# "Sino-Russian Interdisciplinary Mathematical Conference-2"

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# Abstracts of talks

**Vladimir Bogachev. Invariant measures of diffusions and solutions of stationary Kolmogorov equations**

We discuss connections between two classical concepts: invariant measures of diffusions and solutions of stationary Kolmogorov equations. Sufficient conditions for existence and uniqueness of such objects will be analysed.

**George Brovko. Binary grading of tensors in the rational theory of continuum mechanics**

Frame reference, related frames. Transformation of tensors under a change of frame reference system, binary grading of vectors and tensors. Objective tensors of different ranks; types of objectivity: material, spatial and mixed types.

Simple commutative diagrams of objective tensors of one and the same rank and different types of objectivity, transitional tensors, interweaving (intertwining) operators. Tensor processes, representations through introspections (retrospections) and the instant of time, transformation under a change of a frame reference.

Mappings between tensors, types of generalized isotropy. Maps between objective tensors: physically the same and mathematically the same — independent of the reference frame. Frame independence criterion (theorem), special constraints on the mathematical form of the map. Examples. The absence of constraints for Ilyushin-type mappings.

Inductor and package of its conductors (mappings). Ilyushin-type inductor generates a package of conductors of the most general kind. Objective derivatives by time of objective tensor processes are the conductors of the Ilyushin-type inductor — the material derivative by time. Lagrangian and Eulerian representation of objective derivatives.

**Victor Buchstaber. The Chern–Dold character in complex cobordisms**

Let $A^{n+1} = \mathbb{C}^{n+1}/\Gamma$ be a principally polarised abelian variety. The space of holomorphic sections of its canonical line bundle $L$ is one-dimensional and generated by the classical Riemann $\theta$-function. According to the Andreotti–Mayer theorem (1967) for a generic principally polarised abelian variety, the theta divisor $\Theta^n \subset A^{n+1}$ given by the equation $\theta(z, \tau) = 0$ is a smooth irreducible algebraic variety of general type.

The talk is focused on the result of V. Buchstaber and A. Veselov, obtained in 2020–2024, which is based on the construction of the Chern–Dold character in the theory of complex cobordisms (Buchstaber, 1970):

*The exponential generating series of the complex cobordism classes of the theta divisors* $[\Theta^n]$, $n = 0, 1, 2, \ldots$, *realizes the exponential of the universal formal group law.*

We will discuss applications of this result to well-known problems in algebraic topology and algebraic geometry, including properties of the denominators of the Todd polynomials calculated by Hirzebruch in 1956 and the hitherto open Milnor problem (1958) on Chern numbers of irreducible smooth algebraic varieties.

**Dayue Chen. Limit theorems for the tagged particle in exclusion processes on regular trees**

We consider exclusion processes on a rooted $d$-regular tree. We start from a Bernoulli product measure conditioned on having a particle at the root, which we call the tagged particle. For $d > 2$, we show that the tagged particle has positive linear speed and satisfes a central limit theorem. We give an explicit formula for the speed. As a key step in the proof, we first show that the exclusion process "seen from the tagged particle" has an ergodic invariant measure. The talk is based on a joint paper with Peng Chen, Nina Gantert, Dominik Schmid.

**Jian Ding. A glimpse at statistical physics from percolation**

I will describe various percolation models in statistical physics, with emphasis on phase transition, critical phenomenon as well as disorder effects. In particular, I will start my talk with Bernoulli percolation, and then present a few important examples of correlated percolation models. While I try to be a "messenger" communicating progress accomplished in the community as a whole, the talk is inevitably biased by my personal interests (for instance, I will spend some time surveying recent progress on the random field Ising model and Gaussian free field related percolation models).

**Huiling Duan. Interfacial flow over slip boundary on underwater hierarchical surface structures**

Interfacial flow is involved in varieties of natural phenomena and plays important roles in industrial applications. Boundary slippage, usually realized by underwater superhydrophobicity, provides a promising method to regulate interfacial flow and even complex bulk fluid transport by controlling the development of boundary layers and changing the near-wall flow structures. In this talk, a systematic study is presented on the stability and flow control of slip boundary with hierarchical surface structures. Basic physical laws underlying the dynamic evolution of the metastable states are revealed, enabling the prediction of plastron longevity and the realization of ultimate stable state both in closed and open systems. Upon the realization of slip boundary, in a confined microfluidics, three-dimensional backflow over liquid-gas interface is discovered, which is demonstrated to be caused by the interfacial tension gradient along the liquid-gas interface. In a turbulent boundary layer flow, slip boundary is implemented to substantially reduce drag. In a separation flow, hierarchical surface structures are demonstrated to be able to effectively control the flow separation. The current work paves the way for practical applications of Navier-slip boundary in flow control.

**Huijun Fan. Differential geometrical method of Landau–Ginzburg models**

In this talk, I will survey the differential geometrical method on the study of LG models. LG models in simplest form is a pair $(M, f)$, where $f$ is a holomorphic function defined on a complex manifold. However, due to the LG/CY correspondence conjecture arising in Gauged linear sigma model, which was used by Witten to explain the mirror symmetry phenomena, this simple models contain enough information about the corresponding Calabi–Yau models. Based on the observation of physicists, I initiated a differential geometrical method to study the LG model since 2011. This method leads to the study of Schrödinger equations and its deformation, $tt^*$-geometrical structures and integral systems.

**Shuai Guo. Enumerative geometry and integrable hierarchy**

Enumerative geometry typically involves counting solutions to geometric problems, often focusing on the enumeration of curves on a given manifold. An integrable hierarchy is a sequence of exactly solvable mathematical equations that describe complex physical phenomena, such as solitons, arising from nonlinear physics. The deep connection between enumerative geometry and integrable systems dates back to Witten's celebrated conjecture. In this talk, we aim to explore the relationships between these two areas and establish a generalization of Witten's conjecture by considering the underlying decorated Riemann surface. This talk is partially based on joint works with Ce Ji, Chenglang Yang, and Qingsheng Zhang.

**Moubin Liu. A semi-resolved CFD-DEM approach for particulate flows with thermal convection**

Particulate flow has a wide range of industrial applications and is frequently modeled with coupled CFD-DEM approaches. In this work, we first identified a simulation gap between the resolved CFD-DEM and unresolved CFD-DEM through a size effect study. We then analyzed the error sources of the conventional unresolved CFD-DEM when modeling particulate flows with comparable mesh size and particle diameter. We finally developed a semi-resolved CFD-DEM model, which combines the advantages of both resolved and unresolved CFD-DEM models. The semi-resolved CFD-DEM uses a drag force model to characterize particle-fluid interaction, while the relative velocity in the drag force model is corrected through kernel-based approximations on the neighboring fluid cells rather than simply taking values in the local cell containing the concerned particle, and the void fraction in the force model is corrected as well. A number of numerical simulations including the sedimentation of single particle, separation of particles in a fluidized bed, spouted bed with heat particles, particulate flows in complex porous media, additive manufacturing (selective laser melting and direct laser deposition) have been conducted. Numerical results from different CFD-DEM approaches are compared together with experimental data. It is shown that the presented semi-resolved CFD-DEM bridges the simulation gap between the resolved CFD-DEM and unresolved CFD-DEM while it is as efficient as the conventional unresolved CFD-DEM and as accurate as the resolved CFD-DEM.

**Ruochuan Liu. Riemann–Hilbert for $p$-adic varieties**

In this talk I will briefly introduce our work on $p$-adic Riemann–Hilbert correspondence for $p$-adic varieties.

**Benshuai Lyu. Application of the Wiener Hopf technique in the modelling of turbulent boundary layer trailing edge noise and supersonic jet screech**

The Wiener Hopf Technique is used to solve two mixed boundary problems in aeroacoustics. In the first problem, an analytical Green's function for the serrated edge wave scattering problem is solved using the Wiener Hopf technique. A closed form analytical Green's function is obtained for piecewise linear serrations and compared with the canonical Green's function for straight edges. The analytical Green's function is verified using the finite element method. Both noise reduction spectra and directivity patterns are studied as a function of source position . Physical mechanism of sound reduction is discussed. In the second problem, the generation of instability waves in a supersonic jet induced by acoustic wave impingement is examined. To obtain the newly excited instability wave, t he scattered sound field d ue to the acoustic impingement is first solved using the Weiner Hopf technique, with the kernel function factored using asymptotic expansions and overlapping approximations. Subsequently, the unsteady Kutta condition is imposed at the nozzle lip, enabling the derivation of the dispersion relation for the newly excited instability wave. A linear transfer function between the upstream forcing and the newly excited instability wave is obtained. T he amplitude and phase delay and their dependence on the frequency are examined. The new model shows improved agreement between the predicted screech frequencies and the experimental data compared to classical model s.

**Andrey Mironov. Integrable Billiards in Cones**

The talk is based on a joined work with Siyao Yin.

The classical Birkhoff conjecture states that if the plane billiards inside a closed smooth convex curve is integrable then the curve is an ellipse. In higher dimensions, all known integrable billiards are inside billiard tables consisting of pieces of quadrics. We study Birkhoff billiards in convex cones in $\mathbb{R}^n$ and prove that billiards in any $C^3$-smooth convex cone are integrable. This provides the first examples of integrable billiard tables in $\mathbb{R}^n$ not related to quadrics.

### Nikolay Nikitin. Secondary flows of Prandtl's second kind. Mechanism of formation and method of prediction

We formulate a mechanism that makes it possible to explain and, in some cases, predict the shape of secondary flows of Prandtl's second kind arising in turbulent flows in straight pipes of non-circular cross-section.

### Dmitry Orlov. Noncommutative algebraic geometry and geometric realisations of algebraic objects

### Gaiane Panina. Concurrent normals problem for convex polytopes

This is a joint work with I. Nasonov.

It is conjectured since long that for any convex body $P \subset \mathbb{R}^n$ there exists a point in its interior which belongs to at least $2n$ normals from different points on the boundary of $P$. The conjecture is known to be true for $n = 2, 3$ (E. Heil, 1985) and $n = 4$ (J. Pardon, 2012).

We treat the same problem for convex polytopes and prove that each simple polytope in $\mathbb{R}^3$ has a point in its interior with 10 normals to the boundary. This is an exact bound: there exists a tetrahedron with at most 10 normals from a point in its interior. The proof is based on Morse–Cerf theory adjusted for polytopes.

### Sergey Pshenichnov. Waves in inhomogeneous viscoelastic materials

The work is devoted to the improvement of one of the methods for constructing solutions to problems of transient wave processes in viscoelastic inhomogeneous bodies. Piecewise homogeneous and functionally graded materials are considered. The disturbances propagations domain is considered limited. The hereditary properties of materials are characterized by linear Boltzmann–Volterra relations with hereditary kernels of various types. The integral Laplace transform in time and the operation of its reversal are used. New forms of representation of solutions to nonstationary viscoelasticity problems in originals are obtained for both regular and singular relaxation kernels. The solutions contain series expansions as well as integrals. Their advantage is the absence of rapidly oscillating functions under the signs of improper integrals. They are convenient for numerical implementation for any point in time. The proposed approach is demonstrated on examples. The results of studies of nonstationary processes in inhomogeneous viscoelastic bodies under specific initial data are presented.

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### Yury Selyutskiy. Dynamics of pendulum systems in flow

Several pendulum systems placed in a flow of resisting medium are considered. The aerodynamic load is described using the quasi-steady approach. Equilibrium positions and conditions of their stability are analyzed. The effect of different parameters (such as the flow speed, stiffness coefficients of mounting springs, etc.) upon the characteristics of oscillations of such pendulums is studied. Potential ways of application of these systems are discussed.

**Yunhe Sheng. Post groups, post-groupoids and the Yang–Baxter equation**

In this talk, first we recall the notion of a post-group. By differentiation of a post-Lie group, one can obtain a post-Lie algebra, which was introduced by Vallette in 2007 and have important applications in numerical integration on manifolds and Martin Hairer's regularity structures. There are close relationships between post-groups, Rota–Baxter groups, skew-left braces and Lie–Butcher groups. In particular, post-groups give rise to matched pairs of groups, and can be used to construct set-theoretical solutions of the Yang–Baxter equation. Then we introduce the notion of a post-groupoid. A post-groupoid is a group bundle equipped with another binary operation. The section space of a post-groupoid is a weak post group. A post-groupoid gives rise to a groupoid and an action on the original group bundle. By differentiation of a post-Lie groupoid, one can obtain a post-Lie algebroid, which was introduced by Munthe–Kaas and Lundervold in the study of geometric numerical analysis. Finally, we show that post-groupoids provide solutions of the Yang–Baxter equation on quivers. This is a joint work with Chengming Bai, Li Guo, Rong Tang and Chenchang Zhu.

**Albert Shiryaev. On direct and inverse Kolmogorov equations for purely jump-like Markov processes and their generalization**

In the work "On analytical methods in probability theory" (1931), A. N. Kolmogorov, starting from the relations called Kolmogorov–Chapman equations, derived for transition probabilities of inhomogeneous stochastically defined systems, or, as is now commonly said, for inhomogeneous Markov random processes (in an expanded meaning), reverse and direct equations in the following three cases:

(A) systems with a finite number of states;

(B) systems with countable number of states;

(C) diffusion-type systems with a continuous set of states.

The report, which is largely of a review nature, considers the cases (A), (B) and the purely jump case for a Markov process with a Borel state space. The report is based on joint work with E. A. Fainberg.

**Constantin Shramov. Automorphism groups of compact complex manifolds**

Automorphism groups of compact complex manifolds may have complicated structure. However, sometimes they satisfy certain nice properties on the level of their finite subgroups. I will survey results and expectations concerning the boundedness of finite subgroups in automorphism groups of compact complex manifolds and some of their quotients.

**Iskander Taimanov. Central extensions of Lie algebras, dynamical systems, and symplectic nilmanifolds**

Euler's equations on central extensions of Lie algebras are discussed. A special infinite sequence of central extensions of nilpotent Lie algebras constructed from the Lie algebra of formal vector fields on the line is considered, and the orbits of coadjoint representations for these algebras are described. By using the compact nilmanifolds constructed from these algebras by I. K. Babenko and the author, it is shown that covering Lie groups for symplectic nilmanifolds can have any rank as solvable Lie groups.

**Xiaomeng Xu. The quantization of irregular Riemann–Hilbert maps**

This talk gives an introduction to the Stokes phenomenon and the Riemann–Hilbert–Birkhoff (RHB) map of meromorphic connections at a $k$-th order pole. It then introduces the quantum Stokes matrices at a $k$-th order pole, and proves that they give rise to a quantization of the RHB map. In the case of second order pole, it becomes a dictionary between the Stokes phenomenon and the theory of quantum groups

**Pavel Yaskov. Toward general spectral theory for large random Gram matrices with dependencies**

**Jianping Zhao, Yanhong Qin. Introduction to College of Mathematics and Systems Science of Xinjiang University**

We will provide a concise overview of Xinjiang University and its College of Mathematics and Systems Science, touching upon their development history, faculty situation, professional training, academic research and workforce strategy.