Conference "SIMC Youth Race"

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НАУКА
И УНИВЕРСИТЕТЫ
НАЦИОНАЛЬНЫЕ
ПРОЕКТЫ
РОССИИ

SIMC
Steklov International Mathematical Center

МИАН

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Abstracts of talks

Contents

1 Algebra, geometry, and topology

Georgy Chernykh. SU-linear operations and c_1-spherical bordism theory

The c_1-spherical bordism theory W^* is an intermediate theory between complex and SU-bordisms and it plays an important role in calculations of the SU-bordism coefficient ring. There is no natural choice of multiplication on the theory W^* (since a direct product of two c_1-spherical manifolds doesn't have to be c_1-spherical). However, it turns out that the theory W^* is a direct summand in complex cobordism theory MU^*, and one can define a multiplication on W^* via projections. I will talk about SU-linear projections and multiplications on the theory W^*, as well as complex orientations of this theory, corresponding formal group laws and its Landweber exactness.

Mikhail Kornev. Dynamics and Multivalued Groups

In 1971, S. P. Novikov and V. M. Buchstaber gave the construction, predicted by characteristic classes. This construction describes a multiplication, with a product of any pair of elements being a non-ordered multiset of n points. It led to the notion of n-valued groups. Soon after that, V. M. Buchstaber gave the axiomatic definition of n-valued groups, obtained the first results on their algebraic structure, and began to develop the theory. At present, a number of authors are developing n-valued (finite, discrete, topological or algebra geometric) group theory together with applications in various areas of Mathematics and Mathematical Physics. In this talk, we will give some key notions of n-valued group theory and discuss the author's recently obtained results, which are related to multivalued discrete dynamical systems. We will describe some connections with famous results on symbolic dynamics, combinatorics on words and constructions of quasicrystals.

Vasilii Rozhdestvenskii. On a lower bound in the problem of realization of cycles

One of the classical problems in topology is the Steenrod problem on realization of cycles: if $x \in H_n(X;\mathbb{Z})$ is an integral n-dimensional homology class of a topological space X, do there exist an oriented smooth n-dimensional manifold M and a map $f\colon M \to X$ such that x is the image of the generator of $H_n(M;\mathbb{Z})$. In 1954, R. Thom proved that an arbitrary n-dimensional integral homology class can be realized in the above sense after multiplication by some positive integer, which can be chosen uniformly for all n-dimensional classes. Let us denote by $k(n)$ the minimal of such positive integers. In my talk, I will give an overview of classical results about the Steenrod problem and tell my own results about boundaries for the number $k(n)$.

Egor Teplyakov. Topological methods in condensed matter physics

Condensed matter physics is a modern area of mathematical physics where one may see the active interplay between math and physics. Modern mathematical methods are applied for describing phenomena in this branch of physics. In this talk we will consider the main ideas of topological approach for investigation of condensed matter systems.

Wenhao Wang. Orders on Free Metabelian Groups

A bi-order on a group G is a total, bi-multiplication invariant order. Such an order is regular if the positive cone associated to the order can be recognised by a regular language. A subset S in an orderable group $(G, \leqslant)$ is convex if for all $f \leqslant g$ in S, every element $h \in G$ satisfying

$f \leqslant h \leqslant g$ belongs to S. In this talk, I will discuss the convex hull of the derived subgroup of a free metabelian group with respect to a bi-order. As an application, I prove that non-abelian free metabelian groups of finite rank do not admit a regular bi-order while they are computably bi-orderable.

Indira Zeinikesheva. Equivariant formality of moment-angle complexes

We compute the equivariant cohomology $H^*_{T_I}(\mathcal{Z}_\mathcal{K})$ of moment-angle complexes with respect to the action of coordinate subtori $T_I \subset T^m$. We give a criterion for the equivariant formality of $\mathcal{Z}_\mathcal{K}$.

2 Algebraic geometry

Aleksei Golota. Finite groups acting on compact complex parallelizable manifolds

A complex manifold X is called parallelizable if its holomorphic tangent bundle is trivial. A classical theorem of H.-C. Wang says that a compact complex parallelizable manifold is isomorphic to a quotient of a connected complex Lie group by a discrete cocompact subgroup. In my talk I will discuss automorphism groups of compact complex parallelizable manifolds from the viewpoint of their finite subgroups. In particular, I will show that these finite subgroups are "almost abelian" (Jordan property for $\mathrm{Aut}(X)$) and examine some related questions from group theory.

Alexandra Kuznetsova. Automorphisms of families of abelian varieties

Complex abelian varieties and their automorphism groups are classical objects; we have a lot of information about their structure. However, if instead of an abelian variety one considers a (maybe degenerate) family of abelian varieties over a projective base and its fiberwise birational automorphism then the situation is much more vague. In particular, it is even not clear whether the birational automorphism of a family of abelian varieties is regularizable or not. I am going to speak about the criterion for regularizability of automorphisms of families which are indecomposable on a general fiber; i.e. such that all iterates of the automorphism do not preserve a decomposition of a general fiber into a product of two abelian varieties. The talk is based on a joint work in progress with C. Favre.

Konstantin Loginov. Regularity of Fano threefolds

A natural way to study Fano varieties is by looking at its (pluri-)anti-canonical divisors. Regularity measures how singular such divisors could be. We explain how to compute regularity of smooth Fano varieties of dimension 3.

Mikhail Ovcharenko. Dolgachev–Nikulin duality for fibers of toric Landau–Ginzburg models of smooth Fano 3-folds (joint work with A. Harder)

Mirror Symmetry corresponds to Fano varieties certain one-dimensional families which are called *Landau–Ginzburg models*. Elements of these families are Calabi–Yau varieties mirror dual to anticanonical sections of Fano varieties.

In particular, in the three-dimensional case we deal with Mirror Symmetry of K3 surfaces. One of its most interesting forms is so called *Dolgachev–Nikulin duality*: it interchanges the lattices of algebraic and transcendental cycles on a K3 surface.

Theory of *toric Landau–Ginzburg models* provides an effective method of constructing Landau–Ginzburg models of Fano varieties. It is natural to expect that Dolgachev–Nikulin duality holds for fibers of toric Landau–Ginzburg models of smooth Fano threefolds.

This conjecture was proved by Ilten–Lewis–Przyjalkowski in the case of Picard number 1. We establish a certain form of Dolgachev–Nikulin duality for all other smooth Fano threefold, and discuss possible generalisations.

Dmitrii Pirozhkov. Towards homological projective duality for $\mathrm{Gr}(2, 2n)$

Given a variety X over the projective space $\mathbb{P}(V)$ and a semiorthogonal decomposition of the derived category of X which is Lefschetz, i.e., compatible in a certain way with the twist by $\mathcal{O}(1)$,

homological projective duality is a way to construct a triangulated category, now over the dual projective space $\mathbb{P}(V^\vee)$, also with a Lefschetz decomposition, that is in many aspects similar to the derived category of X and enjoys many useful properties. This "dual" category can be constructed in a formal way, but the relations with the derived category of X become much more interesting if the dual category is also described geometrically, for example in terms of some variety over the dual projective space. I will talk about basic notions of homological projective duality and I'll give a conjectural description of the dual category to $\mathrm{Gr}(2, 2n)$ in its Plücker embedding, motivated by the description Kuznetsov gave in 2005 for the case $n = 3$. This is a work in progress.

Andrey Soldatenkov. The mapping class group action on Teichmüller spaces of hyperkähler manifolds

The central result in the moduli theory of compact hyperkähler manifolds is Verbitsky's global Torelli theorem. It describes the Teichmüller space parametrizing complex structures on such manifolds as a non-Hausdorff covering of a certain homogeneous space called the period domain. The Teichmüller space carries a natural action of the mapping class group, i.e. the group of diffeomorphisms modulo diffeomorphisms isotopic to the identity. One can use Ratner's theory to describe the closures of the orbits of this action. In my talk I will try to give a general introduction into this circle of ideas and then explain how one can apply them. I will present some new results from my joint work with Sibony and Verbitsky concerning rigid currents on hyperkähler manifolds.

Anastasia Vikulova. Birational automorphism groups of Severi–Brauer surfaces over the field of rational numbers

In this talk we will discuss finite subgroups in the group of birational automorphisms of Severi–Brauer surfaces. We will prove that the only non-trivial finite subgroups of birational automorphisms group of a non-trivial Severi–Brauer surfaces over the field of rational numbers are $\mathbb{Z}/3\mathbb{Z}$ and $(\mathbb{Z}/3\mathbb{Z})^2$. Also we will discuss 3-subgroups in the birational automorphisms group for a Severi–Brauer surface over any field of characteristic zero.

Bin Wang. Higgs Bundles with Nilpotent Structures

We will talk various Hitchin systems with nilpotent structures at marked points. We will start with a relation between singularities of spectral curves and Kazhdan–Lusztig maps for nilpotent orbits. Then we will show a relation between duality of generic fibers and Springer correspondence for nilpotent orbits for Langlands dual groups Sp_{2n} and SO_{2n+1}.

3 Analysis, differential equations, number theory, and applications

Andrey Dukov. Hyperbolyc polycycles and appearing multiple limit cycles

Consider a 2-dimensional oriented manifold M and a smooth vector field v_0 on M.

Definition 1. A directed graph γ imbedded to M is called a *hyperbolic polycycle* of a vector field if it satisfies the following properties:

- its vertices are hyperbolic saddles of the vector field;
- its edges are separatrix connections; the time determines the direction;
- the graph γ is Eulerian (there exists a path that visits each edge exactly once).

If a polycycle has a monodromy map from a transversal section to itself then the polycycle called *monodromic*.

Let γ be a hyperbolic polycycle formed by n separatrix connections. After a small perturbation of the polycycle γ some limit cycles appear. The *characteristic number* of a saddle is the modulo of the ratio of its eigenvalues, the negative one is in the numerator. A limit cycle is *of multiplicity* m if after any its generic perturbation it splits into not greater than m hyperbolic limit cycles.

The main result is formulated in the following two theorems.

Theorem 1. *For any positive integer n there exists a non-trivial polynomial $\mathcal{L}_n(\lambda_1, \dots, \lambda_n)$ such that the following statement holds. Let v_0 be a vector field with a hyperbolic polycycle γ and the characteristic numbers $\lambda_1, \dots, \lambda_n$ of the saddles satisfy the inequation*

$$\mathcal{L}_n(\lambda_1, \dots, \lambda_n) \neq 0. \tag{1}$$

Then for any C^∞-smooth finite-parameter family the multiplicity of any appearing limit cycle is not greater than n.

The following theorem is opposite to the previous one.

Theorem 2. *Let γ be a monodromic hyperbolyc polycycle formed by n saddles and n separatrix connections. Denote by $\lambda_1, \dots, \lambda_n$ the characteristic numbers of the saddles, suppose $\lambda_1 \dots \lambda_n = 1$. Let V be a generic C^∞-family perturbing the polycycle γ. Then $n+1$-multiple limit cycle appears ($n+1$ limit cycles appear) in the family V.*

These two theorems connect to a polynomial system that discribes the behavior of the perturbed polycycle. Hence, their proofs use the theory of the commutative algebra.

Mikhail Garbuz. On the motion of a spinning heavy plate in an ideal fluid

We consider the problem of free fall of a heavy solid plate in an infinite volume of an ideal fluid. At the initial moment the plate is horizontal, it has a vertical rotation, and its center of mass has

the horizontal velocity. Under the influence of gravity, the plate goes down and rotates around a lateral axis perpendicular to the directions of the initial velocity and initial rotation. At the same time, a gyroscopic effect motivate rotation of the plate around the velocity of the center of mass, and as a result a lateral movement of a plate.

Alexander Kalmynin. Distribution of values of quadratic forms

Study of values of integral quadratic forms is a classical area in number theory, dating back to Fermat, Euler, Gauss and Lagrange. Quantitative questions on distribution of values of quadratic forms arise naturally in many different contexts, including, for instance, the distribution of primes in arithmetic progressions. In this talk, we are going to discuss some classical and modern results on distribution of values of quadratic forms in long and short intervals, connections of this area with L-functions, character sums and modular forms.

Irina Limonova. Dense weakly lacunary subsystems of orthogonal systems

Let $\Phi = \{\varphi_i\}_{i=1}^N$ be an orthogonal system of functions defined on a probability space (X, μ). Let $p > 2$. A deep result by J. Bourgain states: Under the additional assumption $\|\varphi_i\|_\infty \le M$, $1 \le i \le N$, we can choose a subsystem $\{\varphi_j\}_{j\in\Lambda}$ in Φ with $|\Lambda| \ge N^{2/p}$ such that $\|\sum_{k\in\Lambda} a_k\varphi_k\|_p \le C(M,p)\|\sum_{k\in\Lambda} a_k\varphi_k\|_2$. We establish analogs of this theorem for the class of Orlicz spaces that are close to L_2. As a consequence we obtain the existence of a large subsystem of Φ with the norm of the maximal partial sum operator being estimated better than the classical Menshov–Rademacher theorem guarantees for general systems.

Elijah Lopatin. On the scalar approach to the weak asymptotics problem for $\mathcal{GN}$-systems

The talk will be devoted to recent achievements in the problem of extending the new approach to describe the weak asymptotics of Hermite–Padé polynomials to a wider class of $\mathcal{GN}$-systems. This approach was suggested by S. Suetin in 2018 and is based on the use of the scalar equilibrium problem on a compact Riemann surface with an external harmonic field. I will also discuss equivalence of the corresponding method with the classical vector potential technique.

Maria Stepanova. What is CR geometry?

CR geometry is a branch of the modern several complex variables theory, which lies at the intersection of differential geometry, the theory of Lie groups and algebras, partial differential equations and many other areas. One of the main objects of study here are real submanifolds in complex space. In the talk we are going to give an outline of the topic and to discuss some interesting ideas, methods and also recent results.

4 Mathematical methods for quantum technologies and probability theory

Gavriil Bakai. Large Deviations for Galton–Watson branching processes with immigration

Let $\{Z_n\}_{n\geq 0}$ be a Galton–Watson branching process with immigration in one particle. By definition, put

$$Z_0 = 0, \quad Z_n = \sum_{j=1}^{1+Z_{n-1}} X_{n,j}, \quad n \in \mathbb{N}.$$

Here random variables $X_{i,j}$ are independent identically distributed taking non-negative integer values. Put

$$S_0 = 0, \quad S_n = \sum_{i=1}^{n} Z_i.$$

We obtain the exact asymptotics of large deviations probabilities for S_n in the local form. In the subcritical case ($\mathbf{E}X_{1,1} < 1$) under small additional restrictions we obtain the local central limit theorem.

Anton Kozubov. Quantum control attack: towards joint estimation of protocol and hardware loopholes

In this paper we present the approach for description of quantum control attack based on combined protocol and hardware loopholes. It consolidates intercept-resend attack and detection node control (detector blinding attack). In the basic version of B92 protocol detection control is not that crucial; however, when one scales the number of states the state imposing plays a significant role. Protocols that operate with arbitrary even symmetric linearly independent nonorthogonal (e.g., coherent) states are of interest. The cornerstone of the considered approach is that we combine both state discrimination by eavesdropper and different methods of state imposing. In principle, detection control allows one to exclude any bit correlations between legitimate users, which are unknown to Eve, and can be considered as the necessary part of most intercept-resend attacks, including a faked- state attack impossible without a hardware loophole. Moreover, the issue related to unified quantum description of the intercept-resend attack was solved by combining the concepts of von Neumann's measurement scheme and ambiguity of square root extraction for operators. We also present a generalized countermeasure based on additional parameter estimation analysis. As an example, with some numerical estimations we investigate the attack on quantum key distribution systems based on utilization of

Vadim Petruhanov. Optimal control of open quantum systems

Quantum control is an essential part of modern quantum technologies that studies how to optimally manipulate closed and open quantum systems. One of the promising directions in the quantum control is control of open quantum systems using the environment as incoherent control. In this talk we give an introduction to the topic discussing some known results as well as our new results about numerical optimization for control of open quantum systems.

Vsevolod Yashin. Integrable Floquet systems related to logarithmic conformal field theory

In a certain a Temperley–Lieb algebra, connected with logarithmic conformal field theories, we find a simple Lie algebra structure for the elements which are invariant under shift by two lattice sites. This enables us to analytically describe a Floquet-integrable system, to find its conserved local charges and Floquet Hamiltonian.

5 Theoretical physics

Aleksandr Belokon. Operator local quenches in massive scalar field theory

In this talk, we introduce the notion of quantum quenches in relativistic quantum field theory (QFT). Generically, quantum quenches correspond to a sudden change in a quantum system, which brings it out of equilibrium. We study the particular type of quantum quenches, which is described by the operator local quench protocol, where a quantum state becomes excited by a local insertion of a field operator at some spacetime point. These out-of-equilibrium processes are well-studied in a two-dimensional QFT, which has an underlying conformal symmetry — conformal field theory (CFT). We give a short but comprehensive overview of the dynamics of excited states in this theory, and then extend the consideration to an arbitrary number of spacetime dimensions, as well as to the simplest non-conformal case — the massive scalar field theory. We also generalize the CFT results to the two-dimensional massive scalar field theory with the compact spatial dimension. By probing different correlation functions, corresponding to various physical observables, we reveal dynamical features of the theory, which cannot be observed in the equilibrium regime, and compare the results with those in $d = 2$ CFT.

Ali Hajilou. Holographic Anisotropic Model for Heavy Quarks in Anisotropic Hot Dense QGP: Magnetic Catalysis

We present a five-dimensional anisotropic holographic model for heavy quarks supported by Einstein-Maxwell-dilaton action with extended warp-factor. We investigate the influence of the external magnetic field on the 5-dim black hole solution and the confinement/deconfinement phase diagram. Our model can describe the phenomenon of Magnetic Catalysis (MC), i.e. direct dependence of transition temperature to the external magnetic field.

Maria Matushko. Anisotropic spin generalization of elliptic Ruijsenaars–Macdonald operators

Using the elliptic Baxter–Belavin R-matrix we construct commuting set of matrix-valued difference operators, which can be considered as anisotropic versions of the quantum spin Ruijsenaars–Macdonald Hamiltonians. We prove that the commutativity of spin operators is equivalent to a set of nontrivial R-matrix identities.

Vasilii Pushkarev. Non-equilibrium dynamics after quenches in quantum field theories

Quantum field theories can be taken out of equilibrium by an abrupt excitation, which is called quantum quench. A particular mechanism of how the excitation is introduced may be different. One type of quenches assumes global modification of a quantum system by the change of field parameters. Probing the evolution of perturbations after the quench reveals dynamical features of the system which cannot be observed in the equilibrium regime. We study examples of global quenches in the exotic fracton quantum field theory, as well as elaborate on the dynamics in the ordinary relativistic quantum field theory but with finite-size background geometry.

Timofei Rusalev. Entanglement entropy of radiation in spherically symmetric black holes

It will be discussed in the talk about the calculation of the entanglement entropy of Hawking radiation for the cases of spherically symmetric four-dimensional eternal Schwarzschild and

Reissner–Nordström black holes and its relationship with the information paradox in black hole physics. A verification of the basic properties of the entanglement entropy of the pure state of the entire composite system is given, and it is shown that, generally speaking, these properties are not satisfied. The island formula is used to study the time dependence of the entanglement entropy for various types of finite regions in which radiation is collected. Depending on the size of finite regions, a qualitatively new behavior arises, such as discontinuous evolution of the entanglement entropy, the absence of island formation, the "Cauchy surface breaking" and asymmetric island solutions.

Marina Usova. Holographic RG flows of 3d $\mathcal{N} = 2$ supergravity model

We study holographic RG flows in a 3d supergravity model with a scalar field (dilaton) and its potential in terms of dynamical system theory. The gravity equations of motion are reduced to an autonomous dynamical system. Its equilibrium points are analyzed for stability and asymptotic solutions are restored therein. We show that some solutions can be interpreted as fixed points of a dual field theory and that there are RG flows between an unstable UV fixed point and a stable IR fixed point.