International conference
“Number-theoretic aspects of linear algebraic groups and algebraic varieties: results and prospects”

dedicated to 85-th anniversary of academician V. P. Platonov

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Victor Buchstaber. n-valued groups and Fermat's Last Theorem

In 1971, V. M. Buchstaber and S. P. Novikov proposed a construction motivated by the theory of characteristic classes. This construction describes a multiplication such that the product of any pair of points is a multiset of n points. An axiomatic definition of n-valued groups, the results of their algebraic theory, and topological applications were obtained in a subsequent series of works by V. M. Buchstaber. Currently, the theory of n-valued (formal, finite, discrete, topological, and algebro-geometric) groups and their applications in various areas of mathematics and mathematical physics are being developed by a number of authors.

In this talk, for each n, the notion of classes of symmetric n-algebraic n-valued groups will be introduced. For $n = 2$ and 3, a description of the universal objects in these classes will be presented.

An important class of n-algebraic n-valued groups is given by the groups $\mathbb{G}_n$ over the field of complex numbers $\mathbb{C}$. We show that the n-valued multiplication $x * y = [z_1, ..., z_n]$ in $\mathbb{G}_n$ is realized in terms of the eigenvalues of the Kronecker sum of the Frobenius companion matrices of the polynomials $t^n - x$ and $t^n - y$ in the variable t. We introduce $(n \times n)$-matrices $W_n(z; x, y)$ such that for any n their determinant is an integer-valued homogeneous symmetric polynomial $p_n(z; x, y)$ defining the operation $x * y$. The matrix $W_n(1; (-1)^n, 1)$ is the classical Wendt matrix, which was introduced in 1894 in connection with Fermat's Last Theorem. Groups $\mathbb{G}_n$ and polynomials $p_n(z; x, y)$ arise and play an important role in various fields of mathematics and mathematical physics.

In this talk we will present results that open up a new approach to the well-known problem of the Fermat equation for the Kummer tower of cyclotomic fields.

This talk is based on the results of the preprint arXiv: 2505.04296, V. Buchstaber, M. Kornev, *n-Valued Groups, Kronecker Sums, and Wendt's (x, y, z)-Matrices.*

Vladimir Chernousov. Applications of the finiteness conjecture on groups with good reduction

In the talk we will discuss connections between the finiteness conjecture for groups with good reduction and J.-P. Serre's question on the classification of Albert algebras via cohomological invariants. Joint work with A. Rapinchuk and I. Rapinchuk.

Gleb Fedorov. On the unboundedness of period lengths of functional continued fractions in a hyperelliptic field

The report is devoted to joint results with V. P. Platonov concerning the problem of the unboundedness of period lengths of continued fractions of elements from a hyperelliptic field. The famous Abel theorem establishes a criterion for the existence of elements in a hyperelliptic field that have a periodic expansion into a continued fraction. Subsequently, a significant number of studies were aimed at studying the problem of periodicity of functional continued fractions, including obtaining upper bounds on possible period lengths. Until now, the problem of the finiteness of the set of possible period lengths of continued fractions for a given hyperelliptic field has remained open. In our report, we will present results that give a negative solution to this problem.

Philippe Gille. Semiglobal fields and Hasse principle

A semiglobal field is a function field in one variable over a complete discrete valued field. We review results on Hasse principle for homogeneous spaces under connected linear algebraic groups over semiglobal fields. The analogy with number fields is sharp, especially for function fields of p-adic curves.

Alex Lubotzky. Uniform stability of lattices in high-rank semisimple groups

Lattices in high-rank semisimple groups enjoy a number of special properties like superrigidity, quasi-isometric rigidity, first-order rigidity, and more. In this talk, we will add another one: uniform (a.k.a. Ulam) stability. Namely, it will be shown that (most) such lattices D satisfy: every finite-dimensional unitary “almost-representation” of D (almost w.r.t. a sub-multiplicative norm on the complex matrices) is a small deformation of a true unitary representation. This extends a result of Kazhdan (1982) for amenable groups and of Burger–Ozawa–Thom (2013) for $\mathrm{SL}(n, \mathbb{Z})$, $n > 2$. The main technical tool is a new cohomology theory (“asymptotic cohomology”) that is related to bounded cohomology in a similar way to the connection of the last one with ordinary cohomology. The vanishing of H^2 w.r.t. a suitable module implies the above stability. The talk is based on a joint work with L. Glebsky, N. Monod, and B. Rangarajan. To appear in the Memoirs of the European Mathematical Society.

Dmitri Orlov. Geometric realizations of algebraic objects and derived noncommutative algebraic geometry

Denis Osipov. Local regluings of families of curves and sheaves on them and the Deligne–Riemann–Roch theorem

Consider the formal punctured disc and the group which is the semidirect product of the group of invertible functions on this disk and the group of automorphisms of this disk. More precisely, this group has to be considered as a group ind-scheme $\mathcal{G}$ that assigns to every commutative ring A the group $\mathcal{G}(A)$ which is the semidirect product of the group of invertible element $A((t))^*$ of the A-algebra of Laurent series $A((t))$ and the group of continuous A-automorphisms of the algebra $A((t))$. The group ind-scheme $\mathcal{G}$ remarkably acts on the moduli space that parameterizes quintets: a projective curve, an invertible sheaf on the curve, a smooth point on the curve, a formal local parameter at the point, a formal trivialization of the sheaf at the point. Besides, there is the Deligne–Riemann–Roch theorem for invertible sheaves on families of smooth projective curves. I will describe the local Deligne–Riemann–Roch theorem as the equivalence of two central extensions of $\mathcal{G}$ by the multiplicative group $\mathbb{G}_m$. Note that one of the key tools to construct one of the two central extensions is the Contou-Carrère symbol that is the bimultiplicative pairing on the group $A((t))^*$ with values in the group $\mathbb{G}_m(A) = A^*$.

Ivan Panin. SK_1 and Grothendieck–Serre conjecture

Bjorn Poonen. Multiples of a subvariety in an algebraic group

I will explain short proofs of the following, which extend results of Bogomolov and Tschinkel. Let X be a subgroup of a commutative algebraic group G over the algebraic closure k of a finite field such that X generates G. Then $\bigcup_{\phi \in \operatorname{End} G} \phi(X(k)) = G(k)$. If G is semiabelian, then we have the stronger conclusion $\bigcup_{n \geq 1} nX(k) = G(k)$.

Vladimir Popov. Coordinate algebras of algebraic groups: generators and relations

For coordinate algebras of Abelian varieties, the problem of finding a presentation by generators and relations canonically determined by the group structure was considered and solved in the classical works of D. Mumford and G. Kempf. Since every connected algebraic group is an extension of a connected linear algebraic group by an Abelian variety, a similar problem naturally arises for connected affine algebraic groups. In geometric terms, it means finding, for each connected linear algebraic group G, an embedding of its group variety into an affine space that is canonically determined by the group structure of G. Although some G (e.g., $GL(n)$, $SL(n)$, $SO(n)$, $Sp(n)$, $Spin(n)$) are defined through embeddings into affine spaces, these embeddings, not being defined exclusively in terms of the group structure, in the indicated respect are accidental. This talk aims to describe a solution to the specified problem. It is based on the solution to two problems posed by D. E. Flath and J. Tauber in 1992. From the point of view of this theory, the usual naive presentation of $SL(n)$ as a hypersurface $\det = 1$ in n^2-dimensional affine space is canonical only for $n = 2$: the canonical one defines $SL(3)$ as the intersection of 2 homogeneous and 2 non-homogeneous quadrics in a 12-dimensional affine space, $SL(4)$ as the intersection of 20 homogeneous and 3 non-homogeneous quadrics in a 28-dimensional affine space, etc.

Yuri Prokhorov. Conic bundle structures on singular Fano 3-folds

We discuss certain sufficient conditions of the existence of birational conic bundle structures on Fano 3-folds with terminal singularities. As an application we prove non-rationality of some such Fano 3-folds, which are very close to rational ones.

Andrei Rapinchuk. On almost strong approximation in reductive algebraic groups

A criterion for strong approximation in algebraic groups was obtained by V. P. Platonov in characteristic zero, and by G. A. Margulis and G. Prasad in positive characteristic. It follows from this criterion that strong approximation never holds for nonsimply connected groups (in particular, algebraic tori) and a finite set of places. We will report of a recent work where we show that a slightly weaker property, which we termed "almost strong approximation" can hold for nonsimply connected reductive groups and some special infinite sets of places. Applying this fact to maximal tori of an absolutely almost simple simply connected group, we generalize some results on the congruence subgroup problem. Joint work with Wojciech Tralle.

Peter Sarnak. Restricted Chebotarev Theorems for $\mathrm{SL}(2,\mathbb{Z})$ and related groups

The "Chebotarev Prime Geodesic Theorem" for primitive conjugacy classes in $\mathrm{SL}(2,\mathbb{Z})$ (which correspond to closed geodesics on the quotient) is well known and can be proved using the Selberg Trace Formula. We examine local versions of this Theorem where the closed geodesics are restricted in various ways and in particular to being simple. Congruence and arithmetic features intervene decisively as well as transitivity properties of the action of associated mapping class groups.

Maxim Vsemirnov. Groups $(2,3,7;n)$, their quotients and related number-theoretic questions

Umberto Zannier. Bounded generation in linear groups and exponential parametrizations

In fairly recent joint work with Corvaja, Rapinchuk, Ren, we applied results from Diophantine S-unit theory to problems of "bounded generation" in linear groups: this property is a strong form of finite generation and is useful for several issues in the setting. Focusing on "anisotropic groups" (i.e. containing only semi-simple elements), we could give a simple essentially complete description of those with the property. More recently, in further joint work also with Demeio, we proved the natural expectation that sets boundedly generated by semi-simple elements (in linear groups over number fields) are "sparse". Actually, this holds for all sets obtained by exponential parameterizations. As a special consequence, this gives back the previous results with a different approach and additional precision and generality.

Vladimir Zhgoon. The finiteness theorems for generalized Jacobians with nontrivial torsion

The questions on the finiteness of the set of rational points on an algebraic curve over algebraic number fields are fundamental questions of arithmetic algebraic geometry. In my talk I shall discuss new questions about the finiteness of the set of generalized Jacobians of a curve C, defined over algebraic number field, that are associated to the modules m such that a fixed class of finite order in the Jacobian of C lifts to the torsion class in the generalized Jacobian J_m. On one hand, such set of generalized Jacobians with the property mentioned above is infinite. On the other hand, we obtained finiteness results under additional assumptions on the support of m or on the structure of the subgroup J_m. These results were applied to the problem of quasi-periodicity of continued rational fractions constructed in the power series $k((1/x))$, for some special class of elements of the field of rational functions on the hyperelliptic curve $y^2 = f(x)$ over an algebraic number field. In particular, for any n we established the finiteness of the set of polynomials $g(x)$ of degree bounded by n, for which the expansion of the elements $g(x)\sqrt{f(x)}$ in the continued fraction is periodic.