Dressing method in stability problem of nonlinear waves

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Abstract

In this talk I present a review of my results devoted to stability of nonlinear waves in the integrable systems including the KDV [1], KP [2], NLS [3–5] and Burgers [6] equations. The dressing method was suggested by A.B. Shabat (1972) to the KDV case on the base of the Marchenko equation and later got development in a series of papers by V.E. Zakharov and A.B. Shabat. First time we together with Sasha Mikhailov [1] applied the dressing procedure to stability analysis of cnoidal wave for the KDV equation. Remarkably that this wave can be represented as soliton lattice that one allows to understand the nonlinear behavior of a the KDV soliton propagating along the cnoidal wave. In fact, propagation of solitons on the cnoidal wave background was a prototype of breather solitons in the NLS. It was the first example of application of the IST to the non-vanishing potentials. Later this idea was exploited in studies of the nonlinear stage of modulation instability in 1D NLS [3] where first time the breather-type soliton solution oscillating on the condensate was constructed.

In papers [2, 4] we demonstrated that the dressing procedure gives a big advantage in linear stability solution. For both KP and NLS equations we showed that the linear stability problem for cnoidal waves reduces to algebraic expressions for growth rates. In particular, for the KP equation with negative dispersion we proved linear stability for cnoidal wave relative to transverse perturbations and got instability in the case of the positive dispersion. For the NLS case in [4] we developed the linear stability analysis for cnoidal wave which can be represented as a soliton lattice also. When the distance l between solitons tends to infinity, this lattice transforms into the one-soliton solution which is by this reason stable. In another limit $l \to 0$, the cnoidal wave transforms into the condensate. In both focusing and defocusing cases we got analytical expressions for the growth rate as a function of quasi-momentum. In the defocusing case the wave is stable. In the focusing case, the wave undergoes the modulation instability. The nonlinear stage of this instability results in the Fermi-Pasta-Ulam recurrence [5]. In the paper with Grisha Falkovich we obtained very interesting result [6] which has a quantum mechanical meaning. Consider the oscillator potential for the 1D Schrodinger operator. As well known the spectrum of this operator is proportional to $n+1/2$. Question: is it possible to find another potential which has the same spectrum and the same asymptotics at large $|x|$? We constructed first time such potential by means of the dressing procedure. It solves stability problem for the self-similar solution for the Burgers equation.

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[1] E.A. Kuznetsov, A.V. Mikhailov, *Stability of stationary waves in nonlinear weakly dispersive media.* ZhETF, **40**, 855, (1974) [Sov. Phys. JETP **40**, 855 (1975)].

[2] E.A. Kuznetsov, M.D. Spector, G.E. Falkovich, *On the Stability of Nonlinear Waves in Integrable Models.* Physica **10D**, 379 (1984).

[3] E.A. Kuznetsov, *Solitons in parametrically unstable plasma*, DAN SSSR **236**, 575 (1977) [Sov. Phys. Dokl., **22**, 507-508 (1977)].

[4] E.A. Kuznetsov, M.D. Spector, *Modulation instability of soliton train in the fiber communication systems*, Teor. Mat. Fiz. (Theor. Math. Phys.), **120**, 222-236 (1999).

[5] E.A. Kuznetsov, *Fermi-Pasta-Ulam recurrence and modulation instability* Pis'ma ZhETF, **105**, no.2 pp. 108-109 (2017) [JETP Letters, **105**, 125–129 (2017)].

[6] E.A. Kuznetsov, G.E. Falkovich, *On the Stability of Self-Similar Solutions in the Burgers Equation.* Phys. Lett. **86A**, 203-204, (1981).