterra chain admits bi-Hamiltonian and bi-quantum structures [2]. The method of quantisation based on the concept of quantisation ideals proved to be successful for quatisation of stationary Korteveg-de–Vries hierarchies [3]. The Toda hierarchy also admits bi-quantum structures and non-deformation quantisation.

References

[1] A. V. Mikhailov. Quantisation ideals of nonabelian integrable systems. *Russian Mathematical Surveys*, 75(5):978–980, 2020.

[2] S. Carpentier, A.V. Mikhailov, J.P.Wang. Quantisations of the Volterra hierarchy *arXiv:2204.03095 [nlin.SI], 2022*

[3] V.M. Buchstaber, A.V. Mikhailov. KdV hierarchies and quantum Novikov's equations *arXiv:2109.06357v2 [nlin.SI], 2021*

Andrei Mironov
(Lebedev Physical Institute of RAS)

Superintegrability in matrix models: present status

We review the present status of superintegrability in matrix models, that is, the phenomenon of character averages being proportional to the same characters at special points. This phenomenon which was originally realized in the Gaussian models can be extended to pair correlators of characters and survives various deformations of the Gaussian model.

Nikita Nekrasov
(Simons Center for Geometry and Physics)

Exact computations in (non)supersymmetric theories using localization: 30 years later

The many-body Calogero system is a continuous source of interesting problems, with deep connections to various branches of mathematical physics. I will review the less known connection to localization and replicas.

Mikhail Olshanetsky
(ITEP)

Classical 2d Integrable Systems and Gauge Theories

We compare constructions of 2d integrable models through two gauge field theories. The first one is the 4d Chern-Simons (4d-CS) theory proposed by Costello and Yamazaki. The second one is the Affine Higgs bundles (AHB). It is 2d generalization of the Hitchin integrable systems. We consider in more detail the latter construction and illustrate it by considering the Landau-Lifshitz model.

Aleksander Orlov
(Shirshov Institute of Oceanology of RAS)

Bipartite graphs and eigenvalue problems

I will tell you how to build matrix models using graphs on a Riemann surface and how various identities with differential operators are obtained from them. In particular, we obtain the Mironov-Morozov-Natanzon cut-and-join equation, which describes the merging of branching points. (A section in joint work with S.M.Natanzon, TMP 2020)

Stanislav Pakuliak
(Joint Institute for Nuclear Research)

Recent Advances in Algebraic Bethe Ansatz

Recent development of the algebraic Bethe ansatz for the quantum integrable models associated with quantum affine algebras (QAA) and Yangian doubles is reviewed. New approach uses expressions for the off-shell Bethe vectors (BV) in terms of Cartan–Weyl generators of these algebras and allows to obtain effective formulas of the action of monodromy matrix entries onto BV and recurrence relations for them. Relations between Gauss coordinates of the fundamental L-operators and currents for classical series Yangians and QAAs are presented.

Nicolai Reshetikhin
(Tsinghua University, UC Berkeley)

Spin Calogero-Moser system and two dimensional Yang-Mills theory with corners

Quantum spin Calogero-Moser system is a quantum superintegrable system. Its spectrum has a natural description in terms of representation theory of the underlying simple Lie group. In the first part of the talk we will overview this quantum system and will show that its multitime propagator is the partition function of two dimensional Yang-Mills theory on a cylinder. In the second part we define the Yang-Mills theory with open Wilson lines and will argue that it is an extended topological quantum field theory in dimensions 2-1-0. This is a joint work with J. Stokman.

Vladimir Roubtsov
(LAREMA, Université d'Angers)

Multiplicative Bessel Kernels: from addition laws to CY periods

We discuss few very recent results of a work in progress (in collaboration with I. Gaiur and D. Van Straten) about interesting properties of multiplicative Bessel kernels, which includes well-known Clausen and Sonin-Gegenbauer formulae of XIX century, special examples of Kontsevich discriminant loci polynomials, raised as addition laws for special two-valued formal groups (Buchshtaber-Novikov) and period functions for CY and Landau–Ginzburg models.

Armen Sergeev
(Steklov Mathematical Institute of RAS)

Topological phases in solid state physics

Topological methods play an important role in the theory of solid states. They were successfully applied to the theory of insulators characterized by a wide energy gap stable under small deformations. The availability of such gap is also important for the theory of topological phases.

To define the topological phases denote by G the symmetry group and consider the set Ham_G of homotopy equivalence classes of G-symmetric Hamiltonians satisfying the energy gap condition. It is possible to introduce a natural stacking operation on this set making Ham_G into Abelian monoid (i.e. Abelian semigroup with neutral element). The group of invertible elements of this monoid is precisely the topological phase.

It turns out that the family (F_d) of d-dimensional topological phases forms an Ω-spectrum. By this we mean a collection of topological spaces F_d having the property that the loop space ΩF_{d+1} is homotopy equivalent to the space F_d. Every Ω-spectrum generates a generalized cohomology theory determined by the functor h^d, associating with the topological space X the set $[X, F_d]$ of homotopy equivalence classes of the maps $X \to F_d$. We give several examples of concrete physical systems which can be described in terms of generalized cohomology theories.

We also discuss relations between topological phases and K-theory. The K-functor is defined using the spectral flattening of Hamiltonians and can be computed in a series of important examples.

Sergey Suetin
(Steklov Mathematical Institute of RAS)

On Some Algebraic Properties of Hermite–Pade Polynomials

We consider a set of m formal power series in non-negative powers of the variable $1/z$, which are in the "general position". For this set of series and corresponding multiindexes depending on an arbitrary natural n, constructions of Hermite-Pade polynomials of the 1st and 2nd types with the following property are given. If $M_1(z)$ and $M_2(z)$ are two polynomial matrices corresponding to Hermite-Pade polynomials of the 1st and 2nd types, then their product is equal to the identity matrix.

The result is motivated by some novel applications of Hermite–Padé polynomials to the investigation of monodromy properties of Fuchsian systems of differential equations.

Leon Takhtajan
(Stony Brook University, USA, and Euler International Institute, SPb, Russia)

New supersymmetric localization principle

I will present new supersymmetric localization principle, with application to trace formulas for a full thermal partition function. Unlike the standard localization principle, this new principle allows to compute the supertrace of non- supersymmetric observables, and is based on the existence of fermionic zero modes. This is a joint work with Changha Choi, arXiv:2112.07942.

Elizaveta Trunina
(Moscow Institute of Physics and Technology)

Multi-pole extension for elliptic models of interacting integrable tops: relativistic and non-relativistic cases.

We review and give detailed description for gl(NM) Gaudin models related to holomorphic vector bundles of rank NM and degree N over elliptic curve with n punctures. We present full classification for this type of integrable systems by summarizing the previously obtained results and describe the most general gl(NM) classical elliptic finite-dimensional integrable system, which includes all the known as particular cases. We also discuss relativistic analogue of these systems. We present a classification for relativistic Gaudin models on GL-bundles over elliptic curve and describe the most general gl(NM) classical elliptic finite-dimensional integrable system. Also, we provide R-matrix description for both relativistic and non-relativistic most general models through R-matrices satisfying associative Yang-Baxter equation.

Mikhail Vasilyev
(Skoltech, Steklov Mathematical Institute of RAS)

TASEP and integrable many-body systems

We consider a new duality between the discrete-time inhomogeneous multispecies TASEP model on the circle and the quantum Goldfish model from the Ruijsenaars-Schneider family. We present the precise map of the inhomogeneous multispecies TASEP system and 5-vertex model, which are the limiting cases of 6-vertex model, to the trigonometric and rational Goldfish models respectively, where the TASEP local jump rates get identified as the coordinates in the Goldfish model. This is a joint work with A. Gorsky and A. Zotov.

Alexander Veselov
(Loughborough University, UK)

In the shadow of Markov and Mordell

Recently Valentin Ovsienko introduced the shadow Markov triples as solutions of Markov-type equation over dual numbers. I will introduce the shadow analogue of Mordell triples and discuss their relations with shadow Markov triples and Conway topograph.

Anton Zabrodin
(Skoltech)

Constrained Toda hierarchy and turning points of the Ruijsenaars-Schneider model

We introduce a new integrable hierarchy of nonlinear differential-difference equations which we call constrained Toda hierarchy. It can be regarded as a subhierarchy of the 2D Toda lattice obtained by imposing a certain constraint connecting the two Lax operators of the latter. We prove the existence of the tau-function of the constrained Toda hierarchy and show that it is the square root of the 2D Toda lattice tau-function. In this and some other respects the constrained Toda is a Toda analogue of the CKP hierarchy. It is also shown that zeros of the tau-function of elliptic solutions satisfy the dynamical equations of the Ruijsenaars-Schneider model restricted to turning points in the phase space. The spectral curve has holomorphic involution which interchange the marked points in which the Baker-Akhiezer function has essential singularities. This is a joint work with I.Krichever.