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ABSTRACTS

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On the damping problem for a multidimensional control system of retarded type

Adkhamova A. Sh.★

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We consider a multidimensional control system governed by retarded-type differential-difference equations with variable matrix coefficients and several delays. A connection is established between the variational problem for a nonlocal functional and the corresponding boundary value problem on an interval for a system of second-order differential-difference equations. The existence, uniqueness, and smoothness of the generalized solution to the boundary value problem on the entire interval are studied.

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Venttsel problems with discontinuous data

Apushkinskaya D. E.★

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In this talk we present new results concerning the Venttsel boundary value problems for second-order elliptic equations. The novelty is related to discontinuity of the principal coefficients of the differential operators acting inside and on the boundary of the underlying domain. Namely, the principal coefficients belong to the class of functions with *vanishing mean oscillation (VMO).* Also the optimal Lebesgue/Orlicz integrability requirements are imposed on the lower-order coefficients.

We consider *strong solutions* belonging to composite Sobolev spaces with optimal exponent ranges and discuss L^p-regularity and solvability theory for such problems.

The talk is based on results obtained in [1–4] in collaboration with Alexander Nazarov, Dian Palagachev, and Lubomira Softova.

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Stationary solutions of the Vlasov–Poisson system in domains with boundary

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The Vlasov–Poisson equations in domains with a boundary describe the kinetics of charged particles of high-temperature plasma in controlled thermonuclear fusion devices. Mixed problems for the Vlasov–Poisson system for a two-component plasma in cylindrical domains describe in particular the kinetics of charged particles of high-temperature plasma in a mirror trap.

We consider the first mixed problem for the Vlasov–Poisson system in the following two cases: an infinite cylinder and a finite cylinder.

In [1], the case of an infinite cylinder with a given homogeneous external magnetic field is investigated. The problem is reduced to a cross-section of the cylinder (an auxiliary problem) by a special substitution. Further, using the method of characteristics for the Vlasov equation and the method of sub- and super-solutions for nonlinear elliptic boundary value problems, we prove that the auxiliary problem has smooth stationary solutions. Distribution functions of constructed solutions are compactly supported. Returning to the original problem by using substitutions of a special kind, we obtain a new class of stationary solutions of the first mixed problem for the Vlasov–Poisson system with a nonzero electric field potential. Supports of distribution functions of constructed solutions lie at a distance from the boundary of the cylinder.

For the case of a finite cylinder [1], an inhomogeneous external magnetic field of a special configuration is considered. The stationary solutions are constructed with supports of distribution functions touching the boundary of the domain only in two small prescribed discs at the top and the bottom of the cylinder.

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Non-concentration for the acoustic operator in layered media

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Consider the operator $A = -\tilde{c}\Delta$ acting in bounded domains $\Omega := \Omega' \times (0, H) \subseteq \mathbb{R}^d \times \mathbb{R}_+$. The diffusion coefficient $\tilde{c} > 0$ depends on one coordinate $y \in (0, H)$ and is bounded but may be discontinuous. This corresponds to the physical model of "layered media" appearing in acoustics, elasticity, optical fibers etc. The Dirichlet boundary conditions are assumed. In general, for each $\varepsilon > 0$, the set of eigenfunctions is divided into a disjoint union of three subsets : $\mathfrak{F}_{NG}$ (non-guided), $\mathfrak{F}_G$ (guided), and $\mathfrak{F}_{res}$ (residual). The residual set shrinks as $\varepsilon \to 0$. The customary physical terminology of *guided/non-guided* is often replaced in the mathematical literature by *concentrating/non-concentrating solutions*, respectively.

For guided waves, the assumption of "layered media" enables us to obtain rigorous estimates of their exponential decay away from concentration zones. The case of non-guided waves has attracted less attention in the literature. It leads to some very interesting questions concerning oscillatory solutions and their asymptotic properties. Classical asymptotic methods are available for $c(y) \in C^2$ but a lesser degree of regularity excludes such methods. The associated eigenfunctions (in $\mathfrak{F}_{NG}$) are oscillatory. However, this fact by itself does not exclude the possibility of "flattening out" of the solution between two consecutive zeros, leading to concentration in the complementary segment. Non-concentration is established if $c(y)$ is of bounded variation, by proving a *"minimal amplitude hypothesis"*. However, the validity of such results when $c(y)$ is not of bounded variation (even if it is continuous) remains an open problem.

Based on a joint work with A. Benabdallah and Y. Dermenjian.

Local regular and singular solutions of the Chipot–Weissler equation

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Here we study the properties of nonnegative solutions of equations in a domain of $\mathbb{R}^N$, of the type

$$-\Delta u - m|\nabla u|^q = u^p, \tag{1}$$

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where $p > 1$ and $m > 0$. We concentrate our analysis on the solutions with an isolated singularity, or in an exterior domain, or in $\mathbb{R}^N$. The existence of such solutions and their behaviour depend strongly on the values of p and q, in particular on the sign of $q - \frac{2p}{p+1}$, and when $q = \frac{2p}{p+1}$, also of the value of the parameter m, which becomes a key element. The description of the different behaviour is made possible by a sharp analysis of the radial solutions of the equation.

Nonlinear kinetic equations of the Boltzmann type and their discrete models

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The known nonlinear kinetic equations (in particular, the wave kinetic equation and the quantum Nordheim–Uehling–Uhlenbeck equations) are considered as a natural generalization of the classical spatially homogeneous Boltzmann equation. To this goal we introduce the general Boltzmann-type kinetic equation that depends on a function of four real variables $F(x, y; v, w)$. The function F is assumed to satisfy certain commutation relations. The general properties of this equation are studied. It is shown that the above mentioned kinetic equations correspond to different forms of the function (polynomial) F. Then the problem of discretization of the general Boltzmann-type kinetic equation is considered on the basis of ideas similar to those used for construction of discrete models of the Boltzmann equation. The main attention is paid to discrete models of the wave kinetic equation. It is shown that such models have a monotone functional similar to the Boltzmann H-function. The existence and uniqueness theorem for global in time solution of the Cauchy problem for these models is proved. Moreover, it is proved that the solution converges to the equilibrium solution as time goes to infinity. The problem of approximation of the wave kinetic equation by its discrete models is also discussed. This work is supported by the Ministry of Science and Higher Education of the Russian Federation (Megagrant, agreement No. 075-15-2022-1115).

Variational worn stones

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We introduce an evolution model à la Firey for a convex stone which tumbles on a beach and undertakes an erosion process depending on some variational energy, such as torsional rigidity, principal Dirichlet Laplacian eigenvalue, or Newtonian capacity. Relying on the assumption of existence of a solution to the corresponding parabolic

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flow, we prove that the stone tends to become asymptotically spherical. Indeed, we identify an ultimate shape of these flows with a smooth convex body whose ground state satisfies an additional boundary condition, and we prove symmetry results for the corresponding overdetermined elliptic problems. Moreover, we extend the analysis to arbitrary convex bodies: we introduce new notions of cone variational measures and we prove that, if such a measure is absolutely continuous with constant density, then the underlying body is a ball.

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Invariants of normal forms of second-order linear partial differential equations in the plane and applications

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The talk is devoted to the theory of normal forms of mixed-type partial differential equation in the plane and problems related.

First normal forms of the wave equation and the Laplace equation were proposed more then 200 years ago to describe the motion of the string and the velocity potential of an incompressible fluid, respectively. The main symbol of the equation $a(x,y)u_{xx} + b(x,y)u_{xy} + c(x,y)u_{yy} = 0$ with smooth coefficients a, b, c is reduced to these forms in a vicinity of any point where the *discriminant* D, $D = b^2 - 4ac$, is positive or negative, respectively, by a smooth change of coordinates and multiplication by an appropriate smooth nonvanishing function.

For a generic triplet (a, b, c), the level $D = 0$ is either empty or is a smoothly embedded curve (known as the *type change line*) in the plane. Near a point of this line, the equation is of *mixed type*. Systematic analysis of mixed-type equations was started in [1], where the ideology and motivation for many studies in the 20-th century were proposed. In this work, Tricomi also grounded the normal form $u_{yy} + yu_{xx}$ for the main symbol of a generic equation near a typical point of the type change line, but his proof had a gap. The correct proof was proposed in [2]. This form and the form above have no parameters.

Complete classification of main symbols for germs of such generic equations was obtained in [3]. All new forms in this classification include real parameters as invariants. The respective model equations were used in applications much earlier [4]. In the beginning of this century, some normal forms for the parametric case were found [5, 6].

In 2007, structural stability of characteristic net for a typical linear second-order mixed-type equation on the plane was proved for a wide class of equations [7]. This result leads to the natural problem of finding appropriate nonlocal normal forms of

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such equations and invariants of these forms. The first such form (besides the Laplace and the wave equations) $u_{rr} - (r-1)u_{\phi,\phi} = 0$ was proposed in [8] for the case of the Cibrario–Tricomi-type behaviour of the characteristic near the circle $r = 1$ with $r \geqslant 1$. Such a behaviour also occurs for the equation of small bending of a revolution surface near the parabolic line and in the theory of momentless steady states of a thin elastic shell of revolution, see [9].

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Dynamics of concentrated vorticities in incompressible Euler flows

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We consider the Euler equations for an incompressible inviscid fluid

$$\begin{cases} v_t + (v \cdot \nabla)v = -\nabla p \quad \text{in } \mathbb{R}^n \times (0,T), \\ \operatorname{div} v = 0, \end{cases} \tag{1}$$

where $n = 2, 3$. The function $\omega = \operatorname{curl} v$ is called the vorticity of the solution. We consider solutions with *highly concentrated vorticity*. More precisely, with vorticity

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concentrated around a finite number of moving points (vortices) when $n = 2$, and around curves when $n = 3$. When $n = 2$, the vorticity-stream formulation of (1) is the scalar problem

$$\begin{cases} \omega_t + (v \cdot \nabla)\omega = 0 \quad \text{in } \mathbb{R}^2 \times (0,T), \\ v(x,t) = \displaystyle\int_{\mathbb{R}^2} K(x-y)\,\omega(y,t)\,dy, \quad K(x) = \frac{1}{2\pi}\frac{1}{|x|^2}(x_2,-x_1). \end{cases} \tag{2}$$

We discuss the construction of solutions to problem (2) of the form

$$\omega(x,t) = \sum_{j=1}^{N} \frac{\kappa_j}{\varepsilon^2} W\left(\frac{x-\xi_j(t)}{\varepsilon}\right) + o(1)$$

in a fixed interval $[0,T]$, where $o(1) \to 0$ as $\varepsilon \to 0$ and $W(y)$ is a positive rapidly decaying profile. For special configurations, for instance *two vortex pairs* travelling in opposite directions, we find solutions of this type with $o(1) \to 0$ as $t \to +\infty$. We discuss the highly nontrivial generalization of these constructions to the generalized SQG equation, $0 < s < 1$,

$$\begin{cases} \omega_t + (v \cdot \nabla)\omega = 0 \quad \text{in } \mathbb{R}^2 \times (0,T), \\ v(x,t) = \displaystyle\int_{\mathbb{R}^2} K_s(x-y)\,\omega(y,t)\,dy, \quad K_s(x) = \frac{c_s}{|x|^{3-2s}}(x_2,-x_1). \end{cases}$$

In the 3-dimensional case, we explain the *vortex filament conjecture* in its connection with the binormal flow of curves and find its first mathematical proof in the helical case. Finally, we discuss the first mathematical justification of the *leapfrogging vortex ring dynamics* first conjectured by Helmholtz in 1858.

These results correspond to collaboration with Juan Davila (Bath), Antonio Fernandez (UAM, Madrid), Monica Musso (Bath), Shrish Parmeshwar (Imperial College London).

Rigorous results in Zakharov–L'vov stochastic model for wave turbulence

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The wave (or weak) turbulence theory (WT) has been intensively developed in physical works since the 1960's. Despite the great interest in the community, mathematical works devoted to its rigorous justification began to appear only in recent years. In these works, significant progress was achieved in justification of the theory but the problem is still poorly understood.

WT can be viewed as the kinetic theory of interacting nonlinear waves, parallel to the famous R. Peierls' kinetic theory, and also as a toy model for the strong

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turbulence theory. From the mathematical point of view, WT is a heuristic approach for studying small amplitude solutions to nonlinear Hamiltonian PDEs with periodic boundary conditions of large period. The fundamental assertion of WT is that one of the main characteristics of the solution, called the *energy spectrum*, approximately satisfies a nonlinear kinetic equation, called the *wave kinetic equation* and dating back to R. Peierls.

I will talk about my the joint works [1–3] with S. B. Kuksin, as well as with S.G. Vleduts and A. Maiocchi, where we completed the first step in a rigorous justification of this assertion for the energy spectrum of the solution to the nonlinear Schrödinger equation subject to a random perturbation on the torus. This stochastic model for WT was proposed by Zakharov and L'vov.

Speaking in more detail, we consider the cubic NLS equation

$$\partial_t u + i\Delta u - i\lambda|u|^2 u = (\text{viscosity} + \text{random perturbation}) \text{ of size} \ll \lambda, \qquad (1)$$

where $u = u(t,x) \in \mathbb{C}$, $x \in \mathbb{R}^d/(L\mathbb{Z}^d)$, $d \geqslant 2$, $L \gg 1$ is the period of the torus, while $0 < \lambda \ll 1$ is a small parameter. The *energy spectrum* of its solution is defined as a function $n_s(\tau)$ on the Fourier-dual lattice $L^{-1}\mathbb{Z}^d \ni s$, given by

$$n_s(\tau) = \mathbb{E}|v_s(\tau)|^2, \qquad s \in L^{-1}\mathbb{Z}^d.$$

Here τ denotes an appropriately rescaled time t, $\mathbb{E}$ stands for the mathematical expectation, while v_s denote rescaled Fourier coefficients of the solution $u(\tau)$. Instead of the exact solution $u(\tau)$ in papers [1, 2] we studied a *quasisolution* $U(\tau)$ given by the sum of the first three terms of the formal series in λ for the exact solution u. We proved that the quasisolution solves equation (1) with small disparity. Then we showed that the energy spectrum of the quasisolution satisfies the main assertion of WT:

Theorem 1. *Under the wave turbulence limit, when first $L \to \infty$ and then $\lambda \to 0$ (or simultaneously $\lambda \to 0$, $L \gg \lambda^{-1}$), the energy spectrum $N_s(\tau)$ of the quasisolution $U(\tau)$ approximately satisfies the wave kinetic equation. Under the opposite wave turbulence limit, when first $\lambda \to 0$ and then $L \to \infty$, the energy spectrum $N_s(\tau)$ of the quasisolution approximately satisfies a non-autonomous version of the wave kinetic equation.*

Now we are working on the justification of the following conjecture, which should complete the rigorous justification of the main assertion of WT for equation (1).

Conjecture. *The quasisolution $U(\tau)$ well approximates the exact solution $u(\tau)$, so the assertion of Theorem 1 is satisfied for the energy spectrum $n_s(\tau)$ of the exact solution.*

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Initial-boundary value problems for the higher-order nonlinear Schrödinger equation

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The higher-order nonlinear Schrödinger equation

$$iu_t + au_{xx} + ibu_x + iu_{xxx} + \lambda|u|^p u + i\beta\big(|u|^p u\big)_x + i\gamma\big(|u|^p\big)_x u = f(t,x) \qquad (1)$$

is considered. Here a, b, λ, β, γ are real constants, $p \geqslant 1$, $u = u(t,x)$ and f are complex-valued functions. This equation is a generalized combination of the well-known nonlinear Schrödinger equation and the Korteweg–de Vries equation. It describes wave propagation in optical fibers. The initial-boundary value problem on the semiaxis $\mathbb{R}_+ = (0,+\infty)$ is set with the initial and the boundary conditions

$$u\big|_{t=0} = u_0(x), \quad u\big|_{x=0} = \mu(t). \qquad (2)$$

The following two cases are considered: either $\mu \equiv 0$ or $p = 1$ and $\gamma = 0$. In both cases results on existence, uniqueness, and continuous dependence on the input data for global weak solutions are established.

The solutions are constructed in weighted spaces (with weight at $+\infty$). In particular, the result for the exponential weights is the following.

Theorem 1. *Let $T > 0$ be arbitrary. Let either 1) $\mu \equiv 0$, $p < 3$ or 2) $\mu \in H^s(0,T)$ for some $s > 1/3$, $p = 1$, and $\gamma = 0$. Assume also that $u_0 e^{\alpha x} \in L_2(\mathbb{R}_+)$ and $fe^{\alpha x} \in L_1(0,T;L_2(\mathbb{R}_+))$ for some $\alpha > 0$. Then there exists a weak solution of problem (1) and (2) such that*

$$ue^{\alpha x} \in C_w([0,T];L_2(\mathbb{R}_+)), \quad u_x e^{\alpha x} \in L_2(0,T;L_2(\mathbb{R}_+)).$$

If, in addition, $p \leqslant 2$ (this is automatically satisfied in the second case), then such a solution is unique in this space and locally Lipschitz continuously depends on the input data u_0, μ (in the second case) and f in the corresponding spaces.

Similar but more complicated results are obtained for the power weights $(1+x)^\alpha$.

Previously, the initial-boundary value problem for equation (1) on a bounded interval with homogeneous boundary conditions was studied in [1] and the initial value problem in the case where $p = 1$, $\gamma = 0$ was studied in [2].

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Nonexistence of monotone solutions to some coercive elliptic inequalities in a half-space

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The problem of finding sufficient conditions for uniqueness of trivial (distinct from zero or some other constant a. e.) solutions to nonlinear elliptic equations and inequalities in respective functional classes is well-known. A method for studying this problem based on the use of special test functions was suggested by S. Pohozaev [1] and developed in a number of later works; see, in particular, monographs [2, 3], and references there. The first results in this direction were obtained for operators with the principal part similar to the Laplacian or p-Laplacian in a certain sense and power-like nonlinearities. Later they were extended to a larger class of operators [4], so that, in particular, the power-like behavior of the zero order nonlinear term was allowed to take place only near zero. However, the operator was still required to satisfy at least the weak Harnack inequality.

Here we modify the test function method in order to obtain sufficient conditions for uniqueness of trivial solutions to some quasilinear elliptic and parabolic inequalities containing nonlinear terms similar to those of [4] but such that the weak Harnack inequality is not required to hold.

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On the relaxed incompressible porous media equation

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We consider the incompressible porous media equation (IPM) describing the evolution of an incompressible fluid in a porous medium subject to gravity. The initial data of our interest consists of a (not necessarily flat) interface separating a heavier fluid with homogeneous density $\rho_+ > 0$ from a lighter fluid with homogeneous density $\rho_- \in (0, \rho_+)$, with the heavier one being above the lighter one. Due to the gravity term this situation is in real world scenarios highly unstable and mathematically ill-posed as an initial value problem. In the talk we will recall the ill-posedness result of L. Székelyhidi [1], as well as a strategy to recover well-posedness on the level of averaged solutions by solving the resulting "relaxed incompressible porous media equation," which can be seen as a nonlocal hyperbolic conservation law. This strategy goes back to F. Otto [2]. In the main part of the talk we will discuss the construction of an entropy solution for the relaxed IPM equation emanating from a real analytic initial interface.

The talk is based on a collaboration with Ángel Castro (ICMAT, Madrid) and Daniel Faraco (UAM/ICMAT, Madrid) [3].

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Regularity results for nondiagonal parabolic systems

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We consider a parabolic system of equations with the nondiagonal principal matrix in a model parabolic cylinder. It is assumed that the components of a solution are interconnected by the Dirichlet- and the Neumann-type boundary conditions through some matrix on the planar boundary Γ of the half-cylinder. We establish the Hölder continuity of the weak solution in a neighborhood of Γ, using a modification of the method of A-caloric approximations adapted to the considered problem.

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Non-collision singularities and super-hyperbolic orbits in Newtonian planar 4-body problem

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In Newtonian N-body problems, the existence of non-collision singularities, which means there are orbits of the system exhibiting the peculiar feature that some particles would escape to infinity in finite time with infinite velocities without the occurrence of collisions (two or more particles occupying the same point in the physical space), has been long speculated for $N \geqslant 4$. This was commonly known as the Painleve conjecture and has been recently resolved by J. Xue [Acta Math., 2020]. The so-called super-hyperbolic orbits are orbits that exist globally in time and the relative speeds between the particles grow to infinity as time goes to infinity. The existence of such orbits was conjectured by Marchal–Saari in the seventies of last century. In this talk we would review several existing models that allow the existence of non-collision singularities and introduce two new models in the planar 4-body problems, both of which exhibit orbits of non-collision singularities. For certain mass ratios, one of them also allows the construction of super-hyperbolic orbits, which solves the conjecture of Marchal–Saari.

This is based on joint works with J. Gerver and J. Xue [2] and with J. Xue [1].

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Einstein machinery for finite difference model and three type of transient flows

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Assume P_M is a numerical solution of the problem in the discretized domain of the flow. Flow itself is generated by the source (sink) being a small disc $B(r_w)$ located in the box B_0 of discretization with size $\Delta >> r_w$.

Problem. Accurately interpret numerical value of p_0 associated to the box B_0 w.r.t. actual (analytical) value of the pressure on the well $\Gamma_w = \partial B(r_w)$.

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Note that discretization of Γ_w is not possible in the intended application.

To solve this problem, we consider sewing machinery between finite difference and analytical solutions defined at different scales: far away and near the source of the perturbation of the flow. One of the essences of the approach is that the coarse problem and the boundary value problem in the proxy of the source model two different flows. We propose a method to glue solution via total fluxes predefined on coarse grid. It is important to mention that the coarse solution "does not see" boundary.

From an industrial point of view, our report provides a mathematical tool for analytical interpretation of simulated data for fluid flow around a well in a porous medium. It can be considered as a mathematical "shirt" on the famous Peaceman well-block radius formula for linear (Darcy) radial flow but can be applied in much more general scenario including Forchheimer flow.

Note that in the literature known to the authors, the rate of production on the well-q is time independent. We developed a method allowing one to determine the Peaceman well block radius R_0 that depends only on stationary parameters and converges to the classical Peaceman radius R_0 in a cylindrical reservoir as external radius of the domain of the flow goes to infinity. This is applicable to a class of dynamic flows widely used in industry.

We will enlarge Einstein approach for three regimes of the Darcy and non-Darcy flows for compressible fluid (time dependent):

I. *Stationary*; **II**. *Pseudo Stationary* (PSS); **III**. *Boundary Dominated* (BD).

Theorem. *For each of three regimes of filtration, using material balance as sewing machinery, we provide an analytical algorithm for the Peaceman equivalent radius* R_0^{SS}, $R_0^{PSS}(r_e)$, *and* $P_0^{BD}(r_e)$ *which is time invariant. Moreover, we rigorously proved stability of our method, namely:*

$$\lim_{r_e\to\infty} R_0^{PSS}(r_e) = \lim_{r_e\to\infty} R_0^{BD}(r_e) = R_0^{SS}$$

Differential-difference equations with incommensurable shifts of arguments

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We consider the boundary-value problem

$$A_R u \equiv -\sum_{i,j=1}^{n} (R_{ij} u_{x_j})_{x_i} = f(x) \quad (x \in Q), \tag{1}$$

$$u(x) = 0 \quad (x \notin Q). \tag{2}$$

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Here Q is a bounded domain in $\mathbb{R}^n$ with smooth boundary ∂Q, $f \in L_2(Q)$, the difference operators $R_{ij} : L_2(\mathbb{R}^n) \to L_2(\mathbb{R}^n)$ are given by

$$R_{ij}u(x) = \sum_{h \in M_{ij}} a_{ijh}(u(x+h) + u(x-h)) \quad (a_{ijh} \in \mathbb{R}),$$

where $M_{ij} \subseteq M$ are finite sets of vectors with incommensurable coordinates. The generalized solution u of boundary-value problem (1), (2) belongs to the Sobolev space $\mathring{H}^1(Q)$.

For elliptic differential-difference equations with integer shifts of the independent variables, the theory of boundary-value problems was created by Skubachevskii (see [1]). The presence of incommensurable shifts of the arguments greatly complicates the study of such boundary-value problems.

However, in the case where the orbit of ∂Q under the shifts present in the difference operator is finite, the methods developed for problems with integer shifts are applicable. In particular, problem (1)-(2) can be reduced to the boundary problem for a differential equation with nonlocal boundary conditions(see [3]).

If the orbit of the boundary under the shifts in the difference operators is infinite, then the nature of the problem changes fundamentally. In particular, its solutions can have an almost everywhere dense set of derivative discontinuities. A method for obtaining the conditions of strong ellipticity (the fulfillment of the Gårding-type inequality) based on the construction of a system of interrelated matrix polynomials is proposed [2]. These conditions are stable relative to small perturbations of the shifts present in the difference operator.

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Smoothness of generalized solutions to the boundary-value problem for a differential-difference equation with mixed boundary conditions

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Let $Q = (0, d)$ be a finite interval with $d = N + \theta$, $N \in \mathbb{N}$, $0 < \theta \leqslant 1$. Consider

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the boundary-value problem

$$-\left(R_Q u'\right)' = f(x), \quad x \in Q, \tag{1}$$

$$u(0) = 0, \tag{2}$$

$$(R_Q u')(d) = 0 \tag{3}$$

with $f \in L_2(Q)$. The operator $R_Q : L_2(Q) \to L_2(Q)$ is defined by the formula $R_Q = P_Q R I_Q$, where $R : L_2(\mathbb{R}) \to L_2(\mathbb{R})$, $I_Q : L_2(Q) \to L_2(\mathbb{R})$, and $P_Q : L_2(\mathbb{R}) \to L_2(Q)$ are defined as follows:

$$(Ru)(x) = \sum_{j=-N}^{N} a_j(x)u(x+j),$$

$$(I_Q u)(x) = u(x), \quad x \in Q, \quad (I_Q u)(x) = 0, \quad x \in \mathbb{R} \setminus Q, \quad (P_Q u)(x) = u(x), \quad x \in Q.$$

Here $a_j(x) \in C^\infty(\mathbb{R})$ are complex-valued functions.

Consider the partition of the interval Q into disjoint subintervals obtained by deleting the orbits of the endpoints of this interval generated by the group of integer shifts. If $\theta = 1$, then we obtain one class of disjoint subintervals $Q_{1k} = (k-1, k)$, $k = 1, \ldots, N+1$. If $0 < \theta < 1$, then we obtain two classes of disjoint subintervals $Q_{1k} = (k-1, k-1+\theta)$, $k = 1, \ldots, N+1$, and $Q_{2k} = (k-1+\theta, k)$, $k = 1, \ldots, N$.

Let $R_s = R_s(x)$, $x \in \overline{Q}_{s1}$, be a matrix of order $(N+1) \times (N+1)$ if $s = 1$ and $N \times N$ if $s = 2$, with the elements $r^s_{ij}(x) = a_{j-i}(x+i-1)$, $x \in \mathbb{R}$.

It is assumed that the condition

$$Re(R_s(x)Y, Y)_{\mathbb{C}^{N(s)}} \geqslant c\|Y\|^2_{\mathbb{C}^{N(s)}}$$

is fulfilled for all $x \in \overline{Q}_{s1}$, $Y \in \mathbb{C}^{N(s)}$, where $c > 0$ does not depend on x and Y.

Under these assumptions, the existence of a unique solution of problem (1)–(3) is proved. The smoothness of generalized solutions of the problem on subintervals and on the entire interval Q is investigated. It is shown that the smoothness of generalized solutions is preserved on subintervals. It is also proved that, if the function on the right-hand side of equation (1) is orthogonal in $L_2(Q)$ to a finite number of linearly independent functions, then a generalized solution from the Sobolev space $W^1_2(Q)$ belongs to the space $W^2_2(Q)$.

Similar results for the first boundary value problem were obtained in [1] and for the second boundary value problem in [2, 3].

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On a fourth-order nonlocal problem with integral conditions

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Consider the equation

$$Au + \lambda^4 u = -a_0(t)u^{(4)}(t) + \sum_{i=1}^{4} a_i(t)u^{(4-i)}(t) + \lambda^4 u = f_0(t) \quad (t \in (0,1)) \quad (1)$$

with the integral conditions

$$B_{\rho k}u = \int_0^1 \Big(g_{\rho,k-1}(t)u^{(k-1)}(t) + h_{\rho k}(t)u^{(k)}(t)\Big)\,dt = f_{\rho k} \quad (\rho = 1,2,\ k = 1,2). \quad (2)$$

Here, a_i $(i = 0,1,2,3,4)$ are real-valued functions, $a_0(t) \geqslant k > 0$ $(0 \leqslant t \leqslant 1)$ and $a_1, a_2, a_3, a_4 \in C[0,1]$, $f_0 \in L_2(0,1)$ is a complex-valued function, $f_{\rho k} \in \mathbb{C}$ $(\rho = 1,2,$ $k = 1,2)$ are constants; $\lambda \in \mathbb{C}$ is a spectral parameter; $g_{\rho,k-1}, h_{\rho k}$ $(\rho = 1,2,\ k = 1,2)$ are linearly independent real-valued functions.

Introduce the following norms depending on the parameter λ in the Sobolev space $W_2^4(0,1)$ and in the space $\mathcal{W}[0,1] = L_2(0,1) \times \mathbb{C}^4$:

$$|||u|||_{W_2^4(0,1)} = \Big(\|u\|^2_{W_2^4(0,1)} + |\lambda|^8\|u\|^2_{L_2(0,1)}\Big)^{1/2},$$

$$|||f|||_{\mathcal{W}[0,1]} = \Big(\|f_0\|^2_{L_2(0,1)} + |\lambda|^7(|f_{11}|^2 + |f_{21}|^2) + |\lambda|^5(|f_{12}|^2 + |f_{22}|^2)\Big)^{1/2},$$

where $f = (f_0, f_{11}, f_{21}, f_{12}, f_{22})$, $|\lambda| \geqslant 1$.

Denote

$$\omega_\varepsilon = \{\gamma \in \mathbb{C} : |\arg\gamma| \leqslant \varepsilon\} \cup \{\gamma \in \mathbb{C} : |\arg\gamma - \pi| \leqslant \varepsilon\},\ \omega_{\varepsilon,q} = \{\gamma \in \omega_\varepsilon : |\gamma| \geqslant q\},$$

where $\varepsilon > 0$, and

$$\Delta_h^1 = \begin{vmatrix} h_{11}(0) & h_{11}(1) \\ h_{21}(0) & h_{21}(1) \end{vmatrix}, \qquad \Delta_h^2 = \begin{vmatrix} h_{12}(0) & h_{12}(1) \\ h_{22}(0) & h_{22}(1) \end{vmatrix}.$$

Theorem 1. *Let $\Delta_h^1 \neq 0$ and $\Delta_h^2 \neq 0$. Then for any $0 < \varepsilon < \pi/4$ there is $q_0 > 1$ such that the inequality*

$$|||u|||_{W_2^4(0,1)} \leqslant C|\lambda|^{1/2}|||f|||_{\mathcal{W}[0,1]} \quad (3)$$

holds for all $\lambda \in \omega_{\varepsilon,q_0}$ and $u \in W_2^4(0,1)$ with $C > 0$ independent of λ, u.

To study problem (1)-(2), we use methods proposed in [3] and developed in [2].

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Strong diffusion approximation in averaging

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It has been known since 1960s (R.Khasminskii) that the slow motion X^ε in the time-scaled multidimensional averaging setup

$$\frac{dX^\varepsilon(t)}{dt} = \frac{1}{\varepsilon}B(X^\varepsilon(t),\, \xi(t/\varepsilon^2)) + b(X^\varepsilon(t),\, \xi(t/\varepsilon^2)), \quad t \in [0,T]$$

converges weakly as $\varepsilon \to 0$ to a diffusion process provided $EB(x,\xi(s)) \equiv 0$, where ξ is a sufficiently fast mixing stochastic process while mixing is considered with respect to the σ-algebras generated by the process itself. The latter reduces substantially applications to dynamical systems (where $\xi(t) = T^t\omega$ for a flow T^t), and more recently I. Melbourne (Warwick) with various co-authors studied weak convergence under assumptions more applicable to dynamical systems, which required the use of the rough paths theory. I will discuss new results (some of them with P. Friz) about strong convergence in the above setups and their discrete time counterparts which yield new applications and some of them also rely on the rough paths theory. As a byproduct of this study we obtain almost sure invariance principles and then laws of iterated logarithm for iterated sums and iterated integrals.

On the absence of solutions of differential inequalities with φ-Laplacian

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Consider the problem

$$\mathcal{L}_\varphi u \geqslant F(x,u) \quad \text{in } \Omega, \quad u|_{\partial\Omega} = 0, \tag{1}$$

where Ω is an unbounded open subset of $\mathbb{R}^n$, $n \geqslant 2$, F is a non-negative function, and

$$\mathcal{L}_\varphi u = \operatorname{div}\left(\frac{\varphi(|\nabla u|)}{|\nabla u|}\nabla u\right)$$

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is the φ-Laplace operator with some increasing one-to-one function $\varphi : [0, \infty) \to [0, \infty)$ such that

$$\left(\frac{\varphi(|\xi|)}{|\xi|} \xi - \frac{\varphi(|\zeta|)}{|\zeta|} \zeta \right) (\xi - \zeta) > 0$$

for all $\xi, \zeta \in \mathbb{R}^n$, $\xi \neq \zeta$.

We obtain conditions guaranteeing that any non-negative weak solution of (1) is identically equal to zero. Details are given in [1].

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Processing of medical images for ischemic stroke localization

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Nowadays, medical images analysis is very much in demand. There are special difficulties associated with the formulation of the problem, which are currently being overcome with the help of neural networks. However, the world scientific community counts the problem of the black box and data sampling for training as the main obstacle in the systematic use of neural networks for a wide range of tasks. Therefore, other approaches based on gradient-free optimization methods are applied in our work.

Ischemic stroke is a serious disease that occurs due to violation of the blood circulation of the brain. Specialists use the MSCT device to determine it during diagnostics. The resulting data is presented in the form of matrices whose cells contain the values of the density of brain tissues. These values range between 0 and 3000. The matrices have a standard dimension of 512 by 512 pixels. The areas affected by the disease have abnormal darkening not typical of the regular anatomical case. These areas are marked by a specialist.

Our goal is to create an algorithm for localization of ischemic stroke. To solve this problem, the following steps have been taken: recalculating of the original data to represent brain tissue; discarding artifacts by cutting high values and the mask method; filtering data by applying the median filter, the Gaussian filter, and the mitigation filter [1], the latter created specially for our task; contrasting values by using a sigmoid-like function; discarding values by analyzing frequency distribution.

To make problem more formal we complete several tasks. Our algorithm uses several filters with the following parameters: window size W for median blur, σ for Gaussian blur, ϵ for mitigation filter, while C is a coefficient reflecting the curvature. Also, the algorithm uses a coefficient L responsible for choosing the size of cropping frequencies. Another parameter is the length K of the nearest neighbor search in the DBSCAN clustering algorithm [2].

The result of the quality function is the value obtained in the following way. After filtering the density matrix we get a matrix with updated entries. All non-zero

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entries are considered as points in the three-dimensional space, with coordinates x, y and recalculated density value. The resulting points are collected in clusters using the DBSCAN algorithm.

The IOU metric [3] is used to determine the measure of coincidence between two spatial regions. It is defined as the ratio between the area of intersection and the union of two areas. We apply this metric for each received cluster and select the highest value. Applying the previous steps for each brain slice matrix and taking the average, we get a value being the measure of quality of our algorithm. By F we denote the function by which this value is found.

Thus we solve the problem

$$min(1 - F(W, \sigma, \epsilon, C, L, K)) \to 0 \quad (1)$$

We use gradient-free optimization methods of COBYLA [4] to solve this problem. The quality was tested on 20 patients data. The coefficients were improved to reduce the minimum on the left-hand side of of (1) to 0.19.

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Modelling of viral infection and inflammation with a reaction-diffusion system of equations

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Respiratory viral infections have a strong influence on public health and societal life. Better understanding of their progression in the human organism is important for their elimination and prevention of their spreading in the population.

After being infected, the organism tries to eliminate viral infection with the mechanisms of the immune response. Inflammation triggered by viral infection represents

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a part of the innate immune response. It leads to the production of inflammatory cytokines acting as signaling molecules at different stages of the immune response. Inflammation of infected cells can cause their death through the mechanisms of apoptosis, necroptosis, and pyroptosis.

Viral infection can be characterized by virus replication number, by its severity and infectivity. Virus replication number determines whether infection spreads in cell culture or tissue, and its severity characterizes the speed of this spreading. Virus infectivity is determined by the rate of disease transmission between the individuals of the population. In the case of respiratory viral infections, it is determined by the viral load in the upper respiratory tract. These characteristics of viral infections are influenced by inflammation. The main goal of this work is to investigate this process at the initial stage of infection progression.

The process of viral infection spreading in cell culture can be described by reaction-diffusion systems of equations. Virus replication number, its severity and infectivity can be characterized analytically and numerically [1].

In this work, we present a new nonlocal and nonlinear reaction-diffusion model describing the propagation of viral infection in cell culture taking into account the process of inflammation. The properties of infection spreading are studied depending on the parameters of the model. An estimate of the wave speed is obtained by the linearization method. The total viral load, that is, the space integral for the virus concentration is determined. The dependence of these characteristics on the intensity of inflammation is investigated, and biological interpretation of the results is discussed.

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On absence of global positive solutions of coercive KPZ-type inequalities

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For inequalities of the kind

$$\Delta u + \sum_{j=1}^{n} g_j(x,u)\left(\frac{\partial u}{\partial x_j}\right)^2 \geqslant \omega(u)$$

such that there exists g from $C(0,\infty)\cap L_{loc}(0,\infty)$ with the property $g_j(x,s)\leqslant g(s)$ in the half-space $\mathbb{R}^n\times(0,\infty)$, $j=1,2,\dots,n$, we find the following condition guaranteeing

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the absence of global positive solutions:

$$\int_1^\infty \frac{d\tau}{\sqrt{\int\limits_1^\tau \frac{\omega[\chi(s)]}{\chi'(s)} ds}} < \infty,$$

where χ is the inverse function to the function

$$f(s) := \int_0^s e^{\int_0^x g(\tau) d\tau} dx.$$

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Lattice equations and semiclassical asymptotics

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We present the results of our recent paper [1]. This paper deals with an important class of linear equations with shifts in the argument, namely, equations on a uniform rectangular lattice with small step h in $\mathbb{R}^n$. In the case of functions of continuous argument, equations with shifts can be written as h-pseudodifferential equations [2] with symbols 2π-periodic in the momenta. This representation also makes sense for functions of a discrete argument, although differentiation operators are not defined for lattice functions. The phase space of such equations is the product $\mathbb{R}^n \times T^n$. Developing Maslov's ideas [2], we construct a canonical operator on Lagrangian submanifolds of this phase space with values in the space of lattice functions. In comparison with the classical version [3] of the canonical operator and the new formulas introduced in [4, 5], the construction involves a number of new features.

As an example, we consider equations on a two-dimensional lattice that arise in quantum theory (the Feynman checkers model [6, 7]) and in the problem on the propagation of wave packets on a homogeneous tree [8].

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Nonlocal reaction-diffusion model of virus evolution

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Virus mutation can essentially influence viral disease progression at the individual level and in the population. As such, high mutation potential of the HIV infection determines its variability and resistance to antiviral treatment, while emergence of new variants of the SARS-CoV-2 infection leads to new epidemic outbreaks and immunity waning.

Virus quasi-species can be considered as a localized density distribution in the space of genotypes with their random mutations described, under some simplifications, by diffusion.

There are different mechanisms leading to evolution of existing virus strains and to the emergence of new ones. In this lecture, we consider the mechanism where new virus variants appear due to the interaction of the cross-reactivity of the immune response and virus escape. It can be modeled with nonlocal reaction-diffusion equations for the virus density distribution in the genotype space.

Similar to the emergence of biological species, it is based on competition, reproduction, and mutations. Compared to the previous studies, we consider a more detailed and biologically realistic model including the concentrations of uninfected cells, infected cells, viruses, and the immune response.

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On the longtime behavior of spatially periodic entropy solutions to scalar conservation laws

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We study entropy solutions (in the Kruzhkov sense [1]) of the Cauchy problem for the multidimensional conservation law

$$u_t + \mathrm{div}_x \varphi(u) = 0, \tag{1}$$

$u = u(t, x)$, $(t, x) \in \Pi = (0, +\infty) \times \mathbb{R}^n$, with the initial condition

$$u(0, x) = u_0(x) \in L^\infty(\mathbb{R}^n). \tag{2}$$

We assume that the flux vector $\varphi(u) = (\varphi_1(u), \ldots, \varphi_n(u))$ is merely continuous, $\varphi(u) \in C(\mathbb{R}, \mathbb{R}^n)$. We suppose in addition that the initial function $u_0(x)$ is periodic with a full-rank lattice of periods $L \subset \mathbb{R}^n$, that is, for each $e \in L$ one has $u_0(x + e) = u_0(x)$ a.e. in $\mathbb{R}^n$. As was noticed in [2], in this case there exists a unique e.s. $u = u(t, x) \in L^\infty(\Pi)$ of problem (1), (2) and this e.s. is spatially periodic with the same lattice of periods L. We denote by $\mathbb{T}^n = \mathbb{R}^n/L$ the corresponding torus (which may be identified with the periodicity cell) equipped with the normalized Lebesgue measure dx and by $L' = \{\xi \in \mathbb{R}^n | \xi \cdot e \in \mathbb{Z}\ \forall e \in L\}$ the lattice dual to L. We also introduce the constant $I = \int_{\mathbb{T}^n} u_0(x)dx$ being the mean value of initial data. For each nontrivial interval $\alpha \subset \mathbb{R}$ we denote by $D(\alpha)$ the linear hull of such vectors $\xi \in L'$ that the function $\xi \cdot \varphi(u)$ is affine in α. Our main result is the following.

Theorem 1. *Suppose that* $\dim D(\alpha) \leqslant 1$ *for any nontrivial interval* $\alpha \subset \mathbb{R}$. *Then there exists a vector* $\xi \in L'$, $\xi \neq 0$, *a constant* $c \in \mathbb{R}$, *and a* 1*-periodic function* $v(y) \in L^\infty(\mathbb{R})$ *such that*

$$\operatorname*{ess\,lim}_{t\to+\infty} u(t, x) - v(\xi \cdot x - ct) = 0 \quad \text{in } L^1(\mathbb{T}^n). \tag{3}$$

Moreover, $\int_0^1 v(y)dy = I$ *and* $\xi \cdot \varphi(u) - cu = \mathrm{const}$ *on the segment* $[a, b]$, *where* $a = \operatorname{ess\,inf} v(y)$, $b = \operatorname{ess\,sup} v(y)$.

In the non-degenerate case where $\xi \cdot \varphi(u)$ is not affine in any neighborhood of I for all $\xi \in L'$, it follows from Theorem 1 that $v \equiv I$, and we recover the decay property

$$\operatorname*{ess\,lim}_{t\to+\infty} u(t, x) = I \quad \text{in } L^1(\mathbb{T}^n)$$

established in paper [3].

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Homogenization of convolution-type functionals

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The talk will focus on the asymptotic behaviour and homogenization of convolution type variational functionals of the form

$$F_\varepsilon(u) = \int_{\mathbb{R}^d} \int_{G_\varepsilon(z)} f_\varepsilon\Big(x, z, \frac{u(x+\varepsilon z) - u(x)}{\varepsilon}\Big) dx dz$$

with

$$G_\varepsilon(z) = \{x \in Q \, : \, x + \varepsilon z \in Q\};$$

here ε is a positive parameter that tends to zero.

We assume that the integrand $f_\varepsilon(x, z, \zeta)$ satisfies p-growth conditions, $1 < p < \infty$, of the form

$$\psi_{1,\varepsilon}(z)|\zeta|^p - \rho_{1,\varepsilon}(z) \leqslant f_\varepsilon(x, z, \zeta) \leqslant \psi_{2,\varepsilon}(z)|\zeta|^p + \rho_{2,\varepsilon}(z),$$

where $\rho_{1,\varepsilon}$ and $\rho_{2,\varepsilon}(z)$ are non-negative functions integrable uniformly in ε, $\rho_{1,\varepsilon}$ and $\psi_{1,\varepsilon}$ are compactly supported, $\psi_{1,\varepsilon}(z) \geqslant c_0 > 0$ for $|z| < r_0$ with $r_0 > 0$, and

$$\int_{\mathbb{R}^d} \psi_{2,\varepsilon}(z)(|z|^p + 1) dz < +\infty.$$

Theorem 1. *Under the above conditions, for any sequence* $\{\varepsilon_j\}$, $\varepsilon_j \to 0$ *as* $j \to \infty$, *there exist a subsequence* $\{\varepsilon_{j_k}\}$ *and a quasiconvex Carathéodory function* $f_0(x, z)$, $f_0 \, : \, Q \times \mathbb{R}^{m\times d} \to [0, +\infty)$,

$$C_0(|z|^p - 1) \leqslant f_0(x, z) \leqslant C_1(|z|^p + 1), \quad C_0 > 0,$$

such that

$$\Gamma - \lim_{\varepsilon_j \to 0} F_{\varepsilon_j}(u) = \begin{cases} \displaystyle\int_Q f_0(x, \nabla u(x)))dx, & \text{if } u \in W^{1,p}(Q; \mathbb{R}^m), \\ +\infty, & \text{otherwise.} \end{cases}$$

A number of homogenization results will be presented for periodic integrands.

This is a joint project with R. Alicandro, N. Ansini, A. Braides, and A. Trabuzio.

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Isothermal coordinates on surfaces with L^2 second fundamental form

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Let Ω be a bounded domain in the space $\mathbb{R}^2$ of points $X = (X_1, X_2)$, and $\Psi : \Omega \to \mathbb{R}^3$ be a Lipschitz immersion. Following Kuwert & Li, [1], and Tristan Riviere, [2], we say that Ψ is a weak immersion with L^2 second fundamental form ($W^{2,2}$ immersion) if its first fundamental form $\{g_{ij}\}$ is uniformly bounded from below and above and the normal vector field $\mathbf{n}(x)$ belongs to the class $W^{1,2}(\Omega)$. Recall that

$$g_{ij} = \partial_i \Psi \cdot \partial_j \Psi, \quad \mathbf{n} = |\partial_1 \Psi \times \partial_2 \Psi|^{-1} \partial_1 \Psi \times \partial_2 \Psi, \quad \partial_i = \partial_{X_i}.$$

The immersion is isothermal if $g_{12} = 0$, $g_{11} = g_{22} = e^{2f}$, where e^f is a conformal factor. Using the local charts, we can extend this definition to the class of immersions of closed Riemannian manifolds Σ to $\mathbb{R}^3$. Let Σ be one of the standard manifolds: sphere, torus, or Riemann surface of negative constant curvature. The common belief is that any $W^{2,2}$ immersion $\Psi : \Sigma \to \mathbb{R}^3$ admits the isothermal bi-Lipshitz parametrization with the uniformly bounded logarithm f of the conformal factor. This statement is considered as an analytic version of the celebrated Toro theorem [3] and is widely used in applications. However, it is incorrect since $W^{2,2}$ immersion may have hidden branch points. The problem is to find conditions which, being imposed on Ψ, provide the absence of such points. The following theorem gives the answer to this question in the case of immersion of tori. Let Γ be a lattice of periods in $\mathbb{R}^2$ generated by linearly independent vectors l_1, l_2. Without loss of generality we may take $l_1 = (a, 0)$, $l_2 = (b, a^{-1})$, $a > 0$. Let Ψ be an arbitrary $W^{2,2}$ immersion of the torus $\mathbb{T}_\Gamma = \mathbb{R}^2/\Gamma$ into $\mathbb{R}^3$. Set

$$\Phi(X) = \mathbf{n}(X) \cdot (\partial_1 \mathbf{n}(X) \times \partial_2 \mathbf{n}(X)), \quad \lambda = \frac{1}{4\pi} \int_{\mathbb{T}_\Gamma} \Phi \, dX.$$

The quantity $\lambda \geqslant 0$ is the topological degree of the mapping $\mathbf{n} : \mathbb{T}_\Gamma \to \mathbb{S}^2$.

Theorem 1. *Under the above assumptions, there exist a lattice of periods Υ generated by vectors $\boldsymbol{\gamma}_1 = (\alpha, 0)$ and $\boldsymbol{\gamma}_2 = (\beta, \alpha^{-1})$, and a bi-Hölder homeomorphism $\varphi : \mathbb{T}_\Upsilon \to \mathbb{T}_\Gamma$ with the following properties. If $\lambda = 0$, then the immersion $\Psi^* = \Psi \circ \varphi$ belongs to the class $W^{2,2}(\mathbb{T}_\Upsilon)$. Moreover, Ψ^* is isothermal and the logarithm f of its conformal factor belongs to the class $L^\infty(\mathbb{T}_\Upsilon) \cap W^{1,2}(\mathbb{T}_\Upsilon)$. If $\lambda > 0$, then there are λ branch points $b_i \in \mathbb{T}_\Upsilon$ with their multiplicities taken into account, such that the isothermal immersion Ψ^* belongs to the class $W^{2,2}_{\mathrm{loc}}(\mathbb{T}_\Upsilon \setminus \{b_i\})$, and f belongs to the class $L^\infty_{\mathrm{loc}}(\mathbb{T}_\Upsilon \setminus \{b_i\}) \cap W^{1,2}_{\mathrm{loc}}(\mathbb{T}_\Upsilon \setminus \{b_i\})$. Each point $\varphi^{-1}(b_i)$ is a hidden branch point of the immersion Ψ.*

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On new types of patterns in the model of nonlinear optical system with matrix Fourier filter

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Fourier filtering, which consists in changing the signal by controlling its Fourier spectrum, is widely used in problems of information processing utilizing nonlinear feedback optical systems [1]. Mathematical models of filter-multipliers acting on each Fourier harmonic separately, were considered in [2]. In [3, 4], new models of matrix Fourier filtering are proposed, in which the Fourier spectrum is transformed by multiplying it by an infinite filter matrix. The corresponding model of the phase modulation dynamics $u(x, t)$ is governed by the periodic boundary value problem for the diffusion FDE in an infinitely thin ring ($x \in [0, 2\pi]$):

$$\partial_t u + u - D\partial^2_{xx} u = K|\Phi_{E+P}(\exp\{iu\})|^2,$$

where $\Phi_{E+P}(f)$ is the matrix Fourier filtering operator that linearly transforms the Fourier spectrum of the function $f \in L_2(0, 2\pi)$ (see details in [4]), E is the identity matrix ($\Phi_E(f) \equiv f$), P is the matrix filter, $D > 0$ is the diffusion coefficient, $K > 0$.

An important applied problem is a construction of solutions to FDE with prescribed properties using an appropriate choice of a matrix filter. The authors attack this problem by the variational method in [3], and by the methods of the Andronov–Hopf bifurcation theory in [4]. Based on the Turing and the Andronov–Hopf bifurcation theory, the report discusses an approach for the construction of matrix Fourier filters that provide excitation of stationary structures and new types of dynamic structures with specified properties. Analytical studies are illustrated by the results of a computer experiment.

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Blow up of multidimensional electrostatic oscillations of an electron plasma

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We consider the classical Cauchy problem for a system of equations describing arbitrary 3D electrostatic oscillations of cold plasma (e.g. [1]),

$$\frac{\partial n}{\partial t} + \operatorname{div}(n\mathbf{v}) = 0, \quad \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\,\mathbf{v} = -\mathbf{E} - [\mathbf{v} \times \mathbf{B}]\,,$$

$$\frac{\partial \mathbf{E}}{\partial t} = n\mathbf{v} + \operatorname{rot}\mathbf{B}, \qquad \frac{\partial \mathbf{B}}{\partial t} = -\operatorname{rot}\mathbf{E}, \qquad \operatorname{div}\mathbf{B} = 0,$$

where n and $\mathbf{v} = (V_1, V_2, V_3)$ are the density and velocity of electrons, $\mathbf{E} = (E_1, E_2, E_3)$ and $\mathbf{B} = (B_1, B_2, B_3)$ are vectors of electric and magnetic fields. All components of a solution depend on $t \in \mathbb{R}_+$ and $x \in \mathbb{R}^3$.

We introduce an iteration procedure to estimate the blow-up time from below. This procedure is constructive provided one succeeds in obtaining a two-sided estimate of an additional quantity depending on the solution. For the particular case of two-dimensional initial data with radial symmetry, refined sufficient conditions for violation and preservation of smoothness in the first period of oscillations are obtained. Moreover, we give an example of estimating the blow-up time for those data for which results of numerics exist and discuss the roughness of our estimate. The talk extends the results of [2].

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On control systems which possess universal approximating property

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The controlled dynamical system approach to the deep learning of artificial neural networks (ANN) has been explored over the last years by a number of researchers. According to this approach the training algorithm for ANN can be set as optimal control problem for a continuous-time controlled dynamic system. Such system can be seen as a network with a continuum of layers, each one labelled by the time variable. The control values at each instant of time are the parameters of the layer. One can apply the necessary optimality condition — Pontryagin's Maximum Principle (PMP) to the problem, as well as numeric algorithms of optimal control.

In the talk (based on joint work with A. Agrachev) we briefly survey the approach to the deep learning and mainly concentrate on the universal approximating property of the controlled dynamical systems, which is crucial for the effectiveness of the learning procedure. We apply Lie algebraic methods of geometric control and results on ensemble controllability for constructing linear control systems possessing the property.

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η-invariants of boundary-value problems

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Atiyah–Patodi–Singer [1] introduced η-invariant $\eta(A)$ of an elliptic self-adjoint operator A on a closed manifold as a regularized number of positive eigenvalues minus the number of negative eigenvalues. The regularization is defined in terms of analytic continuation of so-called η-function of the operator defined in terms of the eigenvalues. This invariant is a spectral invariant and has numerous applications and generalizations. For instance, it appeared as a contribution of the boundary in index formulas for Dirac operators on manifolds with boundary.

Melrose [2] introduced η-invariant $\eta(D(p))$ for elliptic parameter-dependent families of operators $D(p)$ on closed manifolds as a regularization of the winding number of the family. This invariant is a generalization of the Atiyah–Patodi–Singer η-invariant and also has applications. For instance, it appears as a contribution of the conical point for general elliptic operators on manifolds with isolated singularities.

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Our aim is to extend the η-invariant of Melrose to parameter-dependent families of boundary value problems. We consider general parameter-dependent boundary value problems elliptic in the sense of Agranovich and Vishik and define η-invariants for such families. The main analytical result necessary to define the η-invariant is the asymptotic expansion of the trace of parameter-dependent families for large values of the parameter.

The reported study was funded by RFBR, project No. 21-51-12006.

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On two notions of distance between homotopy classes in $W^{1/p,p}(S^1, S^1)$

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It is known that maps in $W^{1/p,p}(S^1, S^1)$ have a well defined degree. This allows one to partition the space into disjoint classes, $W^{1/p,p}(S^1, S^1) = \bigcup_{d\in\mathbb{Z}} \mathcal{E}_d$. It follows from a result of Brezis and Nirenberg that the $W^{1/p,p}$-distance between any two of these classes, i.e.,

$$\mathrm{dist}(\mathcal{E}_{d_1}, \mathcal{E}_{d_2}) = \inf\{|u - v|_{W^{1/p,p}} \,:\, u \in \mathcal{E}_{d_1}, v \in \mathcal{E}_{d_2}\}$$

is zero.

This reflects the fact that the degree in $W^{1/p,p}(S^1, S^1)$ is continuous but not uniformly continuous. However, in a joint work with Mironescu we proved that the distance between different classes is positive provided we assume a bound of the norm of the maps. There is also another notion of distance which is of interest, namely

$$\mathrm{Dist}(\mathcal{E}_{d_1}, \mathcal{E}_{d_2}) = \sup_{u\in\mathcal{E}_{d_1}} \inf_{v\in\mathcal{E}_{d_2}} |u - v|_{W^{1/p,p}}.$$

We proved that $\mathrm{Dist}(\mathcal{E}_{d_1}, \mathcal{E}_{d_2})$ equals the minimal $W^{1/p,p}$-energy in $\mathcal{E}_{d_1-d_2}$.

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Kifer's criterion for large deviations and applications

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In 1990, Kifer established a sufficient condition for the validity of large deviation principle (LDP) for a sequence of random probability measures on a compact metric space [3]. Kifer's result is a far reaching generalisation of the Gärtner–Ellis theorem and allows one to treat a number of problems from a unified point of view. We shall recall Kifer's theorem, present the main steps of its simplified proof, and show how it can be applied to discrete-time Markov processes with a compact phase space [1]. In particular, we discuss the Donsker–Varadhan type LDP for the motion of a particle immersed into the Navier–Stokes flow [2].

All the results of this talk are obtained in collaboration with V. Jakšić (McGill University, Canada), V. Nersesyan (New York University in Shanghai, China), and C.-A. Pillet (University of Toulon, France). This work is supported by the Ministry of Science and Higher Education of the Russian Federation (Megagrant, agreement No. 075-15-2022-1115)

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Very singular and large solutions of semilinear parabolic and elliptic equations

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Exact conditions for the existence of non-negative very singular (v.s.) solutions, i.e. solutions being more singular at some points of the boundary than any solution of the corresponding linear equation, as well as the structure of these solutions, were first obtained in the work by H. Brezis, L. Peletier, and D. Terman (1986) devoted to the Cauchy problem for a semilinear heat equation with a nonlinear absorption

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term $f(u)$. The study of another important class of strongly singular solutions (called large (l.) solutions) taking an infinite value on the entire boundary or on some part of the boundary of the domain, was initiated by L. Bieberbach (1916) who established the existence of large solutions for a semilinear elliptic equation with the nonlinear absorption term $f(u) = b^2 exp(u)$ in a bounded smooth 2-dimensional domain D. The uniqueness of the (l.) solution was proved for the first time by C. Loevner and L. Nirenberg (1974) in the case of a bounded smooth n-dimensional domain D and the nonlinear absorption $f(u) = u^{(n+2)(n-2)^{-1}}$.

We study the existence and uniqueness conditions, as well as qualitative and asymptotic properties, of (v.s.) and (l.) solutions to various classes of semilinear parabolic and elliptic equations with "nonhomogeneous" absorption of the form $f(t, u)$ or $f(x, u)$ degenerating on the boundary (or on a part of the boundary) of the domain or on some manifold in the domain whose boundary has a non-empty intersection with the boundary of the domain. Exact conditions are established for this degeneracy guaranteeing the existence or non-existence of (v.s.) or (l.) solutions. It is shown in some problems that these conditions are necessary and sufficient for the existence of solutions under discussion. In particular, it is shown in the elliptic case with the absorption $f(x, u) = g(x)u^p$, $p > 1$ degenerating at the boundary of the domain, that the obtained new condition on the degeneracy of $g(x)$ is close to the exact sufficient condition for the uniqueness of the (l.) solution and, surprisingly, to the sufficient and necessary condition for the existence of the (v.s.) solution. This connection justifies a parallel study of the discussed classes of solutions.

Some of these results were published in [1–7].

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Global weak solutions with compact supports to mixed problems for the Vlasov–Poisson system and plasma confinement

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The mixed problem for the Vlasov–Poisson system of equations is the simplest model describing kinetics of high temperature plasma in a fusion reactor. A situation where a sufficiently large number of particles reaches the reactor wall leads to destruction of the reactor. Therefore it is necessary to provide plasma confinement at some distance from the reactor wall. It can be achieved with the help of external magnetic field [1]. Mathematically speaking, we must find conditions for this magnetic field which provide the existence of solutions compactly supported inside the domain. In this lecture we obtain sufficient conditions for the existence of global weak solutions with compact supports to the mixed problem for the Vlasov–Poisson system with external magnetic field.

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On existence of solutions to elliptic differential difference equations with operators having a semibounded variation

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Let $Q \subset \mathbb{R}^n$ be a bounded domain with boundary $\partial Q \in C^\infty$, or $Q = (0, d) \times G$, where $G \subset \mathbb{R}^{n-1}$ is a bounded domain (with boundary $\partial G \in C^\infty$ if $n \geqslant 3$). If $n = 1$, then $Q = (0, d)$. We consider the following problem with an essentially nonlinear operator A:

$$ARu(x) = -\sum_{1 \leqslant i \leqslant n} \partial_i A_i(x, Ru(x), \nabla Ru(x)) = f(x) \qquad (x \in Q), \tag{1}$$

$$u(x) = 0 \qquad (x \notin Q). \tag{2}$$

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Here $f \in W_q^{-1}(Q)$, $1/q + 1/p = 1$, $1 < p < \infty$, and R is a linear difference operator given by the formula

$$Ru(x) = \sum_{h \in \mathcal{M}} a_h u(x+h), \tag{3}$$

where $a_h \in \mathbb{R}$ and $\mathcal{M} \subset \mathbb{Z}^n$ is a finite set of vectors with integer (or commensurable) coordinates.

We suppose that the differential operator A is of Nemytsky type and AR satisfies the strong ellipticity condition and the coercivity condition. Then we prove that there exists at least one generalized solution $u \in \mathring{W}_p^1(Q)$ to problem (1) and (2).

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Homogenization of a nonselfadjoint nonlocal convolution type operator

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In $L_2(\mathbb{R}^d)$, we consider a bounded operator $\mathbb{A}_\varepsilon$, $\varepsilon > 0$, given by

$$(\mathbb{A}_\varepsilon u)(x) := \varepsilon^{-d-2} \int_{\mathbb{R}^d} a((x-y)/\varepsilon)\mu(x/\varepsilon, y/\varepsilon)(u(x) - u(y))\, dy.$$

Such operators appear in mathematical biology and population dynamics.

It is assumed that $a(x)$ is a non-negative function from $L_1(\mathbb{R}^d)$, $\|a\|_{L_1} = 1$ and $\int_{\mathbb{R}^d} |x|^3 a(x)\, dx < \infty$. Suppose that $\mu \in L_\infty(\mathbb{R}^{2d})$, $0 < \mu_- \leqslant \mu(x,y) \leqslant \mu_+ < \infty$ and $\mu(x+m, y+n) = \mu(x,y)$ for all $x, y \in \mathbb{R}^d, m, n \in \mathbb{Z}^d$.

Under these assumptions, the operator $\mathbb{A}_\varepsilon$ is bounded and its spectrum is contained in the right half-plane $\{\lambda \in \mathbb{C} : \operatorname{Re} \lambda \geqslant 0\}$. We find approximation of the resolvent $(\mathbb{A}_\varepsilon + I)^{-1}$ in the operator norm on $L_2(\mathbb{R}^d)$ for small ε. In the selfadjoint case the resolvent $(\mathbb{A}_\varepsilon + I)^{-1}$ converges to the resolvent of the effective operator $\mathbb{A}^0$ as $\varepsilon \to 0$.

Theorem 1. *Suppose in addition that $a(x) = a(-x)$ and $\mu(x,y) = \mu(y,x)$. Then*

$$\left\|(\mathbb{A}_\varepsilon + I)^{-1} - (\mathbb{A}^0 + I)^{-1}\right\|_{L_2 \to L_2} \leqslant C\varepsilon.$$

Here $\mathbb{A}^0 = -\operatorname{div} g^0 \nabla$, where g^0 is a positive definite matrix given in terms of solutions of some auxiliary problems on the cell $\Omega = [0,1)^d$.

Approximation is more complicated in the nonselfadjoint case.

Theorem 2. *Under the above assumptions we have*

$$\left\|(\mathbb{A}_\varepsilon + I)^{-1} - (\mathbb{A}^0 + \varepsilon^{-1}\vec{\alpha} \cdot \nabla + I)^{-1}[q_0^\varepsilon]\right\|_{L_2 \to L_2} \leqslant C\varepsilon.$$

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Here $\mathbb{A}^0 = -\operatorname{div} g^0\nabla$; the matrix g^0 and the vector $\vec{\alpha}$ are given in terms of solutions of some auxiliary problems on the cell Ω; $[q_0^\varepsilon]$ is the operator of multiplication by the function $q_0(x/\varepsilon)$, where q_0 is also a periodic solution of some auxiliary problem.

To prove the results, we modify the operator-theoretic approach for the case of nonselfadjoint operators.

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Solvability of elliptic functional differential equations with orthotropic contractions in weighted spaces

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We study solvability of elliptic functional differential equations with contractions and expansions of the arguments in the principal part:

$$A_R u \equiv -\sum_{i,j=1}^{2} (R_{ij} u_{x_i})_{x_j} = f(x_1, x_2),$$

$$R_{ij}v(x) = a_{ij0}v(x_1, x_2) + a_{ij1}v(q^{-1}x_1, px_2) + a_{ij,-1}v(qx_1, p^{-1}x_2), \quad p, q > 1.$$

The weighted spaces introduced by V. A. Kondratiev are of great importance in the study of elliptic problems in domains with angular or conical points. Solutions to boundary value problems for functional differential equations can also have power singularities on the boundary or inside a bounded domain, so it is natural to consider them in weighted spaces.

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Boundary singular problems for the Chipot–Weissler equation

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Let Ω be a bounded C^2 domain and $p > 1$, $1 < q < 2$, $M > 0$. We study the boundary behaviour of positive functions satisfying

$$\mathcal{L}_{p,q,M}u := -\Delta u - u^p + M|\nabla u|^q = 0 \tag{1}$$

in Ω, which is determined by the competition between the absorption term $|\nabla u|^q$ and the source reaction term u^p.

The study is carried out in the following several directions:

1. description of the solutions with a boundary isolated singularity;
2. existence of solutions with boundary measure data;
3. existence of a boundary trace.

We also touch on the existence of critical exponents in p and q and of critical values for M.

Reaction-diffusion models in the theory of biological evolution

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Reaction-diffusion equations have been widely used to describe biological populations since the classical KPP work on the propagation of a dominant gene. More recently, nonlocal reaction-diffusion models have been developed to describe the emergence of biological species or virus quasi-species and for characterization of other evolutionary patterns [1, 2].

Biological species can be considered as groups of individuals with similar morphological characteristics. Population distributions are relatively stable with respect to some morphological parameter and can be viewed as stationary in appropriate time scale. Therefore, population distributions can be described as stable stationary solutions to some relevant models. It appears however that conventional population models do not have such solutions. In this lecture, we will discuss novel models for

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population distributions with respect to the genotype (and not phenotype, i.e., morphology). We will study the existence and stability of solutions of the corresponding equations and will discuss their biological interpretations.

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Smoothness of generalized eigenfunctions of differential-difference operators on a finite interval

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We consider the smoothness of generalized eigenfunctions to the Dirichlet problem for a differential-difference operator on an interval $(0, d)$. Necessary and sufficient conditions for the existence of generalized eigenfunctions whose smoothness is violated inside the interval are obtained. We give an example of a positive-definite differential-difference operator possessing countably many smooth eigenfunctions and countably many generalized eigenfunctions whose smoothness is violated.

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Determining functionals and finite-dimensional reduction for dissipative PDEs revisited

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We discuss the properties of linear and nonlinear determining functionals for dissipative dynamical systems generated by PDEs. The main attention will be payed to the lower bounds for the number of such functionals. In contradiction to the common paradigm, we will see that the optimal number of determining functionals (the so-called determining dimension) is strongly related to the proper dimension of the set of equilibria of the considered dynamical system rather than to the dimensions of the global attractors and the complexity of the dynamics on it. In particular, in the generic case where the set of equilibria is finite, the determining dimension equals one (in a complete agreement with the Takens delayed embedding theorem) no matter how complex the underlying dynamics is. The obtained results will be illustrated by a number of explicit examples.

Правосторонняя обратимость функциональных операторов и индивидуальная дихотомия

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В докладе рассматривается одно обобщение свойства экспоненциальной дихотомии, опишем его в случае одного из наиболее простых примеров.

Пусть $a(x)$ есть заданная ограниченная обратимая непрерывная матрично-значная функция на прямой и $Bu(x) = a(x)u(x+h)$ – оператор взвешенного сдвига в пространстве вектор-функций $L_2(\mathbb{R}, \mathbb{C}^m)$. Рассматриваются уравнения $Bu - \lambda u = v$, т.е. функциональные уравнения вида

$$a(x)u(x+h) - \lambda u(x) = v(x), \ x \in \mathbb{R}. \tag{1}$$

При исследовании таких уравнений в первую очередь возникает вопрос об условиях обратимости оператора $B - \lambda I$, т.е. о спектре оператора B. Оказалось, что обратимость оператора $B - \lambda I$ эквивалентна существованию экспоненциальной дихотомии решений однородного уравнения. С рассматриваемым уравнением связано отображение β_λ пространства $E = \mathbb{R} \times \mathbb{C}^m$ в себя, действующее по формуле

$$E = \mathbb{R} \times \mathbb{C}^m \ni (x, \xi) \to (x+h, \lambda[a(x)]^{-1}\xi) \in \mathbb{R} \times \mathbb{C}^m,$$

называемое *линейным расширением* отображения $\alpha : x \to x + h$. Говорят, что β_λ допускает экспоненциальную дихотомию, если существует непрерывная проек-

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торно-значная функция $p(x)$, задающая разложение E (как векторного расслоения) в прямую сумму $E = E^s \oplus E^u$ устойчивого и неустойчивого подрасслоений.

Для дифференциальных уравнений связь экспоненциальной дихотомии с разрешимостью была обнаружена еще Перроном, в дальнейшем это свойство было предметом исследований многих авторов.

Но вопрос об условиях существования решения из заданного пространства (без требования единственности) в общей постановке остается открытым. В операторной терминологии это вопрос о существовании правого обратного к оператору $B - \lambda I$. Ряд результатов в этом направлении для конкретных классов функциональных уравнений получен в работах А.Б. Антоневича, Ю.Якубовской, А.А. Ахматовой, Е.В. Пантелеевой, часть из них изложена в [1]. Было обнаружено, что условием правосторонней обратимости является новое свойство линейного расширения β_λ, которое можно назвать *индивидуальной дихотомией,* т.к. оно заключается в существовании для каждого элемента $(x, \xi) \in E$ разложения $\xi = \xi^s(x) + \xi^u(x)$ на устойчивый и неустойчивый элементы.

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О резольвентной сходимости для общих операторов высокого порядка с малыми переменными сдвигами

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Рассматривается несколько классов многомерных эллиптических операторов высокого порядка, младшие члены которых носят нелокальный характер из-за наличия в них малых переменных сдвигов общего вида. Такого сорта операторы рассматриваются во всём пространстве и в произвольной области. Во втором случае на границе ставятся краевые условия общего вида, которые также являются нелокальными. Рассматриваются вопросы резольвентной сходимости для таких операторов в случае, когда переменные сдвиги в подходящем смысле стремятся к нулю. Показано, что тогда предельным будет соответствующий классический дифференциальный оператор, получающийся из исходного в случае, когда сдвиги равны нулю, а именно, имеет место равномерная резольвентная сходимость. Помимо доказательства самого факта сходимости, удалось получить и эффективные оценки её скорости сходимости.

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Метод итераций для эволюционных задач с нелокальным временным условием

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Наше сообщение посвящено одному классу нелокальных задач для эволюционных дифференциальных уравнений. Общая постановка выглядит так. В банаховом пространстве E при $t \in [0, T]$ рассмотрим соотношения

$$\frac{du(t)}{dt} = Au(t), \qquad \int_0^T u(t)\eta(t)\,dt = u_1. \tag{1}$$

Считаем, что A — линейный замкнутый оператор в E с плотной областью определения $D(A) \subset E$, порождающий полугруппу $U(t)$ класса C_0 (см. [1]). Весовая скалярная функция $\eta(t)$ имеет ограниченную вариацию на $[0, T]$ и нужную нормировку $\eta(0) = \eta(0+0) = 1$. Требуется подобрать начальное состояние $u_0 \in E$ так, чтобы функция $u(t) = U(t)u_0$ обладала заданным усреднением $u_1 \in D(A)$.

Отметим следующие основные моменты.

1. Задача (1) эквивалентна операторному уравнению $u_0 - Bu_0 = f$ с элементом $f = -Au_1$ и известным линейным ограниченным оператором B.
2. При некоторых дополнительных ограничениях возможно достаточно точное описание спектра оператора B.
3. Найдены новые эффективные оценки спектрального радиуса B, что позволяет находить элемент u_0 методом итераций при помощи ряда Неймана.
4. Для проверки указанной общей схемы проведены вычислительные эксперименты на примерах нелокальных задач двумерной теплопроводности.

Наши результаты дополняют прежнее исследование [2]. Более подробные формулировки представлены в работе [3]. Исследование поддержано Московским центром фундаментальной и прикладной математики (грант 075-15–2019-1621).

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Псевдодифференциальные операторы в асимптотических задачах для систем дифференциальных и разностных уравнений

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На примерах задач об асимптотиках систем дифференциальных уравнений с частными производными с локализованными правыми частями и асимптотиках многомерных ортогональных полиномов типа полиномов Эрмита мы показываем, как с помощью псевдодифференциальных операторов и операторного исчисления Фейнмана—Маслова можно свести исходные векторные задачи к скалярным, которые с точки зрения квазиклассического приближения оказываются существенно проще исходных. Излагаемые соображения позволяют написать эффективные асимптотические формулы для ряда важных физических задач.

Работа выполнена при поддержке Российского научного фонда (проект № 21-11-00341).

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Эта-инварианты для операторов с параметром, ассоциированных с действием дискретной группы

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В работе исследуются эта-инварианты (см., напр., [1]) для класса нелокальных операторов с параметром, ассоциированных с изометрическим действием дискретной группы степенного роста на гладком замкнутом многообразии.

Пусть X — гладкое замкнутое риманово многообразие, а Γ — подгруппа группы изометрий многообразия X. Будем предполагать, что Γ является группой полиномиального роста в смысле Громова [2]. Через $\Psi_p^m(X)$ обозначим пространство классических псевдодифференциальных операторов (ПДО) с параметром $p \in \mathbb{R}$ (см. напр., [3]) порядка $\leqslant m$. На X рассматриваются семейства операторов

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вида
$$D(p) = \sum_{(\gamma,k)\in\Gamma\times\mathbb{Z}} D_{\gamma,k}(p) T_\gamma e^{2\pi i k p} \colon C^\infty(X) \longrightarrow C^\infty(X), \tag{1}$$
где $D_{\gamma,k} \in \Psi_p^m(X)$, а $T_\gamma u(x,p) = u(\gamma^{-1}(x),p)$ — представление группы Γ операторами сдвига, индуцированное действием на X. В продолжение работы [4], используя подход Мельроуза, мы определяем η-инвариант обратимого семейства операторов (1) как некоторую регуляризацию числа вращения и устанавливаем его основные свойства. В частности, доказано, что η-инвариант обладает логарифмическим свойством, а также получена формула для производной η-инварианта семейства операторов по параметру.

Работа выполнена при частичной финансовой поддержке конкурса «Молодая математика России», а также РФФИ и Немецкого научно-исследовательского сообщества (проект № 21-51-12006).

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О разрешимости модели движения растворов полимеров с памятью

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В ограниченной области $\Omega \subset \mathbb{R}^n$, $n = 2, 3$, на отрезке времени $[0, T]$, $T > 0$, рассматривается следующая начально-краевая задача:
$$\frac{\partial v}{\partial t} + \sum_{i=1}^n v_i \frac{\partial v}{\partial x_i} - \nu\Delta v - \varkappa\frac{\partial \Delta v}{\partial t} - 2\varkappa \mathrm{Div}\Big(\sum_{i=1}^n v_i \frac{\partial \mathcal{E}(v)}{\partial x_i}\Big) - 2\varkappa \mathrm{Div}\Big(\mathcal{E}(v)W_\rho(v) -$$
$$- W_\rho(v)\mathcal{E}(v)\Big) - \frac{\mu_1}{\Gamma(1-\beta)} \mathrm{Div} \int_0^t (t-s)^{-\beta} \mathcal{E}(v)\big(s, z(s;t,x)\big)\, ds + \mathrm{grad}\, p = f;$$
$$z(\tau; t, x) = x + \int_t^\tau v\big(s, z(s;t,x)\big)\, ds, \quad t, \tau \in [0, T], \quad x \in \Omega,$$
$$\mathrm{div}\, v = 0, \quad (t,x) \in (0,T)\times\Omega; \quad v(x,0) = v_0(x), \quad x \in \Omega; \quad v|_{\partial\Omega\times[0,T]} = 0.$$

Здесь v — вектор-функция скорости, p — функция давления среды, f — плотность внешних сил, $z(\tau; t, x)$ — траектория частицы среды, указывающая в момент времени τ расположение частицы среды, находящейся в момент времени t в

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точке x, $\alpha > 0$ — скалярный параметр, $\mu_0 > 0$, $\mu_1 \geqslant 0, 0 < \beta < 1$ — некоторые константы. $\Gamma(\beta)$ — гамма-функция Эйлера, $\mathcal{E} = (\mathcal{E}_{ij}(v))$, $\mathcal{E}_{ij}(v) = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$, $i, j = \overline{1, n}$ — тензор скоростей деформации, $W_\rho(v) = \int_{\mathbb{R}^n} \rho(x-y) W(t, y)\, dy$ — сглаживание тензора завихренности $W = (W_{ij}(v))$, $W_{ij}(v) = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i}\right)$, $i, j = \overline{1, n}$,, где $\rho : \mathbb{R}^n \to \mathbb{R}$ — гладкая функция с компактным носителем, такая что $\int_{\mathbb{R}^n} \rho(y)\, dy = 1$ и $\rho(x) = \rho(y)$ для x и y с одинаковыми евклидовыми нормами, $\mathrm{Div}\, A$ — дивергенция тензора A, то есть вектор $\mathrm{Div}\, A = \left(\sum_{j=1}^{n} \frac{\partial a_{1j}(t,x)}{\partial x_j}, \cdots, \sum_{j=1}^{n} \frac{\partial a_{nj}(t,x)}{\partial x_j}\right)$.

Данная начально-краевая задача описывает движение растворов полимеров (см. [1–3]). Для изучаемой задачи на основе аппроксимационно-топологического метода изучается существование слабых решений.

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О существовании слабых решений начально-краевой задачи для неоднородной несжимаемой модели Кельвина—Фойгта

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В докладе рассматривается модель неоднородной несжимаемой жидкости, которую также называют моделью несжимаемой жидкости с переменной плотностью. Результаты доклада были получены совместно с М. В. Турбиным.

Подобные модели активно исследуются с середины прошлого века до наших дней. Первая постановка задачи о слабых решениях для несжимаемой системы Навье—Стокса с переменной плотностью была предложена А. В. Кажиховым (Докл. АН СССР, 1974). В его работе предполагается, что начальное условие на плотность ρ отделено от нуля, то есть существует константа $m > 0$ такая, что $\rho(x, 0) = \rho_0(x) \geqslant m$, $\rho_0 \in L_\infty(\Omega)$.

В работе Симона (SIAM J. Math. Anal., 1990) для слабой постановки задачи неоднородной несжимаемой системы Навье–Стокса была предпринята попытка ослабить условие отделимости от нуля начального условия на плотность. А именно, предполагается, что $\rho_0(x) \geqslant 0$, $\rho_0 \in L_\infty(\Omega)$, $1/\rho_0 \in L_{6/5}(\Omega)$.

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В монографии П.-Л. Лионса (Mathematical Topics in Fluid Mechanics, 1996) было доказано существование слабого решения начально-краевой задачи для неоднородной несжимаемой системы Навье—Стокса при предположениях: $\rho_0(x) \geqslant 0, \rho_0 \in L_\infty(\Omega)$ с условием $\rho u(x,0) = m \in L_2(\Omega)$ и $m = 0$ при почти всех $x \in \Omega$, при которых $\rho_0(x) = 0$, а также $|m|^2/\rho_0 \in L_1(\Omega)$.

При этом ещё с середины 19-го века известно достаточно большое число сред, которые не удовлетворяют ньютоновскому реологическому соотношению. В докладе рассматривается одна из таких сред, которая называется моделью Кельвина—Фойгта. В докладе рассматривается существование слабых решений начально-краевой задачи для неоднородной несжимаемой модели Кельвина—Фойгта для любой начальной плотности $\rho_0 \geqslant 0$, $\rho_0 \in L_\infty(\Omega)$ (см. [1, 2]).

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Численная оценка влияния нерегулярности границы области на решение краевой задачи для уравнения Лапласа

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Доклад посвящен оценке влияния нерегулярности границы области на решение задачи Дирихле–Неймана для уравнения Лапласа

$$\Delta u\,(x,y) = 0, (x,y) \in \Pi, \tag{1}$$

в прямоугольной области $\Pi = (x,y) \times (0, 2\pi)$, ограниченной горизонтальным отрезком $\{y = h,\ 0 \leqslant x \leqslant 2\pi\}$, вертикальными отрезками ($\{x = 0,\ -h \leqslant y \leqslant h\}$, $\{x = 2\pi,\ -h \leqslant y \leqslant h\}$) и липшицевой кривой γ, соединяющей точки $(0, -h)$ и $(2\pi, -h)$. При этом $u\,(x,y)$ является 2π-периодической функцией по переменной x, удовлетворяет условию Дирихле с функцией $\varphi\,(x)$ на верхней границе и однородному условию Неймана на γ.

Такие задачи возникают в гидродинамике, например при моделировании цунами и «волн-убийц». При этом остаётся важный вопрос о влиянии нерегулярности границы на вид решения $u\,(x,y)$. Ранее была получена аналитическая оценка разности решений возмущённой и невозмущённой задач в норме пространства

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Соболева H^1 на общей области их определения [1]. В данной работе получены численные решения рассматриваемой задачи с помощью сеточной (конечно-разностной) аппроксимации. Проведено сравнение численных и аналитических решений с помощью полученных в [1] оценок в зависимости от вида границы γ и точности её сеточной аппроксимации [2].

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Об одной конструкции схемы Русанова для численного решения уравнений специальной релятивистской магнитной гидродинамики

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Схема Русанова является одной из наиболее робастных в классе схем численного решения задачи Римана. Основной недостаток схемы — повышенная диссипация численного решения на разрывах. При этом было показано, что использование кусочно-полиномиальных реконструкций позволяет получить малодиссипативную схему, соответствующую схемам типа Рое и HLL при использовании аналогичной реконструкции. В случае уравнений специальной релятивистской магнитной гидродинамики полное спектральное разложение для задачи Римана не имеет аналитического решения. В статье предлагается распространение схемы Русанова с использованием кусочно-параболического представления решения на уравнения специальной релятивистской магнитной гидродинамики. Проведена верификация разработанной схемы на классических задачах о распаде произвольного разрыва.

Исследование выполнено за счет гранта Российского научного фонда № 23-11-00014.

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Математическая модель термотока в материале дивертора термоядерного реактора

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Движение расплава является одним из самых разрушительных последствий развития неустойчивостей на современных установках для изучения термоядерной плазмы. При оплавлении и дальнейшем разогреве материала стенок, лимитеров или дивертора они начинают испаряться. Место контакта расплава и испарённого газа приводит в движение расплав под действием термоэлектрических эффектов из-за большого магнитного поля, необходимого для удержания плазмы. Подробное моделирование поможет разобраться в механизмах развития термотоков и позволит разработать методы их подавления. В докладе представлена модель распределения тока в образце вольфрама и испаряемом веществе при нагреве поверхности электронным пучком [1]. Модель в аксиально-симметричной постановке основана на решении уравнений электродинамики и двухфазной задачи Стефана. Уравнение электродинамики на основе рассчитанных значений температуры решается в области образца и в области над образцом. Проведен анализ модели в упрощенной постановке при постоянных значениях электрического сопротивления и термоэдс в газе и металле. Показана зависимость амплитуды и изолиний термотоков от распределения температуры на поверхности образца. Показано, что подробность учета коэффициентов задачи Стефана оказывает большое влияние на результаты решения уравнения электродинамики. Рассмотрен случай переменных значений электрического сопротивления и термоэдс в газе и металле. Использованы упрощенное выражение для электрического сопротивления как зависимости от степени ионизации и оценка коэффициента Зеебека для расчета термоэдс. Степень ионизации определяется как корень уравнения Саха. Параметры модели взяты из экспериментов на стенде Beam of Electrons for materials Test Applications (BETA), созданного в ИЯФ СО РАН [2]. Расчет проведен для анализа и планирования натурных экспериментов с целью определения влияния сил Ампера на динамику вещества. Дальнейшее развитие модели предполагает уточнение расчета удельной электропроводности газа и термоэдс через интеграл по энергии электронов, в том числе включение учета зависимости этих параметров от плотности газа.

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Prediction of the parameters of the trap for plasma confinement in helical magnetic field

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Для решения задач управляемого термоядерного синтеза необходимо удерживать высокотемпературную плазму достаточной плотности в ограниченной области пространства. Основным методом термоизоляции плазмы, рассматриваемым на сегодняшний день, является её удержание в магнитном поле с различными конфигурациями. Наибольший прогресс достигнут в системах с тороидальной топологией магнитного поля. Альтернативным подходом является удержание плазмы в открытых магнитных системах, где поле близко к осесимметричному. Плюсами данного подхода являются более эффективное использование энергии магнитного поля, масштабируемость и инженерная простота системы. Удержание плазмы магнитным полем с винтовой симметрией было предложено в качестве развития метода многопробочного удержания [1]. В системе отсчёта вращающейся плазмы движение магнитных возмущений имеет компоненту скорости, сонаправленную с магнитным полем, что позволяет передавать импульс запертым частицам. Столкновения между пролётными и запертыми частицами обеспечивают эффективную силу, действующую на плазму в целом и способствующую возврату ионов в область удержания. Установка СМОЛА (Спиральная Магнитная Открытая Ловушка) разработана и построена в 2017 году в Институте ядерной физики СО РАН им. Будкера для экспериментальной проверки этой идеи [2]. В докладе представлен метод прогнозирования параметров устройства для удержания плазмы в спиральном магнитном поле, основанный на математическом моделировании [3].

Математическая модель переноса вещества в винтовом магнитном поле построена на основе стационарного уравнения переноса и параметров установки СМОЛА. Математическое моделирование процесса было проведено впервые в работе [3]. Проведена валидация модели и определение влияния параметров эксперимента на эффект удержания. Для численной реализации используется метод установления и более экономичный метод верхней релаксации. Для калибровки модели используются данные измерений. Цель работы состоит в определении диффузии и прогнозе параметров ловушки.

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Спектральный анализ и представление решений вольтерровых интегро-дифференциальных уравнений в гильбертовых пространствах

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Работа посвящена исследованию вольтерровых интегро-дифференциальных уравнений с операторными коэффициентами в гильбертовых пространствах. Главная часть рассматриваемых уравнений представляет собой абстрактное гиперболическое уравнение, возмущенное слагаемыми, содержащими вольтерровы интегральные операторы. Указанные интегро-дифференциальные уравнения могут быть реализованы как интегро-дифференциальные уравнения в частных производных, возникающие в теории вязкоупругости, теории распространения тепла в средах с памятью и ряде других важных приложений. Для широкого класса ядер интегральных операторов установлены результаты о существовании и единственности классических решений указанных уравнений, полученные на основе подхода, связанного с применением теории полугрупп операторов. Проводится спектральный анализ генераторов полугрупп операторов, порождаемых указанными интегро-дифференциальными уравнениями. На основе полученных ранее результатов устанавливается связь между спектрами оператор-функций, являющихся символами указанных интегро-дифференциальных уравнений и спектрами генераторов полугрупп операторов. На основе спектрального анализа генераторов полугрупп операторов и соответствующих оператор-функций получены представления решений рассматриваемых интегро-дифференциальных уравнений (см. [1, 2]).

Работа выполнена при финансовой поддержке Минобрнауки РФ в рамках реализации программы Московского центра фундаментальной и прикладной математики (Соглашение № 075-15-2022-284).

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Об (отсутствии) непрерывной зависимости решений эллиптических функционально-дифференциальных уравнений от параметров

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Рассматриваются эллиптические функционально-дифференциальные уравнения с преобразованиями, включающими сжатия (растяжения) аргументов старших производных неизвестной функции. В ряде случаев свойства таких уравнений демонстрируют неустойчивость по отношению к величине коэффициентов сжатия. Это происходит, в частности, в следующих ситуациях.

1) Уравнения, в которых все параметры сжатия (растяжения) мультипликативно соизмеримы (являются целыми степенями, как положительными, так и отрицательными, одного и того же параметра $q > 1$). «Предельное» дифференциальное уравнение, в котором положено $q = 1$, может быть сильно эллиптическим, но исходное функционально-дифференциальное уравнение не будет сильно эллиптическим ни при каком значении q, сколь угодно близком к 1.

2) Уравнения, в которых в старшей части присутствуют слагаемые с различными параметрами сжатия $p > 1$ и $q > 1$. Сильная эллиптичность устойчива по отношению к малым возмущениям этих параметров в окрестности их мультипликативно несоизмеримых значений, и неустойчива в противном случае. Возмущение одного из этих параметров может привести к существенному изменению свойств краевой задачи (появлению бесконечномерного ядра и негладких решений) независимо от того, насколько мал коэффициент при соответствующем слагаемом в уравнении.

3) Уравнения, в которых присутствуют комбинации сжатия и сдвигов аргументов старших производных. Условия, обеспечивающие однозначную разрешимость и гладкость решений краевой задачи, формулируются при помощи спектрального радиуса соответствующего функционального оператора (условия носят характер достаточных, однако их нарушение также может привести к возникновению бесконечномерного ядра и негладких решений). Оказывается, значение этого спектрального радиуса зависит от того, является ли коэффициент сжатия числом трансцендентным или алгебраическим, а в случае алгебраического числа — от того, каковы коэффициенты его минимального многочлена.

Автору оказана финансовая поддержка Минобрнауки РФ в рамках государственного задания (номер проекта FSSF-2023-0016).

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Математическое моделирование вирусной инфекции

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Целью данной работы является разработка и исследование многомерной модели респираторной вирусной инфекции, в которой рассматриваются с одной стороны инфицированная ткань, а с другой — весь организм человека. Математическая модель состоит из нелокальных уравнений реакции–диффузии с запаздыванием.

Показано, что распространение вирусной инфекции по инфицированной ткани может быть описано реакционно-диффузионной волной. Изучено влияние врожденного и адаптивного иммунного ответа на скорость распространения волны и вирусную нагрузку. Проведено численное моделирование этой задачи и показано, что результаты численных расчетов согласуются с аналитическими результатами.

Работа выполнена при поддержке Министерства науки и высшего образования Российской Федерации (Мегагрант, соглашение № 075-15-2022-1115).

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Геометрия преобразований Мутара двумерных операторов Дирака

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Мы изложим преобразование Мутара для двумерных операторов Дирака, объясним его геометрический смысл с помощью спинорных представлений поверхностей в четырехмерном пространстве и укажем некоторые применения для построения точных решений двумерных солитонных уравнений (Дэви–Стюартсона II и модифицированного Веселова–Новикова).

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Кинетическое состояние и возникновение марковской динамики в точно решаемых моделях открытых квантовых систем

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В фундаментальной работе Н. Н. Боголюбова [1] предложены глубокие идеи по выводу кинетического уравнения Больцмана из уравнений механики. Ключевые предположения состояли в том, что после начального периода динамики наступает так называемая кинетическая стадия, а также имеет место ослабление корреляций при свободной динамике (т.е., если «выключить» столкновения частиц). Кинетическая стадия характеризуется тем, что форма многочастичной функции распределения всецело определяется одночастичной. К сожалению, эти предположения остаются не только недоказанными, но даже и сформулированными лишь на «физическом» уровне строгости. Сам Н. Н. Боголюбов признавал, что точные формулировки следует вывести в будущем [2]. Строгое обоснование этих предположений для газа взаимодействующих частиц остается открытой задачей, но интересна также проверка этих предположений в более простых моделях [3]. В этой работе в качестве последней берется модель открытой квантовой системы [4]. Изучение динамики открытых квантовых систем важно и с практической точки зрения.

Рассматривается динамика в гильбертовом пространстве $\mathcal{H} = \mathcal{H}_\mathrm{S} \otimes \mathcal{H}_\mathrm{R}$, где $\mathcal{H}_\mathrm{S} = \mathbb{C}^2$ — гильбертово пространство системы, а $\mathcal{H}_\mathrm{R} = \mathcal{F}_\mathrm{b}\big(L^2([0,\infty))\big)$ — гильбертово пространство резервуара, бозонное фоковское пространство над $L^2([0,\infty))$. Гамильтониан имеет вид

$$H = \Omega(P_1 - P_0) + H_\mathrm{R} + \lambda H_\mathrm{I},$$

где $\Omega > 0$, P_0 и P_1 — проекторы на элементы некоторого ортонормированного базиса пространства $\mathbb{C}^2$, H_R — гамильтониан свободного бозонного квантового поля, H_I — гамильтониан взаимодействия. Тогда состояние системы и резервуара (оператор плотности — ядерный положительный оператор в $\mathcal{H}$ с единич-

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ным следом) в момент времени t имеет вид $\rho(t) = e^{-itH}\rho_0 e^{itH}$, где ρ_0 — начальное состояние. Состояние системы имеет вид $\rho_{\mathrm{S}}(t) = \mathrm{Tr}_{\mathrm{R}}\rho(t)$ — оператор плотности в $\mathcal{H}_{\mathrm{S}}$, где Tr_{R} — частичный след по резервуару, действующий по правилу $\mathrm{Tr}_{\mathrm{R}}(A \otimes B) = A \,\mathrm{Tr}\, B$.

Утверждение о предрелаксации имеет вид

$$\lim_{t\to\infty} \mathrm{Tr}\{O[\rho(t) - \mathcal{R}\rho_{\mathrm{S}}(t)]\} = 0 \tag{1}$$

для всех $O \in \mathcal{O}$, где $\mathcal{O}$ — некоторый класс операторов, а $\mathcal{R}$ — отображение, ставящее в соответствие оператору плотности системы $\rho_{\mathrm{S}}(t)$ полный оператор плотности $\rho(t)$. Другими словами, кинетическое состояние $\mathcal{R}\rho_{\mathrm{S}}(t)$ характеризуется тем, что полное состояние системы и резервуара полностью определяется оператором плотности исключительно системы. Это аналог предположения Боголюбова о кинетическом состоянии для данной модели. В докладе будет рассказано о математически строгом обосновании утверждений типа (1), а также утверждений об ослаблении корреляций для двух моделей открытой квантовой системы (для двух разных гамильтонианов взаимодействия).

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