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Vladimir Bogachev: Nonlinear Kantorovich problems and Hausdorff distances between plans

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We discuss recent investigations of nonlinear Kantorovich problems of optimal transportation involving cost functions that can depend on transportation plans, in particular, on conditional measures of transportation plans. Some existence results will be presented. In addition, we consider Hausdorff distances on sets of transportation plans associated with the Kantorovich and Kantorovich–Rubinshtein metrics on spaces of probability measures. Such Kantorovich-type Hausdorff distances admit simple bounds via distances between marginal distributions. Some open problems in this area are also mentioned. More details on this direction can be found in the forthcoming paper [1].

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Alexander Bufetov: The Gaussian multiplicative chaos for the sine-process

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To almost every realization of the sine-process one naturally assigns a random entire function, the analogue of the Euler product for the sine, the scaling limit of ratios of characteristic polynomials of a random matrix. The main result of the talk is that the square of the absolute value of our random entire function converges to the Gaussian multiplicative chaos. As a corollary, one obtains that almost every realization with one particle removed is a complete and minimal set for the Paley-Wiener space, whereas if two particles are removed, then the resulting set is a zero set for the Paley-Wiener space. Quasi-invariance of the sine-process under compactly supported diffeomorphisms of the line plays a key rôle.
In joint work with Qiu, the Patterson-Sullivan construction is used to interpolate Bergman functions from a realization of the determinantal point process with the Bergman kernel, in other words, by the Peres-Virág theorem, the zero set of a random series with independent complex Gaussian entries. The invariance of the zero set under the isometries of the Lobachevsky plane plays a key rôle. Conditional measures of the determinantal point process with the Bergman kernel are found explicitly (cf. arXiv:2112.15557, Dec. 2021).

Marat Burnashev: On Stein's lemma in a general case

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Introduction. Let $\mathbf{P}$ and $\mathbf{Q}$ be probability measures on a measurable space $(\mathcal{X}, \mathcal{B})$. We consider testing of simple hypothesis $\mathcal{H}_0$ versus simple alternative $\mathcal{H}_1$, based on observation $\mathbf{x} \in \mathcal{X}$:

$$\mathcal{H}_0 : \mathbf{x} \sim \mathbf{P}, \qquad \mathcal{H}_1 : \mathbf{x} \sim \mathbf{Q}. \tag{1}$$

If for testing a decision (acceptance) region $\mathcal{D} \in \mathcal{X}$ is chosen, such that

$$\mathbf{x} \in \mathcal{D} \Rightarrow \mathcal{H}_0, \qquad \mathbf{x} \notin \mathcal{D} \Rightarrow \mathcal{H}_1, \tag{2}$$

then type-I error probability $\alpha(\mathcal{D})$ and type-II error probability $\beta(\mathcal{D})$ are defined by formulas, respectively,

$$\alpha(\mathcal{D}) = \mathbf{P}(\mathbf{x} \notin \mathcal{D} | \mathcal{H}_0), \qquad \beta(\mathcal{D}) = \mathbf{P}(\mathbf{x} \in \mathcal{D} | \mathcal{H}_1). \tag{3}$$

We consider the case when type-I error probability α, $0 < \alpha < 1$, is fixed, and we are interested in the minimal possible type-II error probability $\beta(\alpha)$

$$\beta(\alpha) = \inf_{\mathcal{D}:\alpha(\mathcal{D}) \le \alpha} \beta(\mathcal{D}). \tag{4}$$

The corresponding optimal decision region $\mathcal{D}(\alpha)$ to this problem is given by Neyman-Pearson lemma. We will need the following notion.

Definition 1. *For probability distributions $\mathbf{P}$ and $\mathbf{Q}$ on $(\mathcal{X}, \mathcal{B})$ the function $D(\mathbf{P}||\mathbf{Q})$ (Kullback–Leibler distance or divergence) is defined as*

$$D(\mathbf{P}||\mathbf{Q}) = \mathbf{E}_{\mathbf{P}} \ln \frac{d\mathbf{P}}{d\mathbf{Q}}(\mathbf{x}) \ge 0, \tag{5}$$

where the expectation is taken over the measure $\mathbf{P}$.

In order to describe Stein's lemma and its relation to the value $\beta(\alpha)$ via the function $D(\mathbf{P}||\mathbf{Q})$, we consider first the case $\mathcal{X} = \mathbf{R}^n$, when $\mathbf{x}$ from (1) has the form $\mathbf{x} = \mathbf{x}_n = (x_1, \dots, x_n) \in \mathbf{R}^n$. We assume that the sample $\mathbf{x}_n$ consists of i.i.d. random variables x_i, $i = 1, \dots, n$ with a distribution $\mathbf{P}$ (in the case of $\mathcal{H}_0$), or with a distribution $\mathbf{Q}$ (in the case of $\mathcal{H}_1$):

$$\mathcal{H}_0 : x_i \sim \mathbf{P}, \qquad \mathcal{H}_1 : x_i \sim \mathbf{Q}, \qquad i = 1, \dots, n. \tag{6}$$

If in model (6) $\mathbf{P} \not\equiv \mathbf{Q}$, then $D(\mathbf{P}||\mathbf{Q}) > 0$ and the value $\beta(\alpha)$ decreases exponentially as $n \to \infty$. Moreover, we have [3, Theorem 3.3]

$$\lim_{\alpha \to 0} \lim_{n \to \infty} \frac{1}{n} \ln \beta(\alpha) = -D(\mathbf{P}||\mathbf{Q}). \tag{7}$$

Relation (7) is called Stein's lemma [1, 2]. [1] In the case of i.i.d. random variables, its proof can be found in [3, Theorem 3.3], [4, Theorem 12.8.1]. It is natural to expect that relation (7) holds not only for i.i.d. random variables as in model (6), but in much more general cases. Some particular

[1] The reference [1] is due to remarks from [2, p. 18] and [3, Theorem 3.3] with attribute to the unpublished paper of C. Stein.

analogs of formula (7) have already appeared for cases of stationary Gaussian [5] and Poisson [6] random processes (with time T instead of n).
Relation (7) is oriented to the case when type-I and type-II errors imply very different losses, and we wish to minimize type-II error probability $\beta(\alpha)$. The case is quite popular in many applications (see, e.g., [5] and references therein). In this paper, for general model (1) some non-asymptotic generalization and strengthening of relation (7) is derived.

Main result. In order to simplify formulation of results and to avoid pathological cases, we assume that the following assumption is satisfied:
I. Probability measures $\mathbf{P}$ and $\mathbf{Q}$ on $(\mathcal{X},\mathcal{B})$ are equivalent and Radon-Nikodim derivative $d\mathbf{P}/d\mathbf{Q}(\mathbf{x})$ is defined, positive and finite for $\mathbf{x}\in\mathcal{X}$.
The main result is the following
Theorem. *If the assumption* **I** *holds, then the minimal possible* $\beta(\alpha)$, $0<\alpha<1$, *satisfies the bounds*

$$\begin{gathered}\ln\beta(\alpha)\geq -\frac{D(\mathbf{P}||\mathbf{Q})+h(\alpha)}{1-\alpha},\\ h(\alpha)=-\alpha\ln\alpha-(1-\alpha)\ln(1-\alpha),\end{gathered} \tag{8}$$

and

$$\ln\beta(\alpha)\leq -D(\mathbf{P}||\mathbf{Q})+\mu_0(\alpha), \tag{9}$$

where $\mu_0(\alpha)$ *is the minimal value* μ_0, *satisfying the relation*

$$\mathbf{P}_{\mathbf{P}}\left\{\ln\frac{d\mathbf{P}}{d\mathbf{Q}}(\mathbf{x})\leq D(\mathbf{P}||\mathbf{Q})-\mu_0\right\}\leq\alpha. \tag{10}$$

With some loss of accuracy we can simplify upper bound (9), using an upper bound for the value $\mu_0(\alpha)$. For example, using Chebyshev inequality, we get
Corollary. *The minimal possible* $\beta(\alpha)$, *satisfies also the upper bound*

$$\ln\beta(\alpha)\leq -D(\mathbf{P}||\mathbf{Q})+\alpha^{-1/2}r_1(\mathbf{P},\mathbf{Q}), \tag{11}$$

where

$$r_1(\mathbf{P},\mathbf{Q})=\left\{\mathbf{E}_{\mathbf{P}}\left[\ln\frac{d\mathbf{P}}{d\mathbf{Q}}(\mathbf{x})\right]^2-D^2(\mathbf{P}||\mathbf{Q})\right\}^{1/2}. \tag{12}$$

Note that all bounds (8), (9) and (11) are pure analytical relations without any limiting operations.

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Mamikon Ginovyan: On the prediction error for singular stationary processes

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Based on the joint works with Nikolay Babayan (Russian-Armenian University, Yerevan, Armenia)

Let $X(t)$, $t \in \mathbb{Z} := \{0, \pm 1, \dots\}$, be a centered discrete-time second-order stationary process. The process is assumed to have an absolutely continuous spectrum with spectral density $f(\lambda)$, $\lambda \in [-\pi, \pi]$. The *"finite" linear prediction problem* is as follows. Suppose we observe a finite realization of the process $X(t)$: $\{X(t),\ -n \le t \le -1\}$, $n \in \mathbb{N} := \{1, 2, \dots\}$. We want to make an one-step ahead prediction, that is, to predict the unobserved random variable $X(0)$, using the *linear predictor* $Y = \sum_{k=1}^{n} c_k X(-k)$.
The coefficients c_k ($k = 1, 2, \dots, n$) are chosen so as to minimize the *mean-squared error (MSE)*: $E\,|X(0) - Y|^2$. If $\widehat{c}_k := \widehat{c}_{k,n}$ are the minimizing constants, then the random variable $\widehat{X}_n(0) := \sum_{k=1}^{n} \widehat{c}_k X(-k)$ is called the *best linear one-step ahead predictor* of $X(0)$ based on the observed finite past: $X(-n), \dots, X(-1)$. The minimum MSE: $\sigma_n^2(f) := E\left|X(0) - \widehat{X}_n(0)\right|^2 \ge 0$ is called the *prediction error* of $X(0)$ based on the past of length n.
One of the main problem in prediction theory is to describe the asymptotic behavior of the prediction error $\sigma_n^2(f)$ as $n \to \infty$.
Observe that $\sigma_{n+1}^2(f) \le \sigma_n^2(f)$, $n \in \mathbb{N}$, and hence the limit of $\sigma_n^2(f)$ as $n \to \infty$ exists. Denote by $\sigma^2(f) := \sigma_\infty^2(f)$ the prediction error of $X(0)$ by the entire infinite past: $\{X(t), t \le -1\}$.
From the prediction point of view it is natural to distinguish the class of processes for which we have *error-free prediction* by the entire infinite past, that is, $\sigma^2(f) = 0$. Such processes are called *deterministic* or *singular*. Processes for which $\sigma^2(f) > 0$ are called *nondeterministic* or *regular*.
Define the *relative prediction error* $\delta_n(f) := \sigma_n^2(f) - \sigma^2(f)$, and observe that $\delta_n(f)$ is non-negative and tends to zero as $n \to \infty$. But what about the speed of convergence of $\delta_n(f)$ to zero as $n \to \infty$? This speed depends on the regularity nature (regular or singular) of the observed process $X(t)$.
In this talk we discuss this question. Specifically, the prediction problem we are interested in is *to describe the rate of decrease of $\delta_n(f)$ to zero as $n \to \infty$*, depending on the regularity nature of the observed process $X(t)$.
It turns out that for nondeterministic processes the asymptotic behavior of the prediction error is determined by the dependence structure of the observed process $X(t)$ and the differential properties of its spectral density f, while for deterministic processes it is determined by the geometric properties of the spectrum of $X(t)$ and singularities of its spectral density f.
In this talk, we discuss the above problem both for deterministic and nondeterministic processes and survey some recent results. We first state some well-known important results for nondeterministic processes. Then, we focus on the less investigated case - deterministic processes. Here we state extensions of Rosenblatt's [6] and Davisson's [5] results concerning asymptotic behavior and upper bounds for the finite prediction error $\sigma_n^2(f)$, obtained in [1]–[4]. Examples illustrate the stated results.

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Alexander Gushchin: Old and new results on sets of joint distributions of certain pairs of stochastic processes

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Based on the joint work with Mikhail Nedoshivin (Lomonosov Moscow State University)
The aim of this talk is twofold. First, we recall known results describing the sets of joint distributions of certain pairs of stochastic processes, namely, a martingale and its maximum [5], and an increasing process and its compensator [2]. We also show how these results are connected with the Skorokhod embedding problem [**?**]. Second, we show how to adapt the Perkins construction [4], [1] in the Skorokhod embedding problem to the above problems and demonstrate some new results.

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Alexander Holevo: Logarithmic Sobolev inequalities and Quantum Gaussian Maximizers

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Based on the joint work with Sergey Filippov (Steklov Mathematical Institute, RAS, Moscow, Russia) Investigations in mathematical theory of quantum communication channels were rapidly developing during the past decades. As distinct from the classical case, quantum channel is characterized by a whole spectrum of different capacities, depending on the type of transmitted information (classical or quantum) and on additional resources used during transmission. Among these capacities, the *classical capacity*, i.e. the ultimate rate of reliable transmission of classical data via a quantum channel, plays a distinguished role, both historically and because of its central importance for quantum communications. A long-standing problem is the energy- constrained classical capacity of bosonic Gaussian channels of various kinds. Hypothesis of Gaussian Maximizers (HGM) states that the full capacity of such channels is attained on quantum Gaussian encodings. The solution of the conjecture even under additional "threshold condition" turned out to be notably difficult (see [1] for a background and survey of previous results). In the present paper we give a proof of HGM for two basic quantum measurement channels which do not satisfy the threshold condition. Remarkably, the proof generates and is based on the new versions of celebrated log-Sobolev inequality [2], [3].

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Rustam Ibragimov: New approaches to robust inference on market (non-)efficiency, volatility clustering and nonlinear dependence

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Many financial and economic variables, including financial returns, exhibit nonlinear dependence, heterogeneity and heavy-tailedness. These properties may make problematic the analysis of (non-) efficiency and volatility clustering in economic and financial markets using traditional approaches that appeal to asymptotic normality of sample autocorrelation functions of returns and their squares. This paper presents new approaches to deal with the above problems. We provide the results that motivate the use of measures of market (non-)efficiency and volatility clustering based on (small) powers of absolute returns and their signed versions. We further provide new approaches to robust inference on the measures in the case of general time series, including GARCH-type processes. The approaches are based on robust t-statistics tests based on the results by Bakirov and Szekely (Zap. Nauch. Sem. POMI, 2005; Journal of Mathematical Sciences, 2006) and new results on their applicability are presented. In the approaches, parameter estimates (e.g., estimates of measures of nonlinear dependence) are computed for groups of data, and the inference is based on t-statistics in the resulting group estimates. This results in valid robust inference under heterogeneity and dependence assumptions satisfied in real-world financial markets. Numerical results and empirical applications confirm the advantages and wide applicability of the proposed approaches.

Linda Khachatryan, Boris Nahapetian: Gibbs measure. Evolution of definition and Dobrushin's thesis

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Based on the joint work with Boris S. Nahapetian (Institute of Mathematics, NAS RA)

In this talk, we give an analytical review of the evolution of the concept of the Gibbs measure, starting from its definition in a finite volume to the definition of a Gibbs random field that does not use the notion of potential. We also touch upon the question of non-Gibbsian random fields and Dobrushin's program on their Gibbsianess (see, for example, [4]).

All known approaches to the definition of a Gibbs random field were based on the concept of the Hamiltonian as the sum of interaction potentials. From the physical point of view, such a definition is very natural since it allows one to construct models of statistical physics with given properties. Meanwhile, from the mathematical point of view, it is necessary to have a definition of the Gibbs random field in terms of its intrinsic properties without the notion of the potential. Such definition was given in the works [2, 3, 5], which made it possible to present the theory of Gibbs random fields in a purely probabilistic manner. Note that, along the way, two important problems were solved: Dobrushin's problem on the description of random fields by systems of consistent one-point conditional distributions (see [1]) and Ruelle's problem on the Gibbsian representation of specification (see [3]).

The presence of probabilistic definition allows investigating general questions for Gibbs random fields such as uniqueness, mixing properties and validity of limit theorems. In particular, it opens the possibility of extending Ildar Ibragimov's limit theorems for dependent random variables to the multidimensional case.

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Alexey Khartov: Quasi-infinitely divisible distributions

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The new class of *quasi-infinitely divisible probability laws* is a natural and rather wide extension of the class of infinitely divisible laws. According to the definition, a probabilistic law on the real line is called a quasi-infinitely divisible if its characteristic function admits the Lévy-Khinchin representation with a real shift parameter and with some non-monotonic (in general case) spectral function having bounded variation on the whole real line. Examples of such laws can be found in the well-known classical monographs by Gnedenko, Kolmogorov [1], and by Linnik, Ostrovsky [2]. However, the definition and the corresponding class were introduced only in 2011 by Lindner and Sato within some problems for stochastic processes. Further, these laws find some applications in the number theory, finance and physics. The first detailed analysis of the class of quasi-infinitely divisible laws was recently made by Lindner, Pan and Sato in [3]. So now this class is actually studied. In the talk, we review basic facts concerning this class and we also present new results for it, which were obtained in [4, 5, 6].

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Dmitry Korshunov: On statistical tests for distinguishing distribution tails

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The talk is devoted to the problem of testing hypothesis about the distribution tail in the framework of statistics of extremes. In many applications of extreme value theory, the distributions with asymptotically power tail (belonging to the Frechet maximum domain of attraction) are used for modelling real data, so the problem of model selection within this domain receives much attention in the literature. However, regular variation of distribution tails is not always appropriate as statistical hypothesis in problems that arise in practice. For example, very often Weibull type distributions (that belong to the Gumbel maximum domain of attraction) are more appropriate for modelling.
In the literature, the statistical methodology for distinguishing distribution tails was developed mostly for distributions from the Frechet maximum domain of attraction, while much less attention was paid to the distribution tails from the Gumbel maximum domain of attraction or for the tails of distributions not belonging to any maximum domain of attraction. To the best of our knowledge the most advanced results in this direction was obtained by I. Rodionov in a series of papers, for the last one see [1].
In this talk we discuss how to construct an asymptotically normal test for distinguishing two distributions with right unbounded supports, say F_0 and F_1 such that their tails are not so close to each other, namely $\overline{F}_0(x) = o(\overline{F}_1(x))$ as $x \to \infty$. We test the asymptotical behaviour of the distribution tail so we consider the null hypothesis $H_0 : \overline{F}(x) \sim \overline{F}_0(x)$ and the alternative $H_0 : \overline{F}(x) \sim \overline{F}_1(x)$. Let $R(x) := -\log \overline{F}(x)$ and $R_0(x) := -\log \overline{F}_0(x)$. Assume that

$$R(x) \quad = \quad R_0(x) + o(\psi(x)) \quad \text{as } x \to \infty,$$

for some known decreasing function $\psi(x) \downarrow 0$. Then the test is based on the statistics

$$\frac{1}{\sqrt{n-m_n}} \sum_{k=m_n+1}^{n} (R_0(X_{(k)}) - R_0(X_{(m_n)}) - 1),$$

where $X_{(k)}$ is the kth order statistics. The main difficulty we discuss is how to make a choice of the number $n - m_n$ of largest observations in terms of $\overline{F}_0(x)$ and $\psi(x)$.

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Vladimir Lotov: Exact formulas in some boundary crossing problems for integer-valued random walks

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Let $X_1, X_2, \ldots$ be i.i.d. integer-valued random variables,

$$S_n = X_1 + ... + X_n, \qquad n \geq 1, \qquad S_0 = 0.$$

We suppose that the distribution of X_1^+ belongs to the class $\mathbf{K}$ that contains distributions of bounded random variables, convolutions of geometric distributions and their mixtures.
For arbitrary $b \geq 1$, introduce

$$\tau = \tau(b) = \inf\{n \geq 1: \ S_n \geq b\}, \qquad \chi = \chi(b) = S_\tau - b.$$

Suppose that $\tau = \infty$ if $S_n < b$ for all n. In this case, the value of χ remains undefined.
The main goal is to find exact formulas:
(i) for $P(\chi(b) = k), \quad k \geq 1$;
(ii) for the renewal function $E\tau(b)$ if $EX_1 > 0$;
(iii) for $P(\sup S_n \geq b)$ if $EX_1 < 0$.
We start with the factorization representation for the double moment generating function $E(z^\tau \mu^\chi; \tau < \infty)$, analyze its structure and demonstrate the way to inverse it.
This work was supported by the Russian Science Foundation (grant no. 22-21-00396).

Alexey Naumov: On Dirichlet boundary crossing

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Based on the joint work with D. Belomestny (Duisburg-Essen University, P. Menard (Otto von Guericke University), E. Moulines (Ecole Polytechnique), S. Samsonov (HSE University), Y. Tang (Deepmind), D. Tiapkin (HSE University) and M. Valko (Deepmind)
In the talk we will provide upper and lower bounds on the deviation of a linear function of Dirichlet random vector (Dirichlet boundary crossing). The second result is devoted to a tight Gaussian lower bound for the distribution of a linear function of Dirichlet random vector. In particular, we extend the normal approximation-based lower bound for Beta distributions by [1]. These problems surprisingly arise in the estimation of regret for many modern reinforcement learning algorithms.

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Alexander Nazarov: Spectral analysis and L_2-small ball asymptotics of some FBM-like processes

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We study the spectral problems for some integro-differential equations arising in the theory of Gaussian processes similar to the fractional Brownian motion. We generalize the method of Chigansky–Kleptsyna [1] and obtain the two-term eigenvalues asymptotics for such equations. Application to the small ball probabilities in L_2-norm is given.

The talk is based on the papers [2], [3] and on some recent results in preparation. The research is supported by the RFBR grant 20-51-12004.

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Ernst Presman: An Inventory Model with Markov Chain Modulated Commodity Prices and long-run average costs

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The following inventory model was considered in [1]: A manufacturer (firm) uses for "production" a "commodity" (resource), which is consumed with unit intensity. The price of the commodity follows a stochastic process, modulated by a continuous time Markov chain with a finite number of states and known transition rates. The firm can buy this commodity at the current price or use "stored" one. The storage cost is proportional to the storage level. The goal of the firm is to minimize the total discounted performance costs. The existence of an optimal "threshold" strategy was proven. The threshold strategy is defined by a vector of levels specifying the minimal commodity amount to keep for a given price. An algorithm was presented to find an optimal "thresholds" and the corresponding value functions. It was shown that as a rule such strategy is unique and all situations were described when it is not unique. In typical optimization problems in continuous time involving Bellman (optimality) equation, a smooth pasting of the first derivatives of the value functions is used. A special feature of the considered model was that: the smooth pasting takes place for the second derivative of the value functions, it is convenient to conduct reasoning not with the price of the game itself, but with its derivative, and the Bellman equation itself is convenient to write out for the imbedded chain.

In this paper we consider the same model but with long-run average costs. To investigate this case, we aim the discount coefficient to zero. As a result, we obtain an analogue of the canonical triplet, well-known in discrete-time Markov chain control problems, which consists of optimal control, the optimal value of the long-run average cost functional and the invariant functional. We prove the existence of optimal threshold strategy and show that the algorithm for finding the optimal thresholds and values of the derivative of the invariant functional does not differ from the case of discounting, but to find the invariant functional itself, it is necessary to analyze the features when the stock level tends to infinity. As a rule, the optimal stationary strategy is again unique, but there may again be cases of non-uniqueness, while in the pre-limit case the optimal strategy is unique.

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Evgeny Prokopenko: Multi-Normex approach for evaluating the sum of heavy tailed random vectors

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Based on the joint work with Marie Kratz (ESSEC Business School, CREAR, France)
Looking for the most accurate possible evaluation of the distribution of the sum of random variables, or vectors, or processes, with unknown distributions, has always been a classical problem in the probabilistic and statistical literature, with various answers depending on the given framework and on the specific application in view. On one hand, (uni- or multivariate) Central Limit Theorems (CLT) or Functional ones prove, under finite variance for the sum components or/and additional conditions, the asymptotic Gaussian behavior of the sum with some rate of convergence, focusing on the 'body' of the distribution. When considering heavy-tailed marginal distributions, Generalized CLT with the convergence to stable distributions, handle the case of infinite variance (see e.g. [1], [2], and references therein), while, in the case of finite variance, an alternative way is to consider trimmed sums, removing extremes from the sample, to improve the rate of convergence; see e.g. [3], [4] and references therein.

When interested in tail distributions, CLTs may give poor results, especially when considering heavy tails. That is why different approaches have been developed, among which: (i) large deviation theorems; (ii) extreme value theorems (EVT) focusing on the tail only; (iii) hybrid distributions combining (asymptotic) distributions for both the main and extreme behaviors when considering independent random variables (see e.g. [5, 6, 7, 8]; we use the name given in [7] for this type of hybrid distribution/method/approach, namely *Normex* distribution/method/approach.
Recall briefly the idea of Normex (for 'Norm(al)-Ex(tremes)') method. It consists of rewriting the sum of random variables as the sum of their ordered statistics, and splitting it into two main parts, a trimmed sum removing the extremes, and the extremes. Using that the trimmed sum of the first $n-k-1$ ordered statistics is conditionnally independent of the k largest order statistics, given the $(n-k)$-th order statistics, we can express the distribution of the sum, integrating w.r.t. to the $(n-k)$-th order statistics and using a CLT for the conditional trimmed sum, and an EVT one for the k largest order statistics. Note that a benefit of Normex approach is that it does not require any condition on the existence of moments, as the CLT applies on truncated random variables.
It is natural to extend the normex approach to a multivariate framework. With this goal of proposing a multi-normex method and distribution, we consider iid random vectors $\mathbf{X}_1, \dots, \mathbf{X}_n$, with parent random vector $\mathbf{X}$ having a heavy-tailed d-dimensional distribution $F_{\mathbf{X}}$ and density $f_{\mathbf{X}}$ (when existing). Note that there are different ways to define multivariate extremes. The chosen way in this paper is w.r.t. the norm $\|\cdot\|$ in R^d, meaning that the ordered (w.r.t. the norm) vector of $(\mathbf{X}_1, \dots, \mathbf{X}_n)$, denoted by $(\mathbf{X}_{(1)}, \dots, \mathbf{X}_{(n)})$, satisfies

$$\|\mathbf{X}_{(1)}\| \le \|\mathbf{X}_{(2)}\| \le \cdots \le \|\mathbf{X}_{(n)}\|.$$

Aiming at proving the benefit of using a multi-normex distribution for a better fit of the whole (unknown) heavy-tailed distribution $F_{\mathbf{X}}$, assuming $F_{\mathbf{X}}$ heavy-tailed in the sense of $\|\mathbf{X}\| \in \mathcal{RV}_{-\alpha}$, we focus analytically on the case $\alpha \in (2;3]$ (when $\|\mathbf{X}\|$ has a finite second moment, but no third moment) to compare the rates of convergence when using the CLT and the multi-normex approach, respectively. Note our focus on heavy tailed distributions (i.e. distributions belonging to the max domain of attraction of Fréchet), where the impact of using Normex distribution will be much stronger than in the light tail case (because of the one big jump principle), in particular for risk analysis and management. We prove that the normex approach leads, as expected, to a better speed of convergence for evaluating the distribution of the sum than the CLT does, for such type of heavy-tailed distributions. When varying the fatness of the tail measured by $\alpha > 0$, we draw this comparison numerically, using geometrical multivariate quantiles.

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Alexander Sakhanenko: On first-passage times for generalized random walks

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The talk is based on the joint works with D. Denisov (University of Manchester, UK) and V. Wachtel (University of Bielefeld, Germany). The research was funded by RFBR and DFG according to the research project 20-51-12007.

Let $S(t)$ be a random process defined for all $t \geq 0$ with $S(0) = 0$ and let $g(t)$ be a non-random function on $[0, \infty)$. Introduce the random variable

$$\tau := \inf \{t > 0 : S(t) \leq g(t)\} = \inf \{t > 0 : S(t) - g(t) \leq 0\},$$

equal to the first moment of the top-down crossing of the level $g(t)$ by our process $S(t)$. We consider in the talk the asymptotic behavior of the upper tail $\mathbf{P}(\tau > T)$ as $T \to \infty$.

For several classes of processes we are going to obtain conditions under which we have asymptotical formulas of the following type:

$$\mathbf{P}(\tau > T) \sim \frac{U(T)}{\sqrt{T}} \quad \text{as} \quad T \to \infty, \tag{1}$$

for some slowly varying functions $U(T)$.
Two such classes of processes may be found in papers [1] and [2]. In these cases

$$U(T) := \sqrt{\frac{2}{\pi}} \mathbf{E}\big[S(\alpha_T) - g(\alpha_T)\,;\, \tau > \alpha_T\big] \tag{2}$$

for specially chosen stopping times α_T such that $\alpha_T/T \to 0$ in probability as $T \to \infty$.

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Stanislav Shaposhnikov: The superposition principle for Fokker–Planck–Kolmogorov equations

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We discuss the superposition principle for probability solutions to the Cauchy problem for the Fokker–Planck–Kolmogorov equation, according to which such a solution μ_t with initial distribution ν is represented by a probability measure P_ν on the path space such that P_ν solves the corresponding martingale problem and μ_t is the one-dimensional distribution of P_ν at time t. We present new sufficient conditions for the superposition principle to be fulfilled in the case of unbounded coefficients. Moreover the superposition principle for equations on a domain is considered.

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Irina Shevtsova: Comparison of central and noncentral Lyapounov fractions with application to Berry–Esseen bounds for Poisson random sums

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Based on the joint work with Vladimir Makarenko (Moscow State University)
For a nondegenerate r.v. X with finite $\mathsf{E}|X|^3$ let us denote

$$L_1(X) = \frac{\mathsf{E}|X|^3}{(\mathsf{E}X^2)^{3/2}}, \quad L_0(X) = L_1(X - \mathsf{E}X) = \frac{\mathsf{E}|X-\mathsf{E}X|^3}{(\mathsf{D}X)^{3/2}}.$$

We find and explicit expression for the least upper bound

$$H(t) := \sup_{\mathsf{E}X = t\sqrt{\mathsf{E}X^2}} \frac{L_1(X)}{L_0(X)}$$

for every $t \in (-1,1)$, as well as its unconditional supremum

$$\sup_{-1<t<1} H(t) = \frac{1}{4}\sqrt{17+7\sqrt{7}} = 1.48997\ldots$$

The numerical result $\sup_{-1<t<1} H(t) < 1.49$ was previously obtained in [2] with the help of a computer. Furthermore, we apply these results to the Berry–Esseen-type inequality for Poisson random sums [1, 3]

$$\Delta_\lambda := \sup_{x\in\mathbf{R}} \left| \mathsf{P}\left(\frac{S_\lambda - \mathsf{E}S_\lambda}{\sqrt{\mathsf{D}S_\lambda}} < x\right) - \Phi(x)\right| \le 0.3031\frac{L_1(X)}{\sqrt{\lambda}}, \quad \lambda > 0,$$

where $S_\lambda = X_1 + \ldots + X_{N_\lambda}$, N_λ being a Poisson random variable with expectation λ, independent of i.i.d. r.v.'s $\{X_k\}_{k\ge1}$, to deduce its analogue involving the non-central Lyapounov fraction

$$\Delta_\lambda \le 0.3031 \cdot H(t)\frac{L_0(X)}{\sqrt{\lambda}} \le 0.4517\frac{L_0(X)}{\sqrt{\lambda}}, \quad \lambda > 0,$$

for every distribution of X_1 with fixed $\mathsf{E}X_1/\sqrt{\mathsf{E}X_1^2} = t \in (-1,1)$. A similar upper bound, uniform with respect to t, was previously obtained in [2].

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Natalia Stepanova: Sparse signal recovery with Subbotin noise

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Based on the joint work with Joshua Miller (Carleton University)
We study the problem of high-dimensional variable selection in a sparse sequence model with Subbotin noise. We derive the regions of exact and almost full variable selection with respect to a Hamming loss function, and propose adaptive procedures that achieve both types of selection. The proposed procedures are shown to be optimal in the asymptotically minimax sense. Our results augment previous work in this area.

Alexander Tikhomirov: Limit theorems for spectrum of random graphs

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We consider the limit of empirical spectral distribution of Laplace matrix and adjacency matrix of generalized random graphs. Applying Stieltjes transform method we proof under general conditions that limit spectral distribution of adjacency matrix is Wigner law and empirical spectral distribution of Laplace matrices convergence to the free convolution of Wigner law and normal law.

Vladimir Ulyanov: Inference via randomized test statistics

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Based on the joint work with Nikita Puchkin (HSE University and IITP RAS)
We show that external randomization may enforce the convergence of test statistics to their limiting distributions in particular cases. This results in a sharper inference. Our approach is based on a central limit theorem for weighted sums. We apply our method to a family of rank-based test statistics and a family of phi-divergence test statistics (see e.g.[1]) and prove that, with high probability with respect to the external randomization, the randomized statistics converge at the rate $O(1/n)$ (up to some logarithmic factors) to the limiting chi-square distribution in Kolmogorov metric. See the details in [2].

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Vladimir Vatutin: Critical branching processes in random environment with immigration: life of a single family

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Based on the joint works [1] and [2] with Smadi Charline (INRAE, Grenoble, France)
We consider a critical branching process in an i.i.d. random environment, in which one immigrant arrives at each generation. We are interested in the event $\mathcal{A}_i(n)$ that all individuals alive at time n are offspring of the immigrant which joined the population at time i n and on the population size of the population at time n given the event $\mathcal{A}_i(n)$ occurs. We consider the cases when n is large and i follows different asymptotics which may be related to n (i fixed, close to n, or going to infinity but far from n). In order to do so, we establish some limit theorems for random walks conditioned to be positive or nonnegative, which are of independent interest.

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Anatoly Vershik: Uniqueness of invariant Markov numerations and tilings of posets

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Partially based on the joint work with Fedya Petrov
We introduce a new class of probabilistic-combinatorial problems: to describe a Markov chain which monotonically numerates the elements of a countable poset, or, for a continual poset, monotonically tiles it by segments. We look for the "most random" way, i.e., the way with the maximal entropy. Such problems have very different applications, from Young diagrams statistics to the spectra of random matrices, etc. In most cases the answer is unique up to intensity of numeration or tiling, and the obtained Markov chains are related to Bernoulli schemes in a simple but non-trivial manner. Even the simplest examples which we discuss give interesting and unusual answers, and the explicit computation of transition probabilities is quite difficult.

Elena Yarovaya: Martingale method for studying branching random walks

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Based on the joint work with Natalia Smorodina (St. Petersburg Department of Steklov Mathematical Institute of Russian Academy of Sciences)
We consider a continuous-time branching random walk on a multidimensional lattice $\mathbb{Z}^d$, $d \in \mathbb{N}$, in which particles can die and produce offspring at any point on the lattice, see, for example, [1]. Let the transport of particles on $\mathbb{Z}^d$ be given by a symmetric, homogeneous and irreducible random walk. A branching intensity at a point $x \in \mathbb{Z}^d$ tends to zero as $\|x\| \to \infty$. Moreover, an additional condition is satisfied for the parameters of the branching random walk, which guarantees exponential growth in time of the average number of particles at each point $x \in \mathbb{Z}^d$. Under these assumptions, we prove the limit theorem about the mean square convergence of the normalized number of particles at an arbitrary fixed point $x \in \mathbb{Z}^d$, as $t \to \infty$. The proof is based on the approximation of the normalized number of particles by a nonnegative martingale [2].
The research was supported in part by the Russian Foundation for Basic Research (RFBR), Project No. 20-01-00487.

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