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058w: Nonlinear Partial Differential Equations

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Международный математический центр «Сириус»

Международная научная конференция

Нелинейные уравнения в частных производных

23–27 июня 2025 года

Программа и аннотации докладов

ФТ «Сириус», 2025

Организаторы

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Тематика конференции охватывает широкий спектр вопросов, связанных с изучением нелинейных уравнений в частных производных (УрЧП), объединенных в следующие основные направления, характеризующиеся актуальностью и активными современными исследованиями:

- *Качественная теория нелинейных УрЧП: существование и единственность решений, а также их свойства, связанные с гладкостью, регулярностью, симметриями и асимптотическим поведением.*
- *Вариационные методы нелинейного анализа: их развитие и применение в теории УрЧП.*
- *Функциональные неравенства: устойчивость, достижимость точных констант, свойства экстремальных функций, и приложения в оптимизационных задачах и теории УрЧП.*
- *Апостериорный анализ: конструктивные оценки мер отклонений приближенных решений от точных решений нелинейных УрЧП и вариационных неравенств.*

Целью конференции является обсуждение новейших достижений, проблем и актуальных вопросов в этих областях, обмен опытом и установление новых плодотворных контактов между научными группами. В рамках конференции представлены доклады как ведущих экспертов, так и активных молодых учёных.

Международный математический центр «Сириус», ФТ «Сириус»

Оглавление

Программа конференции

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10^{00} — 10^{30} Скубачевский Александр Леонидович (A.L. Skubachevskii)
On smoothness of eigenfunctions for differential-difference operators

10^{30} — 11^{00} Шейпак Игорь Анатольевич (I. Sheipak)
Wavelet representation of singular function. Some applications to the spectral problems

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12^{00} — 12^{30} Растегаев Никита Владимирович (N. Rastegaev)
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$10^{00} - 10^{30}$ Кожевникова Лариса Михайловна (L.M. Kozhevnkova)
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Кофе-пауза

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Payne nodal set conjecture for the Riesz fractional p*-Laplacian in Steiner symmetric domains*

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$12^{30} - 12^{45}$ Казимиров Данил Дмитриевич (D. Kazimirov)
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TBA

Кофе-пауза

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Obstacle problem for the p-Laplacian and its a posteriori analysis

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12^{45} — 13^{00} Борунов Семён Сергеевич (S. Borunov)
Verification of a posteriori estimates for solutions to the thin obstacle problem

Обед

14^{30} — 15^{00} T.V. Anoop
On reverse Faber-Krahn inequalities

15^{00} — 15^{30} Тедеев Анатолий Фёдорович (A. Tedeev)
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15^{30} — 16^{00} Бобков Владимир Евгеньевич (V. Bobkov)
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TBA

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10^{30} — 11^{00} Шишков Андрей Евгеньевич (A.E. Shishkov)
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12^{30} — 12^{45} Апаев Мурад Рауф оглы (A.R. Apayev)
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12^{45} — 13^{00} Каляев Тимур Джанбулатович (T. Kalyayev)
Homogenization problem in a Neumann sieve-type domain with “light” concentrated masses. Subcritical case

Обед

14^{30} — 15^{00} Боровских Алексей Владиславович (A.V. Borovskikh)
Group analysis of the one-dimensional kinetic equation and the closure problem for the momentum system

15^{00} — 15^{30} Муравник Андрей Борисович (A. Muravnik)
Nonlocal problems for singular parabolic equations with gradient nonlinearities

Аннотации докладов

26.06
14:30-15:00

On reverse Faber-Krahn inequalities

T. V. Anoop, Jiya Rose Johnson

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This talk discusses the reverse Faber-Krahn inequality for the largest eigenvalue, $\tau_1(\Omega)$, of the Logarithmic potential operator $\mathcal{L}$ on a bounded open set $\Omega \subset \mathbb{R}^2$. We further discuss monotonicity of $\tau_1(\Omega \setminus \mathcal{O})$ with respect to certain translations and rotations of an obstacle $\mathcal{O}$ within Ω.

[AJ25] T.V. Anoop and J.R. Johnson, *Reverse Faber-Krahn inequalities for the Logarithmic potential operator*, 2025, arXiv: 2501.13569.

Homogenization problems of Navier-Stokes equation for the domain perforated along the boundary

27.06
12:30-12:45

Murad Apayev

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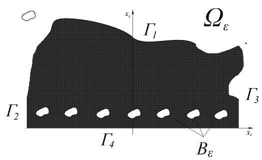
We describe the domain. Let Ω denote a bounded domain in $\mathbb{R}^2$, lying in the upper half-plane, whose boundary Γ is piecewise smooth and consists of several parts: $\Gamma = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup \Gamma_4$, where Γ_4 is the segment $\left[-\frac{1}{2}; \frac{1}{2}\right]$ on the abscissa axis, Γ_2 and Γ_3 belong to the lines $x_1 = -\frac{1}{2}$ and $x_1 = \frac{1}{2}$, respectively, $\Gamma \backslash \Gamma_4$ is smooth. Here and throughout $\varepsilon = \frac{1}{2\mathcal{N}+1}$ is the small parameter, $\mathcal{N}$ is a natural number, $\mathcal{N} \gg 1$.

We use the following notation. Let G be an arbitrary two-dimensional domain with a smooth boundary lying in the circle $K = \{\xi : \xi_1^2 + (\xi_2 - \frac{1}{2})^2 < a^2\}$, $0 < a < \frac{1}{2}$. Let us denote $G_\varepsilon^j = \{x \in \Omega : \varepsilon^{-1}(x_1 - j, x_2) \in G\}$, $j \in \mathbb{Z}$, $G_\varepsilon = \bigcup\limits_j G_\varepsilon^j$, $\Gamma_\varepsilon = \partial G_\varepsilon$. We define the domain Ω_ε as $\Omega \setminus \overline{G_\varepsilon}$ (see Fig. 1).



We consider the following problem:

$$\begin{cases} \dfrac{\partial u_\varepsilon}{\partial t} - \nu \Delta u_\varepsilon + (u_\varepsilon, \nabla) u_\varepsilon = g(x), & x \in \Omega_\varepsilon, t > 0, \\ (\nabla, u_\varepsilon) = 0, & x \in \Omega_\varepsilon, t > 0, \\ \nu \dfrac{\partial u_\varepsilon}{\partial n} = 0, & x \in \Gamma_4, t > 0, \\ u_\varepsilon = 0, & x \in \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup \Gamma_\varepsilon, t > 0, \\ u_\varepsilon = U(x), & x \in \Omega_\varepsilon, t = 0. \end{cases} \tag{1}$$

Here $u_\varepsilon = u_\varepsilon(x,t) = (u_\varepsilon^1, u_\varepsilon^2)$, $g = g\,(x_1, x_2) = (g^1, g^2), g_j \in L_2(\Omega)$, n is the outer normal vector to the boundary and $\nu > 0$. Also consider the problem

$$\begin{cases} \dfrac{\partial u_0}{\partial t} - \nu \Delta u_0 + (u_0, \nabla) u_0 = g(x), & x \in \Omega, t > 0, \\ (\nabla, u_0) = 0, & x \in \Omega, t > 0, \\ u_0 = 0, & x \in \Gamma, t > 0, \\ u_0 = U(x), & x \in \Omega, t = 0. \end{cases} \tag{2}$$

We continue the solutions of the problem (1) by zero inside the pores.

Theorem. *Let u_ε be a solution to problem (1), u_0 be a solution to problem (2). Then, as the small parameter ε tends to zero, we have: $u_\varepsilon \to u_0$ strongly in $L_2((0,T), L_2(\Omega))$.*

Obstacle problem for the p-Laplacian and its a posteriori analysis

26.06
10:30-11:00

Darya Apushkinskaya

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We discusses the functional error identity and estimates, which are performed for measures of deviations from exact solutions of the obstacle problem for the p-Laplace operator. They are fulfilled for any function from the admissible (energy) functional class that contains the generalised solution of the problem. In doing so, no special properties of the approximations or numerical procedures are used. Also, no informa- tion about the exact configuration of the of the coinsidence set is needed. The right-hand side of the identity and estimates contains only known functions and can be explicitly computed, while the left one represents a certain measure of the deviation of the approximate solution from the exact one. The obtained functional relations allow us to estimate the error of any approximations of the problem irrespective of the method of their obtaining. In addition, they allow us to compare the exact solutions of problems with different data, which makes it possible to to estimate the errors of mathematical models, for example, those that that arise when simplifying the coefficients of a differential of a differential equation.

The paper is based on results obtained in [ANR24] jointly with A.A. Novikova and S.I. Repin with the support of the Russian Science Foundation (grant 24-21-00293).

[ANR24] D.E. Apushkinskaya, A.A. Novikova, and S.I. Repin, *A posteriori error estimates for approximate solutions to the obstacle problem for the p-Laplacian*, Differential Eq. **60**:10 (2024), pp. 1476–1490.

26.06
15:30-16:00

On maximum and comparison principles for parabolic problems with the p-Laplacian

Vladimir Bobkov

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We will discuss strong and weak versions of maximum and comparison principles for solutions of a class of problems

$$\begin{cases} \partial_t u - \Delta_p u &= \lambda |u|^{p-2}u + f(x,t), \quad (x,t) \in \Omega_T, \\ u(x,0) &= u_0(x), \quad x \in \Omega, \\ u(x,t) &= 0, \quad (x,t) \in \partial\Omega \times (0,T), \end{cases} \tag{$\mathcal{P}$}$$

where $\lambda \in \mathbb{R}$, $\Omega_T := \Omega \times (0,T)$ is a parabolic cylinder, $\Omega \subset \mathbb{R}^N$ $(N \geq 1)$ is a bounded domain with Lipschitz boundary $\partial\Omega$, and $T \in (0,+\infty)$.

It is well-known that in the linear case $p = 2$ any (classical) solution u of ($\mathcal{P}$) satisfies the weak maximum principle, that is, the assumptions $u_0 \geq 0$ in Ω and $f \geq 0$ in Ω_T imply that $u \geq 0$ in Ω_T. Moreover, the additional assumption $u(x_0,t_0) = 0$ for some $(x_0,t_0) \in \Omega_T$ yields $u \equiv 0$ in Ω_{t_0}, i.e., the strong maximum principle holds. At the same time, analogous principles for $p \neq 2$ cannot be satisfied, in general, without additional assumptions on the parameter λ, initial and source data; they are significantly different for the *fast diffusion* (singular case, $p < 2$) and *slow diffusion* (degenerate case, $p > 2$).

Several related counterexamples are given. The talk is based on the works [BT14, BT19].

[BT14] V. E. Bobkov and P. Takáč, *A strong maximum principle for parabolic equations with the p-Laplacian*, Journal of Mathematical Analysis and Applications **419**:1 (2014), pp. 218–230.

[BT19] V. Bobkov and P. Takáč, *On maximum and comparison principles for parabolic problems with the p-Laplacian*, Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas **113** (2019), pp. 1141–1158.

Group analysis of the one-dimensional kinetic equation and the closure problem for the momentum system

27.06
14:30-15:00

Aleksei Borovskikh

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The problem of obtaining the continuous medium equations from kinetic equations based on group methods of analysis is discussed. The main idea is to calculate the symmetry group of the kinetic equation, transfer its action to moment variables, find the invariants of this group as functions of moment variables and, by setting invariant relations between them, cut and close the infinite system of moment equations, obtaining a finite system of continuous medium equations.

This idea is realized [PB21, PB18] on the simplest one-dimensional kinetic equation

$$f_t + cf_x + (Ff)_c = 0. \tag{1}$$

Group analysis of equation (1) is carried out in the class of diffeomorphisms of the space of all variables t, x, c, f (and also F, in the case of an equivalence group), which satisfy the following three conditions:

- the condition of invariance under these transformations of the relations
$$dx = c\,dt, \qquad dc = F\,dt, \tag{2}$$
which expresses the conservation of the relation between the physical quantities (t, x, c, F);
- the condition of invariance of the family of lines
$$dx = dt = 0, \tag{3}$$
necessary to preserve the physical meaning of the moment variables;
- condition of invariance under changes of variables of the quantity
$$(1 + c\theta_x + F\theta_c)f(t, x, c)dxdc, \tag{4}$$
on any surface $t = \theta(x, c)$, which expresses the independence of the number of particles from the choice of the coordinate system.

It was established that the group of point transformations of the space of variables (t, x, c, f, F) leaving relations (2), (3) and quantity (4) invariant coincides with the group of diffeomorphisms of the space of variables (t, x) and the transformations of the remaining variables generated by them; the equivalence group of equation (1) coincides with this group.

A group classification of equations (1) in the specified class of transformations was carried out. For the obtained symmetry groups, the action of these groups on moment variables was calculated and invariants were found.

In the case of $F = 0$, the differential invariant led to the system $\rho_t + (\rho u)_x = 0, \qquad u_t + uu_x = 0$, which is well known as the equations of "pressureless hydrodynamics". In this case, each solution $(\rho(t,x), u(t,x))$ with initial values $\rho_0(x)$ and $u_0(x)$ corresponds to the distribution $f(t,x,c) = \rho_0(x-ct)\delta(c-u_0(x-ct))$.

For equations with three-dimensional (submaximal in dimension) symmetry groups, the original statement of the problem was transformed into the following: we must first solve the system from equation (1), on the one hand, and from the equations representing the condition of the invariant expression of f through the first two moment variables from the other, and construct after for these f equations of a continuous medium that relate precisely these two moment variables.

The work was carried out with the financial support of the Ministry of Science and Higher Education of the Russian Federation. Agreement No. 075-02-2025-1530.

[PB18] K.S. Platonova and A.V. Borovskih, *Group analysis of the one-dimensional Boltzmann equation: III. Condition for the moment quantities to be physically meaningful*, Theoretical and Mathematical Physics **195**:3 (2018), pp. 886–915.

[PB21] K.S. Platonova and A.V. Borovskih, *Group analysis of the one-dimensional Boltzmann equation. Invariants and the problem of moment system closure*, Theoretical and Mathematical Physics **208**:3 (2021), pp. 1165–1181.

Verification of a posteriori estimates for solutions to the thin obstacle problem

26.06
12:45-13:00

Semyon Borunov

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Problems with a thin obstacle arise in financial mathematics [CT06] or when modeling a membrane stretched over a thin object. In this paper we considered the problem with a thin obstacle for the Laplace operator in the square domain, found its numerical solution, and realized estimates for the distance from the approximate solution to the exact one. Also a majoranta optimization algorithm is proposed for a more realistic estimation of the quality of the solution.

Let $\Omega \in \mathbb{R}^n$ – an open, connected, bounded set with a Lipschitz boundary $\partial\Omega$. $\mathbf{M}$ – is a smooth $(n-1)$-dimensional manifold that divides Ω into two Lipschitz subdomains Ω_+, Ω_-. The function $\psi : \mathbf{M} \to \mathbb{R}$ – an obstacle, $\varphi : \partial\Omega \to \mathbb{R}$ such that $\varphi \geq \psi$ on $\mathbf{M} \cap \partial\Omega$. The function ψ is smooth, $\varphi \in H^{1/2}(\partial\Omega)$. The problem with a thin obstacle for the Laplace operator is the problem of minimization of the functional

$$J(v) = \frac{1}{2}\int_\Omega |\nabla v|^2 dx$$

on the set $K = \{v \in H^1(\Omega) : v \geq \psi \text{ on } \mathbf{M}, v = \varphi \text{ on } \partial\Omega\}$.

It is known that for such a problem there exists a single solution [LS67].

We will consider $\Omega = [0, a] \times [0, a]$, $\psi \equiv 0$, $\mathbf{M} = \{(x_1, x_2) : x_1 = x_2\}$.

The problem is solved numerically on a regular grid by affine approximation and coordinate relaxation (also known as coordinate descent). For each interior point of the grid – v_{ij}, we choose 6 adjacent triangles, consruct an affine approximation of function v and computes $J(v)$. The same operation is performed for $v_{ij} \pm h$. The new value at node ij will be one of the set $\{v_{ij},\, v_{ij}+h,\, v_{ij}-h\}$, yielding the smallest value of the functional $J(v)$.

We verify two estimates, obtained in [AR18]. $\forall v \in K$:

$$\|\nabla(u-v)\|\|_\Omega \leq \|\nabla v - q^*\|_\Omega + \sqrt{2}\int_{\mathbf{M}} \lambda(v-\psi)d\mu + C_{F_+}\|\ \operatorname{div} q^*\|_{\Omega_+} + C_{F_-}\|\operatorname{div} q^*\|_{\Omega_-} + C_{Tr_{\mathbf{M}}}\|\lambda - [q^*\cdot n]\|_{\mathbf{M}}, \quad (1)$$

$$
\begin{aligned}
\|\nabla(u-v)\|^2_\Omega \leq (1+\beta_1)\|\nabla v - q^*\|^2_\Omega \\
&+ (1+\beta_1^{-1})(1+\beta_2)\left[C_{F_+}\|\operatorname{div} q^*\|_{\Omega_+} + C_{F_-}\|\operatorname{div} q^*\|_{\Omega_-}\right]^2 \\
&+ (1+\beta_1^{-1})(1+\beta_2^{-1})C^2_{Tr_{\mathbf{M}}}\|\lambda - [q^*\cdot n]\|^2_{\mathbf{M}} \\
&+ 2\int_{\mathbf{M}} \lambda(v-\psi)d\mu,
\end{aligned} \tag{2}
$$

where $C_{F_\pm}$ – the Friedrichs consants for $\Omega_\pm$,
$C_{Tr_{\mathbf{M}}}$ – is a trace constant, $q^* \in H(\Omega_\pm, \operatorname{div})$, $\lambda \in \Lambda$, $\beta_1, \beta_2 > 0$.
$H(\Omega_\pm, \operatorname{div}) = \{q^* \in L_2(\Omega, \mathbb{R}^n) : \operatorname{div} q^* \in L_2(\Omega_\pm), [q^* \cdot \mathbf{n}] \in L_2(\mathbf{M})\}$,
$\Lambda = \{\lambda \in L_2(\mathbf{M}), \lambda(x) \geq 0$ a.e on $\mathbf{M}\}$.

Estimation (2) admits optimization on q^*. The optimization algorithm is similar to the solution-finding algorithm, but now we are looking for the minimum of the right-hand side of (2). The table provides the distances from the exact to the numerical solutions that were constructed on the adaptive grid and the values of the majorant (1). The $\mathfrak{M}(y^*)$ denotes the right-hand side of (1) at $q^* = y^*, \lambda = [y^* \cdot \mathbf{n}]$, q^*_{opt} – the result of majorant optimization.

n points	$\|\nabla(u-v)\|_\Omega$	$\mathfrak{M}(\nabla u)$	$\mathfrak{M}(\nabla v)$	$\mathfrak{M}(q^*_{opt})$
57	0.084817	0.220160	0.599901	0.418466
113	0.085171	0.181408	0.760170	0.459660
225	0.085420	0.153645	1.033083	0.444953

The paper is based on results obtained jointly with D.E. Apushkinskaya and S.I. Repin.

[AR18] D.E. Apushkinskaya and S.I. Repin, *Thin obstacle problem: Estimates of the distance to the exact solution*, Interfaces and free boundaries **20** (2018), pp. 511–531.

[CT06] R. Cont and P. Tankov, *Financial modelling with jump processes*, Chapman & Hall / CRC Press, 2006.

[LS67] J. L. Lionis and G. Stampacchia, *Variational inequalities*, Communications on pure and applied mathematics **20** (1967), pp. 493–519.

The third type (Robin) boundary condition for a quasilinear problem with critical growth of the right-hand side

23.06
12:30-12:45

Danil Bystrov

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We consider the existence of a positive solution to the following problem

$$\begin{cases} -\Delta_p u = u^{p^*-1} & \text{in } \Omega, \\ |Du|^{p-2}\partial_{\vec{n}} u + \lambda u^{p-1} = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ with $\partial\Omega \in \mathcal{C}^2$, $n \geq 2$, $1 < p < n$, $\lambda > 0$, $\Delta_p u = \operatorname{div}(|Du|^{p-2}Du)$ is the p-Laplacian operator and $p^* = \frac{np}{n-p}$ stands for the critical Sobolev exponent.

In the case of $p = 2$, some results on the solvability of (1) were obtained in [Wan91].

Since the embedding $W_p^1(\Omega) \hookrightarrow L_{p^*}(\Omega)$ is not compact, the standard variational method cannot be applied directly. We use a variant of the concentration-compactness method by P.-L. Lions and give some sharp sufficient conditions for the solvability of the problem (1).

The talk is based on joint work with A. I. Nazarov [BN24].

The work was supported by the Theoretical Physics and Mathematics Advancement Foundation «BASIS».

[BN24] D.V. Bystrov and A.I. Nazarov, *The Robin problem for quasilinear equations with critical growth of the right-hand side*, Zapiski Nauchnykh Seminarov POMI **536** (2024), pp. 126–139.

[Wan91] X.-J. Wang, *Neumann problems of semilinear elliptic equations involving critical Sobolev exponents*, Journal of Differential Equations **93**:2 (1991), pp. 283–310.

Lavrentiev–Bitsadze equation in partially perforated domain

25.06
10:00-10:30

Gregory A. Chechkin

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We consider the equation

$$-u^{\varepsilon}_{yy} - (\operatorname{sign} y)\, u^{\varepsilon}_{xx} = f(x, y)$$

in a semi-perforated domain D_{ε}, the perforated part of which is located in the half-plane $y > 0$ and has a locally periodic structure with a characteristic size ε, and the part lying in the lower half-plane $y < 0$ has a homogeneous structure. On the outer boundary of the domain, the homogeneous Dirichlet condition is imposed, while on the boundary of the cavities, a boundary condition of the third kind (Robin condition) is imposed with a parameter ε^{α}, responsible for energy dissipation. The asymptotic behavior of the solution is investigated as the small parameter ε tends to zero. We assume that $f \in C^1(\mathbb{R}^2)$ and vanishes when $y < 0$.

Three different cases are studied: $\alpha > 1$ (subcritical case), $\alpha = 1$ (critical case) and $\alpha < 1$ (supercritical case).

Nonlinear inverse problems for fractional differential equations

26.06
12:00-12:30

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Nonlinear inverse problems for fractional differential equations in Banach spaces are considered. An equation is solved with respect to the higher-order fractional derivative and contains a nonlinear operator which depends on the lower-order derivatives of a solution and an unknown time-dependent element of the inverse problem. In addition to the initial conditions, an overdetermination condition is set. The cases of fractional derivatives of Gerasimov — Caputo, Riemann — Liouville, Dzhrbashyan — Nersesyan are considered [FBI22, Fed+22, FPM23]. The existence and uniqueness of local and global solutions are proved. Abstract results are used in the study of initial boundary value problems for partial differential equations with unknown coefficients.

[FBI22] V.E. Fedorov, L.V. Borel, and N.D. Ivanova, *Nonlinear inverse problems for a class of equations with Riemann–Liouville derivatives*, Zapiski Nauchnykh Seminarov POMI **519** (2022), pp. 264–288.

[Fed+22] V.E. Fedorov, N.D. Ivanova, L.V. Borel, and A.S. Avilovich, *Nonlinear inverse problems for fractional differential equations with sectorial operators*, Lobachevskii Journal of Mathematics **43**:11 (2022), pp. 3125–3141.

[FPM23] V.E. Fedorov, M.V. Plekhanova, and D.V. Melekhina, *Nonlinear inverse problems for equations with Dzhrbashyan–Nersesyan derivatives*, Fractal and Fractional **7**:6 (2023), p. 464.

Inverse iteration method for higher eigenvalues of the p-Laplacian

23.06
12:45-13:00

Timur Galimov

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Let $1 < p < \infty$ and let $\Omega \subset \mathbb{R}^D$ be a domain of finite measure, $D \geqslant 1$. Consider the problem of finding an *eigenvalue* $\lambda \in \mathbb{R}$ and an *eigenfunction* $u \in W_0^{1,p}(\Omega) \setminus \{0\}$ such that

$$\int_\Omega |\nabla u|^{p-2}\langle \nabla u, \nabla v\rangle\, dx = \lambda \int_\Omega |u|^{p-2} uv\, dx \quad \text{for any } v \in W_0^{1,p}(\Omega).$$

This is a weak form of the Dirichlet eigenvalue problem for the p-Laplace operator

$$\Delta_p(\cdot) := \operatorname{div}(|\nabla(\cdot)|^{p-2}\nabla(\cdot)),$$

turning into the classical Dirichlet Laplace eigenvalue problem for $p = 2$. Eigenfunctions are precisely critical points of the Rayleigh quotient functional $u \mapsto R[u]$ defined on $W_0^{1,p}(\Omega) \setminus \{0\}$ as

$$R[u] := \frac{\int_\Omega |\nabla u|^p\, dx}{\int_\Omega |u|^p\, dx},$$

and eigenvalues are critical levels of R.

It is known (see, for example, [Lin]) that there exists the smallest eigenvalue $\lambda_1(\Omega, p)$, called the *first* eigenvalue, and that $\lambda_1(\Omega, p)$ is also the only eigenvalue admitting sign-constant eigenfunctions (all of which being constant multipliers of one-another). The properties of $\lambda_1(\Omega, p)$ allow to develope different numerical methods for its numerical approximation, among which algorithms based on inverse iteration schemes play an important role. All the other eigenvalues are called *higher*, and they are significantly less studied from both the theoretic and the numeric viewpoints.

We propose a novel algorithm using inverse iterations to approximate some higher eigenvalue alongside the corresponding eigenfunctions. Given an arbitrary initial guess $u_0 \in L^\infty(\Omega)$ such that

$$\min\left\{\|u_0^+\|_p, \|u_0^-\|_p\right\} > 0 \quad \text{and} \quad \Big|\{x \in \Omega :\ u_0(x) = 0\}\Big| = 0,$$

this algorithm generates a sequence $\{u_{k+1}\} \subset W_0^{1,p}(\Omega) \setminus \{0\}$ by solving consecutive p-Poisson equations with the property that their solutions u_{k+1} satisfy

$$R[u_{k+1}] = R[u_{k+1}^+] = R[u_{k+1}^-],$$

where for any $u\colon\ \Omega \to \mathbb{R}$ we let $u^\pm := \max\{\pm u, 0\}$. We prove the following convergence results for $\{u_{k+1}\}$.

Theorem 1. *Let $\mathcal{U}$ be the set of all strong limit points of the sequence $\{u_k\}$ in $W_0^{1,p}(\Omega)$. Then the following assertions hold:*

1. *The sequence $\{R[u_k]\}$ monotonically decreases towards a higher eigenvalue R^*:*

$$R^* := \inf_{k \geqslant 1} R[u_k] = \lim_{k\to\infty} R[u_k].$$

2. *$\mathcal{U} \neq \varnothing$ and it is a subset of the (sign-changing) eigenfunctions corresponding to the eigenvalue R^*.*
3. *Any subsequence $\{u_{k_n}\}$ contains a sub-subsequence having a strong limit in $W_0^{1,p}(\Omega)$. If $\{u_{k_n}\}$ converges strongly to some $u \in \mathcal{U}$, then, for any $i \in \mathbb{N}$, the shifted subsequence $\{u_{k_n+i}\}$ also converges strongly to u.*
4. *$\lim\limits_{k\to\infty} \rho(u_k, \mathcal{U}) = 0$, where ρ stands for the distance function in $W_0^{1,p}(\Omega)$.*
5. *$\mathcal{U}$ either consists of a single eigenfunction or has no isolated elements. In particular, if R^* is a simple eigenvalue, then the whole sequence $\{u_k\}$ converges to the corresponding eigenfunction.*

The talk is based on the paper [BG25].

[BG25] V. Bobkov and T. Galimov, *Inverse iteration method for higher eigenvalues of the p-Laplacian*, 2025, arXiv: 2504.12836.

[Lin] P. Lindqvist, *A nonlinear eigenvalue problem*, Topics in mathematical analysis, pp. 175–203.

25.06
9:30-10:00

Minimax bifurcation formula for analyzing saddle-node bifurcations

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I will discuss bifurcations in nonlinear equations, focusing on a novel approach based on the minimax variational formula for identifying saddle-node bifurcations.This formula offers three significant advantages:

(i) it allows direct detection of saddle-node bifurcations without the need to find solutions to the equations;

(ii) it provides quantitative analysis of bifurcation values;

(iii) offers a practical framework for calculating bifurcations.

In addition, by this formula, one can prove that the bifurcations of finite element approximations of equations converge with the corresponding bifurcations of infinite-dimensional systems.

The method will be illustrated by several examples, including:

- Identifying saddle-node bifurcation points for solutions of nonlinear elliptic systems, particularly when alternative approaches are challenging.
- Identifying bifurcation points in nonlinear finite-dimensional systems, in particularly those relevant to managing complex electrical networks.
- Extension of the Perron-Frobenius theory to a set of arbitrary matrices, including those that are neither irreducible nor essentially positive, and do not preserve the cone.

The talk is based on [Ily21, Ily24, IV24, PDP20].

[Ily21] Y. Ilyasov, *Finding saddle-node bifurcations via a nonlinear generalized Collatz–Wielandt Formula*, International Journal of Bifurcation and Chaos **31**:01 (2021), p. 2150008.

[Ily24] Y. Il'yasov, *A finding of the maximal saddle-node bifurcation for systems of differential equations*, Journal of Differential Equations **378** (2024), pp. 610–625.

[IV24] Y. Il'yasov and N. Valeev, *An extension of the Perron-Frobenius theory to arbitrary matrices and cones*, The Electronic Journal of Linear Algebra **40** (2024), pp. 788–802.

[PDP20] P.D.P. Salazar, Y. Ilyasov, L.F.C. Alberto, E.C.M. Costa, and M.B.C. Salles, *Saddle-node bifurcations of power systems in the context of variational theory and nonsmooth optimization*, IEEE Access **8** (2020), pp. 110986–110993.

Homogenization problem in a Neumann sieve-type domain with “light” concentrated masses. Subcritical case

27.06
12:45-13:00

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In this paper we study the problem with ‘light’ concentrated masses in Neumann sieve-type domain. A domain with a periodic structure Ω_ϵ is given by the equations:

$$\Omega_0 = \{x \in \mathbb{R}^2 : x_1 \in (0;a), x_2 \in (0;\gamma_1(x_1))\},$$

$$\Gamma_1 = \{x \in \mathbb{R}^2 : x_2 = \gamma_1(x_1); x_1 \in [0;a]\},$$

$$\Omega_{1,\epsilon} = \{x \in \mathbb{R}^2 : x_1 \in (0;a), x_2 \in (\gamma_2(x_1); -\epsilon l)\},$$

$$G_j(\epsilon) = \{x \in \mathbb{R}^2 : |x_1 - \epsilon(j + \frac{1}{2})| < \frac{\epsilon h}{2}; x_2 \in [-\epsilon l; 0]\},$$

$$G_\epsilon = \bigcup_{j=0}^{N-1} G_j(\epsilon) \qquad \Omega_\epsilon = \Omega_0 \cup \Omega_{1,\epsilon} \cup G_\epsilon,$$

where a, h, $l \in \mathbb{R}$, $0<h<1$ and $N \in \mathbb{N}$, while $\epsilon=\frac{a}{N}$.
Auxiliary designations for the rest of the domain boundary are also introduced:

$$\gamma_{||,\epsilon} = \partial G_\epsilon \cap \{x_2 = (-\epsilon l; 0)\}, \qquad \gamma_{=,\epsilon} = \partial\Omega_\epsilon \cap (\{x_2 = 0\} \cup \{x_2 = -\epsilon l\}),$$

$$\Gamma_\epsilon = \gamma_{||,\epsilon} \cup \gamma_{=,\epsilon}, \quad \Gamma_{2,\epsilon} = \Omega_\epsilon \backslash (\Gamma_1 \cup \Gamma_\epsilon), \quad I_0 = \{x \in \mathbb{R}^2 : x_1 \in [0;a]; x_2 = 0\},$$

$$I_{0,\epsilon} = \{x \in (\partial\Omega_0 \setminus (\partial G_\epsilon \cup \Gamma_1)) \cap \{x_2 = 0\}\},$$

$$I_{l,\epsilon} = \{x \in (\partial\Omega_{1,\epsilon} \setminus \partial G_\epsilon) \cap \{x_2 = -\epsilon l\}\}.$$

We investigate the following spectral problem:

$$\begin{cases} -\Delta_x u(\epsilon, x) = \lambda(\epsilon)\rho_\epsilon(x) u(\epsilon, x), & x \in \Omega_\epsilon, \\ -\partial_\nu u(\epsilon, x) = 0, & x \in \partial\Omega_\epsilon \setminus \Gamma_1, \\ u(\epsilon, x) = 0, & x \in \Gamma_1, \\ [u] = [\partial_{x_2} u] = 0, & x \in G_\epsilon \cap (\{x_2 = 0\} \cup \{x_2 = -\epsilon l\}). \end{cases}$$

Here $\partial_\nu = \frac{\partial}{\partial \nu}$ is the outward normal derivative; $\partial_{x_2} = \frac{\partial}{\partial x_2}$; the brackets the jump of the enclosed quantities; the density $\rho_\epsilon(x)$ defined as follows:

$$\rho_\epsilon(x) = \begin{cases} 1, & x \in \Omega_0 \cup \Omega_{1,\epsilon}, \\ \epsilon^{-\alpha}, & x \in G_\epsilon. \end{cases}$$

The method of matching asymptotic expansions is applied and homogenized boundary value problems for leading terms of the asymptotic expansion of eigenvalues and eigenfunctions are derived. When $\alpha = 1$ we get:

$$\begin{cases} -\Delta_x v_0^+(x) = \lambda_0 v_0^+(x), & x \in \Omega_0, \\ -\Delta_x v_0^-(x) = \lambda_0 v_0^-(x), & x \in \Omega_{1,0}, \\ \partial_\nu v_0^+(x) = 0, & x \in \Gamma_2, \\ v_0^+(x) = 0, & x \in \Gamma_1, \\ v_0^+(x_1, 0) = v_0^-(x_1, 0), & x_1 \in (0, a), \\ \partial_{x_2} v_0^+(x_1, 0) - \partial_{x_2} v_0^-(x_1, 0) = -\lambda_0 h l v_0^+(x_1, 0), & x_1 \in (0, a), \end{cases}$$

and

$$\begin{cases} -\Delta_x v_1^+(x) = \lambda_0 v_1^+(x) + \lambda_1 v_0^+(x), & x \in \Omega_0, \\ -\Delta_x v_1^-(x_1, x_2) = \lambda_0 v_1^-(x_1, x_2) + \lambda_1 v_0^-(x_1, x_2), & x \in \Omega_{1,0}, \\ \partial_\nu v_1^+(x) = 0, & x \in \Gamma_2 \\ v_1^+(x) = 0, & x \in \Gamma_1, \\ v_1^+(x_1, 0) - v_1^-(x_1, 0) = 2(C_1^{(0)} v_0^+(x_1, 0) + C_1^{(2)} \partial_{x_2} v_0^+(x_1, 0)), & x \in I_0, \\ \partial_{x_2} v_1^+(x_1, 0) - \partial_{x_2} v_1^-(x_1, 0) = -\mu_0 \partial^2_{x_1 x_1} v_0^+(x_1, 0) & \\ \qquad -\lambda_0 \int\limits_{\Pi_l} Z_1^{(2)}(\eta) \mathrm{d}\eta \partial_{x_2} v_0^+(x_1, 0) & \\ \qquad -(\lambda_1 h l + \lambda_0 \int\limits_{\Pi_l} Z_1^{(0)}(\eta) \mathrm{d}\eta) v_0^+(x_1, 0), & x \in I_0. \end{cases}$$

We also consider a case for $\alpha \in (0, 1)$ in this research and similar boundary problems are derived.

Sharp estimates of functions in Sobolev spaces with uniform norm

24.06
12:30-12:45

Danil Kazimirov

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Consider the Sobolev space $\mathring{W}_p^n[0;1]$ $(1 \leqslant p \leqslant \infty)$, which consists of real-valued functions f having absolutely continuous derivatives up to the order $n-1$ inclusively, such that $f^{(n)} \in L_p[0;1]$ and satisfying the boundary conditions $f^{(j)}(0) = f^{(j)}(1) = 0$, $0 \leqslant j \leqslant n-1$. The space $\mathring{W}_p^n[0;1]$ is endowed with the natural norm $\|f\|_{\mathring{W}_p^n[0;1]} := \|f^{(n)}\|_{L_p[0;1]}$.

For each $a \in (0;1)$ and $0 \leqslant k \leqslant n-1$, the goal is set to study the functions $A_{n,k,p}(a)$ that are minimum possible in the inequalities

$$|f^{(k)}(a)| \leqslant A_{n,k,p}(a)\|f^{(n)}\|_{L_p[0;1]},$$

and evaluate the global maximum of the function $A_{n,k,p}$ over $(0;1)$:

$$\Lambda_{n,k,p} := \max_{a\in(0;1)} A_{n,k,p}(a).$$

The number $\Lambda_{n,k,p}$ is the exact (sharp) constant of the embedding of the space $\mathring{W}_p^n[0;1]$ into the space $\mathring{W}_\infty^k[0;1]$, $0 \leqslant k \leqslant n-1$:

$$\Lambda_{n,k,p} = \sup\{\|f^{(k)}\|_{L_\infty[0;1]} : \ \|f^{(n)}\|_{L_p[0;1]} = 1\}.$$

The novel result concerns the calculation of the embedding constant $\Lambda_{n,k,\infty}$ for the case $p=\infty$, n is an odd and k is an even integer, $0 \leqslant k < n$. The main proved theorem provides the formula for evaluating $\Lambda_{n,k,\infty}$ in terms of either the Peano kernel $V_n(t) := \frac{(-1)^n}{(n-1)!}\int_{-1}^t (t-u)^{n-1}\cdot \operatorname{sgn}(U_n(u))du$ of order n or a hypergeometric function ${}_{n+2}F_{n+1}$ of the type $(n+2, n+1)$:

Theorem. *Let n be an odd integer, k an even integer* $(0 \leqslant k \leqslant n-1)$. *The following equality holds for the exact embedding constants $\Lambda_{n,k,\infty}$*

of the Sobolev spaces $\mathring{W}^n_\infty[0;1] \hookrightarrow \mathring{W}^k_\infty[0;1]$*:*

$$\Lambda_{n,k,\infty} \equiv \max_{a\in(0;1)} A_{n,k,\infty}(a) = A_{n,k,\infty}\left(\frac{1}{2}\right) = \frac{|V_n^{(k)}(0)|}{2^{n-k}} =$$

$$= \frac{n+1}{2^{2(n-k-1)}\pi(n-k)!}\int_0^1 \frac{(1-u^2)^{n-k}u^k}{1-u^{2(n+1)}}du =$$

$$= \frac{(n+1)\,(k-1)!!}{2^{n-k-2}\,\pi\,(2n-k+1)!!}\,{}_{n+2}F_{n+1}\left[\begin{matrix}1,\ \frac{k+1}{2n+2},\ \frac{k+3}{2n+2},\ \ldots,\ \frac{2n+k+1}{2n+2}\\ \frac{2n-k+3}{2n+2},\ \frac{2n-k+5}{2n+2},\ \ldots,\ \frac{4n-k+3}{2n+2}\end{matrix};\ 1\right].$$

Payne nodal set conjecture for the Riesz fractional p-Laplacian in Steiner symmetric domains

24.06
11:30-12:00

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Let $p \in (1, +\infty)$, $s \in (0,1)$ and let Ω be a bounded domain in $\mathbb{R}^N$, $N \geq 1$. Consider the eigenvalue problem with Dirichlet boundary condition, i.e. the boundary value problem

$$\begin{cases} (-\Delta)_p^s\, u = \lambda |u|^{p-2} u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{$\mathcal{D}$}$$

where $(-\Delta)_p^s$, $p > 1$, is the Riesz, or semi-restricted, fractional p-Laplacian defined for sufficiently regular functions as

$$(-\Delta)_p^s u(x) = \text{const} \cdot \lim_{\varepsilon \to 0+} \int_{\mathbb{R}^N \setminus B(0,\varepsilon)} \frac{|u(y) - u(x)|^{p-2}(u(y) - u(x))}{|y - x|^{N+ps}}\, dy.$$

The second eigenfunction, defined per standard Lyusternik-Shnirelmann argument, is a sign-changing function. The Payne nodal set conjecture for Steiner symmetric domains asserts that the nodal set of any second eigenfunction intersects the boundary $\partial\Omega$. In the local settings with $s = 1$, the conjecture was proven in [Pay73] and [Dam00] in the linear case $p = 2$ and in [BK19] in the nonlinear case $p \in (1, +\infty)$. We extend and generalize these results to the nonlocal nonlinear case $s \in (0,1)$ and $p \in (1, +\infty)$.

Theorem 1. *Assume that Ω is Steiner symmetric with respect to the hyperplane $H_0 := \{(x_1, \ldots, x_N) \in \mathbb{R}^N : x_1 = 0\}$. Let u be a second eigenfunction of ($\mathcal{D}$). Then*

$$\operatorname{dist}(\operatorname{supp} u^-, \partial\Omega) = 0 \quad \textit{and} \quad \operatorname{dist}(\operatorname{supp} u^+, \partial\Omega) = 0.$$

A similar result is obtained for least energy nodal solutions of the equation $(-\Delta)_p^s\, u = f(u)$ under nonlocal Dirichlet boundary conditions, where the model case of f being a subcritical and superlinear nonlinearity, i.e. $f(u) = |u|^{q-2}u$ with $q \in (p, p_s^*)$.

The proof is based on properties of polarization specific to the nonlocal case $s \in (0,1)$. Most importantly, in a strong contrast to the local case, the polarization strictly decreases certain strong functionals associated with the Slobodetski seminorm of a given function unless the polarized function coincides with either the original function or its reflection. Curiously, we do not require any assumptions on smoothness of $\partial\Omega$, and even connectedness and boundedness of Ω can be weakened.

[BK19] V. Bobkov and S. Kolonitskii, *On a property of the nodal set of least energy sign-changing solutions for quasilinear elliptic equations*, Proceedings of the Royal Society of Edinburgh Section A: Mathematics **149**:5 (2019), pp. 1163–1173.

[Dam00] L. Damascelli, *On the nodal set of the second eigenfunction of the laplacian in symmetric domains in $\mathbb{R}^N$*, Atti della Accademia Nazionale dei Lincei. Classe di Scienze Fisiche, Matematiche e Naturali. Rendiconti Lincei. Matematica e Applicazioni **11**:3 (2000), pp. 175–181.

[Pay73] L.E. Payne, *On two conjectures in the fixed membrane eigenvalue problem*, Zeitschrift für angewandte Mathematik und Physik ZAMP **24** (1973), pp. 721–729.

23.06
9:30-10:00

On the blow-up criterion for solutions of second-order quasilinear elliptic inequalities

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We consider the inequality

$$-\operatorname{div} A(x, u, \nabla u) \geq f(u) \quad \text{in } \mathbb{R}^n, \tag{1}$$

where $n \geq 2$ and A is a Carathéodory function such that

$$(A(x, s, \zeta) - A(x, s, \xi))(\zeta - \xi) \geq 0,$$

$$C_1 |\xi|^p \leq \xi A(x, s, \xi), \quad |A(x, s, \xi)| \leq C_2 |\xi|^{p-1}, \quad n > p > 1,$$

with some constants $C_1, C_2 > 0$ for almost all $x \in \mathbb{R}^n$ and for all $s \in \mathbb{R}$ and $\zeta, \xi \in \mathbb{R}^n$. The function f on the right-hand side of (1) is assumed to be non-negative and non-decreasing on the interval $[0, \varepsilon]$ for some $\varepsilon \in (0, \infty)$.

By a solition of (1) we mean a function $u \in W^1_{p,loc}(\mathbb{R}^n)$ such that $f(u) \in L_{1,loc}(\mathbb{R}^n)$ and

$$\int_{\mathbb{R}^n} A(x, u, \nabla u) \nabla \varphi \, dx \geq \int_{\mathbb{R}^n} f(u) \varphi \, dx$$

for any non-negative function $\varphi \in C_0^\infty(\mathbb{R}^n)$.

Theorem 1. *Inequality (1) has a positive solution such that*

$$\operatorname*{ess\,inf}_{\mathbb{R}^n} u = 0$$

if and only if

$$\int_0^\varepsilon \frac{f(t) \, dt}{t^{1+n(p-1)/(n-p)}} < \infty.$$

The proof is given in [KS24, KSS25].

[KS24] A.A. Kon'kov and A.E. Shishkov, *On global solutions of second-order quasilinear elliptic inequalities*, Math. Notes **116** (2024), pp. 1014–1019.

[KSS25] A.A. Kon'kov, A.E. Shishkov, and M.D. Surnachev, *On the existence of global solutions of second-order quasilinear elliptic inequalities*, Math. Notes (to appear in 2025).

23.06
15:30-16:00

Classical Leray problems on steady-state Navier–Stokes system: recent advances and new perspectives

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In recent years, using the geometric and real analysis methods, essential progress has been achieved in some classical Leray's problems on stationary motions of viscous incompressible fluid: the existence of solutions to a boundary value problem in a bounded plane and three-dimensional axisymmetric domains under the necessary and sufficient condition of zero total flux; the uniqueness of the solutions to the plane flow around an obstacle problem in the class of all D-solutions, the nontriviality of the Leray solutions (obtained by the "invading domains" method) and their convergence to a given limit at low Reynolds numbers; and, more generally, the existence and properties of D-solutions to the boundary value problem in exterior domains in the plane and three-dimensional axisymmetric case, etc. A review of these advances and methods will be the focus of the talk. Most of the reviewed results were obtained in our joint articles with Konstantin Pileckas, Remigio Russo, Xiao Ren, and Julien Guillod, see, e.g., the recent survey paper [KR23].

[KR23] M. Korobkov and X. Ren, *Stationary solutions to the Navier–Stokes system in an exterior plane domain: 90 years of search, mysteries and insights*, Journal of Mathematical Fluid Mechanics **25**:3 (2023), p. 55.

Nonlinear elliptic variational inequalities with contacting and non-contacting measurable bilateral obstacles

27.06
10:00-10:30

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We consider variational inequalities with invertible operators $\mathcal{A}_s\colon W_0^{1,p}(\Omega) \to W^{-1,p'}(\Omega)$, $s \in \mathbb{N}$, in divergence form and constraint set $V \subset W_0^{1,p}(\Omega)$ defined by a measurable lower obstacle $\varphi\colon \Omega \to \overline{\mathbb{R}}$ and a measurable upper obstacle $\psi\colon \Omega \to \overline{\mathbb{R}}$, where Ω is a nonempty bounded open set in $\mathbb{R}^n$ $(n \geqslant 2)$ and $p > 1$.

It is assumed that the sequence $\{\mathcal{A}_s\}$ G-converges to an invertible operator $\mathcal{A}\colon W_0^{1,p}(\Omega) \to W^{-1,p'}(\Omega)$. For the obstacles φ and ψ, some different cases are considered.

In the first case, it is assumed that, for every nonempty open set ω in $\mathbb{R}^n$ with $\overline{\omega} \subset \Omega$, there exist functions $\varphi_\omega, \psi_\omega \in W_0^{1,p}(\Omega)$ such that $\varphi \leqslant \varphi_\omega \leqslant \psi_\omega \leqslant \psi$ a.e. in Ω and $\varphi_\omega < \psi_\omega$ a.e. in ω. In this case, we have $\operatorname{meas}\{\varphi = \psi\} = 0$.

In the second case, it is assumed that the following conditions are satisfied:

(C_1) $\operatorname{int}\{\varphi = \psi\} \neq \varnothing$ and $\operatorname{meas}(\partial\{\varphi = \psi\} \cap \Omega) = 0$;

(C_2) there exist functions $\bar{\varphi}, \bar{\psi} \in W_0^{1,p}(\Omega)$ such that $\varphi \leqslant \bar{\varphi} \leqslant \bar{\psi} \leqslant \psi$ a.e. in Ω and $\operatorname{meas}(\{\varphi \neq \psi\} \setminus \{\bar{\varphi} \neq \bar{\psi}\}) = 0$.

In this case, we have $\operatorname{meas}\{\varphi = \psi\} > 0$.

Finally, in the third case, it is assumed that $\varphi \leqslant 0$ and $\psi \geqslant 0$ a.e. in Ω. This case admits both possibilities: $\operatorname{meas}\{\varphi = \psi\} = 0$ and $\operatorname{meas}\{\varphi = \psi\} > 0$. Therein, an additional condition on the coefficients of the operators $\mathcal{A}_s$ is required.

We expose our recent results showing that in all the described cases, the solutions u_s of the considered variational inequalities converge weak-ly in $W_0^{1,p}(\Omega)$ to the solution u of a similar variational inequality with the operator $\mathcal{A}$ and the constraint set V. We note that in the first and third cases, $\mathcal{A}_s u_s \to \mathcal{A}u$ strongly in $W^{-1,p'}(\Omega)$, while in the second case, this is not true in general. Furthermore, in the second case, the sequence of energy integrals $\langle \mathcal{A}_s u_s, u_s\rangle$ does not converge to $\langle \mathcal{A}u, u\rangle$ in general.

24.06
10:00-10:30

Local renormalized solution of anisotropic elliptic equation with variable exponents in nonlinearities in $\mathbb{R}^n$

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M. F. Bidaut-Veron [Bid03] introduced the concept of a local renormalized solution for the following equation with the p-Laplacian, absorption, and a Radon measure μ:

$$-\Delta_p u + |u|^{p_0-2}u = \mu, \quad p \in (1,n), \quad 0 < p-1 < p_0. \tag{1}$$

In particular, M. F. Bidaut-Veron proved the existence of a local renormalized solution in $\mathbb{R}^n$ of the equation (1) with $\mu \in L_{1,\mathrm{loc}}(\mathbb{R}^n)$.

In the present work, the concept of local renormalized solution is adapted to the anisotropic elliptic equation of the second order with variable growth exponents and locally integrable function f:

$$-\operatorname{div} \mathrm{a}(\mathrm{x}, \nabla u) + b(\mathrm{x}, u, \nabla u) = f, \quad \mathbb{R}^n. \tag{2}$$

Denote $C^+(\mathbb{R}^n) = \{p \in C(\mathbb{R}^n) \mid 1 < \overline{p} \le \widehat{p} < +\infty\}$, where $\overline{p} = \inf_{\mathrm{x}\in\mathbb{R}^n} p(\mathrm{x})$ and $\widehat{p} = \sup_{\mathrm{x}\in\mathbb{R}^n} p(\mathrm{x})$. Denote $\overrightarrow{\mathrm{p}}(\cdot) = (p_1(\cdot), p_2(\cdot), \ldots, p_n(\cdot)) \in (C^+(\mathbb{R}^n))^n$, $\overrightarrow{\mathbf{p}}(\cdot) = (p_0(\cdot), \overrightarrow{\mathrm{p}}(\cdot)) \in (C^+(\mathbb{R}^n))^{n+1}$ and

$$p_+(\mathrm{x}) = \max_{i=\overline{1,n}} p_i(\mathrm{x}), \quad p_-(\mathrm{x}) = \min_{i=\overline{1,n}} p_i(\mathrm{x}), \quad \mathrm{x} \in \mathbb{R}^n.$$

Assumption P. We assume that functions

$$\mathrm{a}(\mathrm{x},\mathrm{s}) = (a_1(\mathrm{x},\mathrm{s}), \ldots, a_n(\mathrm{x},\mathrm{s})) : \mathbb{R}^{2n} \to \mathbb{R}^n, \quad b(\mathrm{x}, s_0, \mathrm{s}) : \mathbb{R}^{2n+1} \to \mathbb{R},$$

included in the equation (2) are Carathéodory functions. Assume that there exist nonnegative functions $\Phi_i \in L_{p_i'(\cdot),\mathrm{loc}}(\mathbb{R}^n)$ and positive numbers $\widehat{a}, \overline{a}$ such that for a.e. $\mathrm{x} \in \mathbb{R}^n$ and all $\mathrm{s}, \mathrm{t} \in \mathbb{R}^n$, the following inequalities hold:

$$|a_i(\mathrm{x},\mathrm{s})| \le \widehat{a}\left(\mathrm{P}(\mathrm{x},\mathrm{s})^{1/p_i'(\mathrm{x})} + \Phi_i(\mathrm{x})\right), \quad i = 1, \ldots, n;$$

$$(\mathrm{a}(\mathrm{x},\mathrm{s}) - \mathrm{a}(\mathrm{x},\mathrm{t})) \cdot (\mathrm{s} - \mathrm{t}) > 0, \quad \mathrm{s} \ne \mathrm{t};$$

$$\mathrm{a}(\mathrm{x},\mathrm{s}) \cdot \mathrm{s} \ge \overline{a}\mathrm{P}(\mathrm{x},\mathrm{s}).$$

Hereinafter, we use the notation $p_i'(\cdot) = p_i(\cdot)/(p_i(\cdot) - 1)$, $\mathrm{P}(\mathrm{x}, \mathrm{s}) = \sum\limits_{i=1}^{n} |s_i|^{p_i(\mathrm{x})}$, $\mathrm{s} \cdot \mathrm{t} = \sum\limits_{i=1}^{n} s_i t_i$, $\mathrm{s} = (s_1, \dots, s_n)$, $\mathrm{t} = (t_1, \dots, t_n)$.

In addition, let there exist a nonnegative function $\Phi_0 \in L_{1,\mathrm{loc}}(\mathbb{R}^n)$, a continuous nonnegative function $\widehat{b} : \mathbb{R}^+ \to \mathbb{R}^+$, and a positive number $\overline{b}$ such that for a.e. $\mathrm{x} \in \mathbb{R}^n$ and all $s_0 \in \mathbb{R}$, $\mathrm{s} \in \mathbb{R}^n$, the following inequalities hold:

$$|b(\mathrm{x}, s_0, \mathrm{s})| \le \widehat{b}(|s_0|)\left(\Phi_0(\mathrm{x}) + \mathrm{P}(\mathrm{x}, \mathrm{s})\right);$$

$$b(\mathrm{x}, s_0, \mathrm{s}) s_0 \ge \overline{b} |s_0|^{p_0(\mathrm{x})+1}, \quad p_+(\cdot) - 1 < p_0(\cdot).$$

Here we assume that

$$\underline{p}(\mathrm{x}) = n \left(\sum_{i=1}^{n} 1/p_i(\mathrm{x}) \right)^{-1} < n, \quad \underline{p}^*(\mathrm{x}) = \frac{n\underline{p}(\mathrm{x})}{n - \underline{p}(\mathrm{x})}.$$

Denote $q_0(\cdot) = \underline{p}^*(\cdot)/\overline{p}_-'$, $\overline{p}_-' = \overline{p}_-/(\overline{p}_- - 1)$, let the following additional assumption be satisfied:

$$p_+(\cdot) - 1 < q_0(\cdot),$$

which is possible provided that $p_+(\cdot) < \underline{p}^*(\cdot)$.

In an anisotropic Sobolev space with variable exponents, the existence and regularity of a local renormalized solution of equation is established, and it is proved that the solution is sign-constant.

Theorem 1. *Let $f \in L_{1,\mathrm{loc}}(\mathbb{R}^n)$ and Assumption P be satisfied. Then there exists a local renormalized solution u of the equation* (2). *If $f \ge 0$ for a.e. $\mathrm{x} \in \mathbb{R}^n$, then $u \ge 0$ for a.e. $\mathrm{x} \in \mathbb{R}^n$.*

In [BD07], conditions on the exponents $p_i(\cdot)$, $i = 0, \dots, n$, sufficient for the uniqueness of a local weak solution of the anisotropic equation (1) were found. For a local renormalized solution without additional restrictions on the growth of the solution at infinity, the uniqueness is not known.

[BD07] M. Bokalo and O. Domanska, *On well-posedness of boundary problems for elliptic equations in general anisotropic Lebesgue-Sobolev spaces*, Mat. Stud **28**:1 (2007), pp. 77–91.

[Bid03] M.F. Bidaut-Véron, *Removable singularities and existence for a quasilinear equation with absorption or source term and measure data*, Advanced Nonlinear Studies **3**:1 (2003), pp. 25–63.

Nonlocal problems for singular parabolic equations with gradient nonlinearities

27.06
15:00-15:30

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In a cylindrical domain, parabolic equations with singular coefficients at nonlinear terms of the KPZ type are considered. Instead of a boundary condition on the lateral surface of the cylinder, an integral nonlocal condition is set. Depending on the relationships between the parameters of the equation and the nonlocal condition, sufficient conditions for the nonexistence of solutions or the solvability of the problem are established.

Joint work with A. A. Grebeneva

On self-similar solutions to a multi-phase Stefan problem for the heat equation in a moving ray

26.06
10:00-10:30

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In the talk, we will discuss a multi-phase Stefan problem for the heat equation on a moving ray $x > \alpha\sqrt{t}$ with the Dirichlet or Neumann conditions on the boundary $x = \alpha\sqrt{t}$. A variational description of self-similar solutions is proposed, on the basis of which the results on the existence and uniqueness of the solution are proved. The case of an infinite number of phase transitions is also considered.

A posteriori estimates for problems with monotone operators

26.06
11:30-12:00

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We propose a method of obtaining a posteriori estimates which are not based on the duality theory and which applies to variational inequalities with monotone operators, without assuming the potentiality of operators. We use this method in various problems driven by nonlinear operators of the p-Laplacian type, including the anisotropic p-Laplacian, polyharmonic p-Laplacian, and fractional p-Laplacian.

These results are joint with V.E. Bobkov (*Institute of Mathematics, Ufa Federal research Centre, RAS*), see [BP25].

[BP25] V.E. Bobkov and S.E. Pastukhova, *A posteriori estimates for problems with monotone operators*, 2025, arXiv: 2504.09931.

27.06
12:00-12:30

On the weighted Neumann eigenvalue problem in Hölder domains

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We consider the weighted Neumann (p,q)-eigenvalue problem [GPU24]:

$$-\mathrm{div}(|x|^\alpha |\nabla u|^{p-2}\nabla u) = \lambda \|u\|^{p-q}_{L_q(\Omega_\gamma)} |u|^{q-2}u \text{ in } \Omega, \quad \frac{\partial u}{\partial \nu} = 0 \text{ on } \partial\Omega_\gamma,$$

in bounded Hölder γ-singularities domains $\Omega_\gamma \subset \mathbb{R}^n$.

In the case $1 < p < \alpha + \gamma$ and $1 < q < p^* = \gamma p/(\alpha + \gamma - p)$ where $0 < \alpha < n(p-1), n \geq 2$, we prove solvability of this eigenvalue problem and existence of the minimizer of the associated variational problem. In addition, we establish some regularity results of the eigenfunctions and some estimates of (p,q)-eigenvalues.

This work was supported by the Ministry of Science and Higher Education of Russia (agreement No. 075-02-2025-1728/2).

[GPU24] P. Garain, V. Pchelintsev, and A. Ukhlov, *On (p,q)-eigenvalues of the weighted p-Laplace operator in outward Hölder cuspidal domains*, Revista Matemática Complutense (2024), pp. 1–24.

Existence of non-radial extremal functions for Hardy–Sobolev inequalities in non-convex cones

23.06
12:00-12:30

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The symmetry breaking is obtained for Neumann problems for equations with p-Laplacian generated by the Hardy–Sobolev inequality:

$$\begin{cases} -\Delta_p u = \dfrac{u^{q-1}}{|x|^{(1-\sigma)q}} & \text{in } \Sigma_D, \\ \dfrac{\partial u}{\partial \nu} = 0 & \text{on } \partial\Sigma_D, \\ u > 0 & \text{in } \Sigma_D. \end{cases}$$

Here $1 < p < n$, $n \geqslant 3$, $0 < \sigma \leqslant 1$, $q = p^*_\sigma = \frac{np}{n-\sigma p}$, $D \subset \mathbb{S}^{n-1}$ and

$$\Sigma_D = \{xt : x \in D, t \in (0, +\infty)\} \subset \mathbb{R}^n$$

is a non-convex cone. Such problems have obvious radial solutions — Talenti–Bliss functions of $|x|$. However, under a certain restriction on the first Neumann eigenvalue $\lambda_1(D)$ of the Beltrami–Laplace operator on D:

$$\lambda_1(D) < (1-\alpha)(n-1-\alpha(p-1)), \quad \alpha = (1-\sigma)\frac{q}{p},$$

we prove this radial solution cannot be the extremal function, therefore minimizer must be non-radial.

In the case $p = 2$, $\sigma = 1$ this problem was investigated in [CPP24].

This research was supported by the Theoretical Physics and Mathematics Advancement Foundation “BASIS”.

[CPP24] G. Ciraolo, F. Pacella, and C.C. Polvara, *Symmetry breaking and instability for semilinear elliptic equations in spherical sectors and cones*, Journal de Mathématiques Pures et Appliquées **187** (2024), pp. 138–170.

26.06
9:30-10:00

A posteriori analysis of nonlinear boundary value problems with monotone operators

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Methods for obtaining fully computable estimates of the distance between a given function and a solution to a boundary value problem are discussed. They are based on special functional identities that hold for boundary value and initial boundary value problems with monotone operators. The left-hand side of the identity represents a certain measure of the distance between the approximate and exact solutions. This measure is dictated by the differential operator. It is a natural characteristic of the closeness of a function to the solution of the problem in question. In particular, minimizing sequences of variational problems converge to the minimizer with respect to precisely this measure. In some cases, the right-hand side of the identity contains only known data of the problem and functions characterizing the approximate solution. Then, the identity can be directly used for measuring errors of numerical approximations. In other cases, the right hand side contains unknown functions, which, however, can be excluded. As a result we obtain guaranteed two–sided error bounds.

Estimates allow us to estimate the error of any approximation of problems regardless of the method by which they have been constructed. In addition, they allow comparison of exact solutions to problems with different data, which makes it possible to evaluate errors in mathematical models. For example, such type errors arise in dimension reduction, homogenization, and simplification of mathematical models based on partial differential equations.

Periodic solutions of the Euler-Bernoulli equation vibrations of a beam subjected to compression, with mixed boundary conditions

24.06
12:00-12:30

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The report will consider the problem of periodic solutions of the quasilinear Euler-Bernoulli equation of beam vibrations with boundary conditions corresponding to the case of rigidly closed left and pivotally supported right ends. For the corresponding Sturm-Liouville problem, asymptotic formulas for eigenfunctions and eigenvalues will be obtained. Based on the variational method, the theorems on the existence of periodic solutions are proved if the nonlinear term has more than linear growth. [JR23, Rud24].

[JR23] S. Ji and I.A. Rudakov, *Infinitely many periodic solutions for the quasi-linear Euler–Bernoulli beam equation with fixed ends*, Calculus of Variations and Partial Differential Equations **62**:2 (2023), p. 66.

[Rud24] I.A. Rudakov, *Periodic solutions of the Euler–Bernoulli quasilinear vibration equation for a beam with an elastically fixed end*, Mathematical Notes **115**:5 (2024), pp. 800–808.

23.06
14:30-15:00

A few observations on the first eigenvalue of the Robin p-Laplace operator

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We obtain shape derivative formulae for the first eigenvalue of the Robin p-Laplace operator. This result is used to study the variation of the first eigenvalue with respect to perturbations of the domain. In particular, we prove that for large values of the boundary parameter, the first eigenvalue is monotonic with respect to domain inclusion for smooth domains. [AMS24].

[AMS24] Ardra A, M. Mallick, and S. Sasi, *On a shape derivative formula for the Robin p-Laplace eigenvalue* (2024), arXiv: 2208.05183.

Spectral and functional inequalities on antisymmetric functions

24.06
12:45-13:00

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We investigate so-called (d, N)-antisymmetric functions that appear in the area of interest as wavefunctions of fermions' systems. Here N and d are natural numbers and we consider $x = (x_1, \dots, x_N) \in \mathbb{R}^{dN}$, where $x_i = (x_{i1}, \dots, x_{id}) \in \mathbb{R}^d$ for all $1 \leqslant i \leqslant N$. Every function u defined on $\mathbb{R}^{dN}$ we call *antisymmetric*, if for all $1 \leqslant i, j \leqslant N$ and $x_1, \dots, x_N \in \mathbb{R}^d$

$$u(x_1, \dots, x_i, \dots, x_j, \dots, x_N) = -u(x_1, \dots, x_j, \dots, x_i, \dots, x_N).$$

As we expected, the constants in classical inequalities are much better for antisymmetric functions. Indeed, one of our main results in [HLS23] is Hardy inequality for antisymmetric functions. It claims that for any $u \in \mathcal{H}^1_A(\mathbb{R}^{dN})$ we have

$$\int_{\mathbb{R}^{dN}} |\nabla u(x)|^2 \, dx \geqslant H_A(dN) \int_{\mathbb{R}^{dN}} \frac{|u(x)|^2}{|x|^2} \, dx \tag{1}$$

The constant in (1) is sharp and $H_A(dN) \sim N^{2+2/d}$ unlike classical N^2.

Similarly, we improve Sobolev inequality restricted to antisymmetric functions. For every $u \in \mathcal{H}^1_A(\mathbb{R}^{dN})$, $dN \geqslant 3$ the following inequality holds

$$\int_{\mathbb{R}^{dN}} |\nabla u(x)|^2 dx \geqslant S_A(dN) \left(\int_{\mathbb{R}^{dN}} |u(x)|^{\frac{2dN}{dN-2}} \, dx \right)^{\frac{dN-2}{dN}}.$$

The constant $S_A(dN)$ is sharp and is $(N!)^{\frac{2}{dN}}$ times bigger than the classical constant $S(dN)$.

The spectrum of Laplace-Beltrami operator restricted to antisymmetric function is fully investigated in [Shc24]. The main theorem claims that for $d = 1$ the eigenvalues of the Laplace-Beltrami operator $-\Delta_\theta$ on antisymmetric functions on sphere equals

$$\lambda_l = \left(l + \frac{N(N-1)}{2}\right)\left(l + \frac{N(N+1)}{2} - 2\right),$$

whose multiplicity κ_l satisfies the inequality

$$\frac{2}{N!(N-2)!}\ell^{N-2} \leqslant \kappa_\ell \leqslant \frac{2}{N!(N-2)!}\left(\ell + \frac{N(N+1)}{2}\right)^{N-2}.$$

Spectral inequalities for Schrödinger operator are presented as Cwikel-Lieb-Rozenblum and Lieb-Thirring inequalities in [LS25]. Let $d = 1$ and $V \geqslant 0$ be a symmetric potential such that $V \in L^{\gamma+N/2}(\mathbb{R}^N)$. Then for any γ satisfying the conditions

$$\begin{cases} \gamma \geqslant 1/2 & \text{if } N = 1, \\ \gamma > 0 & \text{if } N = 2, \\ \gamma \geqslant 0 & \text{if } N \geqslant 3. \end{cases}$$

we have

$$\operatorname{Tr}(-\Delta - V)_-^\gamma \leqslant \frac{L_{\gamma,N}}{N!} \int\limits_{\mathbb{R}^N} V^{\gamma+N/2}\,dx,$$

where $L_{\gamma,N}$ is classical LT constant.

[HLS23] T. Hoffmann-Ostenhof, A. Laptev, and I. Shcherbakov, *Hardy and Sobolev inequalities on antisymmetric functions*, Bull. Math. Sci. (2023), p. 2350010.

[LS25] A. Laptev and I. Shcherbakov, *Spectral and functional inequalities on antisymmetric functions*, Ufa Math. Journal **17**:1 (2025), pp. 142–153.

[Shc24] I. Shcherbakov, *Spectrum of the Laplace–Beltrami operator on antisymmetric functions*, Springer, Journal of Mathematical Sciences **279**:1-3 (2024), pp. 563–572.

On the Hénon problem with different fractional Laplacians

23.06
11:30-12:00

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Let B be a unit ball $\mathbb{R}^n$ $(n \geq 2)$, $q > 2$ and $\alpha > 0$. The Dirichlet problem

$$\begin{cases} -\Delta u = |x|^\alpha |u|^{q-2} u \\ u > 0 \text{ in } B \\ u\Big|_{\partial\Omega} = 0 \end{cases} \tag{1}$$

was introduced by M. Hénon in [Hén73] . This problem was proposed as a model for spherically symmetric stellar clusters and was investigated numerically for some definite values of q and α. In the last decades it became evident that the Hénon equation and its modifications exhibits very interesting features concerning existence, multiplicity and symmetry properties of solutions.

First, for $s \in (0,1)$ we consider the problem

$$(-\Delta)^s_{\mathcal{D}} u = |x|^\alpha |u|^{q-2} u \quad \text{in } B, \qquad u \in \widetilde{H}^s(B), \tag{2}$$

where $(-\Delta)^s_{\mathcal{D}}$ stands for the restricted or spectral Dirichlet fractional Laplacian. We use the notation $\widetilde{H}^s(B) = \Big\{ u \in H^s(\mathbb{R}^n) \ : \ \text{supp}(u) \subset \overline{B} \Big\}$, where $H^s(\mathbb{R}^n)$ is the standard Sobolev–Slobodetskii space.

It was shown in [Shc20] that for sufficiently large α, problem (2) has an arbitrary number of different positive solutions (not obtained from each other by orthogonal transformations). For original problem (1) similar results were obtained in [SSW02]. A similar problem driven by p-Laplacian was considered in [Naz01, KN07].

Further, we consider the problem

$$(-\Delta)^s_{\mathcal{N}} u + u = |x|^\alpha |u|^{q-2} u \quad \text{in } B, \qquad u \in H^s(B), \tag{3}$$

where $(-\Delta)^s_{\mathcal{N}}$ is the spectral Neumann fractional Laplacian.

A problem (3) driven by standard Laplacian $(s = 1)$ was investigated in [GS08]. A similar problem with p-Laplacian was considered in [Shc18].

We show that for $q \in \left(2; \dfrac{2(n+\alpha)}{n-2s}\right)$ the problem (3) has a positive radial solution. However, for $s > 1/2$, $q \in \left(\dfrac{2(n-1)}{n-2s}, \dfrac{2n}{n-2s}\right)$ and α sufficiently large there exists also a nonradial positive solution of (3).

We also show that for q sufficiently close to 2, the radial function is a local minimizer of the energy functional corresponding to equation (3) in the space $H^s(B)$.

[GS08] M. Gazzini and E. Serra, *The Neumann problem for the Hénon equation, trace inequalities and Steklov eigenvalues*, Ann. I. H. Poincaré **25** (2008), pp. 281–302.

[Hén73] M. Hénon, *Numerical experiments on the stability of spherical stellar systems*, Astronomy and Astrophysics **24** (1973), pp. 229–238.

[KN07] S.B. Kolonitskii and A.I. Nazarov, *Multiplicity of solutions to the Dirichlet problem for generalized Hénon equation*, J. Math. Sci. **144**:6 (2007), pp. 4624–4644.

[Naz01] A.I. Nazarov, *On the symmetry of extremal in the weight embedding theorem*, J. Math. Sci. **107**:3 (2001), pp. 3841–3859.

[Shc18] A.P. Shcheglova, *The Neumann problem for the generalized Hénon equation*, J. Math. Sci. **235**:3 (2018), pp. 360–373.

[Shc20] A.P. Shcheglova, *Multiplicity of positive solutions for the generalized Hénon equation with fractional Laplacian*, Zap. nauchnyh seminarov POMI **489** (2020), pp. 207–224.

[SSW02] D. Smets, J. Su, and M. Willem, *Non radial ground states for the Hénon equation*, Comm. Contemp. Math. **4** (2002), pp. 467–480.

Wavelet representation of singular function. Some applications to the spectral problems

23.06
10:30-11:00

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We consider wavelet representation of some class of functions. In particular, the Weierstrass function and the Takagi–Landsberg family of functions allow such a representation. Special attention is paid to the functions of Takagi–Landsberg family. It is shown that the wavelet representation of Takagi–Landsberg functions is directly related to affine self similarity with with lower triangular self-similarity matrices.

The Hölder exponent of Takagi–Landsberg functions is calculated. For some parameters of wavelet representation (affine parameters of self similarity) the asymptotics of eigenvalues for corresponding spectral problem for string equation, where weight is generalized derivative of Takagi–Landsberg functions is obtained.

27.06
10:30-11:00

Very singular and large solutions of semilinear elliptic and parabolic equations

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The history of the study of two classes of singular solutions of the stationary and non-stationary diffusion-nonlinear absorption type equations is discussed. The first class are large solutions, i.e. non-negative solutions in bounded domain that take infinitely large value on the entire boundary of the corresponding domain. The existence of such solutions and the study of their properties were initiated by the works of L. Bieberbach (1916), J.B. Keller (1957), R. Osserman (1957), C. Loevner-L. Nirenberg (1974). The second class of solutions are very singular solutions, i.e. solutions that have singularities on the boundary of the corresponding domain that are stronger than those admissible for the corresponding linear equations. Such solutions were first established in the semilinear heat equations by H. Brezis-L. Peletier-D. Terman (1986). The studies of these classes of solutions were continued by many leading mathematicians.

In the recent decades, the properties of large and very singular solutions have been actively studied in the case of general absorption nonlinearities degenerating on the boundaries of corresponding domains. Exact conditions on the mentioned degeneration have been found, ensuring the existence of solutions from the described classes. Exact conditions for the uniqueness of large solutions have been obtained, which surprisingly "almost" coincide with the conditions for the existence of very singular solutions. The report will present new results on the existence, uniqueness and asymptotic properties of mentioned classes of solutions.

On smoothness of eigenfunctions for differential–difference operators

23.06
10:00-10:30

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Unlike ordinary differential equations, smoothness of generalized solutions to boundary value problems for neutral differential–difference equations on a finite interval $(0, d)$ can be violated and preserves only on some subintervals. However, for generalized eigenfunctions of differential–difference operators problem of smoothness remained open. We obtain the necessary and sufficient conditions of smoothness for eigenfunctions of differential–difference operators. We construct an example of differential–difference operator having a countable set of non–smooth eigenfunctions and a countable set of smooth eigenfunctions [VS23].

[VS23] R.Yu. Vorotnikov and A.L. Skubachevskii, *Smoothness of generalized eigenfunctions of differential–difference operators on a finite interval*, Mathematical Notes **114**:5 (2023), pp. 1002–1020.

On different statements of boundary value problems for linear and nonlinear elliptic differential–difference equations

24.06
10:30-11:00

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The relevance of study of elliptic differential-difference equations is closely connected with their important and numerous applications. Firstly, they are related to variational problems arising in the theory of multilayer plates and shells, in the theory of control of systems with aftereffect, etc. Secondly, in some cases elliptic differential–difference equations are caused by elliptic problems with non–local boundary conditions arising in plasma theory (Bitsadze-Samarskii problem), in the theory of diffusion processes, etc. For example, when considering the problem of minimum for a quadratic functional containing a function and its derivatives with shifts in spatial variables, the corresponding Euler equation has the form

$$R_Q^* A R_Q u = f, \qquad x \in Q, \tag{1}$$

where $Q \subset \mathbb{R}^n$ is a bounded domain with boundary $\partial Q \in C^\infty$, A is a linear strongly elliptic differential operator with constant coefficients of the second order, R_Q is a linear difference operator with constant coefficients containing shifts in spatial variables. In the case of homogeneous Dirichlet conditions $u|_{\partial Q} = 0$, this equation reduces to the form

$$A(R_Q^* R_Q)u = f, \qquad x \in Q. \tag{2}$$

Assuming $w(x) = R_Q^* R_Q u(x)$, we reduce equation (2) to the form

$$Aw = f, \qquad x \in Q. \tag{3}$$

However, the new function $w(x)$ must satisfy non–local boundary conditions when the traces of the function w on some pieces of the boundary are equal to linear combinations of traces of w on the shifts of these pieces inside the region. If equation (1) is derived from the problem of the minimum of a functional containing a function and its derivatives with shifts in spatial variables to the degree of $p > 1, p \neq 2$, the differential operator A is nonlinear and will not commute with the difference operator. Therefore, equation (1) will not be equivalent to equation

(2), i.e. it cannot be reduced to a non-local elliptic boundary value problem. Thus, in the nonlinear case, we obtain two different types of differential-difference equations (1) and (2), which are investigated by different methods and have different applications.

25.06
10:30-11:00

Chow-Rachevsky theorem for Sobolev vector fields

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We prove a weak version of the Chow-Rachevsky theorem for vector fields having only Sobolev regularity and generating suitable flows as selections of solutions to the respective ODEs, for a.e. initial datum.

Hodge decomposition in variable exponent Lebesgue and Sobolev spaces

24.06
9:30-10:00

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We shall discuss classical boundary value problems for the Hodge Laplacian in variable exponent Lebesgue and Sobolev spaces and related results such as Hodge decomposition, gauge fixing, div-rot type systems. This is a joint work with A. Balci (Bielefeld Uni) and S. Sil (Indian Institute of Science).

26.06
15:00-15:30

The Cauchy problem for doubly degenerate parabolic equations with weights

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We consider the Cauchy problem in the Euclidean space for a doubly degenerate parabolic equation with space-dependent exponential weights. We assume here that the solutions of the Cauchy problem to be globally integrable in space in appropriate weighted sense. Under suitable assumptions, we prove for the solutions sup estimates, i.e., the decay rate at infinity, the property of finite speed of propagation, and support estimates. All our estimates are given explicitly in terms of the weight appearing in the equation.

The talk is based on the work [AT25] supported by North-Caucasus Centre of Mathematical Research of the Vladikavkaz Scientific Centre of the Russian Academy of Sciences, agreement 075-02-2023-914.

[AT25] D. Andreucci and A.F. Tedeev, *The Cauchy problem for doubly degenerate parabolic equations with weights*, Nonlinear Differential Equations and Applications NoDEA **32**:2 (2025), p. 26.

Optimization inverse spectral problems and nonlinear differential operators

27.06
11:30-12:00

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The report addresses a new formulation of the inverse spectral problem for self-adjoint semi-bounded differential operators (both for partial and ordinary derivatives) — the optimization inverse spectral problem.

In the optimization inverse spectral problem, the spectral data of the operator $L(q_1, q_2, \ldots, q_n)$, where $q_1(x), q_2(x), \ldots, q_n(x)$ are the coefficients of the corresponding differential expression, consist of only a part of the spectrum, specifically a finite number of eigenvalues:

$$\lambda_1^* < \lambda_2^* < \cdots < \lambda_m^* \in \mathbb{R}.$$

Based on these spectral data, it is required to find new coefficients $\hat{q}_1, \hat{q}_2, \ldots, \hat{q}_n$ that are minimally distant (in some metric) from the given $q_1, q_2, \ldots, q_n$, such that:

$$\lambda_k(L(\hat{q}_1, \hat{q}_2, \ldots, \hat{q}_n)) = \lambda_k^*, \quad k = 1, \ldots, m.$$

Such problems are hereafter referred to as "optimization inverse spectral problems with incomplete data" (abbreviated as OISP).

The formulation of the optimization inverse spectral problem naturally arises in various mathematical models for the identification and construction of linear dynamical systems with specified resonance characteristics.

A notable and certainly noteworthy feature of optimization inverse spectral problems is that each such problem is equivalent to some nonlinear differential operator. Moreover, by selecting the linear operator $L(q_1, q_2, \ldots, q_n)$ and the functional spaces for the coefficients $q_1(x), q_2(x), \ldots, q_n(x)$ in the formulation of OISPs, one can "construct" functionals that generate various nonlinear equations. In particular, it is possible to obtain well-known nonlinear equations and operators in mathematical physics: nonlinear (systems of) Schrödinger equations, Gross-Pitaevskii equations, Hartree-Fock-type systems, p-Laplace operators, and others.

Using the example of Schrödinger operators, the report will present the general ideas and approaches to studying the optimization inverse spectral problem, as well as results on the existence of solutions, cases of solution uniqueness, and the representation of optimal coefficients. The main focus will be on describing the nontrivial connection between the investigated inverse problem and the nonlinear Schrödinger equation (NLSE), the Gross-Pitaevskii equation, Manakov-type systems, and the study of certain properties of these equations (existence, uniqueness, nodality, and others).

[IV19] Y.Sh. Ilyasov and N.F. Valeev, *On nonlinear boundary value problem corresponding to N-dimensional inverse spectral problem*, Journal of Differential Equations **266**:8 (2019), pp. 4533–4543.

[IV21] Y. Ilyasov and N. Valeev, *Recovery of the nearest potential field from the m observed eigenvalues*, Physica D: Nonlinear Phenomena **426** (2021), p. 132985.

[Sad24] Sadovnichii, V.A. and Sultanaev, Ya.T. and Valeev, N.F., *Optimization inverse spectral problem for the one-dimensional Schrödinger operator on the entire real line*, Differential Equations **60**:4 (2024), pp. 465–471.

[SSV22] V.A. Sadovnichii, Ya.T. Sultanaev, and N.F. Valeev, *Optimization inverse spectral problem for a vector Sturm–Liouville operator*, Differential Equations **58**:12 (2022), pp. 1694–1699.

[VI20] N.F. Valeev and Y.Sh. Ilyasov, *Inverse spectral problem for Sturm–Liouville operator with prescribed partial trace*, Ufa Math. J **12**:4 (2020), pp. 20–30.

Equations with Riemann–Liouville derivative and subordination principle

26.06
12:30-12:45

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The principle of subordination is investigated for linear equations in Banach spaces that are resolved with respect to the Riemann – Liouville derivative, involving a closed linear operator at the unknown function. It is shown that if a strongly continuous resolving family of operators exists for an equation of order α_1, then an analytic resolving family also exists for every such equation of order $\alpha_2 < \alpha_1$ [FV25]. Two representations of the analytic family are constructed from the original strongly continuous family through the Stanković and Mellin transforms. This fact allows to consider new classes of quasilinear equations.

[FV25] V.E. Fedorov and D.A. Vershinina, *Strongly continuous resolving families of equations with Riemann–Liouville derivative*, Journal of Mathematical Sciences **287**:1 (2025), pp. 52–68.

On the existence and uniqueness of the Burgers' equation based on Ellis rheological model

23.06
15:00-15:30

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In this work, we investigate the existence and uniqueness of solutions for the viscous Burgers' equation for the isothermal flow of Ellis fluids

$$\partial_t u + u\partial_x u = \nu\partial_x \left(\phi^{-1}\left(\partial_x u\right)\right)$$

with the initial condition $u(0,x) = u_0$, $0 < x < l$, and the boundary condition $u(t,0) = u(t,l) = 0$, $0 < t < T$, where $\phi(\tau) = \left[1 + c\,|\tau|^{\alpha-1}\right]\tau$, $0 < c < \infty$, $0 < \alpha < \infty$, ν is the kinematic viscosity, τ is the shear stress, u is the velocity of the fluid and $l, T > 0$. We proved the existence and uniqueness of solution $u \in L^2\left(0,T; {W_0}^{1,p}(0,l)\right)$ for $u_0 \in W_0^{1,p}(0,l)$, where $p = 1 + \frac{1}{\alpha}$.

Non-Newtonian fluids, such as polymer solutions, colloidal suspensions, and biological substances, have viscosity that varies with the applied shear rate, requiring advanced models to accurately describe their flow characteristics [BAH87]. Incorporating non-Newtonian properties into Burgers' equation enables a more precise representation of fluid behavior which is crucial for applications in industrial processes, biomedical engineering and materials science. Recent studies have explored some mathematical properties of Burgers' equation with non-Newtonian viscosity, providing insight into the existence and stability of solutions for such models [Shu15]. Ellis rheological model effectively captures shear-thinning effects, where viscosity decreases as shear rate increases, a phenomenon commonly observed in polymeric and biological fluids.

[BAH87] R.B. Bird, R.C. Armstrong, and O. Hassager, *Dynamics of polymeric liquids. Vol. 1: Fluid mechanics*, John Wiley and Sons Inc., New York, NY, 1987.

[Shu15] Y. Shu, *Numerical solutions of generalized Burgers' equations for some incompressible non-Newtonian fluids*, University of New Orleans: Theses and Dissertations, 2015.