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INTERNATIONAL CONFERENCE

«Differential Equations and Related Topics»,

dedicated to the Centenary Anniversary of

IVAN G. PETROVSKII

(1901-1973)

XX Joint Session of Petrovskii Seminar and Moscow Mathematical Society

BOOK of ABSTRACTS

Moscow
May 22 – 27, 2001

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PLENARY LECTURES

Atiyah M.
(University of Edinburgh)

Geometry and Physics in the 20th century

In the past twenty years the theoretical physics of quantum field theories and strings has developed into a remarkable structure. Although only partially understood and by no means rigorous it has already used vast amounts of modern geometry and has in return had a dramatic impact on the field. The challenge for the 21st century is to understand the significance of this theory, to put it on to firm foundations and of course to relate it ultimately to the real world. I will attempt to give an overvierw of these exciting ideas.

Faddeev L.D.
(Steklov Math. Institute, St. Petersbourg)

Topological solitons in 3+1 dimensional space-time

The term soliton in general sence is used for localized static or periodic solutions of nonlinear evolution equa- tions. For one dimensional space there exists a very well developed theory of solitons in connection with integrable models. For higher dimensions we know only examples. In my talk I want to concentrate on a particular model allowing for solitons, localized in the vicinity of a knot. Corresponding topological charge is the Hopf invariant. Some applications in physics will be indicated.

Novikov S.P.
(Moscow State University)

Topological Phenomena in the Quantum Solid State Physics

Investigations of the conductivity tensor of normal metals in the strong magnetic field leads to the highly nontrivial topological problems. New observable interger-valued topological characteristics of this tensor were revealed as a result of these investigations.

Palis J.
(Rio de Janeiro, Brazil)

A Scenario for Dissipative Dynamics and Recent Results.

Sadovnichii V.A.
(Moscow State University)

Outstanding Mathematician and Head of MSU (to Centenary Anniversary of I.G.Petrovskii)

KEY LECTURES

Ball J.M.
(University of Oxford)

Microgeometry and phase transformations

The talk will describe joint work with C. Carstensen and with R.D. James concerning compatibility of gradients $Dy(x)$ for maps $y : \Omega \to \mathbf{R}^m$, where $\Omega \subset \mathbf{R}^n$ is open. In particular generalizations of the Hadamard jump condition will be described, and also results relating compatible and incompatible sets of matrices to Young measures. The principal applications are to models of displacive phase transformations in solids.

Bensoussan A., Boccardo L., Frehse J.
(France)

Nonlinear systems of elliptic equations with natural growth conditions and sign conditions

Following the work of Landes, we present some new ideas to treat the following system of equations

$$-div(a(x, u, Du)) + g(x, u, Du) = h$$

with Dirichlet boundary conditions. The first order nonlinear term has natural growth conditions and the right hand side is not smooth. However sign conditions permit to prove the existence of a solution.

Bojarski B.
(Math. Institute, Warsaw)

Geometry of the Riemann-Hilbert transmission problem

Geometry of the Riemann–Hilbert problem of holomorphic function theory — more generally the geometry of the Cauchy data for solutions of elliptic p.d.e. — will be discussed in the context of T.Kato Fredholm pairs of subspaces of a Hilbert space, Birman-Solomyak elliptic fans and relations with K-Theory and bordisms.

Bolibruch A.A.
(Steklov Mathematical Institute)

The Riemann-Hilbert problem and isomonodromic deformations

The problem of construction of a Fuchsian system of linear differential equations

$$\frac{dy}{dz} = \left(\sum_{i=1}^{n} \frac{B_i}{z-a_i}\right) y$$

on the Riemann sphere from a given representation

$$\chi : \pi_1(\bar{C} \setminus \{a_1, \ldots, a_n\}, z_0) \longrightarrow GL(p; C)$$

(which is called the Riemann-Hilbert problem) has in general a negative solution (see [1]). Nevertheless, it turns out that every irreducible representation still can be realized as the monodromy of some Fuchsian system ([1]). We give new sufficient conditions for the positive solvability of the RHP which generalize the one mentioned above. It does not depend on the location of singular points, it has a combinatorial nature, and it is formulated in terms of stability of a vector bundle F with a logarithmic connection ∇ constructed from the representation χ. Recall that the pair (F, ∇) is called *stable* if for every subbundle $F' \subset F$ invariant with respect to ∇ one has that the slope $\deg(F')/\mathrm{rk}(F')$ of this subbundle is smaller than the slope of the whole bundle F. We prove that *the Riemann-Hilbert problem has a positive solution in the class of Fuchsian systems with irreducible sets of coefficients if and only if at least one of vector bundles with logarithmic connections constructed from χ is stable in the sense mentioned above.* We also prove that *for every stable pair* (F, ∇) *there exists an isomonodromic deformation of the pair which leads to a vector bundle with the splitting type* $(c_1, \dots, c_p)$, *such that* $\forall i : c_i - c_{i+1} \leqslant 1$.

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Bolsinov A.V.
(Moscow State University)

Integrability and non-integrability of geodesic flows on smooth manifolds

The aim of the talk is to discuss the two following general problems in Riemannian geometry: 1) Which smooth compact manifolds admit Riemannian metrics with integrable geodesic flows? 2) What are topological obstructions to integrability of geodesic flows on compact Riemannian manifolds? In particular, we want to mention two results recently obtained.

Theorem 1. (I.A.Taimanov & A.B. [1]) *There exists a real analytic Riemannian 3-manifold* (M^3, g) *such that 1) the geodesic flow on* (M^3, g) *is completely integrable; 2) the topological entropy of this geodesic flow is positive; 3) the growth of the fundamental group* $\pi_1(M)$ *is exponential.*

This result shows that, in general, topological entropy cannot be considered as a topological obstruction to integrability of geodesic flows.

Theorem 2. (B.Jovanovic & A.B. [2]) *Let* G/H *be the homogeneous space of a compact Lie group* G *endowed with the natural bi-invariant metric (i.e., the submersion metric corresponding to the bi-invariant metric on* G*). Then the geodesic flow on* G/H *is completely integrable in non-commutative sense by means of polynomial integrals and in Liouville sense by means of* C^∞ *smooth integrals. Moreover, the same result is true for any bi-quotient space* $K\backslash G/H$ *of a compact Lie group.*

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Dubrovin B.A.
(SISSA, Trieste)

On Normal Forms of Integrable PDEs

In the talk we will address the problem of classification of *integrable systems* on the loop space $\mathcal{L}(M^n)$ of a smooth manifold M^n of the form

$$\frac{\partial u^i}{\partial t} = \sum_{j=1}^{n} A^i_j(u) \frac{\partial u^j}{\partial x} \qquad (*)$$

and their integrable perturbations in the class of 1+1 evolutionary PDEs. The following two features of integrability are adopted as the basis for the classification programme developed recently by Youjin Zhang and the author: 1) bihamiltonian structure and 2) existence of a τ-function. An appropriate extension of the group of diffeomorphisms of M^n naturally acting on the loop space $\mathcal{L}(M^n)$ is involved in the classification of deformations. The first classification result says that, under certain genericity assumption, hierarchies of integrable systems of the form (*) on $\mathcal{L}(M^n)$ are labelled by Frobenius manifold structures on M^n or by their degenerations. Frobenius manifolds were introduced by the author in the beginning of 90s as the geometric setup of the so-called equations of associativity discovered by physycists E.Witten, R.Dijkgraaf, E. and H. Verlinde. For mathematicians it is best known appearance of Frobenius manifolds in the theory of the genus zero Gromov - Witten invariants of compact symplectic manifolds, although Frobenius manifolds arise also in other branches of mathematics. Remarkably, the relationship between integrable hierarchies and Gromov - Witten invariants persists also in the classification of *integrable deformations* of systems (*), but the Gromov Witten invariants of higher genera become involved. At the moment this conjectural relationship remains to look rather misterious. However, we found some evidences supporting this conjecture. In particular, we proved that the first order deformation is always described in terms of the theory of elliptic Gromov - Witten invariants. In certain cases we were able to go beyond the first order to reproduce the known identities for the genus 2 Gromov - Witten invariants starting from integrable hierarchies.

E Weinan
(Princeton University)

Mathematical Problems Related to the Theory of Turbulence

Recently there has been major advance in the understanding of simple models of turbulence. Two most notable examples are the Burgers turbulence and the passive scalar turbulence. Progress has also been made on understanding the interplay between dissipation and the scaling behavior at small scales. In this talk, we will review some of these results, with emphasis on the ones that are likely to be of direct relevance to hydrodynamic turbulence. We will end with a comparison between weak and strong turbulence.

Hida T.
(Meijo University, Japan)

Laplacians in white noise analysis and Petrovskii's method of reducing the second variation to canonical form.

Many Laplacians have appeared in white noise analysis and those operators play different important roles, respectively. Among others, the Levy Laplacian, which is formally speaking a Cesaro limit of ordinary second order differential opperators, enjoys significant properties. One of its haracterizations is that it is essentially infinite dimensional and entirely different from neither the number operator nor the infinite dimensional Laplace-Beltrami operator. This fact can be illustrated by observing the method of approximation of functionals, for which the domain of functions (in fact, they are the variables of functionals) is divided into subintervals of equal length. The topology that defines the limit towards the Levy Laplacianis is, of course, not the ordinary one. Note that this method is very much different from the Fourier series expansion of functions. In the course of such an approaximation we shall appeal to the Petrovskii's method introduced in reducing the second variation to canonical form by triangular transformations. We refer to his paper appeared in Ucheniye Zepiski Moscow Univ. (1934), 5-16. By doing so, we can see various profound properties of the Levy Laplacian. In particular, we recognize the reason why it acts effectively on the space of generalized white noise functionals, where interesting connection with quantum dynamics can be found explicitly.

Ilyashenko Yu.S.
(Steklov Mathematical Institute)

Restricted versions of the Hilbert 16th problem

The strongest form of the Hilbert 16th problem (second part) is: what is the maximal number limit cycles for a planar polynomial vector field of degree n? There are some simplifications of the problem: the same question is asked not for all polynomial vector fields but for some special class: say, Abel or Lienard equations. Even in this setting the problem is still open. The restricted version considers the class that satisfies some additional assumptions. The talk presents upper estimates on the number of limit cycles of *Abel and Lienard equations* through the degree of the polynomials in the right hand side and the magnitude of their coefficients. The estimate for Lienard equation is obtained in a joint work with A.Panov. Another counterpart of Hilbert 16th problem is the *infinitesimal* one: how many limit cycles may be generated by the ovals of the Hamiltonian polynomial vector field of degree n under a perturbation of the same degree. The estimate is obtained for *special* Hamiltonians of arbitrary degree n. It is an exponential of a polynomial in n. This result is obtained in a joint work with A.Glutsuk. and improves estimates of Novikov and Yakovenko (unpublished).

Jaeger W.
(University of Heidelberg)

Navier Stokes Flow and Laws at Interfaces and Rough Boundaries

In this lecture a survey is given covering the results of asymptotic analysis for problems arising in flow along a rapidly oscillatory surface or in a partially porous medium. The transmission laws connecting the free flow and the filtrations flow in a porous media will be discussed, effective boundary conditions on an approximating "smooth" boundary surface replacing a rough one will be derived. The dependence of the drag force on the scale of roughness is analysed in the nonturbulent situation. The effective terms and quantities can be numerically computed, errors of the approximations are estimated. The results are in agreement with experimental measurements. The report is dealing with results obtained by Mikelic, N.Neuss and Jaeger.

Karasev M.B.
(MIEM, Moscow)

Quantum Method of Characteristics

There is the well known asymptotical method of characteristics developed by Petrovskii, Lax, Keller, and in a general operator setting by Maslov, as well as by many other mathematicians and physisists, for semiclassical solving partial differential equations. As it occures, this method has an exact nonasymptotical version which could be called the quantum method of characteristics. The quantum version is related to the construction of quantum submanifolds and mappings, and also it uses the technique of symplectic groupoids and membrane amplitudes. An interesting analitical result which follows from this quantum geometry is the appearence of new formulas for solutions of certain classes of differential equations, e.g. The exact solutions and the global asymptotics obtained by this method are based on model "special functions" (for instance, the hypergeometric or theta-functions, or others), which are associated with irreducible leaves and polarizations of a dynamical algebra of the equation. In the talk there will be explaned the general ideas of the method, and examples of asymptotics associated with quantum surfaces of revolution will be given.

Katok A.B.
(Pennsylvania State University)

Ergodic theory and dynamics for muldimensional time

Kobelkov G.M.
(Moscow State University)

Parabolic approximations of the Navier–Stokes equations

A parabolic system of partial differential equations

$$\mathbf{v}_t - \nu\Delta\mathbf{v} - \frac{1+\gamma}{\varepsilon}\nabla\mathrm{div}\,\mathbf{v} + v^k\mathbf{v}_{x_k} + \frac{1}{2}\mathrm{div}\,\mathbf{v}\cdot\mathbf{v} + \nabla q = 0$$

$$\varepsilon^2 q_t + \varepsilon q - \mathrm{div}\,\mathbf{v} = 0 \tag{1}$$

$$\mathbf{v}|_{\partial\Omega\times[0,T]} = 0, \quad \mathbf{v}(x,0) = \mathbf{u}_0(x), \quad q(x,0) = q_0(x)$$

with $\gamma > 0$ and $\varepsilon \to 0$ is proposed to approximate the system of nonstationary Navier–Stokes equations:

$$\mathbf{u}_t - \nu\Delta\mathbf{u} + u^k\mathbf{u}_{x_k} + \nabla p = 0$$

$$\mathrm{div}\,\mathbf{u} = 0, \quad \mathbf{u}|_{\partial\Omega\times[0,T]} = 0, \quad \mathbf{u}(x,0) = \mathbf{u}_0(x) \tag{2}$$

For $\gamma > 0$ and a small initial value of $\mathbf{u}_0(x)$ in the norm of the Sobolev space $\mathbf{H}_0^1$, it is proved that

$$\max_t \|\mathbf{v}(t) - \mathbf{u}(t)\|^2 + \int_0^\infty \|\mathbf{v}(t) - \mathbf{u}(t)\|_1^2\,dt \leqslant c\varepsilon\left(\|\mathbf{u}_0\|_1^2 + \|\mathbf{u}_t(0)\|^2 + \varepsilon\|q_0\|^2\right) \tag{3}$$

here $\|\cdot\|$ means the norm of $\mathbf{L}_2$ and $\|\cdot\|_1$ means the norm of $\mathbf{H}_0^1$. A comparison with the classic approximation of problem (2) (to get it we have to omit the term

∇div in the first equation of (1), change the sign in the front of "div" in the second equation of (2) and omit the time derivative in the second equation of (2)) is carried out. In particular, it is proved that the norm $\|q - \frac{1+\gamma}{\varepsilon}\nabla \operatorname{div} \mathbf{v}\|$ is *uniformly bounded in time. So there is a hope that for a small perturbation in the* initial pressure function $\left(\|q(0) - p(0)\| \leqslant \delta\right)$, the uniform convergence of q to p holds as well. Results of numerical experiments are discussed.

Kozlov V.V.
(Moscow State University)

Hamyltonian systems, statistical mechanics and balanced thermodynamics

New ideas of heat balance theory connected with modern theory of dynamical systems will be discussed in the lecture. We develop a new approach to get the canonical Gibbs distribution using the results on nonintegrability of general form canonical systems and not using ergodic hypothesis (which has not been yet proved for systems of Bolzman–Gibbs gas type, and even not valid for some cases). We obtain new criteria which garantee the absence of an additional integral of the motion equation of mechanical systems with toric configuration space. In particular, the system of connected pendulums does not admit integrals which are independent from energy integral. These results allow to develop a thermodynamical description of mechanical systems with finite degree of freedom.

Ladyzhenskaya O.A.
(Steklov Mathematical Institute, St.-Petersburg)

Known and unknown facts on the Navier-Stokes and Modified Navier-Stokes equations

1) Known results on the solvability of the principal boundary-value problems for the stationary Navier-Stokes equations in a fixed domain. 2) On some interesting unsolved problems for the stationary Navier-Stokes equations. 3) What we know and do not know on the solvability of the initial-boundary value problems for the non-stationary Navier-Stokes equations. What new results on the Navier-Stokes equations could merit for a high prize? 4) On results and problems for the modifications of the Navier-Stokes equations giving a deterministic description of the dynamics of fluids without any restrictions of the values of some norms of data.

Manin Yu.I.
(Max-Planck Institute for Mathematics in the Sciences)

On the distribution of continued fractions and modular symbols

This is a report on our joint work with Matilde Marcolli. We prove an extension of the classical Gauss–Kuzmin theorem about the distribution of continued fractions, which in particular allows one to take into account some congruence properties of consecutive convergents. Using this theorem, we study the statistical behavior of geodesics on the modular surfaces ("asymptotic modular symbols"). We also prove a series of identities which can be interpreted in terms of function theory on the tower of "non-commutative modular curves", in the spirit of Connes' geometry.

Mitidieri E.
(Dipartimento di Scienze Matematiche, Universita di Trieste)

On Some Degenerate and Singular Evolution Problems

In recent years, many researchers have generalized the classical results of Fujta–Hayakawa to several classes of equations and systems of partial differential equations.

A common fact in many investigations is that the proof that the critical exponent of a given problem belongs to the blow-up case is based on the knowledge of the fundamental solution of the differential operator or on its sharp asymptotic estimates. In general this fact does not allow to extend this kind of results to quasilinear problems. In this talk we shall present an approach developed by Stanislav I. Pohozaev and the author that does not require any information on the fundamental solution of the problem under investigation. We shall show that this approach, when applied to concrete PDE's, gives sharp results.

Nikolski N.
(Steklov Institute of Mathematics, St.Petersburg)

Recent results on the similarity problem

This is a survey of results and methods concerning the problem of similarity to a normal operator: from Friedrichs Γ-equations, the Wermer calculus approach and the harmonic analysis approach (S.Naboko and van Casteren), to recent results based on the function model for Hilbert space operators.

A resolvent similarity criterion is obtained. It says that the linear growth of the resolvent towards the spectrum is necessary and sufficient for a Hilbert space contraction with finite rank defect operators and the spectrum not covering the unit disc to be similar to a normal operator. Similar results are proved for operators having a spectral set bounded by a Dini-smooth Jordan curve and possessing a normal dilation of finite spectral multiplicity. In particular, a dissipative operator with finite rank imaginary part is similar to a normal operator if and only if its resolvent grows linearly towards the spectrum. The main technical tools used for the proof are the Sz.-Nagy-Foias function model and estimates of spectral projections via matrix valued corona theorem.

Relevant results on the insufficiency of linear resolvent growth which is not followed by a smallness of defect operators or the contractivity hypothesis are presented.

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[3] N. Nikolski, *Operators, Functions, and Systems*, vol.1&2, AMS Monographs and Surveys, 2001.

Plotnikov P.I.

(Institute of Hydrodynamics, Novosibirsk)

Gradient flows of marginal functions and nonlinear elliptic-parabolic equations

We deal with mathematical problems concerned the slow dynamics of a multiphase thermodynamical system. It is supposed that a continuum occupies a bounded domain $\Omega \subset R^3$ and its evolution is described in terms of the "inverse temperature" $\vartheta : \Omega \times (0,T) \to R$ and the "order parameter" $\varphi : \Omega \times (0,T) \to R^n$. If the relaxation time is equal to 0, then the governing equations can be written in the form of a "parabolic-elliptic" system of nonlinear equations

$$\Omega \times (0,T) : \quad \partial_t(\vartheta + \varphi_1) = \Delta\vartheta, \quad -\varepsilon^2\Delta\varphi + \nabla_\varphi\Phi(\varphi) = \varepsilon\vartheta e_1,$$

which are supplemented with the boundary and initial data

$$\partial\Omega \times (0,T) : \quad \frac{\partial\vartheta}{\partial n} + \lambda\vartheta = 0, \quad \frac{\partial\varphi}{\partial n} + \mu\varphi = 0,$$

$$\Omega : \quad -\vartheta(0) - \varphi_1(0)) = u_0 \in H_0(\Omega).$$

Here the thermodynamical potential Φ has the representation $\Phi = g(\varphi) - \frac{1}{2}Q\varphi\cdot\varphi$ in which Q is an arbitrary $n \times n$ matrix and $g : R^n \to R^1$ is a convex function satisfying the conditions

$$c^{-1}|\varphi|^4 \leqslant g(\varphi) \leqslant c(|\varphi|^4 + 1), \quad |\nabla_\varphi g(\varphi)| \leqslant c(|\varphi|^3 + 1).$$

Note that the second equation is the Euler equation for the the "free energy" functional

$$F(\vartheta, \varphi) = \int\limits_\Omega \left[\frac{\varepsilon|\nabla\varphi|^4}{2} + \frac{1}{\varepsilon}\Phi(\varphi) - \frac{\vartheta^2}{2} - \vartheta\varphi_1\right] dx + \frac{\mu}{2}\int\limits_{\partial\Omega} |\varphi|^2 ds.$$

We show that for any positive ε this problem has a solution which generates a gradient flow for marginal function $S(u) = \max\limits_{\varphi\in H_1(\Omega)} W(u,\varphi)$ of the entropy functional

$$W(u,\varphi) = \langle\Theta(u,\varphi), u\rangle - F(\Theta(u,\varphi),\varphi), \quad \Theta(u,\varphi) = -u - \varphi_1.$$

We also prove that this solution satisfies the maximum entropy and minimum entropy production principles and investigate its behavior when the small parameter ε tends to 0.

Pohozaev S.I.
(Steklov Mathematical Institute)

Blow-up solutions to nonlinear hyperbolic problems

The study of existence and nonexistence of global solution to semilinear wave equations has been initiated in seventies and intensively developed. We prove the nonexistence of global solutions of a very wide class of nonlinear hyperbolic type inequalities and systems of such inequalities. Our approach is based on an adequate choice of test functions and dimensional analysis. We do not use any facts from corresponding linear theory such as comparison theorems and the explicit form of fundamental solution. This approach was developed by the author jointly with E. Mitidieri [1, 2, 3], A. Tesei [4], L. Veron [5]. As application of the approach we consider nonlinear higher-order hyperbolic equations and inequalities and semilinear degenerate hyperbolic inequalities. Using our approach, we obtain for the first time the sufficient conditions of complete and instantaneous blow-up for singular semilinear and nonlinear hyperbolic inequalities in a bounded domain.

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Ramis J.P.
(Universite de Toulouse (Paul Sabatier) and Institut Universitaire de France)

Differential Galois theory and nonintegrability of Hamiltonian Systems

Sinai Ya.G.
(Princeton University and Landau Institute)

New finite dimensional approximation of hydrodynamic type equations

The difficulties arising in proving the existence and uniqueness theorems for equations of Navier–Stokes type are connected with the absence of the understanding of the diffusion mechanism. We propose a new approach to get finite dimensional systems of ordinary differentional equations having a similar diffusion mechanism. The lecture is based on the joint work with Dinaburg and Posvyanski.

Skrypnik I.V.
(Institute of Applied Mathematics and Mechenics of NAS Ukraine)

Topological degree theories for (S_+) operators and applications

The lecture is devoted to topological degree theories for different classes of mappings involving operators of type (S_+). A degree theory for operators defined on open set of Banach spaces was developed in the monograph [1]. Extensive applications of this degree to nonlinear elliptic problems were given in monograph [2]. At last time [3] general initial-boundary value problem for essentialy nonlinear parabolic equations was studied by using this degree. Namely we reduced such problem to nonlinear operator equation $Au = 0$ with A satisfying condition (S_+). The second type of operators of our considerations is densely defined operator $A : \mathcal{D}(A) \subset X \to X^*$ satisfying generalized condition (S_+). We introduce degree and we give the applications to Dirichlet problem for the equation

$$\sum_{i=1}^{u} \frac{\partial}{\partial x_i} \left\{ \rho^2(u) \frac{\partial u}{\partial x_i} + a_i \left(x, u, \frac{\partial u}{\partial x} \right) \right\} = \sum_{i=1}^{u} \frac{\partial f_i(x)}{\partial x_i} \tag{1}$$

with very weak growth assumption for the function $\rho(u)$. The third type of our operators is densely defined operator $M + A : \mathcal{D}(M + A) \subset X \to X^*$. We assume that the operator M satisfies a variant of the maximal monotonicity condition and the operator A satisfies (S_+) condition with respect to the operator M. We introduce a notion of degree for such operators and we give the application of this degree to the study of the Cauchy-Dirichlet problem for the nonlinear parabolic equation

$$\frac{\partial u}{\partial t} - \sum_{i=1}^{n} \frac{\partial}{\partial x_i} a_i \left(x, t, u, \frac{\partial u}{\partial x} \right) + \rho(x, t, u) = \sum_{i=1}^{n} \frac{\partial}{\partial x_i} f_i(x, t) \tag{2}$$

in conditions that the operator corresponding to $\rho(x, t, u)$ is not compact. We study the computation of the index of a critical point for nonlinear densely defined operators of second type and give the application of the index formula to the operator corresponding to the Dirichlet problem for the equation (1). Results on degree theories for densely defined operators were established in [4,5].

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Temam R.M.

(University Paris-Sud)

Mathematical problems in meteorology and oceanography

We will review the primitive equations of the atmoshere and the ocean and their coupling. We will descibe some mathematical poblems that they raise, some recent results and some less recent ones.

Tesei A.

(Universita' Di Roma "La Sapienza")

Semilinear Elliptic and Parabolic Inequalities with First Order Terms[1]

We discuss some recent results, obtained jointly with S. I. Pohozaev, concerning instantaneous blow-up of solutions to semilinear parabolic problems of the following type:

$$\begin{cases} \frac{\partial u}{\partial t} - \Delta u \geqslant \lambda |x|^{-\mu}(x, \nabla u) + |x|^{-\alpha}u^q & \text{in } \Omega \times (0,T) \\ u \geqslant 0 & \text{in } \Omega \times (0,T). \end{cases} \tag{0.1}$$

Here $\Omega \subseteq I\!R^n$, $n \geqslant 3$ is a bounded smooth domain which contains the origin, $q > 1$ and λ, μ, α are real parameters. By $(\cdot,\cdot)$ we denote the scalar product in $I\!R^n$. We also investigate the related problem of nonexistence of solutions to semilinear elliptic problems corresponding to (0.1), namely:

$$\begin{cases} -\Delta u \geqslant \lambda |x|^{-\mu}(x, \nabla u) + |x|^{-\alpha}u^q & \text{in } \Omega \\ u \geqslant 0 & \text{in } \Omega. \end{cases} \tag{0.2}$$

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Vassiliev V.A.
(Moscow Center of Continuous Mathematical Education)

Theory of lacunas and Petrovskii condition for hyperbolic operators

Lacunas of a hyperbolic differential operator are the domains of the complement of its wave front in which the principal fundamental solution coincides with a regular function. I.G. Petrovskiĭ in his seminal 1945 work has related this analytical property with topological properties of the set of complex zeros of the symbol of the operator and established a topological criterion of the existence of lacunas: a certain integration cycle (called now the Petrovskiĭ cycle) should be homologous to zero. In the works of A.M. Davydova, V.A. Borovikov, and J. Leray the local analog of this property was investigated, namely the local regularity (sharpness) of the principal fundamental solution close to particular points of wave fronts. In the ≈1970 works of M.F. Atiyah, R. Bott and L. Garding a local version of the topological Petrovskiĭ condition was found, and the fact that it is sufficient for the sharpness was proved. The necessity of this condition (conjectured also by Atiyah, Bott and Garding) was proved in 1983 for almost all hyperbolic operators; however for very degenerate operators this conjecture is false. For all simple (of types A_k, D_k or E_k) singularities of wave fronts the localized Petrovskiĭ condition ($\simeq$ the sharpness) has a transparent geometrical characterization, which allows us to enumerate all regularity domains close to all such singularities, in particular close to *all* points of wave fronts of generic operators in spaces $\mathbb{R}^n$, $n \leqslant 7$. (For the simplest simple singularities, of types A_2 and A_3, this enumeration was done by Garding about 1976). The main tools of this study come from the local singularity theory of smooth maps, especially from the Picard–Lefschetz theory of hypersurface singularities. In the talk main concepts of the theory of lacunas, ideas of proofs, and some further results, links and perspectives will be described.

Véron L.
(Department of Mathematics, Univ. of Tours)

Boundary trace and removability for nonlinear elliptic equations

We study the boundary value problem

$$\begin{aligned} -\Delta u + |u|^{q-1}u &= 0, \quad \text{in } \Omega, \\ u &= \nu, \quad \text{on } \partial\Omega \end{aligned} \qquad (0.1)$$

where Ω is a smooth bounded domain in $\mathbb{R}^N$ and $\nu \in \mathfrak{M}(\partial\Omega)$ (= the space of bounded Borel measures on $\partial\Omega$). A function u is a solution of this problem if

$u \in L^1(\Omega) \cap L^q(\Omega; \delta\, dx)$ (where $\delta(x) = \operatorname{dist}(x, \partial\Omega)$)and

$$\int_\Omega (-u\Delta\zeta + |u|^{q-1}u\zeta)\, dx = \int_{\partial\Omega} \zeta\, d\nu, \tag{0.2}$$

for every function $\zeta \in W^{2,\infty}(\Omega)$ such that $\zeta = 0$ on $\partial\Omega$.

In the subcritical case, $1 < q < q_c := (N+1)/(N-1)$, the problem has a unique solution forevery measure ν (Gmira and Veron). In the supercritical case, $q \geqslant q_c$, this is no longer true; for instance,the problem has no solution if the measure ν is concentrated at a single point. A measure $\nu \in \mathfrak{M}(\partial\Omega)$ such that (0.1) is solvced a *q-trace*. It is known that, for every q-trace, the solution of (0.1) is uniqueand that the solution depends monotonically on the boundary data.

The characterization of q-traces has been the subject of several studies: Le Gall, Dynkin and Kuznetsov for $q_c \leqslant q \leqslant 2$,and Marcus and Veron for $q > 2$.In these works, the following result was established:

> *If $q \geqslant q_c$, problem (0.1) has a solution if and only ifν vanishes on everu Borel set $E\{\partial\}\Omega$ such that$C_{2/q,q'}(E) = 0$, $1/q + 1/q' = 1$.*

Here $C_{2/q,q'}$ denotes Bessel capacity on $\partial\Omega$. We observethat, for $1 < q < q_c$, $C_{2/q,q'}(E) = 0$ only if $E = \varnothing$.

The characterization of q-traces is closely related to the characterizationof removable boundary singularities. A closed set $K \subset \partial\Omega$ is a q-removable boundary singularity, if the equation $-\Delta u + |u|^{q-1}u = 0$ has no positive solution such that $u \in C(\bar\Omega \setminus K)$and $u = 0$ on $\partial\Omega \setminus K$. Note that in this definition, nothing is assumed concerning the behaviour of thesolution near K. In particular, it is not assumed that the solution possesses a boundary trace $\nu \in \mathfrak{M}(\partial\Omega)$. The following result was proved by Le Gall for $q = 2$, Dyand Kuznetsov for $1 < q < 2$ and Marcus and Véron for $q > 2$:

> *A closed set $K \subset \partial\Omega$ is a q-removable boundary singularity if and only if $C_{2/q,q'}(K) = 0$*

In Le Gall and Dynkin-Kuznetsov's papers the basic approach is probabilistic while in Marcus-Véron's the method is purely analytic. The probabilistic approach imposes the restriction $q \leqslant 2$. On the other hand, some of the techniques impose the restriction $q > 2$.

In the talk (joint work with M. Marcus), we provide a unified proof of theresults quoted above, applying to all q in the supercritical range.The method, which is once again purely analytic, is based on a careful study of a special class ofq-traces, namely the set of q-admissible measures. We say thata measure $\nu \in \mathfrak{M}(\partial\Omega)$ is q-admissible if the Poisson potential of $|\nu|$ isin $L^q(\Omega; \delta\, dx)$. Recall that the Poisson potential of a measure ν is given by

$$\mathbb{P}(\nu) = \int_\Omega P(x,y)\, d\nu(y), \tag{0.3}$$

where P is the Poisson kernel of Ω. The set of q-traces will be denotedby $\mathfrak{M}_q(\partial\Omega)$ while the set of q-admissible measures will be denoted by $\mathfrak{D}_q(\partial\Omega)$.

Every q-admissible measure is a q-trace. Since $\mathfrak{M}_q(\partial\Omega)$ is closed in $\mathfrak{M}(\partial\Omega)$ with respect to the total variation norm, it follows that

$$\mathrm{cl}_{\mathfrak{M}}\, \mathfrak{D}_q(\partial\Omega) \subseteq \mathfrak{M}_q(\partial\Omega), \tag{0.4}$$

where $\mathrm{cl}_{\mathfrak{M}}$ denotes closure inthis topology. We note that, for $q \geqslant q_c$, $\mathfrak{D}_q(\partial\Omega)$ is not closed with respect to thetotal variation norm and consequently *there exist q-traces that are not q-admissible.*

Remark 1 *Since $L^\infty(\partial\Omega) \subset \mathfrak{D}_q(\partial\Omega)$, it follows that $L^1(\partial\Omega) \subset \mathrm{cl}_{\mathfrak{M}}\, \mathfrak{D}_q(\partial\Omega)$.*

In addition it was shown that, for $q > 2$, every non-negative, bounded measure in $W^{-2/q,q}(\partial\Omega)$ is q-admissible.

Our first result provides a characterization of positive q-admissible measures in terms of Bessel spaces.

Theorem A 1 *Suppose that $q \geqslant q_c$. Then:*
(a) If ν is a q-admissible measure then $\nu \in W^{-2/q,q}(\partial\Omega)$.
(b) If $\nu \in (W^{-2/q,q})^+(\partial\Omega)$ then ν is q-admisible.
(c) There exists a constant $C = C(q)$ such that, for every $\nu \in (W^{-2/q,q})^+(\partial\Omega)$,

$$C^{-1}\|\nu\|_{W^{-2/q,q}(\partial\Omega)} \leqslant \|\mathbb{P}(\nu)\|_{L^q(\Omega;\delta dx)} \leqslant C\|\nu\|_{W^{-2/q,q}(\partial\Omega)}. \tag{0.5}$$

Employing this result we are able to improve (0.4) by establishing the following.

Theorem B 1 *For every $q > 1$,*

$$\mathrm{cl}_{\mathfrak{M}}\, \mathfrak{D}_q(\partial\Omega) = \mathfrak{M}_q(\partial\Omega). \tag{0.6}$$

Te results lie at the core of our proof of the capacitary characterization of q-traces and q-removable boundary singularities described above. The main ingredients in our proof are: (a) capacitary results, (b) various interpolation theorems involving Besov or Sobolev spaces,, (c) a useful result of concerning measures which do not charge sets of $C_{\gamma,q}$-capacity zero and (d) a construction ofa new 'optimal' lifting from $W^{2/q,q'}(\partial\Omega)$ into a weighted Sobolev spacein Ω.

Viana M.

(Inst. de Mat., Rio de Janeiro)

Dynamics: hyperbolicity and beyond

There has been much recent progress in the theory of dynamical systems, from which an understanding of the typical behaviour of very general systems is emerging, extending the scope of the classical theory of hyperbolic dynamics. I'll discuss some of these developments. In particular, I'll report on very recent results in the theory of Lyapunov exponents of smooth systems, unveiling a sharp dichotomy between some form of uniform hyperbolicity and total absence of expanding/contracting behaviour (zero Lyapunov exponents).

Zhikov V.V.
(Vladimir State Pedagogical University)

Some remarks about linear degenerate elliptic equations [2]

I. Divergent equations with Partially Muckenhoupt weights Under a certain *additional condition* solution are Holder continuity but

$$\left.\begin{array}{l}\text{Harnack inequality}\\ \text{Sobolev inequality}\\ \text{double condition inequality}\end{array}\right\}\text{ fail.}$$

Fabes – Birolli – Serapioni model example. Some open problems.

II. Elliptic equation $\operatorname{div}(\nabla u + B\nabla u) = f$ **with skew symmetrical matrix** $B \in L^2$ Uniqueness problem. Analogy with Krylov's problem. "Diffusion in turbulent flow". "Variational" way to choose a solution. Statement of Homogenization problem.

III. Divergent equation with Lavrentiev phenomenon H-solution and W-solution. Variational solutions. Accessible or approximation solutions. How to describe the set of all accessible solutions? Model examples. Some open problems.

[2]This research was partially supported by Russian Foundation for Basic Research under grant No 99-01-00072

SECTIONAL TALKS

A'Campo Norbert
(University of Basel)

Real deformations and complex topology of singularities

Let $\gamma : \mathbb{C} \to \mathbb{C}^2$ be a polynomial map such that $\gamma(0) = 0$ and that the image of γ is a curve X with an isolated singularity at 0. A real number $\epsilon > 0$ is called a subcritical radius for the singularity of X, if the boundary of every Euclidean ball $B(0, \epsilon') \subset \mathbb{C}^2$ with center at 0 and radius $0 < \epsilon' \leqslant \epsilon$ intersects the smooth part of X transversally. By definition, the local knot $K(X)$ at 0 of the singularity of the curve X at 0 is the pair $(\partial B(0,\epsilon), X \cap \partial B(0,\epsilon))$, which up to homeomorphism does not depend upon the subcritical radius ϵ. A deformation $\gamma_t : \mathbb{C} \to \mathbb{C}^2$ of the mapping γ is a polynomial mapping $(z,t) \in \mathbb{C} \times \mathbb{C} \mapsto \gamma_t(z) \in \mathbb{C}^2$ with $\gamma_0 = \gamma$. The deformation γ_t will be called small relative to the choice of a subcritical radius ϵ if for all $t \in [0,1]$ the intersection of the curve $X_t := \gamma_t(\mathbb{C})$ with $\partial B(0,\epsilon)$ is a transversal intersection of submanifolds in $\mathbb{C}^2$. During a small deformation the type of the local knot in $\partial B(0,\epsilon)$ does not change. We say that a deformation γ_t is generic if for all $t \in [0,1]$ the curve $X_t \cap B(0,\epsilon)$ has only ordinary double point singularities. Generic deformations exist and the number of ordinary double point singularities of X_t is the genus $\delta(X)$ of the local knot $K(X)$. From now on we will assume that the mapping γ and and its small polynomial deformation γ_t are given by real polynomials. We say that the generic real deformation γ_t is a M-deformation if for every $t \in [0,1]$ all singularities of the curve $X_t \cap B(0,\epsilon)$ have real coordinates. The intersection $D(0,\epsilon) \cap X_t, t \in]0,1]$, is the image of a generic relative immersion with $\delta(X)$ crossings of an interval $[a,b]$ in the ϵ-disk $D(0,\epsilon)$ of $\mathbb{R}^2$. We call the immersion $D(0,\epsilon) \cap X_t \subset D(0,\epsilon)$ a divide $P \subset D(0,\epsilon)$ for the singularity at 0 of X. The knot $K(P)$ of a divide $P \subset D(0,\epsilon)$ is by definition the intersection in the tangent space $T(\mathbb{R}^2) = \mathbb{R}^4$ of the ϵ-ball in $\mathbb{R}^4$ and the space of tangent vectors to the divide P. We now briefly state some results. The first explains the title of our talk:

Theorem 1 *For the plane curve singularity X and any of its divides P the knots $K(X)$ and $K(P)$ are homeomorphic.*

Not every divide, i.e. not every image P of a generic relative immersion of the interval $[0,1]$ in the unit disk D, is a divide of a plane curve singularity. However, knots of divides share properties with knots of plane curve singularities.

Theorem 2 *The knot $K(P)$ of a divide $P \subset D$ is a fibred knot.*

Theorem 3 *The knot of a divide is transversal to the tight contact structure of S^3. The contact class in the sense of Emmanuel Giroux of the knot of a divide is tight.*

Theorem 4 *The unknotting number of the knot $K(P)$ of a divide P equals the number of double points of the immersed curve P.*

Theorem 5 *A knot of a divide is the closure of a strongly quasi-positive braid. Knots of ping-pong divides are closures of positive braids.*

Theorem 6 *Knots of singularities are knots of ping-pong divides.*

The topology and the monodromy of divide knots are very special with many interesting properties. To planar trees we associate so called slalom divides and knots. The arithmetic properties of slalom knots are related to the theory of "Dessin d'enfants" of Alexander Grothendieck and to the generalized Tchebichev polynomials of George Shabat.

Adamyan V., Tkachenko I.M.
(Odessa State University)

Inverse Stefan Problem for the Inhomogenious Heat Equation

The boundary problem for the inhomogeneous heat equation in linear, spherical and cylindrical geometrical setups with mobile interfaces and sources located on low-dimensional manifolds is considered. The problem of the heat release time dependence determination by a given interface speed is solved.

Andrei Afendikov
(Keldysh Institute of Applied Mathematics, Moscow)
Alexander Mielke
(Math. Institut A, Universität Stuttgart)

On the unfolding of reversible vector fields with SO(2)-symmetry and a non-semisimple eigenvalue 0

We consider four-dimensional ordinary differential equations depending on a vector-valued parameter λ in a neighborhood of the origin. For $\lambda = 0$ the origin is supposed to be an equilibrium whose linearization has a fourfold non-semisimple eigenvalue 0. Moreover, we assume that the vector fields are SO(2)-invariant and reversible. Such systems occur typically from spatial dynamical systems in physics near the instability threshold, where the evolution variable is obtained from a one-dimensional axial direction. The reversibility is then associated to a reflection symmetry and the SO(2)-invariance might arise for instance from an additional spatial variable in which periodicity is assumed, see [1, 2, 5, 6] for such applications. Our aim is to describe the generic unfoldings of such a singularity. It turns out that there are two cases. In Case 1 the SO(2) action and the reversor commute, and in Case 2 they do not commute. In both cases the lowest order terms which are derived via quasihomogeneous truncation lead to the steady one-dimensional Ginzburg-Landau equation

$$\frac{d^2}{dx^2}A + a(\lambda)A + b(\lambda)\frac{d}{dx}A + d|A|^2A = 0.$$

In Case 1 the reversibility acts like $(x, A) \mapsto (-x, A)$; and we have $b(\lambda) = 0$ and general coefficients $a(\lambda), d \in \mathbb{C}$. Then, we get the complex Ginzburg-Landau equation (cGL). In Case 2 the reversibility is $(x, A) \mapsto (-x, \overline{A})$ which implies $a(\lambda), ib(\lambda), d \in \mathbb{R}$. We demonstrate that this equation is completely integrable while (cGL) in general has complicated dynamics. For instance, there are cascades of n-homoclinic orbits in (cGL) [3, 4]. Under the certain restriction on parameters of (cGL) in an arbitrary small neighbourhood of the origin it is possible to prove the existence of a horseshoe.

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Agoshkov V.I.
(Institute of Numerical Mathematics, RAS, Moscow)

On a new method for solving the Stokes problem

We examine in this study the following approach to investigation and solution of problems of a given class, exemplified by a boundary value problem for the Stokes system: the function of pressure is regarded as an "additional" unknown to the "main" compo nent of a problem's solution, and the continuity condition from the Stokes system is considered as one of "observation conditions" ("observation data", etc.), which is prescribed to close the system. Then, the problem, which is considered as an inverse on e, is included to the family of optimal control problems that depend on a regularization parameter. In the following, the problems of optimal control are examined and solved with classic methods. At a conclusion stage, we must show that, for a trivial reg ularization parameter, we obtain the original problem and corresponding results (possibly, with additional restrictions on the initial data of problem). We consider some examples of iteration processes for the solution of problems in question, justify them, estimate their convergence rates, and present some results of numerical experiments.

Agranovich M.S.
(Moscow State Institute of Electronics and Mathematics)

Spectral Problems for the Dirac System with Spectral Parameter in Local Boundary Conditions

Let Ω be a bounded domain in R^3 with C^∞ connected boundary Γ. We consider the stationary Dirac system in Ω (for a free particle) written in the form

$$a(D)u - b_1 v = 0, \quad a(D)v - b_2 u = 0.$$

Here u and v are 2-dimensional vector-valued functions, $a(D) = \sum \sigma_j D_j$, $D_j = -i\partial/\partial x_j$, σ_j $(j = 1, 2, 3)$ are Pauli matrices, and b_1, b_2 are real constants. The boundary condition on Γ has the form

$$ia(\nu)v^+ = \lambda u^+,$$

where λ is the spectral parameter, u^+ and v^+ are the boundary values of u and v, and the matrix $a(\nu)$ is obtained from $a(D)$ by replacing D_j by the components of the outward unit normal ν. This problem has been communicated to the author by a Polish physicist Dr. R. Szmytkowski [1]. Assume that $k^2 = b_1 b_2 \neq 0$ and that the homogeneous Dirichlet problem for the equation $\Delta u + k^2 u = 0$ has no nontrivial solutions. Then we reduce the problem to the equation

$$Lu^+ = \lambda u^+$$

on the boundary. The reduction is equivalent only on C^∞ functions. Here L is a nonelliptic first order pseudodifferential operator on Γ, but it has an inverse L^{-1} on C^∞ functions, which is again a nonelliptic first order pseudodifferential operator. However, the linear combination $S = b_1 L - b_2 L^{-1}$ is an elliptic first order operator with positive principal symbol. This permits us to conclude that the operator L has a discrete spectrum but with two points of accumulation of eigenvalues, 0 and ∞. There exists an orthonormal basis in $L_2(\Gamma)$ consisting of C^∞ eigenfunctions, and it remains to be un unconditional basis in all Sobolev spaces $H^s(\Gamma)$. We also indicate the asymptotic behavior of eigenvalues tending to zero and to infinity. Similar problems for the Maxwell system have been considered long ago, see [2] and references therein. We also consider 1) the case of $b_1 \neq 0$, $b_2 = 0$, 2) the case of a particle in a smooth electromagnetic field, 3) generalizations to other dimensions of the space and other numbers of unknown functions, 4) other problems for the Dirac system similar to other problems for the Maxwell system considered in [2].

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Agranovich Yu. Ya.

(Voronezh State Technical University)

Spectral properties of integral operators and a conditional variant of Hilbert's 16-th problem

This talk is based on a joint work with Bugakova O.A. and Golovin A.V. As we consider some problems of object localization in Spatial Analysis it is natural to measure the information quantity by the angle value under which the object in consideration is seen from the given point. In this connection we consider the following model problem: *let two given segments be fixed on the plane in arbitrary manner. It is necessary to find all points of the plane from which these segments are seen under equal angles.* The desired geometric locus of points forms a real algebraic curve corresponding an inhomogeneous polynomial of the 6-th order of two variables. The coefficients of this polynomial depend on geometric and metric parameters of the problem: the length of the segments, the angle among the segments and the coordinates of their centers. Thus we get a special family of polynomials of the 6-th order, which defines the solutions of the problem in consideration. The obtained solutions permit to consider the cosine of the angles under which the segments are seen from the points of the found curves. Using these functions as kernels of integral operators acting in the Hilbert space of signals of finite power, we obtain the possibility to investigate the structure of unitary invariants of these operators: the spectrum and Hellinger types in accordance with the parameters of the problem. Thus we can see how the continuous change of the parameters of the problem leads to the discrete change of some integer characteristics of the operators and their invariant subspaces. The integral operators defined above can also be used as window transforms or as filters in Spatial Analysis. It is clear from the statement of this problem that the topic in the consideration is closely related to Hilbert's 16-th problem [1] and to well-known results of Petrovsky [2].In the talk it will present the following results: (1) the factorization of 6-th order polynomials on polynomial factors; (2) definition of the generic disposition of segments on the plane. The solving of the problem on segments in the case of the generic disposition; (3) The solving of the problem in the all degeneral cases: in the type of the geometrical degeneration and in the type of the metric degeneration; (4) determination of the kernels of integral operators and the presentation of the kernels of integral operators corresponding to the problem in the form of convergent power series with coefficients in the form of trigonometrical polynomials; (5) the matrix presentation of integral operators in the basis of the Hilbert space.

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Ahmedov H.I., Mamedov Yu.A.
(Baku State University)

On almost regularity by A.A.Shkalikov one spectral problem and expansion formula by its root elements

For the equation

$$y'' + q(x)y = \lambda^2 y, \quad 0 < x < 1, \tag{1}$$

almost regular (a.r.) (order no less than one in sense of definition of [2]) problems arise only in the boundary conditions as the form

$$U_1(y) \equiv \alpha_{11}y'(0) + \alpha_{10}y(0) + \beta_{11}y'(1) + \beta_{10}y(1) = 0,$$

$$U_2(y) \equiv \alpha_{20}y(0) + \beta_{20}y(1) = 0, \tag{2}$$

where $q(x)$ is a complex-valued function, α_{ij} and β_{ij} are complex numbers, $|\alpha_{11}| + |\alpha_{11}| > 0$, $|\alpha_{20}| + |\beta_{20}| > 0$. In terms of coefficients of equation and boundary conditions we'll find a.r-ity criterian order $m \geqslant 0$ of the problem (1),(2).

Theorem1. *Let $q(x) \in C^m[0,1]$, $m \geqslant 0$. Then for the a.r.-ity order m of the problem (1), (2) it is necessary and sufficient*
$\alpha_{11}\beta_{20} + \alpha_{20}\beta_{11} \neq 0$, *when* $m = 0$ *(regularity)*;
$\alpha_{11}\beta_{20} + \alpha_{20}\beta_{11} = 0$, $\alpha_{10}\beta_{20} + \alpha_{20}\beta_{10} \neq 0$, *whenm* $= 1$;
$\alpha_{11}\beta_{20} + \alpha_{20}\beta_{11} = 0$, $\alpha_{10}\beta_{20} + \alpha_{20}\beta_{10} = 0$, $\alpha_{11}\beta_{20} \neq 0$,
$q^{(k)}(0) = (-1)^k q^{(k)}(1)$ $(k = \overline{0, m-3})$, $q^{(m-2)}(0) \neq (-1)^{m-2} q^{(m-2)}(1)$,
when $m \geqslant 2$. The statement of decomposability by root element is also proved by methods of [1]. The problem as (1),(2) is a.r. order m.

Theorem2. *Let $q(x) \in C^m[0,1]$, $m \geqslant 0$, and let the problem (1),(2) be almost regular order m. Then for any function $f(x)$ such that*

$$f(x) \in C^{m+2}[0,1],\ U_i\left(\left[\frac{d^2}{dx^2} + q(x)\right]^k f(x)\right) = 0,\ (i = 1,2;\ k = 0,1,\ldots,\left[\frac{m}{2}\right]),$$

the expansion formula

$$f(x) = -\sum_{\nu=1}^{\infty} \underset{\lambda_\nu}{res\lambda} \cdot \int_0^1 G(x,\xi,\lambda) f(\xi) d\xi, \tag{3}$$

where $G(x,\xi,\lambda)$- is Green's function of the problem (1),(2), $\{\lambda_\nu\}_{\nu=1}^{\infty}$ is set of its poles, the series in (3) converges uniformly (in general with brackets) at $x \in [0,1]$, is true. Note that a denumerable set of Green function's poles of the problem (1),(2) in the case $m \geqslant 1$ is proved in [2].

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Ajiev S.S.

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On some properties of nonseparable spaces of differentiable functions.

G. M. Fichtenholtz and L.V. Kantorovich ([3], 1934) showed the noncomplementability of $C([0,1])$ in $L_\infty([0,1])$. R.C. Phillips (1940) proved that there is no any linear bounded operator (in particular, projection) from l_∞ to itself with the kernel c_0. Here are analogues of Phillips theorem obtained for anisotropic Nikol'skii, Besov ($B^s_{p,\infty}(G)$), and Lizorkin-Triebel ($L^s_{p,\infty}$) spaces of functions, defined on open subsets of n-dimentional Euclidean space, as well as for the spaces, closely related to the above spaces, that are defined by local best approximations by polinomials in different metrics (for example the spaces of De Vore and Sharpley, $BMO(G)$, $BMO(G) \cap L_\infty(G) = L_\infty(G)$). Let $X(G)$ be one of the spaces under consideration then let $\overset{\circ}{X}(G)$ be its subspace which coinsides in most cases with the closure of C^∞ in $X(G)$ (we are using another explicit but more long definition). The case of (quasi)seminormed $X(G)$ that is so called homogeneous variant of any of the above mentioned spaces is considered too. The following is a short form of the first theorem.

Theorem. *Suppose that G is an open subset of n-dimentional Euclidean space, a linear topological space Y has a countable total set of continuous linear functionals. Then, for an arbitrary continuous linear operator $F : X \to Y$, $Ker(F) \neq \overset{\circ}{X}(G)$. Or, in other words, $\overset{\circ}{X}(G)$ can not be represented as as an intersection of a countable number of hiperplanes of the space X. Every above space is fitted to be Y.*

As particular cases, the pairs $(X(G), \overset{\circ}{X}(G))$ of zero smothness, such as anisotropic $(L_\infty(G), L_\infty \cap VMO(G))$, $(BMO(G), VMO(G))$, $(L_\infty(G), C(\bar{G}), diam(G) < \infty)$, are also covered. To compensate negative consequence, namely, the noncomplementness of $\overset{\circ}{X}(G)$ in $X(G)$, the norms of the quotient spaces $X(G)/\overset{\circ}{X}(G)$ are calculated (partly, if G is a domain satisfying flexible λ-horn condition). The main tools of the proofs are Phillips theorem by itself, construction of convenient copies of l_∞ (implying nonseparability) in these spaces, and the theory of function spaces developed in [1,2].

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On the coefficients of eigenfunctions-series expansion for problems with an eigenvalue parameter in boundary conditions

We consider the eigenvalue problem

$$l(y,\lambda)=\sum_{\nu=0}^{n}\lambda^{\nu}\,l_{\nu}(y)=0,\qquad U_j(y,\lambda)=\sum_{\nu=0}^{m}\lambda^{\nu}\,U_j^{\nu}(y)=0. \tag{0.1}$$

Here $j=1,\,2,\,\ldots,\,n,\quad l_{\nu}(y)=\sum_{s=\nu}^{n}p_{\nu s}(x)\,y^{(n-s)}(x)=0,$
$U_j^{\nu}(y)=\sum_{k=0}^{n-1}\left(\alpha_{jkl}\,y^{(k)}(0)+\beta_{jkl}\,y^{(k)}(1)\right)$. For the regular eigenvalue problem (0.1) A.A. Shkalikov proved that derivative chains of eigen and associated elements $\widetilde{y}_k^{\,h}$ are the basis in the Hilbert space $\mathcal{W}_{2,U}^{r+1}$. ($\mathcal{W}_{2,U}^{r+1}$ is subspace of $W_2^{n+r+1}\times W_2^{n+r}\times\cdots\times W_2^{r+1}$, where $W_2^s=W_2^s(0,\,1)$ is Sobolev space.)

Let $\widetilde{v}=\{v_0,\,v_1,\,\ldots,\,v_{n-1}\}\in\mathcal{W}_{2,U}^{r+1}$ and $\widetilde{v}=\sum_{k=1}^{\infty}\sum_{h=0}^{g_k}c_{kh}\,\widetilde{v}_k^{h}$.

Theorem. *Suppose* $\int_0^1 l_0(y)\cdot\overline{z}\,dx=\int_0^1 y\cdot\overline{l_0^*(z)}\,dx+P_0(y,z)$,
$P_0(y,z)=U_1^0(y)\cdot\overline{V_{2n}^0(z)}+U_2^0(y)\cdot\overline{V_{2n-1}^0(z)}+\cdots U_{2n}^0(y)\cdot\overline{V_1^0(z)}$;
$U_j^0(y)$, $V_j^0(z)$ $(j=1,2,\ldots,2n)$ *are homogeneous linearly independent forms; there exist numbers* $\xi_{ij}^{\nu}\in\mathbb{C}$ $(i,j=1,2,\ldots,n)$, *and homogeneous forms* $\overline{\gamma_1^{\nu}(z)},\overline{\gamma_2^{\nu}(z)},\ldots,\overline{\gamma_n^{\nu}(z)}$, *such that*
$U_i^{\nu}(y)=\sum_{j=1}^{n}\xi_{ij}^{\nu}\,U_{n+j}^0(y),\qquad \nu=1,2,\ldots,m,$
$\int_0^1 l_{\nu}(y)\cdot\overline{z}\,dx-\int_0^1 y\cdot\overline{l_{\nu}^*(z)}\,dx=\sum_{j=1}^{n}\overline{\gamma_j^{\nu}(z)}\cdot U_{n+j}^0(y),\quad \nu=1,\,2,\,\ldots,\,n.$
Then $c_{kh}=p_{kh}/q_{kh}$, *where*
$p_{kh}=\sum_{s=0}^{n-1}\sum_{\nu=0}^{n-1-s}\left(l_{\nu+1+s}(v_{\nu}),\,z_k^{p_k-h,s}\,)\right)_{L_2}-$

$$-\sum_{i=1}^{n}\sum_{s=0}^{m-1}\sum_{\nu=0}^{m-1-s} U_i^{\nu+1+s}(v_\nu)\cdot\overline{V_{2n+1-i}^{0}\left(z_k^{p_k-h,s}\right)},$$

$$q_{kh}=\sum_{s=0}^{n-1}\sum_{\nu=0}^{n-1-s}\left(l_{\nu+1+s}(y_k^{h,\nu}),\, z_k^{p_k-h,s}\right)_{L_2}-$$

$$-\sum_{i=1}^{n}\sum_{s=0}^{m-1}\sum_{\nu=0}^{m-1-s} U_i^{\nu+1+s}(y_k^{h,\nu})\cdot\overline{V_{2n+1-i}^{0}\left(z_k^{p_k-h,s}\right)}.$$

Here $s=n,\ldots,m-1$, and v_s is $(n-1)$-th component of $H^{s+1-n}\widetilde{v}$ (H is Shkalikov's linearizator).

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Accelerated convergence method for solving Sturm-Liouville Problem.

Many problems in mechanics, theory of oscillations and stability, control theory, mathematical and theoretical physics, hydrodynamics, acoustics, dynamics of the ocean and atmosphere, theory of elasticity, etc., lead to generalized boundary-value problems of determining natural frequencies and modes of oscillations. It is required to construct a solution of the generalized Sturm-Liuoville problem, in which the coefficients of the equation are arbitrary non-linear functions of the desired parameter. To fix our ideas, we will consider the following eigenvalue problem

$$(p(x,\lambda)u')'+r(x,\lambda)u=0,\quad 0\leqslant x\leqslant \ell<\infty;\quad u(0)=u(\ell)=0$$

$$0<p_1\leqslant p\leqslant p_2<\infty,\quad 0<r_1\leqslant r\leqslant r_2<\infty\quad \lambda\in\Lambda$$

We will formulate the problem of obtaining real values of λ for which non-trivial solutions of the equation with the boundary conditions exist. As compared with the classical case, the behavior of the eigenvalues λ_n and the functions $u_n(x)$ — as a rule turns out to be extremely unusual, and a detailed study of it is difficult. The properties of the "spectrum" as a function of the order number n (and x for eigenfunctions) may dramatically differ from the generally known properties, obtained for the classical problem. Calculations of the eigenvalues and eigenfunctions with a required degree of accuracy encounter, essential difficulties: there are no effective algorithms. A highly effective numerical-analytical method for solving the problem, which posses the property of accelerated convergence, is described in this paper. It is based on differential relation, which we have established, between the eigenvalue λ_n and the length ℓ of the interval. Here is some model examples:

$$1)\ \ell=p=1;\qquad r=(\lambda+x^2)^{-2}$$

On the basis of our algorithm we have obtained:

$$\lambda_1=0,1655405;\quad \frac{\Delta\lambda}{\lambda_1}\leqslant 10^{-4};\quad \text{exact}\quad \lambda_1=0,165643.$$

$$2)\ \ell = p = 1; \quad r(x,\lambda) = (\lambda + 0,1 \sin \pi x)^{-2},$$

$$\lambda_1 = 0,235283, \quad \frac{\Delta\lambda}{\lambda_1} \leqslant 4 \cdot 10^{-6}; \quad \lambda_2 = 0,097163, \quad \frac{\Delta\lambda}{\lambda_2} \leqslant 4 \cdot 10^{-5}$$

We have solved a number of some applied problems in mechanics, theory of oscillations and stability, and hydrodynamics.

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Mass exchange and diffusion in porous materials

1. Introduction The paper contains a macroscopic continuum model of adsorption in porous materials consisting of three components. We suppose physical adsorption processes which means that particles of the adsorbate stick to the skeleton due to weak van der Waals forces (see: [2]). In the flow process of a fluid/adsorbate mixture through channels of a solid component the fluid serves as carrier for an adsorbate whose mass balance equation contains a source term. It consists of two parts: first a Langmuir contribution which is connected with bare sides on internal surfaces (see: [3]) and becomes the Langmuir isotherm in equilibrium. The second one is revealed with changes of the internal surface driven by the source of porosity which is a parameter of the balance equation for porosity (see: [4]). A simple numerical example which describes the transport of pollutants in soils illustrates the coupling of adsorption and diffusion. We use a linear regular perturbation method and Laplace transforms to find an analytical approximate solution of the problem. In order to get numerical solutions for the inverse Laplace transform we use a Fortran-solver. The results show that after some period of time arises a maximum for the amount of adsorption for a certain range of fluid/adsorbate velocities.

2. Three-dimensional adsorption/diffusion model

We investigate a flow of a fluid-adsorbate mixture through channels of a porous medium. Particles of adsorbate settle down on the surface of the skeleton so that their kinematics changes from that of the fluid to that of the skeleton. Before fluid and adsorbate flow with a common velocity $\mathbf{v}^F$ through the skeleton which has the velocity $\mathbf{v}^S$. Fluid, adsorbate and skeleton have the current mass densities ρ^F, ρ^A and ρ^S, respectively. The mass source of the adsorbate is denoted by $\hat{\rho}^A$. With the definitions for the mass density of the liquid $\rho^L := \rho^F + \rho^A$, the adsorbate concentration in the fluid $c := \frac{\rho^A}{\rho^L}$ and the source of concentration

$\hat{c} := \frac{\hat{\rho}^A}{\rho^L}$ the classical balance equations have the following form

$$\frac{\partial \rho^S}{\partial t} + \text{div}\,\left(\rho^S \mathbf{v}^S\right) = -\rho^L \hat{c},$$
$$\frac{\partial \rho^L}{\partial t} + \text{div}\,\left(\rho^L \mathbf{v}^F\right) = \rho^L \hat{c},$$
$$\frac{\partial c}{\partial t} + \mathbf{v}^F \cdot \text{grad}\, c = (1-c)\,\hat{c},$$
$$\frac{\partial \rho^S \mathbf{v}^S}{\partial t} + \text{div}\,\left(\rho^S \mathbf{v}^S \otimes \mathbf{v}^S - \mathbf{T}^S\right) = \hat{\mathbf{p}},$$
$$\frac{\partial \rho^L \mathbf{v}^F}{\partial t} + \text{div}\,\left(\rho^L \mathbf{v}^F \otimes \mathbf{v}^F + p^L \mathbf{1}\right) = -\hat{\mathbf{p}}.$$

According to Dalton's law we expected $p^A \cong cp^L$ with $p^L = p^F + p^A$. Furthermore $\mathbf{T}^S$ denotes the partial Cauchy stress tensor in the skeleton, and $\hat{\mathbf{p}} = \pi\left(\mathbf{v}^F - \mathbf{v}^S\right) - \rho^L \hat{c} \mathbf{v}^F$ is the momentum source in the liquid where π denotes the permeability coefficient. For the scalar field of porosity we have an additional balance equation as introduced in e.g. [4]. For small deformations of the skeleton it has the form

$$\frac{\partial n}{\partial t} + \mathbf{v}^S \cdot \text{grad}\, n + n_E \text{div}\,\left(\mathbf{v}^F - \mathbf{v}^S\right) = \hat{n} = -\frac{\Delta}{\tau}.$$

Here $\Delta = n - n_E$ is the deviation of the porosity n from its equilibrium value n_E, τ is the relaxation time of porosity and $\hat{n}$ is the source of porosity.

3. Specification of the mass source

On the macroscopic level of description we denote the normalized fraction of occupied sides per unit volume by ξ, i.e. the fraction of bare sides is $1-\xi$. Furthermore we denote the internal surface area of the pores by f_{int} and the mass of adsorbate per unit area of this surface by m_A. Then the mass source is given by the relation

$$\hat{\rho}^A = -\frac{m^A}{V}\frac{d\left(\xi f_{int}\right)}{dt} = -\frac{m^A}{V}\left(f_{int}\frac{d\xi}{dt} + \xi\frac{d f_{int}}{dt}\right), \tag{1}$$

The first contribution on the right-hand side of this equation describes the change of the fraction of occupied sides. It is specified by the Langmuir evolution equation

$$\frac{d\xi}{dt} = a\,(1-\xi)\,p^A - b\xi e^{-\frac{E_b}{kT}},$$

where p^A is the partial pressure of the adsorbate in the fluid phase and a and b are material parameters. The energy barrier E_b for particles adsorbed on the skeleton is assumed to be constant. Furthermore k denotes the Boltzmann constant and T

is the absolute temperature. The right hand side again exists of two terms: the adsorption rate (first term) and the desorption rate (second term). In full phase equilibrium they are equal so that the time change of occupied sides is equal to zero. In this case we get the well-known *Langmuir isotherm* of occupied sides. The other part of (1) describes the change of the internal surface. We assume that this change is coupled with relaxation of the porosity n, which is described by the balance equation of porosity. The source of porosity $\hat{n}$ describes the intensity of dissipative changes of porosity per unit time and volume of the porous material. Motivated by elementary considerations about changes of the internal surface and of the porosity in a porous medium yielding film adsorption (see [1]) we assume $\frac{1}{f_{int}}\frac{df_{int}}{dt} \propto \frac{\hat{n}}{n}$.

4. Results

The most important investigation of this work is that the results show the coupling of adsorption and diffusion. This means that the amount adsorbed (absolute value of the concentration source) depends on the relative velocity of the components. We can show that there is a region of relative velocities where the rate of adsorption reaches a maximum value. In the case of very small and very large diffusion the adsorption rate decays much faster than it is the case for moderate diffusion.

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Generalized solutions of Maxwell equations. Shock electromagnetic waves

By the study of processes of spreading of nonstationary electromagnetic waves we meet some difficulties, in accordance with arising the shock electromagnetic waves, for which is distinctive breach of continuity of fields on wave fronts. Last one prevents to using classical methods of mathematical physics or increases the requirements on smoothness of electromagnetic fields that greatly limits a class of solved problems. Entering a notion of *generalized solutions* of the equations and applying the methods of theory of distributions enable to cope with these problems[1].

We consider the solutions of Caushy problem and two boundary value problem (BVP) in the space of generalized functions $D_6'(R^4)$ for the Maxwell equations:

$$-\varepsilon\partial_t E + rot H = j^E, \quad \mu\partial_t H + rot E = j^H \tag{0.1}$$

where electrical and magnetic permeability ε, μ are constant. The initial field (E_0, H_0) and electric and magnetic currents $j^E(x,t)$, $j^H(x,t)$ are known. At the boundary S, which is Lyapunov surface, there are next conditions on the shallow currents: $E \times n = j_H^S(x,t)$ $(first\ BVP)$, $H \times n = j_E^S(x,t)$ $(second\ BVP)$. Here n is unit vector of external normal, $a \times b$ and (a,b) are vector and scalar product consequently. Eqs.(1) are hyperbolic.

Theorem 1. *The solution of BVP, which is continues and differentiable every where besides waves fronts F_t, is its generalized solutions in $D_6'(R^4)$ only if $[E]_{F_t} = (\mu/\varepsilon)^{1/2}[H]_{F_t} \times m$, $[H]_{F_t} = -(\mu/\varepsilon)^{-1/2}[E]_{F_t} \times m$. Also $[W]_{F_t} = (m, [P]_{F_t})$.* Here $[f]_{F_t}$ is the gap of $f(x,t)$ on F_t, $m = gradF_t/\|gradF_t\|$ is wave vector, $P = c^{-1}[E,H]$ is Poiting vector, $W = 0,5(\varepsilon\|E\|^2 + \mu\|H\|^2)$ is the density of electromagnetic energy.

Corollary. *At the front of shock waves F_t* $([E]_{F_t}, m) = 0$, $([H]_{F_t}, m) = 0$. It means that the shock electromagnetic waves are *transverse* ones.

Next theorem is proved.

Theorem 2. *If the solution of BVP satisfies to conditions of theorem 2, it is unique.*

Using Green's tensor [2] the generalized solutions, their regular integral representation and singular boundary integral equations for I and II BVP have been constructed. For solving BVP here the method of generalized function are developed analogous to [1,3]. Also the problem of strong shock waves expansion has been solved as Caushy problem.

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On the correct solvability conditions of boundary-value problems for a class of the odd order operator-differential equations with discontinuous coefficients

In a separable Hilbert space H the following boundary-value problem is studied for a class of the odd order operator-differential equations with discontinuous coefficients:

$$A_0 u^{(4k+1)}(t) + \rho(t) A^{4k+1} u(t) + \sum_{j=1}^{4k+1} A_j(t) u^{(4k+1-j)}(t) = f(t), t \in R_+ = [0; \infty), \quad (1)$$

$$u^{(j)}(0) = 0, \quad j = \overline{0, n}, \quad n < 4k + 1, \quad (2)$$

where $f(t) \in L_2(R_+; H)$, $u(t) \in W_2^{4k+1}(R_+; H)$ ([1]), $A_0 = \pm E$ (E is an unit operator in H), A is a selfadjoint positively-defined operator in H ($A = A^* >> cE$, $c > 0$), $A_j(t)$, $j = \overline{1, 4k+1}$ are linear, generally speaking, unbounded operators, defined almost for every $t \in R_+$, and $\rho(t)$ is a scalar bounded function, defined in the following way: $\rho(t) = \begin{cases} \alpha_1, & 0 \leqslant t \leqslant T_1, \\ \alpha_2, & T_1 < t \leqslant T_2, \\ \dots\dots\dots & \\ \alpha_s, & T_{s-1} < t < +\infty, \end{cases}$ where α_j, $i = \overline{1, s}$ are positive numbers, generally speaking, not equal to each other, and $\inf_t \rho(t) = \alpha_1$, $\sup_t \rho(t) = \alpha_s$; $n = 2k$ or $n = 2k - 1$ according to the choice of the operator A_0. Here we take the derivative $u^{(j)} \equiv \frac{d^j u}{dt^j}$ in the sense of generalized functions theory. The boundary-value problem $\{(1); (2)\}$ is studied in [2] for $\rho(t) \equiv 1$, $t \in R_+$. In the work the following theorem holds true as well.

Theorem. *Let $A_0 = E$, $n = 2k$, $A = A^* > cE$, $c > 0$, the operators $B_j(t) = A_j(t) A^{-j}$, $j = \overline{1, 4k+1}$ are bounded in H and the inequality takes place*

$$\sum_{j=1}^{4k+1} \left[\left(\frac{4k+1-j}{4k+1} \right)^{\frac{4k+1-j}{8k+2}} \cdot \left(\frac{j}{4k+1} \right)^{\frac{j}{8k+2}} \cdot \beta_j \cdot \alpha_1^{-\frac{4k+1+j}{8k+2}} \times \right.$$

$$\left. \times \alpha_s^{\frac{4k+1-j}{8k+2}} \sup_t \|B_j(t)\|_{H \to H} \right] + \alpha_1^{-1} \sup_t \|B_{4k+1}(t)\|_{H \to H} < 1,$$

where

$$\beta_j = \begin{cases} 2^{\frac{4k+1-j}{4k+1}(2k-1)k}, & j \leqslant 2k-1, \\ 2^{\frac{j}{8k+2}[(4k+1)(2k-j)+4k^2-1]}, & j > 2k-1 \end{cases}$$

Then the boundary-value problem $\{(1);\ (2)\}$ *at any* $f(t)$ *from space* $L_2(R_+;H)$ *has a unique solution from* $W_2^{4k+1}(R_+;H)$. Corresponding theorem also takes place when $A_0 = -E$ and $n = 2k-1$.

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On the optimization problem for a remainder of some class of quadrature formulae

Let's consider the ordinary differential equation

$$Ty \equiv y^{(2r)}(x) - \lambda y(x) = f(x) \tag{1}$$

with boundary conditions

$$y^{(\sigma_i)}(0) = y^{(\gamma_i)}(1) = 0 \quad (i = 1,2,\dots,r), \tag{2}$$

where $f(x)$ is a continuous function on the interval $[0;1]$, $\sigma_i < \sigma_{i+1}$, $\gamma_i < \gamma_{i+1}$ $(i = 1,2,\dots,r-1)$, σ_i, $\gamma_i \in \{0,1,2,\dots,2r-1\}$ and $\lambda = 0$ is not an eigenvalue of the corresponding homogenous problem. In the work the minimization problem is solved for a remainder $R_{nN}(y)$ of the following quadrature formula

$$\int_0^1 y(x)dx = \int_0^1 y_n(x)dx + \sum_{k=1}^{N}\sum_{\ell=1}^{2r-2} A_k^{(\ell)}\left[y^{(\ell)}(x_k) - y_n^{(\ell)}(x_k)\right] + R_{nN}(y), \tag{3}$$

on the set of solutions of the boundary-value problem $\{(1);\ (2)\}$. Note that here $0 < x_1 < x_2 < \ldots < x_N < 1$, $y_n(x)$ an approximate solution of the boundary-value problem $\{(1);\ (2)\}$, which is searched in the following form:

$$y_n(x) = \sum_{k=0}^{n} C_{kn} u_{kn}(x),$$

and found by colocation method. Note that the known functions $u_{kn}(x)$ $(k = 0,1,2,\ldots$

$\dots, n)$ are solutions of the equation $u_{kn}^{(2r)}(x) = \ell_{kn}(x)$ $(k = 0, 1, 2, \dots, n)$ with boundary conditions (2), $\ell_{kn}(x)$ $(k = 0, 1, 2, \dots, n)$ are Lagrange fundamental polynomials. As colocation nodes we choose the roots of polynomial $\omega_n(x)$, where $\{\omega_n(x)\}$ is a system of orthogonal polynomials with respect to the weight $p(x)$ on [0;1]. Note that both $p(x)$ and $1/p(x)$ are nonnegative summable functions on [0; 1]. Differential equation (1) as a functional, is considered in the space $L_{2,p}$ of functions $y(x)$, continuous on [0; 1] together with their derivatives up to the $2r$-th order inclusive. The norm is:

$$\|y\|_{L_{2,p}} = \left(\int\limits_0^1 p(x)|y^{(2r)}(x)|^2 dx \right)^{1/2}$$

In the work the optimal nodes and coefficients are found on the set of solutions of the problem $\{(1); (2)\}$ for quadrature formulae (3). Estimates from below and from above are found for their remainder $R_{nN}(y)$ as well.

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L_p – solvability of the Dirichlet problem for Heat equation in noncylindrical domains

In the talk the Dirichlet problem is investigated in Sobolev spaces with Muckenhoupt weights for some class of model domains. We give the complete description of the weights for unique solvability of the problem together with the corresponding coercive estimate. In particular the Dirichlet problem in a ball B_R of radius R is studied. We formulate this result for a classical Sobolev space $W_{p,0}^{2,1}(B_R)$, $p > 1$. Our considerations take place in the coordinate space E_{n+1} of ponits $(t, x) = (t, x_1, \dots, x_n)$. The solvability of the Dirichlet problem

$$Lu = f \text{ in } B_R,\ f \in L_p(B_R),\ u \in W_{p,0}^{2,1}(B_R) \quad (0.1)$$

for heat conductivity operator $L = \Delta - \partial/\partial t$ closely connect to the first eugenvalue $\mu = \mu(R)$ of the problem

$$\sum_{i=1}^{n} \left(\exp(-|\xi|^2/4) v_{\xi_i}\right)_{\xi_i} + \mu \exp(-|\xi|^2/4) v = 0 \text{ in } Q_{2R},\ v|_{\partial Q_{2R}} = 0$$

in n - dimensional ball $Q_{2R} = \{\xi : |\xi| < 2R\}$.

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Theorem. *The solution of the problem (0.1) is unique. This problem is solvable with the coercive estimate*

$$\| u \|_{W^{2,1}_{p,0}(B_R)} \leqslant C(n,p,R) \| u \|_{L_p(B_R)}$$

if and only if $\mu(R) > (p - n/2 - 1)/p$. For instance if $p \in (1, (n+2)/2)$ then the problem (0.1) is solvable in a ball of arbitrary radius R. It should be noted that the criterions solvability of the problem (0.1) for various parabolic operators L with constant coefficient are different. In particular for operator $L = \Delta - \alpha \cdot \partial/\partial t$, $\alpha > 0$, the necessary and sufficient conditon of solvability is $\mu(\alpha R) > (p - n/2 - 1)/p$. The first observation of this kind belongs to I.G. Petrovskii.

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Finite-difference scheme to the equations of a one-dimensional viscous real gas and the equations of one-dimensional nonlinear thermoviscoelasticity with nonsmooth data

The initial-boundary value problem

$$D_t\eta = Du \quad \text{in } Q; \tag{1}$$
$$D_t u = D\sigma + g, \quad \sigma = \nu(\eta)\rho Du - p(\eta,\theta), \quad \rho = 1/\eta \quad \text{in } Q; \tag{2}$$
$$D_t e(\eta,\theta) = D\pi + \sigma Du + f, \quad \pi = \lambda(\eta,\theta)\rho D\theta \quad \text{in } Q; \tag{3}$$
$$\eta|_{t=0} = \eta^0(x), \quad u|_{t=0} = u^0(x), \quad e(\eta,\theta)|_{t=0} = e^0(x) \quad \text{on } \Omega; \tag{4}$$
$$u|_{x=0} = u_0(t), \quad u|_{x=X} = u_X(t) \quad \text{on } (0,T); \tag{5}$$
$$-\pi|_{x=0} = \chi_0(t), \quad \pi|_{x=X} = \chi_X(t) \quad \text{on } (0,T), \tag{6}$$

describes one-dimensional motions of a viscous real gas and a thermoviscoelastic body of the Voigt type. The unknown functions $\eta(x,t), u(x,t), \theta(x,t)$ are defined on $Q = \Omega \times (0,T)$, where $\Omega = (0, X)$. The existence of a global weak solution of problem (1)-(6) with nonsmooth data was proved in [1], [2]. For solving this problem, we propose the new nonlinear finite-difference scheme [3], [4]. Under almost the same assumptions as in [1] on the data, we obtain *a priori* estimates, global with respect to time and the problem data, for the finite-difference solutions and prove the existence of these solutions. Besides, a theorem concerning the convergence of some subsequence of finite-difference solutions to a weak solution of the initial-boundary value problem (1)-(6) is proved. This work was supported by the Russian Foundation for the Basic Research, projects 00-01-00207 and 01-01-00700.

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Approximation and asymptotic behaviour of one-parameter semigroups of contractions in a Hilbert space

Let (R, V) consist of a one-parameter semigroup of contractions $R = (R_t)_{t\in\mathbf{R}_+}$ in a Hilbert space H and its minimal unitary dilation $V = (V_t)_{t\in\mathbf{R}}$ in a Hilbert space H_u, $H \subset H_u$. We claim $R = R^1 \oplus R^2$, where R^1 is a completely nonunitary contraction semigroup of type $C_{0\cdot}$, i.e. $R_t^1 \to 0$, when $t \to +\infty$ in strong toplogy, and R^2 is a unitary semigroup. We shall call by canonical this decomposition of R. Note that a minimal isometrical dilation of R coincides with V because $R^1 \in C_{0\cdot}$. Let H_{is} be a Hilbert space of minimal isometrical dilation of R^*, $H \subset H_{is}$, $H_{is} \subset H_u$. Denote $\mathcal{V} = \mathcal{V}(H, H_{is}, H_u)$ a set of all pairs (R, V) defined by semigroups R having isometrical and unitary dilations in H_{is} and H_u correspondingly. Consider the minimal isometrical dilation $Y = V^*|_{H_{is}}$ of R^*. Define the index of R as a number of orthogonal solutions (1-cocycles) of the equation $f_{t+s} = f_t + Y_t f_s$, $f_t \in H_{is}$, $s, t \geqslant 0$. Let $d_R, r = (d_R + I)(d_R - I)^{-1}$ and $d_Y, y = (d_Y + I)(d_Y - I)^{-1}$ be generators and cogenerators of the semigroups R and Y correspondingly. Then the deficiency number n of r is defined by the formula $n = dim[(I - r^* r)^{1/2} H]$. The deficiency index of D is a number of solutions of the equation $d_Y^* f = -f$ in the Hilbert space H_{is}. Notice that all these definitions yield to the same number that we call the index of R. We introduce the following notion of approximation preserving the index (see also [1-2]).

Definition. *Two pairs (R, V), $(T, U) \in \mathcal{V}$ are said to be approximating each other if (i) $V_t - U_t \in s_2$, (ii) $V_t f = U_t f$, $\forall f \in H_u \ominus H_{is}$, $t \in \mathbf{R}$.*

Notice that condition (*ii*) of the definition garantees the equality of the indecies of R and T. Moreover every pair (R, V) defines a quantum stochastic process $B(V)$ and an associated completely positive semigroup $B(R)$ on the pair of hyperfinite factor (M', M) generated by the representations of the algebras of canonical anti-commutation relations over H_u and H correspondingly. Two pairs approximating each other in the sence of our definition are cocycle conjugate by a Markovian cocycle (see [2] for the proof and [3] for the definitions and notations).

Theorem. *Let $R^2 \equiv 0$ in the canonical decomposition of R and R included in the pair (R, V). Then for every norm-continuous unitary semigroup S in a Hilbert space K, there exist a pair $(T, U) \in \mathcal{V}$ such that the part T^2 in the canonical decomposition of T is unitary equivalent to S and (T, U) is approximating (R, V).* The part T^2 determines the asymptotic behaviour of $T = (T_t)_{t \in \mathbb{R}_+}$, when $t \to +\infty$. Hence Theorem implies that approximating arbitrary pair from $\mathcal{V}$, one can reach the required asymptotic dynamics.

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Evaluation of structural characteristics of flows

The evolution of complex dynamic systems is highly determined by structure of locally maximal invariant sets and their characteristics, such as Lyapunov exponents, the Morse Spectrum, the Conley Index, the topological entropy and dimension. The authors are developing some methods of applied symbolic dynamics, which gives one an opportunity to obtain an information about the qualitative behavior of a system without any preliminary information about the system. These methods are based on the synthesis of theoretical results and computer-oriented algorithms for the qualitative studying of dynamic systems. The first step consists in constructing the so-called symbolic image of a system, which is an approximation of this system. We consider the oriented graph, which vertices are the elements of some closed finite covering $\{D_i\}_{i=1}^{s}$. The vertices i and j are connected by an oriented edge $i \to j$ iff the image of the cell D_i intesects the cell D_j. It is proved that there exists a correspondence between pseudo-trajectories of a dynamic system and admissible paths in its symbolic image. An analisys of the symbolic image is based on various methods of studying oriented graphs. For example, it is possible to find a cell for which there exists a closed path passing through it. In particular, the problem of localization of periodic trajectoies, the construction of neighborhoods of chain recurrent components, attractors and their domains, filtrations and isolative sets, are solved. If the pair (K, L), where K is an isolating neighborhood and L is an exit set, has a non-trivial homotopy type (the Conley Index), than the set K contains a non-empty invariant set. The non-triviality of homology groups $H_k(K, L)$ provides the sufficient condition. Consequently, it is highly interesting to find an effective algorithm for computation of the homology groups of the pair of sets, each of these is a union of some cubes. Currently, the authors are testing an algorithm for the plane sets.

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Loop Equations in Noncommutative Field Theories

Noncommutative analog of loop equations in the form of Levy-Laplace equation for the Yang-Mills theory will be presented.

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On the strong nonliner nondiagonal parabolic systems with q-growth

Let Ω be a domain in $\mathbb{R}^2$, $Q = \Omega \times (0,T)$, the number $T > 0$ is fixed arbitrarily. We consider a solution $u : Q \to \mathbb{R}^N, N > 1$, $u = (u^1, ..., u^N)$, of the parabolic system

$$u_t - \frac{d}{dx_\alpha} a_\alpha(x,u,u_x) + b(x,u,u_x) = 0, \quad (x,t) \in Q. \tag{1}$$

We assume that functions $a^k_\alpha(x,u,p)$ and $b^k(x,u,p)$, $k = 1,...,N$, are smooth enough, and for a fixed $q > 1$

$$\frac{\partial a^k_\alpha}{\partial p^l_\beta}(x,u,p)\theta^k_\alpha \theta^l_\beta \geqslant \nu(1+|p|)^{q-2}|\theta|^2, \quad \left| \frac{\partial a^k_\alpha}{\partial p^l_\beta}(x,u,p) \right| \leqslant \mu(1+|p|)^{q-2}, \tag{2}$$

where $p, \theta \in \mathbb{R}^{2N}, \nu, \mu = const$. Firstly, we consider systems with $a_\alpha = a_\alpha(x,u_x)$ and $b = 0$ under assumption $q > 1$ in (2). For these systems, local regularity of weak solutions *inside* of Q were stated by J.Necas, V.Sverac for the parameter $q \geqslant 2$ [1], and by J.Frehse, G.Seregin for $q \in (1,2)$[2]. We prove global in time classical solvability of Cauchy-Diriclet (C-D) and Cauchy- Neumann (C-N) problems for such type systems with any $q > 1$ in condition (2). Next, we study systems (1) in the case when the elliptic operator of the system is the Euler operator of a functional $E[u] = \int\limits_\Omega f(x,u,u_x)dx$, i.e., $a^k_\alpha = f_{p^k_\alpha}(x,u,u_x)$, $b^k = f_{u^k}(x,u,p)$. We assume the "natural" growth of b:

$$| b(x,u,p) | \leqslant b_0(1+|p|)^q, \quad b_0 = const > 0.$$

No smallness conditions on b_0 are assumed. For such type systems with $q = 2$, we proved the existence of the global in time almost everywhere smooth solutions in $\overline{Q}$ for C-D and C-N problems. The solutions have at most finitely many singular points [3]. It is appeared that in the case $q > 2$ there exists a smooth solution in $\overline{Q}$. We shall also discuss some other author's results devoted to systems (1), (2).

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Sectorial operators in divergent form and its m-accretive extensions

According to T.Kato [1], a linear operator S acting in a complex Hilbert space H and defined on a linear manifold $\mathcal{D}(S)$ is called sectorial with a semiangle $\alpha \in [0, \pi/2)$ and with the vertex at the origin if the numerical range is contained in the sector of the complex plane: $\Theta(\alpha) = \{\lambda \in \mathbb{C} : |\arg \lambda| \leqslant \alpha\}$. We assume that a densely defined closed sectorial operator S has a divergent form $S = L_2^* Q L_1$, where linear operators L_1, L_2 and Q satisfy the following conditions:

(a) $L_1 \subset L_2$ are two closed densely defined operators in H taking values in a Hilbert space $\mathfrak{H}$,

(b) $Q \in \mathcal{L}(\mathfrak{H})$ and Re $(Qf, f) \geqslant m\|f\|^2$, $f \in \mathfrak{H}$, $m > 0$,

(c) the linear manifold $\mathcal{D}(L_1) \cap \mathcal{D}(L_2^* Q L_2)$ is dense in the Hilbert space $\mathcal{D}(L_1)$ equiped by the graph norm.

In this talk we give a description of all m-accretive and m-sectorial extensions of such operator S in terms of abstract boundary conditions and consider applications to differential operators. A key role plays a "boundary" triplet $\{\mathcal{H}_+ \subseteq \mathcal{H} \subseteq \mathcal{H}_-, \Phi, \Gamma\}$, consisting of a rigged Hilbert space $\mathcal{H}_+ \subseteq \mathcal{H} \subseteq \mathcal{H}_-$ and linear operators $\Phi : \mathcal{D}(L_1^*) \to \mathcal{H}_-$, $\Gamma : \mathcal{D}(L_2) \to \mathcal{H}_+$ such that
$\mathcal{R}(\Phi) = \mathcal{H}_-$, $\mathcal{R}(\Gamma) = \mathcal{H}_+$, $\ker \Phi = \mathcal{D}(L_2^*)$, $\ker \Gamma = \mathcal{D}(L_1)$, with the Green's identity:

$$(L_1^* f, u)_H - (f, L_2 u)_{\mathfrak{H}} = (\Phi f, \Gamma u)_{\mathcal{H}} \quad \text{for all} \quad f \in \mathcal{D}(L_1^*) \quad \text{and all} \quad u \in \mathcal{D}(L_2).$$

We consider two cases: 1) $\dim(\mathcal{D}(L_2)/\mathcal{D}(L_1)) < \infty$ (this condition implies (c)) and
2) $\dim(\mathcal{D}(L_2)/\mathcal{D}(L_1)) = \infty$, L^{-1} is a bounded. In the first case the boundary space $\mathcal{H}$ is a finite dimensional.

Theorem. *Let* $\dim(\mathcal{D}(L_2)/\mathcal{D}(L_1)) < \infty$, $\mathcal{P}$ *be the projection in* $\mathfrak{H}$ *onto* $\overline{\mathcal{R}(L_1)}$ *w.r.t. the decomposition* $\mathfrak{H} = \overline{\mathcal{R}(L_1)} \dot{+} Q^{*-1}(\ker L_1^*)$ *and let* $\{\mathcal{H}, \Phi, \Gamma\}$ *be a boundary triplet. Then the relations*

$$\mathcal{D}(\widetilde{S}) = \left\{ \begin{array}{l} u \in \mathcal{D}(L_2), \\ Q\mathcal{P}L_2 u + 2Q_R^{1/2}\widetilde{X}\Gamma u \in \mathcal{D}(L_1^*) \\ \Phi\left(Q\mathcal{P}L_2 u + 2\mathcal{P}^* Q_R^{1/2}\widetilde{X}\Gamma u\right) \in \widetilde{\mathbf{W}}(\Gamma u) \end{array} \right\},$$

$$\widetilde{S}u = L_1^*\left(Q\mathcal{P}L_2 u + 2Q_R^{1/2}\widetilde{X}\Gamma u\right)$$

give a one-to-one correspondence between all m-accretive extensions of $S = L_2^* Q L_1$ *and all pairs* $\left\langle \widetilde{\mathbf{W}}, X \right\rangle$, *where* $\widetilde{\mathbf{W}}$ *is an m-accretive linear relation in* $\mathcal{H}$, $\widetilde{X} : \mathcal{D}(\widetilde{\mathbf{W}}) \to Q_R^{1/2}\overline{\mathcal{R}(L_1)}$ *is a linear operator such that* $\left\|\widetilde{X}e\right\|^2 \leqslant \operatorname{Re}\left(\widetilde{\mathbf{W}}(e), e\right)_{\mathcal{H}}$, $e \in \mathcal{D}(\widetilde{\mathbf{W}})$. *The operator* $\widetilde{S}$ *is an m-sectorial iff* $\widetilde{\mathbf{W}}$ *is an m-sectorial linear relation and* $\left\|\widetilde{X}e\right\|^2 \leqslant \delta^2 \operatorname{Re}\left(\widetilde{\mathbf{W}}(e), e\right)_{\mathcal{H}}$, $e \in \mathcal{D}(\widetilde{\mathbf{W}})$ *for some* $\delta \in [0, 1)$.

This result is applied for Sturm-Liouville operators. In the second case under the additional conditions we give an abstract version of Vishik's boundary operators [2] and use the corresponding result for the minimal operator generated by a second order uniformly elliptic differential expression in the bounded domain in $\mathbb{R}^n$.

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On the boundedness of solutions of one class of second order differential equtions in Hilbert space

В гильбертовом пространстве $\mathfrak{H}$ рассматривается дифференциальное уравнение

$$u''(t) + Bu'(t) + (T + iS)u(t) = 0, \quad u(0) = u_0,\ u'(0) = u_1,\ t \in \mathbb{R}_+, \quad (0.1)$$

T, B – самосопряженные, положительно определенные операторы, S – замкнутый симметрический оператор, $\mathcal{D}(B) \supset \mathcal{D}(T^{\frac{1}{2}})$ и $\mathcal{D}(S) \supset \mathcal{D}(T)$. Уравнение данного класса возникает при изучении уравнения ($\mathbf{v} = \{v_x, v_y\}$; $a_1, a > 0$)

$$w_{tt} + a_1 w_t + \Delta^2 w + a(\mathbf{v}, \operatorname{grad} w) = 0, \quad w = w(t, x),\ x \in \Omega \subset \mathbb{R}^2, \quad (0.2)$$

описывающего малые колебания пластины в потоке газа ([1], [2]). Обозначим

$$\inf_{x \in \mathcal{D}(B)} \big((Bx, x)/(x, x)\big) = m$$

и

$$q(x) = m(Tx, x) - \big((2B - m)^{-1} Sx, Sx\big),\ x \in \mathcal{D}(T).$$

$\mathfrak{H}_s$ – шкала пространств, построенная по оператору T.

Theorem 1 *Пусть $q(x)$ полуограничена снизу (т.е. $q(x) \geqslant c(x,x)$, $\forall x \in \mathcal{D}(T)$). Тогда $\forall(u_1,u_0) \in \mathfrak{H}_{\frac{1}{2}} \times \mathfrak{H}_1$ задача Коши для уравнения (0.1) корректно разрешима в пространстве $\mathfrak{H} \times \mathfrak{H}_{\frac{1}{2}}$. Если $q(x) \geqslant 0$, $\forall x \in \mathcal{D}(T)$, то все решения ограниченны на полуоси $\mathbb{R}_+$ в пространстве $\mathfrak{H} \times \mathfrak{H}_{\frac{1}{2}}$*

Если Ω – ограниченная область с липшицевой границей, то для уравнения (0.2) доказана корректная разрешимость задачи Коши и найдены достаточные условия ограниченности решений в пространстве $W_2^2(\Omega)$. Если область Ω – многоугольник, то решения уравнения (0.2) с краевыми условиями $w\mid_{\partial\Omega}= \Delta w\mid_{\partial\Omega}= 0$ ограничены если $|v|^2 \leqslant (a_1^2\lambda_1)/a^2$, λ_1 – наименьшее собственное значение задачи Дирихле для оператора Лапласа на Ω.

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Methods of elimination in conformity to inverse problems for ODE systems

Consider a problem of determination of the parameters θ_i of the system of ODE

$$\dot{x} = f(x,y,\theta) \tag{0.1}$$

$$\dot{y} = g(x,y,\theta) \tag{0.2}$$

$$x(0) = x_0,\ y(0) = y_0$$

on given a vector of the solutions $x(t)$, $t \in [0,T]$. Here $\theta \in R^s$; $x \in R^n, n \geqslant 1$; $y \in R^m, m \geqslant 1$ is the vector of the unknown solutions, $f_i(x,y,\theta), g_j(x,y,\theta)$ are the polinomials on appropriate variables. We have an inverse problem (IP) of determination of parameters of an ODE system in conditions of an incompleteness of the information. The problems of a similar kind arises in modeling processes of chemical and biological kinetics, systems of automatic control and other. It is known [1], that the incompletness of the information on dynamic characteristics of process results to nonuniqueness of the IP solutions. Reparametrizate the initial system, eliminating variables $y_j, \dot{y}_j$, to investigate a number of IP solutions of the system (1)-(2). Write the obtained system in the form

$$\sum_{l=1}^{q_i} \alpha_{il}\varphi_{il}(x, \ldots, x^{(m)}) = \varphi_{i0}(x, \ldots, x^{(m+1)}), \qquad (0.3)$$

$$x_i^{(j)}(0) = x_{ij,0}(x_0, y_0, \theta), \quad i = 1, \ldots, n, j = 1, \ldots, m$$

where $\alpha_{il} = \alpha_{il}(\theta)$ are some rational functions on parameters; φ_{ij} are linear - independent monomials on appropriate variables. For the constructive variables elimination it is essential polinomial dependence of the right-hand sides of the system (1) - (2), that permits to use advanced algorithms and methods of computer algebra. It is shown, that at some assumptions the direct (in a sense of the solutions $x(t)$) and, as a consequence, the inverse problems for the systems (1) - (2) and (3) are equivalent and IP of determination of the parameters α_{il} of the system (3) has the unique solution. The number and the form of parametrical functions $\alpha_{il}(\theta)$ defines a number of the IP solutions for the system (1) - (2).

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Microlocal Analysis and Multiwavelets

Microlocal filtering is performed with adapted orthonormal multiwavelets wich are derived from several scaling functions. Microlocal filtering can also be considered to be the action of pseudo-differential operators whose symbols are smooth functions with compact supports in Fourier space. Expansion of functions or signals in terms of a tight frame multiwavelet gives a rough estimate of their microlocal content.

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On Asymptotic Properties of the Emden–Fowler Type Equation with Complex-Valued Coefficient

Asymptotic behaviour of all solutions to the differential equation

$$y''(x) = p(x)|y(x)|^m y(x) \tag{0.1}$$

is described provided $m > 0$, $p(x) \to p_0 \in \mathbb{C} \setminus \mathbb{R}$.

Theorem. *Let $p(x)$ be a continuous complex-valued function, $p(x_0) = p_0 \in \mathbb{C} \setminus \mathbb{R}$, $m > 0$. Let $y(x)$ be an inextensible solution of (0.1) defined on (x_1, x_0) or (x_0, x_2) with $-\infty \leqslant x_1 < x_0 < x_2 \leqslant +\infty$. Then*

$$|y(x)| = \sqrt[m]{\frac{m^2}{8Q(x-x_0)^2}}\,(1+o(1)),$$

$$\arg y(x) = \pm\frac{m\ \mathrm{Im}\, p_0}{(2m+8)Q}\ \ln|x-x_0|\,(1+o(1)),$$

as $x \to x_0$, with the sign "+" on (x_0, x_2) or "−" on (x_1, x_0) and the constant

$$Q = \frac{\mathrm{Re}\, p_0 + \sqrt{|p_0|^2 + \dfrac{m}{m+4}(\mathrm{Im}\, p_0)^2}}{m+2}. \tag{0.2}$$

Theorem. *Let $p(x)$ be a continuous complex-valued function, $p(x) \to p_0 \in \mathbb{C} \setminus \mathbb{R}$ as $x \to \epsilon\infty, \epsilon = \pm 1$, $m > 0$. Let $y(x)$ be a solution of (0.1) defined near $\epsilon\infty$. Then*

$$|y(x)| = \sqrt[m]{\frac{m^2}{8Qx^2}}\,(1+o(1)),$$

$$\arg y(x) = \frac{\epsilon m\ \mathrm{Im}\, p_0}{(2m+8)Q}\ \ln|x|\,(1+o(1)),$$

as $x \to \epsilon\infty$, with the constant Q given by (0.2). Some proves can be found in [1], [2].

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Riesz bases of exponential divided differences and controllability problems

The method of moments is a powerful tool in control theory of partial differential equations. It is based on properties of exponential families (usually in space $L^2(0,T)$) such as minimality and the Riesz basis property. In recent years interest in the method of moments in control theory has increased, and this is connected with investigations into new classes of distributed parameter systems such as hybrid systems, structurally damped systems and systems with singularities in control or equation. The other challenging subject concerns simultaneous controllability of several systems. Control problems for these systems have raised a number of new difficult problems in the theory of exponential families. The principal questions which we consider in this talk are connected with the basis property of linear combinations from exponentials $e^{i\lambda_n t}$ in the case when the distance between some points λ_n tends to zero and therefore the family of exponentials $\{e^{i\lambda_n t}\}$ does not form a Riesz basis in $L^2(0,T)$. Using a new approach we have generalized the classical Ingham inequality for the case when the set $\{\lambda_n\}$ is the union of a finite number of separated sets [1]. Moreover, we obtained a full description of Riesz bases of special kinds linear combinations of exponentials — generalized divided differences [2]. We applied our results to problems of simultaneous and partial controllability of several elastic strings and beams [1]–[3]. The talk is based on joint papers with S. Ivanov and W. Moran.

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On the Riesz basicity of eigenvectors of twice continuity differentiable operator functions

This talk is based on a joint work with A. Dijksma and L.I. Sukhocheva. Let $L : [a, b] \to \mathbf{L}(\mathcal{H})$ be a continuous function which values are selfadjoint bounded operators on a Hilbert space $\mathcal{H}$. We set

$$\mathcal{E}(L) = \text{c.l.s.}\,\{\ker L(\lambda) \mid \lambda \in [a, b]\},$$

$\kappa_+(L(a))$ ($\kappa_-(L(b))$, resp.) the number of positive (negative) eigenvalues of $L(a)$ ($L(b)$, resp.) counting multiplicity. By the definition a differentiable operator function L satisfies the condition (S) if there exist positive numbers ε and δ such that for every $x \in [a, b]$ and $f \in \mathcal{H}$, $\|f\| = 1$,

$$|(L(x)f, f)| < \varepsilon \quad \Rightarrow \quad (L'(x)f, f) > \delta.$$

The main result is the following proposition:

Theorem 1. *Let L be a twice continuously differentiable selfadjoint operator function, defined on $[a, b]$, such that $L(a)$ and $L(b)$ are Fredholm operators, $\kappa_+(L(a)) < \infty$, $\kappa_-(L(b)) < \infty$, the conditions* S *and*

$$\int\limits_{(0)} \frac{\omega(t, L'')}{t} dt < \infty,$$

hold, where $\omega(t, L'')$ is the modulus of continuity of L'' in the Markus-Matsaev sense:

$$\omega(t, L'') = \max\{\|L''(x + t) - L''(x)\| \mid x \in [a, b - t]\}, \; t \in [0, b - a].$$

If $\sigma(L)$ is at most a countable set, then every union of orthonormal bases of $\ker L(x)$, $x \in [a, b]$, *is a Riesz basis in $\mathcal{E}(L)$ and*

$$\operatorname{codim} \mathcal{E}(L) = \kappa_+(L(a)) + \kappa_-(L(b)).$$

If L satisfies the condition (S) and there is a number $c \in (a, b)$ such that $L(c)$ is a compact operator, then $\sigma(L)$ is at most a countable set, $\kappa_+(L(a)) < \infty$, $\kappa_-(L(b)) < \infty$, and $L(x)$ is Fredholm for $x \neq c$. Theorem 2 is in some sense an inverse result:

Theorem 2. *Let* $\dim \mathcal{H} = \infty$ *and let* $L : [a, b] \to \mathbf{L}(\mathcal{H})$ *be a continuous selfadjoint operator function which satisfies the conditions:*

(i) $f \neq 0$, $(L(x)f, f) = 0 \Rightarrow (y-x)(L(y)f, f) > 0$, $y \neq x$,
(ii) $\kappa_+(L(a)) < \infty$, $\kappa_-(L(b)) < \infty$,
(iii) *there is a* $c \in (a,b)$ *such that* $L(x)$ *is Fredholm for* $x \neq c$.
Then
(a) $\kappa_+(L(x)) < \infty$ *for* $x \in [a,c)$ *and* $\kappa_-(L(x)) < \infty$ *for* $x \in (c,b]$,
(b) $L(c)$ *is compact*,
(c) $\dim \mathcal{E}(L) = \dim \mathcal{H}$,
(d) $\operatorname{codim} \mathcal{E}(L) \geqslant \kappa_+(L(a)) + \kappa_-(L(b)$, *and*
(e) *if* $\sigma(L) \cap [a,c)$ $(\sigma(L) \cap (c,b])$ *is a finite set consisting of* $\kappa_{a,c}$ $(\kappa_{c,b})$, *say, eigenvalues counting multiplicity, it holds*

$$\kappa_+(L(c)) = \kappa_+(L(a)) + \kappa_{a,c} \quad (\kappa_-(L(c)) = \kappa_-(L(b)) + \kappa_{c,b}).$$

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Babich M.
(Steklov Math. Institute, St. Petersburg)

Symmetries of the 4-matrices Schlesinger Systems and the algebraic surfaces

By the every set of parameters of 4-matrices Schlesinger System we construct the algebraic surface. This surface keep all the information about the parent Schlesinger System. Systems, connected by a symmetry transformation generate the isomorphyc algebraic surfaces. All types of symmetries, such as a permutation of parameters, Schlesinger and Okamoto transformations are considered. The new form of Schlesinger system and the action of the modular group on the corresponding linear system are presented.

Babich V.M., Dement'ev D.B., Samokish B.A., Smyshlyaev V.P.
(Steklov Mathematical Institute, St. Petersburg)

Scattering of plane electromagnetic wave by a conducting body with a conical singularity

The problem of scattering plane electromagnetic wave by axisymmetrical perfectly conducting body is considered. It is assumed that the body has conical singular point on its axis and this point is in the shadow zone. It is possible using method of matching of asymptotic expansions to obtain an analytical expression for the creeping wave propagating along the body surface. The scattering of the creeping wave by the conical singular point is considered (as axisymmetrical as well

non-symmetrical cases). The problems were reduced to solution of Fredholm integral equations. The numerical procedure to calculate the corresponding diffraction coefficients is suggested. Appropriate calculations were performed.

Bakhvalov N.S., Eglit M.E.
(Moscow State University)

About properties of higher order equations for elastic waves in stratified media [4]

The equations for wave propagation in stratified media with periodic structure are considered. They have the form

$$\rho\left(\frac{x_1}{\varepsilon}\right)\frac{\partial^2 \mathbf{u}}{\partial t^2} = \frac{\partial}{\partial x_i}\left(A_{ij}\left(\frac{x_1}{\varepsilon}\right)\frac{\partial \mathbf{u}}{\partial x_j}\right)$$

A scalar ρ and matrices A_{ij} are periodic functions of a coordinate x_1 with period ε. All dimensions are supposed to be scaled by a typical wave length. Long waves are considered so the dimensionless period ε is small. At $\varepsilon << 1$ it is possible to describe the process approximately by the so-called averaged equations. In [1] the method of two-scaled asymptotic expansions had been used to obtain the averaged equation for a certain $\mathbf{w}$ in the following form

$$\hat{\rho}\frac{\partial^2 \mathbf{w}}{\partial t^2} \sim \sum_{m\geqslant 2}^{\infty} \varepsilon^{m-2} h_{l_1 l_2 l_3}\frac{\partial^m \mathbf{w}}{\partial x_1^{l_1}\partial x_2^{l_2}\partial x_3^{l_3}}, \quad m = l_1 + l_2 + l_3,$$

where $\hat{\rho}$ is the averaged over the period value of ρ, the matrices $h_{l_1 l_2 l_3}$ are constant and the following relations are true

$$(h_{l_1 l_2 l_3})^T = (-1)^m h_{l_1 l_2 l_3}, \quad \mathbf{u} \sim \sum_{q,l_1,l_2,l_3=0}^{\infty} \varepsilon^{q+m} N^q_{l_1 l_2 l_3}\left(\frac{x_1}{\varepsilon}\right)\frac{\partial^{q+m}\mathbf{w}}{\partial q^m \partial x_1^{l_1}\partial x_2^{l_2}\partial x_3^{l_3}}.$$

The matrices $h_{l_1 l_2 l_3}$ at $l_1 + l_2 + l_3 \geqslant 3$ are of special interest here. These matrices determine, in particular, dispersion of waves in microinhomogeneous media. Matrices $h_{l_1 l_2 l_3}$ which are contained in the equations for waves propagating orthogonally to the layers have been investigated in [2]. Here the waves propagating along the layers are considered. The averaged equation can be written in the form (x_2 - axis is parallel to the wave velocity)

$$\hat{\rho}\frac{\partial^2 \mathbf{w}}{\partial t^2} = H_2\frac{\partial^2 \mathbf{w}}{\partial x_2^2} + \varepsilon H_3\frac{\partial^3 \mathbf{w}}{\partial x_2^3} + \varepsilon^2 H_4\frac{\partial^4 \mathbf{w}}{\partial x_2^4} + O(\varepsilon^3)$$

[4]The work was supported by Russian Foundation for Basic Research projects 99-01-01146, 99-01-01153).

Similar equation describes long wave propagation in a stratified plate with the surfaces free of forces. A parameter ε is then the ratio of the plate thickness to a typical wave length. Calculations of the matrices H_2, H_3, H_4 have been done for different structures. In particular, unbounded periodic media consisting of a repeating system of two homogeneous isotropic layers as well as three-layered plates with a plane of symmetry have been considered. In all studied cases it was found that $H_3 = 0$, H_2, H_4 were diagonal, $H_2 \geqslant 0$ ($H_2 > 0$ for an unbounded medium), and H_4 was not definite. Therefore the dispersion has different signs for different components of a displacement vector **w**. Calculations for media consisting of a repeating system of three homogeneous isotropic layers and those for two-layered plates gave $H_3 = -H_3^T \neq 0$; H_2, H_4 were diagonal as before, $H_2 \geqslant 0$ ($H_2 > 0$ for an unbounded medium), and H_4 was not definite. The authors thank E.V.Chizhonkov for his help in the organizing calculations.

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Barsukov A.I.

(Voronezh Military Institute for Radioelectronics)

Similarity of a J-dissipative operator to a Hilbert space dissipative operator

This talk is based on a joint work with T.Ya. Azizov, A. Dijksma and P. Jonas.

Let $\mathcal{H}$ be a Hilbert space with the scalar product $(\cdot,\cdot)$ and $\mathcal{H} = \mathcal{H}_+ \oplus \mathcal{H}_-$ be its orthogonal decomposition such that $dim\mathcal{H}_- =\leqslant \infty$. The operator $J = P^+ - P^-$, where $P^\pm$ is the orthogonal projection onto $\mathcal{H}^\pm$, is called the fundamental symmetry corresponding to this decomposition. The space $\mathcal{H}$ with the inner product $[\cdot,\cdot] = (J\cdot,\cdot)$ is called Pontryagin space and is called by the symbol Π_κ. Closed densely defined operator $A : dom\, A\Pi_\kappa$ is J-dissipative if $\mathrm{Im}\,[Ax, x] \geqslant 0$, $x \in dom\, A$. We shall give the criteria for the similarity of a dissipative operator A on a Pontryagin space Π_κ to a Hilbert space dissipative operator in terms of the location of its spectrum and an uniform bound on the growth of the resolvent of A. The main result is the following

Theorem. *Let A be a maximal dissipative operator in a Pontryagin space and let $\lambda_1, \lambda_2, \ldots, \lambda_m$ be the point in $\mathcal{H}$ for which* $\ker(A - \lambda_j I)$ *contains at least one nonpositive nonzero vector, $j = 1, 2, \ldots, m$. The following statements are* *equivalent: (1) $\sigma(A) \subset C^+$ and for each $j = 1, 2, \ldots, m$, there is a sequence $\{\lambda_{ij}\}$ in $\rho(A) \cap C^-$ with $\lambda ij \to \lambda j$ such that*

$$\sup_n |\lambda_{ij} - \lambda_j| |\operatorname{Im} \lambda_{jn}|^{-1} < \infty, \sup_n |\operatorname{Im} \lambda| \|(A - \lambda_{jn} I)^{-1}\| < \infty.$$

(2)$\sigma(A) \subset C^+$ and for each $j = 1, 2, \ldots, m$, there is a sequence λ_{jn} in $\rho(A)$ with $\lambda_{jn} \to \lambda_j$ such that

$$\sup_n |\lambda_{jn} - \lambda_j| \cdot \|(A - \lambda_{jn} I)^{-1}\| < \infty.$$

(3) A is similar to a maximal dissipative operator on Hilbert space. (4) $\sigma(A) \subset^+$ and

$$\sup\{|\operatorname{Im} \lambda^k| \|(A - \lambda I)^{-k}\| : k \in N, \lambda \in C^-\} < \infty.$$

(5) $\sigma(A) \subset C^+$ and

$$\sup_{\{} |\operatorname{Im} \lambda| \|(A - \lambda I)^{-1}\| : \lambda \in C^-\} < \infty.$$

Moreover, if A is also symmetric or selfadjoint in Pontryagin space, then (1), (2), (4) and (5) hold if and only if A is similar to a symmetric and maximal dissipative or selfadjoint operator in a Hilbert space, respectively.

Beklaryan L.A.
(Central Institute of Economics and Mathematics of Russian Academy of Science, Moscow)

The complexity of structure of the group of homeomorphisms of the line and the connected problem of classification of the differential equations with a deflecting argument[5]

We consider differential equations with the deflecting argument

$$\dot{x}(t) = f(t, x(g_1(t)), \ldots, x(g_s(t))), \quad t \in \mathbb{R},$$

where $f : \mathbb{R} \times \mathbb{R}^{\kappa\sim}$ is a continuous mapping and $g_i : \mathbb{R} \longmapsto \mathbb{R}$, $i = 1, \ldots, s$ are the homeomorphisms, preserving orientation. We outline the equations with canonical type of the argument deflection to which the given equation can be reduced using transformation of the time variable. Such problem occurs to be equivalent to the problem of classification of the groups $G = < g_1, \ldots, g_s >$ of homeomorphisms

[5]in Russian

of the line, given by the functions of the argument deflection. The classification of the groups of homeomorphisms, preserving orientation, in its time, is closely connected with different topological, algebraic and combinatorical characteristics of group and also with the properties of the metric invariants.

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Belishev M.I.

(Steklov Mathematical Institute, St.-Petersburg)

Dynamical systems with boundary control: models and characterization of inverse data[6]

An abstract version of the BC-method is proposed as a chapter of the linear system theory dealing with dynamical systems with boundary control (DSBC). A characterization of response operator (transfer operator function) of DSBC is given; a set of models (canonical realizations) of DSBC determined by response operator is presented. As application, a conditional existence theorem characterizing the dynamical Dirichlet-to-Neumann map of the Riemannian manifold is obtained. An abstract analog of the Gelfand–Levitan–M.Krein–Marchenko equations is derived.

Belokolos E.D.

(Institute of Magnetism, National Academy of Sciences of Ukraine)

The integrability and the structure of atom

We have proved in the Hartree-Fock approximation that an electron configuration of atomic ground state is defined by a resonance condition $qn_r + pl = const.$, where n_r, l are radial and orbital quantum numbers of a single-electron state with an energy $E = 0$, and q, p are positive integers. If we insert in the resonance con-

[6]This work supported by the grant RFBR N 99-01-00107

dition the semiclassical Bohr-Sommerfeld expression for the quantum number n_r in terms of the atomic potential $V(r)$ we obtain an integral Abel equation for this potential. In terms of the Langer variable the Abel integral equation transforms into an equation for spectrum of a one-dimensional Schrödinger equation and as a result of that the atomic potential appears to be a soliton or its isospectral deformation. In terms of the radial variable r this soliton is nothing else as the well known Tietz potential $V(r) = -Z/r(1+(r/R))^2$, where Z is a nucleus charge and R is a parameter. The Schrödinger equation with the Tietz potential for a radial part of the electron wave function is an ordinary differential equation with rational coefficients having two regular and one irregular singular points. In semiclassical approximation the spectrum of Schrödinger operator with the Tietz potential is universal and is described in terms of elliptic integrals. For the Mendeleev periodic system of elements we have proved the Madelung empirical rule $(n+l, n)$, in accordance to which the electrons complete in the atom with growth of the charge Z at first the states with the least possible value of the quantum number $n+l$ and at a given value of the number $n+l$ complete the states with the least possible quantum number n. We introduce in natural way a notion of the Mendeleev electron shell and calculate its different properties. It is shown also that the deformations of the Tietz potential are many-well potentials for which we can calculate effectively the eigenstates and eigenvalues. These many-well potentials can be used in order to explain an existence of the d-, f-, and g- transition series of elements in the periodic system of elements.

Belokurov V.V. Soloviev Yu.P., Shavgulidze E.T.
(Moscow State University)

General approach to calculation of functional integrals and summation of divirgent series

Belov V.V., Maksimov V.A.
(Moscow Institute of Electronics and Mathematics)

Quasimodes of helium atom corresponding to stable orbits

It is well known [1,2] that to closed phase trajectories of a hamiltonian system that are *nondegenerate* orbitally stable in the linear approximation, one can assign quasimodes of the corresponding $(mod\hbar^{3/2})$ quantum problem within the framework of Maslov's complex germ theory [2]. But in the resonant case, orbitally stable trajectories are *degenerate*, and asymptotic eigenvalues are also degenerate in order up to $O(\hbar^2)$, $\hbar \to 0$. In the present talk we consider the problem of constructing quasimodes $(mod\hbar^{5/2})$ corresponding to *degenerate* orbitally stable closed trajectories $\Lambda^1(E)$ for a physically important non-integrable system (the model of helium atom with quadrupole momentum) with hamiltonian

$$H = \frac{p_1^2}{2} + \frac{p_2^2}{2} - \frac{Z}{r_1} - \frac{Z}{r_2} + \frac{1}{r_{12}} + \gamma\left(\frac{1}{r_1^3} + \frac{1}{r_2^3}\right),$$
$$p_i \in \mathbf{R}^3,\ r_i = |x_i|,\ r_{12} = |x_1 - x_2|,\ x_i \in \mathbf{R}^3,\ Z \geqslant 1 - \text{const}.$$

For $\gamma = 0$, such trajectories are known in the physical literature (e.g., "frozen planet" configuration, Langmuir orbit). For $\gamma \neq 0$, we found another class of these orbits – relative equilibria (RE) of the corresponding classical system. Using the method of reduction by the diagonal action of $SO(3)$ group, we constructed all RE of the system and proved their orbitally stability. The scheme of semiclassical quantization for degenerate trajectories $\Lambda^1(E)$ is the following: 1) in the neighborhood of $\Lambda^1(E)$, we construct a one-parametric family of k-dimensional invariant isotropic tori $\Lambda^k(E)$ by shifting along the vector fields of hamiltonian's symmetries $L_1, \ldots, L_{k-1}$ transversal to $\Lambda^1(E)$ $(2 \leqslant k \leqslant 5)$; 2) it generates a k-parametric family of almost invariant isotropic tori $\Lambda^k(E, J)$, $J \in \mathbf{R}^{k-1}$ with complex germ; 3) according to [3], in the neighborhood of $\Lambda^k(E, J)$ we construct new canonic variables of action-angle type and the normal form of hamiltonian in these variables. After quantization of $\Lambda^k(E, J)$ in the new variables, we obtain a correction of order $O(\hbar^2)$ to the formulae of degenerate spectrum corresponding to the degenerate trajectory $\Lambda^1(E)$.

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Ben Amara J.
(Faculté des Sciences de Tunis)

Sturm-Liouville Problem with spectral parameter in boundary conditions

In this work, we study spectral problems of the form

$$-u''(x) + q(x)u(x) = \lambda u(x), \qquad 0 < x < \pi \tag{0.1}$$

$$\begin{cases} u'(0) = 0 \\ u'(\pi) = m\lambda u(\pi); \end{cases} \tag{0.2}$$

Here $q(x)$ is a real integrable function, λ is a spectral parameter and m is a given real parameter. The sign of m plays an essential role. In the case $m > 0$, this problem can be interpreted as a spectral problem for a selfadjoint operator in a well chosen Hilbert space (see[1]). Here we mainly interested by the case $m < 0$, where the problem is realised as a spectral problem for a selfadjoint linear pencils. We give sufficents conditions on the parameter m and the potential $q(x)$ for which all the eigenvalues are real and simple, and we establish the oscillation theorem for the eigenfunctions. These results are obtained essentially by using the Pontriaguin space properties and developping the analytic Sturm theory.

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Berest Yu.

(Cornell University)

Lacunas For Hyperbolic Operators with Variable Coefficients

The question of characterizing linear differential systems which propagate waves without diffusion (also known as the problem of lacunas) goes back to the classical work of Hadamard and Petrovsky. In early 70s Atiyah, Bott and Garding developed a profound and nearly complete theory of lacunas for hyperbolic operators with constant coefficients. Extending this theory to operators with variable coefficients presents a major challenge, and the problem of lacunas still remains largely open in this general case. The talk will review some recent results and ideas (mostly from harmonic analysis and algebraic geometry) that may hopefully lead to a new approach to this difficult problem.

Berezansky Yu.M.

(Institute of Mathematics NASU, Kyiv)

Some Generalization of the Classical Moment Problem

In the talk will be given some generalization of classical moment problem, which connected with spectral theory of special family of commuting selfadjoint operators.These operators act in the Hilbert space constructed by positive definite kernel.As result, we find the conditions on sequence of measures, which guaranteed that it is a sequence of correlation measures of some measure on configuration space.

Berkovich L.M.
(Samara State University)

On a method of exact linearization and on higher order nonlinear evolutionary equations

Last thirty years the activity in the study of nonlinear processes has been blowing up. The most effective development takes place in the sphere of exact analytic solutions. The talk is dedicated to the method of exact **linearization** [1] of nonlinear autonomous ordinary differential equations (ODE). The new class of nonlinear evolutionary equations (NEE) of higher order, which is different to the higher analogues of KdV equations, is constructed.

Theorem. *In order NEE*

$$\frac{\partial y}{\partial t} = F(y, \frac{\partial y}{\partial x}, \dots, \frac{\partial^n y}{\partial x^n}) \tag{1}$$

to be transformed by substitution

$$z = \beta \int \varphi^{\frac{n^2-n+2}{2n}} \exp(\int f dy) dy, \; ds = \varphi dx, \; \beta = const \text{ is normalizing factor.}$$

to the linear evolutionary equation

$$\frac{\partial z}{\partial t} = \frac{\partial^n z}{\partial s^n} + \sum_{k=1}^{n} \binom{n}{k} b_k \frac{\partial z^{n-k}}{\partial s^{n-k}},$$

it is nessesary and sufficient that (1) can be represented in the form factorization

$$\frac{\varphi^{\frac{n^2-n+2}{2n}} \exp(\int f dy)}{\int \varphi^{\frac{n^2-n+2}{2n}} \exp(\int f dy) dy} \frac{\partial y}{\partial t} =$$

$$= \prod_{k=1}^{n} \left[\frac{1}{\varphi} D - \left(\frac{1}{y\varphi} - \frac{\varphi^{\frac{n^2-3n+2}{2n}} \exp(\int f dy)}{\int \varphi^{\frac{n^2-n+2}{2n}} \exp(\int f dy) dy} \right) y_x - r_k \right] y. \tag{2}$$

Let us demonstrate NEE of orders $n = 2$ and $n = 3$ of general form belonging to the class (1),(2):

$$\varphi^2 y_t = y_{xx} + f y_x^2 + 2b_1 \varphi y_x + b_2 \varphi \exp(-\int f dy) \int \varphi \exp(\int f dy) dy;$$

$$\varphi^3 y_t = y_{xxx} + 3f y_x y_{xx} + (\frac{1}{3}\frac{\varphi_{yy}}{\varphi} - \frac{5}{9}\frac{\varphi_y^2}{\varphi^2} - \frac{1}{3}f\frac{\varphi_y}{\varphi} + f^2 + f_y)y_x^3 +$$

$$+3b_1\varphi[y_{xx} + (f + \frac{1}{3}\frac{\varphi_y}{\varphi})y_x^2] + 3b_2\varphi^2 y_x + b_3\varphi^{5/3}\exp(-\int f dy)\int \varphi^{4/3}\exp(\int f dy)dy.$$

Example [2]. The equation $y_t = h/2y_{xx} - 1/2y_x^2$ by the transformation $z = \exp(-y/h)$, $ds = \sqrt{2/h}dx$ is linearized to $z_t = z_{ss}$. For constructed classes NEE we find the invariant solutions and also general solutions on the base of principles of nonlinear superposition. Other NEE admit different type of factorization (see, for example., [3]).

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Biryuk A.E.

(*Moscow State University; Heriot-Watt University*)

On generalised equations of Burgers type with small viscosity

Consider the one dimensional non-linear parabolic equation of Burgers type

$$\frac{\partial}{\partial t}u + \frac{\partial}{\partial x}f(u) = \nu u_{xx}. \tag{0.1}$$

Here ν is positive real parameter (viscosity).

Theorem 1. *Suppose that $f \in \mathrm{Lip}(\mathbb{R})$ and the initial state $u_0 \in L_\infty(\mathbb{R})$; then the solution to the Cauchy problem for equation (0.1) satisfies*

$$\left|\frac{\partial u}{\partial x}(t,\cdot)\right|_{C^0(\mathbb{R})} \leqslant 5|u_0|_{L_\infty}\max(\frac{Lip(f)}{\nu}, \frac{1}{\sqrt{t\nu}})\,. \tag{0.2}$$

As a corollary for this Theorem we have:

$$\text{if } t \geqslant \frac{\nu}{Lip(f)^2} \text{ then } \left|\frac{\partial u}{\partial x}(t,\cdot)\right|_{C^0(\mathbb{R})} \leqslant 5|u_0|_{L_\infty}\frac{Lip(f)}{\nu}\,.$$

Theorem 2. *Given $f \in C^k(\mathbb{R})$, $(k \geqslant 1)$ and an initial state $u_0 \in L_\infty(\mathbb{R})$; then there exist ν-independent constants C_k and $\tilde{c}_k$ such that for any $\nu > 0$ and for any $t \geqslant \tilde{c}_k \nu$ we have*

$$\left|\frac{\partial^k u}{\partial x^k}(t,\cdot)\right|_{C^0(\mathbb{R})} \leqslant C_k \frac{1}{\nu^k}. \tag{0.3}$$

Now we turn our attention to the case of small viscosity, i.e., $\nu \ll 1$. It turns out that if the viscosity ν is small then its power in (0.3) is optimal (in sense of Theorem 3 bellow). Consider the so-called integrable case of equation (0.1), i.e., $f(u) = au^2$ with $a \neq 0$. In this case equation (0.1) can be reduced to the heat equation by so-called Hopf–Cole transformation $(u = -\frac{\nu}{a}\frac{\partial}{\partial x}\ln\varphi)$. We remark that this transformation was known by V. Florin (see [1]) before Hopf and Cole. Using the explicit formula for the solution one can get

Theorem 3. *Suppose $f(u) = au^2$ with $a \neq 0$. Given $u_0 \in C_b^1(\mathbb{R})$ such that the Cauchy problem*

$$\begin{cases} u_t + 2auu_x = 0 \\ u(0,x) = u_0(x) \end{cases}$$

develops shocks for $t > T$. Then for any $t > T$ we have

$$\limsup_{\nu\to 0}\left|\nu^k \frac{\partial^k u}{\partial x^k}(t,\cdot)\right|_{C^0(\mathbb{R})} > 0. \tag{0.4}$$

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Biroli M.

(Politecnico di Milano)

Nonlinear subelliptic Schrödinger type problems

Let $X_i = \sum_{j=1}^N a_{ij} D_{x_j}$, i = 1,...,m, $a_{ij} \in C^\infty(R^N)$, be vector fields on R^N satisfying the Hörmander condition (i.e. the vector fields X_i and their commutators up to the order k span all the directions in R^N). We denotes $X_i^* = -\sum_{j=1}^N D_{x_j} a_{ij}$. In the following Xu denotes the vector $(X_1 u, ..., X_m u)$.
A distance relative to the vector fields X_i can be defined as

$$d(x,y) = sup\{\phi(x) - \phi(y);\ \phi \in C_0^\infty(R^N),\ |X\phi| \leqslant 1 \text{ a.e.}\} \tag{1}$$

(see for ex. NAGEL A., STEIN E. and WEINGER S., Balls and metrics defined by vector fields I: Basic properties, *Acta Math.*, **155**, pp. 103-147, (1985)).
We recall that, for $d(x,y) \leqslant 1$, there exists a constant $c \geqslant 1$ such that

$$\frac{1}{c}|x-y| \leqslant d(x,y) \leqslant c|x-y|^{\frac{1}{k+1}}$$

so the topology defined by d on R^N is equivalent to the euclidean one.
In the following we denote by $B(x,r)$ (B_r,B) the ball of center x and radius r relative to the distance d.
We now define the Sobolev spaces relative to the vector fields X_i. We fix a ball $B_{2\overline{R}}$ of radius $2\overline{R}$; let $\Omega \subseteq B_{\overline{R}}$ be an open set in R^N ($B_{\overline{R}}$ is the ball with the same center as $B_{2\overline{R}}$ and radius $\overline{R}$); the space $W^{1,p}(\Omega, X)$, $p \in (1,+\infty)$, is defined as the completion of the space $C^\infty(\overline{\Omega})$ for the norm

$$||u||_{1,p} = (||u||^p_{L^p(\Omega)} + ||Xu||^p_{(L^p(\Omega))^m})^{\frac{1}{p}}.$$

By $W^{1,p}_{loc}(\Omega, X)$ we denote the space of the functions in $W^{1,p}(A,X)$ for every open set A such that $\overline{A} \subseteq \Omega$. The space $W^{1,p}_0(\Omega,X)$, $p \in (1,+\infty)$, is defined as the closure of the space $C^\infty_0(\overline{\Omega})$ in $W^{1,p}(\Omega,X)$. We denote by $W^{-1,q}(\Omega,X)$, $\frac{1}{p}+\frac{1}{q}=1$, the dual space of $W^{1,p}_0(\Omega,X)$ considered as a subspace of the distributions on Ω.
We observe that a natural notion of variational p-capacity of a set E with respect to an open set Ω containing the closure of E (denoted by $p-cap(E,\Omega)$)is associated with the vector fields X_i.

DEFINITION 1. *A Radon measure μ on Ω is in the Kato space $K^q(\Omega)$, $\frac{1}{p}+\frac{1}{q}=1$, iff $lim_{r\to 0}\ \eta(r)=0$, where*

$$\eta(r) = sup_{x\in\Omega} \int_0^r (\frac{\delta^p}{m(B(x,\delta))}|\mu|(B(x,\delta)\cap\Omega))^{\frac{1}{p-1}}\frac{d\delta}{\delta}$$

the Radon measure μ is in $K^q_{loc}(\Omega)$ if it is in $K^q(A)$ for every open set $A \subseteq \overline{A} \subseteq \Omega$.

We can prove that every measure in $K^q(\Omega)$ is in $W^{-1,q}(\Omega,X)$.
We are now ready to give the results that are the object of this paper.
Consider the problem

$$\sum_{i=1}^{m} X_i^*(|Xu|^{p-2}X_iu) + \mu|u|^{p-2}u = 0 \quad \text{in } \Omega \tag{2}$$

where $\mu \in K^q(\Omega)$.

THEOREM 1. *Let u be a weak local positive solution in Ω of (2). Then for every ball $B(x,r) \subseteq B(x,8r) \subseteq \Omega$ with $r \leqslant R_1$ (R_1 suitable) we have*

$$sup_{B(x,r)} u \leqslant C \; inf_{B(x,r)} u$$

where C is a constant depending only on p.

From Theorem 1. we easily deduce results on the local regularity of solutions of (2).

THEOREM 2. *Let u be a weak local solution in Ω of (2). Then $u \in C(\Omega)$. Moreover if $|\mu|(B(x,r)) \leqslant r^{n-p+\epsilon}$, $r \leqslant \overline{R}_0$, u is locally Hölder continuous.*

We also investigate the boundary behavior of solutions of (2).

DEFINITION 2. *A point $x_0 \in \partial\Omega$ is a regular point for the Schrödinger problem if for every neigbourhood U of x_0 the solution of the Dirichlet problem for (2) in $U \cap \Omega$ with a boundary data continuous at x_0 is continuous at x_0.*

DEFINITION 3. *A point $x_0 \in \partial\Omega$ is a Wiener point if*

$$\int_0^1 \left(\frac{p-cap(\Omega^c \cap B(x_0,\rho), B(x_0,2\rho))}{p-cap(B(x_0,\rho), B(x_0,2\rho))}\right)^{\frac{1}{p-1}} \frac{d\rho}{\rho} = +\infty$$

where $\Omega^c = R^N - \Omega$

We have the following result:

THEOREM 3. *A point $x_0 \in \partial\Omega$ is a regular point for the Schrödinger problem iff it is a Wiener point. Then the set of regular points for the Schrödinger problem does not depend on the measure $\mu \in K^q(\Omega)$ and it is the same as in the case $\mu = 0$.*

We end by observing that the results of Theorems 1,2 and 3 seem to be new also in the euclidean case ($X_i = D_{x_i}$, $i = 1, 2, .., N$).

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Circle patterns and integrable systems

Circle patterns can be considered as discrete analogs of conformal mappings. We study patterns with combinatorics of the square or hexagonal grid (as shown in Figures) and prescribed intersection angles and show that they are described by integrable systems. In particular one can assume that the circles intersect orthogonally (square grid) [1] or with the angle $\pi/3$ (hexagonal grid) [4]. Circle patterns with the hexagonal grid such that six intersection points $z_1, \ldots, z_6$ on each circle have multi-ratio

$$\frac{(z_1 - z_2)(z_3 - z_4)(z_5 - z_6)}{(z_2 - z_3)(z_4 - z_5)(z_6 - z_1)} = -1$$

are also shown to be integrable [3]. The corresponding Lax representations are defined on regular lattices different from the standard square lattice Z^2. In all these cases using the isomonodromic solutions we construct discrete analogs of holomorphic mappings z^a and $\log z$.

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Some method of estimating the L_1-norm of exponential sums and ever non-converging Fourier series

On the basis of the characterization of the Hardy Space H^1 performed by the author, using the Valle-Pussen expansion a new method of obtaining the lower multiplicative estimates of L_1-norm of the exponential sum of the common form was derived. In particular, some estimates for the sum of exponents with the coefficients of 0 and 1 that take into account both density and arithmetic properties of the spectrum were obtained. For each type of spectrum with certain arithmetic properties, for example class of convex spectrums, there is a precise at the edges scale of estimates, that differentiate the spectrums of that class by their density properties. An ever non-converging Walsh-Paley series of class $\varphi(L)$, in case $\varphi(n) = o(\sqrt{\log n})$ as $n \to \infty$, and the one of class H_1^ω for every continuity module $\omega(n)$, satisfying the condition

$$\int_0^{\frac{1}{2}} \frac{\omega(n)}{n\sqrt{\log \frac{1}{n}}} dn = \infty.$$

These result reduce twice the gap between the upper estimate of Sjölin derived by Carleson method and the present-day lower estimates based on the Kolmogorom structures.

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Efficient algorithms for stiff elliptic problems with large parameters

A finite element approximation and range of iterative algorithms for solving a stiff elliptic boundary value problem with large parameters of the higher derivative are considered. The convergence rate of the algorithms is independent of the spread in coefficients and the discretization parameter. Numerical experiments are observed for algorithm comparison in different conditions.

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Singularities of fronts of linear waves

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New results for the II-nd part of 16-th Hilbert's problem[7].

1. The main determinant ([1],[2],[3]). Let us consider map from $(-\varepsilon,\varepsilon)\times\mathbb{R}^{+}=\{(t,\tau)\}$ on the plane

$$(t,\tau)\mapsto x(t)+\tau \mathbf{v}(x(t)), dx(t)/dt=\mathbf{v}(x(t)). \tag{0.1}$$

Lemma. For any point (t,τ) and $\{(t,\tau_j)\}$ of general position we have

$$\sum_{j=1}^{n}\mu_j(\mathbf{x})L_{\mathbf{V}}(\mathbf{x})(f_j)=0,\ f_j=\ln\left|(\tau-\tau_j)\big/(\hat{\tau}-\tau_j)\right|, \tag{0.2}$$

where $\hat{\tau},\tau_j$ (t_j time on τ_j) is some phase curves of polynomial (degree n) vector field $\mathbf{v}$ and $\hat{\tau}\neq\tau_j$, $j=0,\dots,n$; $\mu_j(\mathbf{x})$ is Vandermonde's determinant of $(\tau_1,\dots,\tau_{j-1},\tau_{j+1},\dots,\tau_n)$, divided on λ_j, such that $\lambda_j dt=dt_j$.

2. Nonlocal (first) integral. *Corollary.* On the semi-affine covering $\{t\times\tau\}$ there exists the first nonlocal integral in the form

$$H\left(\tau,\tau_0,\tau_1,\dots,\tau_n,\hat{\tau},t\right)=\sum_{j=1}^{n}s_j(\tau_j)f_j\Bigg|_{t_j=g_j(t)} \tag{0.3}$$

where $g_j(t):t\to t_j$ are suitable diffeomorphisms, and $s_j=(+1)$ or (-1), depending from sign of relative $\mu_j(\mathbf{x})$ and orientation τ_j, along decreasing t_j. Proof of corollary. Denote by ψ_j (or $g_t(\mathbf{x}_0):\mathbb{R}\to\mathbb{R}^2$) diffeomorphism of phase curve, such that $(\psi_j)_*\mathbf{V}=\mu_j\mathbf{V}$, where $\psi_j(\mathbf{x}_0)=\mathbf{x}_0$ (solution (1) on the phase curve τ). Then we have

$$\sum_{j=1}^{n}\mu_j L_{\mathbf{V}}(f_j)=\sum_{j=1}^{n}L_{\mu_j\mathbf{V}}(f_j)\Bigg|_{\mathbf{y}}=\sum_{j=1}^{n}L_{\psi_{j*}\mathbf{V}}(f_j)\Bigg|_{\mathbf{y}=\psi_j(\mathbf{x}_j)}=$$

$$=\sum_{j=1}^{n}L_{\mathbf{V}}\left(\psi_j^*f_j\right)\Bigg|_{\mathbf{x}_j=\psi^{-1}(\mathbf{y})}=\sum_{j=1}^{n}L_{g_{t_j*}\mathbf{V}(\mathbf{x}_0)}\left(\psi_j^*f_j\right)=$$

[7]The work partially is supported by RFFI grant .No 01-01-00538. About Gilbert's problem see [3], the full text of proof see in [1].

$$= \sum_{j=1}^{n} L_{\mathbf{V}(\mathbf{x}_0)} \left(g^*_{t_j} \circ \psi^*_j f_j \right) = L_{\mathbf{V}(\mathbf{x}_0)} \left(\sum_{j=1}^{n} g^*_{t_j} \circ \psi^*_j f_j \right) =$$

$$= L_{\mathbf{V}(g_\epsilon(\mathbf{x}_0))} \left(\sum_{j=1}^{n} g^*_{t_{j-\epsilon}} \circ \psi^*_j f_j \right) = 0$$

Remark. For suitable cases from (3) follows that a nest of limit circles of v admit linear upon n estimation (in contradictory with common opinion ([6] and commentary in it) about quadratic one).

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New symmetries of the MHD equilibrium equations

In this paper, we introduce new intrinsic symmetries of the magnetohydrodynamics equilibrium equations. As known [1], the intrinsic symmetries (or Backlund transforms) play an important role for the soliton equations of plasma physics such as the Korteweg - de Vries equation, the Kadomtsev - Petviashvili equation and the Davey - Stewartson equation. Another exceptional nonlinear equations with intrinsic symmetries are the Sine - Gordon equation and the Liouville equation. All these equations are the model single equations which are applicable only at some approximations. They depend on a part of spatial variables (one for KdV and two for KP and DS equations). In spite of their detailed investigation in the literature, all these soliton equations are not fundamental equations of physics. We present in this paper the intrinsic symmetries of the fundamental system of eight nonlinear equations of magnetohydrodynamics equilibria, that depend on all three spatial variables x, y, z and are given by the explicit formulas [2]. For the ideal gas plasma, the symmetries depend on an arbitrary function $a(x)$ that is constant on magnetic surfaces. For the subsonic plasma flows (Mach number $M << 1$) with $\operatorname{div} \mathbf{V} = 0$ and with variable plasma density $\rho(x)$, the symmetries depend on two arbitrary functions $a(x)$, $b(x)$ which are constant on magnetic surfaces. The symmetries generate an infinite family of new MHD equilibria from any known one [3]. In [4], Grad conjectured that only exceptional and isolated plasma equilibria could exist. In this paper, we show that MHD equilibria are not isolated and that any MHD equilibrium is contained in a smooth family of equilibria which are obtained by the action of the newly discovered intrinsic symmetries. The two discovered symmetries form infinite-dimensional abelian Lie groups.

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Second species solutions of Poincaré for the restricted 3-body problem

The second species solutions of Poincaré for the n-body problem are solutions shadowing a chain of collision orbits. By using variational methods, we prove a result concerning symbolic dynamics of such solutions for Lagrangian systems with weak Newtonian singularities, i.e. with the Lagrangian of the form $L = L_0(q, \dot{q}) + \varepsilon U(q)$, where L_0 is smooth, U has a finite set of Newtonian singularities, and $\varepsilon > 0$ is small. As an application, chaotic almost collision solutions are constructed for the 3-body problem when one of the masses is small with respect to another. The talk is based on a work with R.S. MacKay.

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Asymptotics of the eigenvalues of the Laplace operator in cylinder with frequently alternating type of boundary condition

Let $x' = (x_1, x_2)$, $x = (x', x_3)$ be the Cartesian coordinates in $\mathbb{R}^2$ and $\mathbb{R}^3$, $\Omega' \subset \mathbb{R}^2$ be the bounded simply connected domain with smooth boundary, $\Omega = \Omega' \times [0, H]$, D_1 and D_2 be the upper and lower basis of the cylinder Ω, Σ be the lateral surface, s be the natural parameter of $\partial\Omega'$. Put $N \gg 1$ an integer number, $\varepsilon = \frac{H}{\pi N}$ a small parameter, $\gamma_\varepsilon = \{x : x' \in \partial\Omega', |x_3 - \varepsilon\pi(j + 1/2)| < \varepsilon\eta f(s), j = \overline{0, N-1}\}$, where $f(s) > 0$ is some function, $f \in C^\infty(\partial\Omega')$, $\eta = \eta(\varepsilon)$, $0 < \eta(\varepsilon) < \pi/2$, $\Gamma_\varepsilon = \Sigma \backslash \overline{\gamma}_\varepsilon$, ν is the outer normal to $\partial\Omega$. We study the singular perturbed eigenvalue problem

$$-\Delta\psi_\varepsilon = \lambda_\varepsilon\psi_\varepsilon, \quad x \in \Omega, \qquad \psi_\varepsilon = 0, \quad x \in D_1 \cup \gamma_\varepsilon, \qquad \frac{\partial\psi_\varepsilon}{\partial\nu} = 0, \quad x \in D_2 \cup \Gamma_\varepsilon,$$

Denote $A = -\lim\limits_{\varepsilon\to 0} (\varepsilon \ln \eta(\varepsilon))^{-1} \geqslant 0$. Analogously to [1]–[3] it can be established the averaging (limit) problems the perturbed problem are as follows

$$-\Delta\psi_0 = \lambda_0\psi_0, \quad x \in \Omega, \quad \psi_0 = 0, \quad x \in D_1, \quad \frac{\partial\psi_0}{\partial\nu} = 0, \quad x \in D_2,$$
$$\psi_0 = 0 \text{ (as } A = \infty) \text{ or } \left(\frac{\partial}{\partial\nu} + A\right)\psi_0 = 0 \text{ (as } A \in [0, +\infty)), \quad x \in \Sigma.$$

Hereinafter λ_0 is a simple eigenvalue of one averaging problems.
Theorem 1. *Let $A = \infty$. Then the asymptotics for the eigenvalue λ_ε converging to λ_0 has the form*

$$\lambda_\varepsilon = \lambda_0 + \varepsilon\lambda_1(\eta) + \dots,$$

where $\lambda_1(\eta)$ is some explicitly calculated function.
Theorem 2. *Let $A \in [0, +\infty)$. Then the asymptotics for the eigenvalue λ_ε converging to λ_0 has the form*

$$\lambda_\varepsilon = \Lambda_0 + \varepsilon\Lambda_1(\mu) + \varepsilon^2\Lambda_2(\mu) + \dots,$$

where $\Lambda_j(\eta)$ are some explicitly calculated function, holomorphic on μ, $\mu = \mu(\varepsilon) = -(A + (\varepsilon \ln \eta(\varepsilon))^{-1})$, $\Lambda_0(0) = \lambda_0$. The author is partially supported by RFBR grant ж 99-01-01143 and grant of Ministry of Education of Russia ж E00-1.0-53.

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Глобальный анализ и топологические характеристики нелинейных отображений

Борисович Ю.Г. в развитие своей работы [Теория фредгольмовых отображений и некоторые задачи оптимального уравления// Итоги науки и техники , серия Современная математика и ее приложения. Тематические обзоры, том 68. - Москва, 1999. - с. 27 -48.] исследовал оператор $H(u,x) = L(u,x) - G(u,x)$, возникающий в проблеме управления нелинейной динамической системой со связями , действующий из пространства абсолютно непрерывных функций в пространство суммируемых. При условиях, обеспечивающих эквивариантность L, G относительно действия тора, вычислена топологическая степень $degH$, что при условии $degH \neq 0$ гарантирует существование управляемой пары (u,x). Борисовичем Ю.Г. совместно с Золотаревым И.Ю. проведены исследования эквивариантных отображений многообразий с точки зрения построения топологических характеристик со значениями в классах G -бордизмов, обобщающих классические характеристики степени отображений Хопфа - Брауэра и вращения М.А.Красносельского. Получены следующие результаты. 1. Для произвольного компактного многообразия X^{n+k} и действующей на нем компактной группы Ли G определены классы эквивариантно- оснащенных бордизмов $GB_n(X^{n+k})$. Рассматривая произвольные замкнуты подгруппы H группы G, определяем классы эквивариантно-оснащенных Н-бордизмов. 2. Для собственного G - отображения $f : X^{n+k} \to Y^k$ компактных G - многообразий и регулярного значения $y_0 \in Y^k$, $G_{y_0} = H \subset G$ определена система $[f^{-1}(y_0), V^k]_L$, $L \subset H$, в которой $[f^{-1}(y_0), V^k]_L \subset LB_n(X^{n+k})$, и которую мы назовем "обобщенной Н-степенью". Имеет место

Теорема. Обобщенная Н-степень является инвариантом относительно Н-гомотопии отображения f , и если хотя бы одна ее L-компонента не бордантна нулю, то прообраз $f^{-1}(y_0)$ не пуст и является L-многообразием. Борисовичем Ю.Г. и Портной Т.В. были получены следующие результаты. Изучались особые точки системы векторных полей с точки зрения теории препятствий. Для Ω - области n-мерного, дифференцируемого, ориентированного многообразия M^n системы n векторных полей $\{f_i\}_{i=1}^n$ заданных на $\overline{\Omega}$ и таких, что на $\partial\overline{\Omega}$ они линейно независимы, были построены следующие гомотопические инварианты особых точек этой системы (точек Ω в которых ситема $\{f_i\}_{i=1}^n$ линено зависма):

1. индекс изолированной особой точки x_0

$$C_f(x_0) \in \pi_{n-1}(V_{n,n}),$$

2. глобальный топологический индекс

$$C_f(\partial\overline{\Omega}) \in \pi_{n-1}(V_{n,n})$$

Доказана теорема о том, что в случае конечного числа особых точек $x_i, i = 1,...,m$ в Ω, $C_f(\partial\overline{\Omega}) = (\sum_{i=1}^{m} C_f(x_i))$. Справедлива следующая теорема:

Теорема Если $C_f(\partial\overline{\Omega}) \neq 0$, то в Ω имеется особая точка системы векторных полей $\{f_i\}_{i=1}^n$. В случае, когда $\partial\overline{\Omega} = \bigcup_{i=1}^{k} B_i$ и на каждом B_i на систему $\{f_i\}_{i=1}^n$ наложены некоторые условия, гомотопический индекс был построен с помощью гомотопических групп $(\kappa + 1)$-ад М.М. Постникова: $C_f(x_0) \in \pi_{n-1}(E^n; M_1, :, M_k)$, где M_i определяются условиями наложенными на систему $\{f_i\}_{i=1}^n$ на B_i соответствено. Получены обобщения на бесконечномерный случай. Рассматривается гильбертово пространство H. Предполагаем, что существует последовательность конечномерных подпространств $H^1 \subset H^2 \subset \cdots \subset H^n \subset \cdots \subset H$ и последовательность непрерывных отображений $\{p_i\}_{i=1}^{\infty}$ таких, что $p_i : H \to H_i$, $\|p_i x - x\| \to 0$, $i \to \infty$ равномерно на каждом множестве $\Omega \subset H$. Пусть Ω -область гильбертова пространства H, такая, что $\Theta \notin \Omega$ и $\overline{\Omega} \bigcap H^i, i = \overline{1,\infty}$ является клеточным комплексом, имеющим только одну клетку старшей размерности. Пусть на Ω задана система векторных полей $\{f_i\}_{i=1}^{\infty}$, таких что на границе $\Gamma = \partial\overline{\Omega}$ подсистема $\{f_i\}_{i=1}^n$ линейно независима для любого $n \in N$ и каждое $f_i : \overline{\Omega} \to H$ является отображением вида $I - K_i$, где K_i - вполне непрерывное и каждое $f_i(x) \neq 0$ на $\Gamma = \partial\overline{\Omega}, i = \overline{1,\infty}$.

Определение 1. Систему векторных полей $\{f_i\}_{i=1}^{\infty}$, в гильбертовом пространстве H будем называть линейно независимой, если подсистема $\{f_i\}_{i=1}^n$, линейно независима для любого $n \in N$.

Определение 2. Точку x_0 будем называть особой точкой системы векторных полей $\{f_i\}_{i=1}^{\infty}$, если существует такое $n \in N$, для которого подсистема $\{f_i\}_{i=1}^n$ линейно зависима.

При сделанных выше предположениях построен глобальный топологический индекс

$$C_f(\partial\overline{\Omega}) \in (\pi_{m-1}(V_{m,m}, \ldots\ldots)), m \to \infty$$

Имеет место теорема о существовании в Ω по крайней мере одной особой точки в случае, когда $C_f(\partial\overline{\Omega}) \neq 0$. Также получен индекс изолированной особой точки. В случае гильбертова пространства H и сферы $S \subset H$ коразмерности 1, для всякой сферы $S_1 \subset S \subset H$ коразмерности 2 для некоторго $m < \infty$ построен глобальный топологический индекс:

$$C_f(\partial\overline{\Omega}) \in (\pi_{m-2}(V_{m,m-1}), \ldots, \pi_{m+5}(V_{m+5,m+4})).$$

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Borovskikh A.V.
(Voronezh State University)

The formula of wave spreading in the one-dimensional medium [8]

Theorem. *Let* $k(x) > 0$ *and the function* $\phi(x) = \dfrac{k'(x)}{2k(x)}$ *is continuously differentiable. Then the common solution of* $k(x)\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial}{\partial x}k(x)\dfrac{\partial u}{\partial x}$, *in the class of functions* $u(t,x)$, *twice continuously differentiable on the set of real numbers and having finite energy* $E(t) = \dfrac{1}{2}\int\limits_{I\!R}\left[\left(\dfrac{\partial u}{\partial t}\right)^2 + \left(\dfrac{\partial u}{\partial x}\right)^2\right]k(x)\,dx$, *is described with*

[8] Results presented in this paper were obtained during the author's visit by invitation to the University of Valenciennes end of Hainait-Cambresis in June 2000, their publication is being carried out with the help of the grants GosComVuz RF N 97-0-1.8-100 in the field of fundamental sciencies and No.11 in the fiels of mathematics

the formula

$$\begin{aligned} u(x,t) &= \sqrt{\frac{k(x-t)}{k(x)}}V(x-t) + \sqrt{\frac{k(x+t)}{k(x)}}W(x+t) + \\ &+ \frac{1}{2}\int\limits_{x-t}^{x+t} \sqrt{\frac{k(y)}{k(x)}} V(y) J\Big(\frac{x+y-t}{2}, \frac{x-y-t}{2}, x\Big)\, dy - \\ &- \frac{1}{2}\int\limits_{x-t}^{x+t} \sqrt{\frac{k(y)}{k(x)}} W(y) J\Big(\frac{x+y+t}{2}, \frac{x-y+t}{2}, x\Big)\, dy + \\ &+ \frac{1}{2}\int\limits_{x-t}^{x+t} \sqrt{\frac{k(y)}{k(x)}} W(y) \tilde{J}\Big(\frac{x+y-t}{2}, \frac{x-y-t}{2}, x\Big)\, dy - \\ &- \frac{1}{2}\int\limits_{x-t}^{x+t} \sqrt{\frac{k(y)}{k(x)}} V(y) \tilde{J}\Big(\frac{x+y+t}{2}, \frac{x-y+t}{2}, x\Big)\, dy \end{aligned}$$

where V, W are any twice continuously differentiable function, such that

$$\int\limits_{I\!R} \Big[\big(V'(x) + \phi(x)V(x) - \phi(x)W(x)\big)^2 + \\ + \big(W'(x) + \phi(x)W(x) - \phi(x)V(x)\big)^2\Big] k(x)\, dx < \infty,$$

and satisfying the following relations with the initial data $u_0(x), u_1(x)$

$$V(x) + W(x) = u_0(x), \quad -[k(x)V(x)]' + [k(x)W(x)]' = k(x)u_1(x).$$

The notations $J(a,b,x)$ and $\tilde{J}(a,b,x)$ are used for the functions that are the solutions of the integral equations

$$J(a,b,x) = -\int\limits_a^x \phi(\sigma - b)\phi(\sigma)\, d\sigma - \int\limits_a^x \int\limits_b^0 \phi(\sigma - b)\phi(\sigma - \tau) J(\sigma, \tau, x)\, d\tau d\sigma$$

and

$$\tilde{J}(a,b,x) = \phi(a) - \int\limits_b^0 \int\limits_a^x \phi(a - \tau)\phi(\sigma - \tau)\tilde{J}(\sigma, \tau, x)\, d\sigma d\tau.$$

Borowiec A.
(Wroclaw University)

Complex geometry and new class of Einstein metrics

We study a class of pseudo-Riemannian manifolds (M^{2n}, g) with a complex orthogonal group $SO(n, C)$ as a structure group. We call them anti-Kählerian manifold. These manifolds have signature (n, n) and admit a parallel complex structure J. ¿From complex point of view they can be identified with complex manifolds admitting holomorphic metric. Complex manifolds of this type, when half-flat, are known from Penrose's non-linear graviton twistor construction. It is proved that all odd Chern numbers of an anti-Kählerian manifold vanish and that complex parallelisable manifolds (in particular the factor space G/D of a complex Lie group G over the discrete subgroup D) are anti- Kählerian manifolds. The complexification of any analytic (in particular, Einstein) metric gives an anti-Kähler (in particular, anti- Kähler-Einstein) metric. This gives a method for constructing of new solutions of Einstein equations. This talk is based on joint works with M. Ferraris, M. Francaviglia and I. Volovich

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Mikhail Borsuk
(University of Warmia and Mazury in Olsztyn)

The behavior of weak solutions to the BVP for elliptic quasilinear equations with triple degeneration near an edge of the domain boundary

We investigate the behavior of weak solutions to the BVP (Dirichlet problem or mixed problem) for equations whose prototype is:

$$-\frac{d}{dx_i}(r^\tau |u|^q |\nabla u|^{m-2} u_{x_i}) + a_0 r^{\tau-m} u|u|^{q+m-2} - \mu r^\tau |u|^{q-1} |\nabla u|^m \operatorname{sgn} u = f(x),$$

$$r^2 = x_{n-1}^2 + x_n^2; \quad n \geqslant 3, \quad 0 \leqslant \mu < 1, \quad q \geqslant 0, \quad m > 1, \quad a_0 \geqslant 0, \quad \tau \geqslant m-2$$

in a neighborhood of an edge on the domain boundary. We establish almost precise exponent of the solution decreasing rate. The basic idea of our investigation is the construction of barrier founction and the using of new comparison principle

Bouzar C.
(Université d'Oran.Algérie)

A generalization of the problem of elliptic iterates

The aim of this work is to find algebraic necessary and sufficient conditions such that the next inclusion, between spaces of Gevrey vectors of systems of linear patial différential operators,

$$G^s\left(\Omega,(P_j)_{j=1}^N\right) \subset G^{s'}\left(\Omega,(Q_j)_{j=1}^L\right) , \tag{0.1}$$

holds. If the system $(Q_j)_{j=1}^L$ is reduced to the elementary system of differential operators $(D_1,..,D_n)$ we obtain then the classical "problem of elliptic iterates". The general problem (1) is completely solved in the case of systems of differential operators with constant coefficients.

Definition 1 *The system $(P_j(x,D))_{j=1}^L$ is sayed N-quasielliptic in Ω if i) $\forall x \in \Omega$, $\digamma(x_0) = \digamma(x)$. ii) $\digamma(x_0)$ is a regular Newton's polyhedron iii) $\forall x_0 \in \Omega, \exists c > 0, \exists r > 0, \forall \xi \in R^n, |\xi| \geqslant r$,*

$$cV(\xi) \leqslant \sum_{j=1}^L |P_j(x_0,\xi)| \quad ,$$

where $V(\xi) = \sum_{\alpha\in S} |\xi^\alpha|$, S is the set of vertices of $\digamma(x_0)$.

In the case of systems of differential operators with variables coefficients we have a general result for N-quasielleptic systems. We give the definitions of the space of Gevrey vectors of the system $(P_j(x,D))_{j=1}^L$, noted $G^s\left(\Omega,(P_j)_{j=1}^L\right)$, and the space of Gevrey classes with respect to the Newton's polyhedron $\digamma$, noted $G^s_\digamma(\Omega)$, and after we prove a theorem resolving the posed problem (1) in the case of differential operators with variable coefficients. The result obtained generalise the theorems of Nelson, Komatsu, Kotake-Narasimhan and others... This work has been done with Rachid CHAÏLI.

Brailov Yu.A.
(Moscow State University)

Geometry of completely integrable systems on the semisimple Lie algebras

There is natural class of completely integrable systems on the semisimple Lie algebras — the systems obtained by the method of "shifting argument" introduced by A.S. Mischenko and A.T. Fomenko. The methods of extracting information about Liouville's foliation on the invariant torii in these systems are developed. Particulary, some properties of bifurcational diagram expressed in terms of algebraic structure.

Bratus A.S., Iassakova E.S.
(Moscow State University)

Stabilizing and Destabilizing Diffusion Influence in the System of Semi-linear Parabolic Equation

Stability of the boundary-value problem for a system of semi-linear parabolic equations is considered. It is known that this problem is connected with the sign of eigenvalues real part for a certain linearized operator. This operator includes Jacoby matrix for the system without diffusion and the differential operator related to the diffusion. Asymptotic formulae for this operators eigenvalue have been obtained. This result permits to investigate influence of the diffusion on stability. In particular case where the diffusion matrix contains Jordan blocks is considered. This case corresponds to existence of the cross-diffusion components in the system. Several questions then arise. If the system without diffusion is stable in some neighborhood of the fixed point, will the system with diffusion also be stable? If yes, then what kind of conditions should be laid upon the elements of the elements of the diffusion matrix? Inversely, if the system without diffusion is unstable is it possible to make it stables there by a proper choice of the corresponding diffusion matrix? As an example the boundary value problems for Brusselator reaction mechanism and Predator-prey system with the diffusions are considered.

Brudnyi A.Yu.
(University of Calgary)

Planar Analytic Vector Fields

In the talk we consider some geometric problems related to the Poincare center-focus problem for the families of planar analytic vector fields with a singular point depending analytically on a parameter.

Brüning J.
(Humboldt University, Berlin)
Geyler V.A.
(Mordovian State University, Saransk)

One-dimensional geometric scattering on compact manifolds

We consider a "horned" topological space $\widehat{X}$ constructed from a compact Riemannian manifold (X, g) of dimension $d \leqslant 3$ by gluing a finite number of semi-axes $\mathbf{R}_j^+$, $j = 1, ..., n$ ("horns"). The Schrödinger operator H on $\widehat{X}$ is defined by the "restriction–extension procedure" [1] applied to the direct sum of the Neumann Laplacians on $\mathbf{R}_j^+$ and a Schrödinger operator on X of the form $-g^{-1/2}(x)(\partial_\mu + iA_\mu(x))g^{1/2}(x)g^{\mu\nu}(x)(\partial_\nu + iA_\nu(x)) + p(x)$ (A_μ and p are sufficiently smooth real-valued functions). For the Hamiltonian H, we prove the existence and uniqueness theorem for the scattering states. Thus, we determine the transition amplitudes from $\mathbf{R}_j^+$ to $\mathbf{R}_k^+$ ($j \neq k$) and the reflection amplitudes in each $\mathbf{R}_j^+$. Also an explicit form for the corresponding scattering matrix S is found and its unitarity is proved. It is shown that the spectrum of H as well as the spectrum of a point perturbation of H can be restored from S even in the case of $n = 1$.

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Brusentzev A.G.
(Belgorod State Academy of Technology)

About self-adjointness in essential of Schroedinger non-semibounded operator with strongly potential

The report is concerned with sufficient condition for self-ajointness of Schroedinger operator in the space $L_2(G)$ with strong singularities of potential on ∂G. These problems were considered in the serie s of known papers related to Schroedinger semi-bounded operator in some special types domains G . The stated results contain generalization of these works in the directions of refusal of operator semi-boundness, of extension of domains classes and, in some cases, of conditions relaxation on potential smoothness. In particular, the report includes the theorem about self-ajointness of many particle Schrodinger Hamiltonian. As distinguished from the known results, the theorem involves the systems, which are in the outer field with nonsemi-bounded potential.

Buchstaber V.M.
(Moscow State University)

Integrable nonlinear differential equations and uniformization of universal spaces of Jacobians

Recent achievements in the theory of Abelian functions have been obtained by developing of classical Klein's ideas about construction of the theory of sigma function for curves of genus $g > 1$ as a theory of function that has all the key properties of theta function of g variables plus an additional and extremely important one of modular invariance likewise the elliptic Weierstrass sigma function. An approach to construction of Kleinian sigma function has been developed by the author together with V.Enolskii and D.Leykin (see [1], [2]). It is based on the use of the models of nondegenerate algebraic curves $V = \{(y,x) \in C^2 : f(y,x) = 0\}$, where $f(y,x) = y^n - x^s - \sum \lambda_{ij} y^i x^j$, $is + jn < ns$, n and s are coprime. The genus of such a curve is $g = \frac{1}{2}(n-1)(s-1)$. As a result, the Kleinian sigma function was constructed and described as an entire function $\sigma(u;\lambda)$, where $u = (u_1, \cdots, u_g)$, $\lambda = (\lambda_{ij})$ on the universal space $\mathcal{U}(n,s)$ of Jacobi varieties of (n,s)-curves that is a fibration with the base given by the coefficients λ of polynomial $f(y,x)$ and a fibre given by Jacobi variety $Jac(V)$. We obtained an explicit realization of the spaces $\mathcal{U}(2, 2g+1)$ (hyperelliptic case) and $\mathcal{U}(3,n)$ (trigonal case), where $n = g+1$ and $g = 3l$ or $3l+1$, as an algebraic subvarieties $\mathcal{U}(n,s) \subset C^N$, where $N = 3g$ for hyperelliptic case and $N = 4g$, if $g = 3l$ or $N = 4g+2$, if $g = 3l+1$ for trigonal case. We uniformaze these varieties with the help of $\wp$-functions of g variables that are derivatives of order > 1 of the logarithm of the σ-function. The above results ensued from application of obtained descriptions of basis of functions for the differential field of meromorphic functions on Jacobians $Jac(V)$ of (n,s)-curves for $n = 2,3$ and algebraic relations over λ between them. Our studies in this direction are motivated by the fact that under obtained uniformization the important nonlinear differential equations (Korteweg-de Vries (KdV), Kadomtsev-Petviashvili (KP), Sine-Gordon, Boussinesq types) appear naturally and explicitly. As it will be shown in the talk, this gives new resources for applications of the theory of Abelian functions.

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Budaev V.D.
(Russian State Pedagogical University, St. Petersburg)

Some Basic Properties of Systems of Root Functions of Nonselfadjoint Differential Operators

Let L be any linear operator (in particular, ordinary or partial differential operator) in Hilbert space H. Define a system of root functions $\{u_n\}_{n=1}^{\infty}$ of L as any system of nontrivial functions which satisfy an equation

$$Lu_n + \lambda_n u_n = \theta_n \varphi(\lambda_n) u_{n-1}$$

where the numbers θ_n ar either equal to 0 (in which case the element u_n is called as eigenfunction) or 1 (in which case we let $\lambda_n = \lambda_{n-1}$ and u_n an associated function), and also $\theta_1 = 0$. The numbers λ_n are called eigenvalues, $\varphi(\lambda)$ is arbitrary nonvanishing function. This definition is similar to V.A.Ilyin's definition of root functions of nonselfadjoint differential operators.

Theorem. *Suppose that the length of every chain of root functions is uniformly bounded. One can construct a system of root functions $\{\tilde{u}_n\}_{n=1}^{\infty}$ satisfies the following conditions:*

1. *All eigenfunctions of systems $\{u_n\}$ and $\{\tilde{u}_n\}$ coincide;*

2. *Every l-order associated fuction $\tilde{u}_n$ is a linear combination of functions u_n, $u_{n-1}, \ldots, u_{n-l}$;*

3. *The estimation*
$$\|\theta_n u_{n-1}\| \leqslant \psi(\lambda_n)\|u_n\|$$
holds with arbitrary preassigned function $\psi(t) > 0$;

4. *If system $\{u_n\}$ is complete in H, then $\{\tilde{u}_n\}$ also is complete in H;*

5. *If system $\{u_n\|u_n\|^{-1}\}$ satisfy the Bessel inequality (for every $f \in H$), then $\{\tilde{u}_n\|\tilde{u}_n\|^{-1}\}$ also satisfy the Bessel inequality.*

Some further properties of this construction will be given at the talk. For the first time the similar results was obtained in [1] for nonselfadjoint ordinary differential operators.

Bufetov A.I.
(Independent University of Moscow and Princeton University)

Markov operators and convergence of spherical averages for actions of free groups

A measure-preserving action of a free group can be viewed as a stationary Markov process. If the group action is ergodic, then the process is ergodic, and if the action does not have nontrivial eigenvalues then the tail sigma-algebra of the process is trivial. Triviality of the tail sigma-algebra gives convergence of spherical averages for actions of free groups, and, in particular, a new proof of the Nevo – Stein theorem.

Butuzov V.F., Nedelko I.V.
(Moscow State University)

On the Global Domain of Attraction of the Stable Solutions with Internal Transition Layers

Consider the problem:

$$\varepsilon^2 \Delta u - u_t = f(u,x,\varepsilon), \quad (x,t) \in D \times (0,+\infty), \quad D \subset R^2, \tag{1}$$

$$\left.\frac{\partial u}{\partial n}\right|_{\partial D} = 0, \quad t \in (0,+\infty), \tag{2}$$

$$u(x,0,\varepsilon) = u_0(x,\varepsilon), \quad x \in \bar{D}, \tag{3}$$

where ε is a small positive parametr, $f(u,x,\varepsilon)$ is the smooth function. Suppose the following assumption holds. (A1). There exist functions $\bar{u}(x)$ and $\hat{u}(x) \in C^2(\bar{D})$ such that $\bar{u}(x) < \hat{u}(x)$, $x \in \bar{D}$, function $f(u,x,0)$ is equal zero in the region $\Pi = \{(u,x) : \bar{u}(x) \leqslant u \leqslant \hat{u}(x),\ x \in \bar{D}\}$ only for $u = \varphi_i(x)$, $i = 0,1,2$, and besides $\bar{u}(x) < \varphi_1(x) < \varphi_0(x) < \varphi_2(x) < \hat{u}(x)$, $f_u(\varphi_i(x),x,0) > 0$, $i = 1,2$, $f_u(\varphi_0(x),x,0) < 0$, $x \in \bar{D}$. (A2). Let the equation $J(x) = \int\limits_{\varphi_1(x)}^{\varphi_2(x)} f(u,x,0)du = 0$ determine a closed smooth curve $\Gamma \subset D$ dividing D into subdomains $D_0^{(+)}$ (inside the curve Γ) and $D_0^{(-)}$ and let $\partial J/\partial n_0 < 0$, $x \in \Gamma$, where $\partial/\partial n_0$ denotes the derivative along the inner normal of Γ. It is well known, if assumptions (?1), (?2) are satisfied then for sufficiently small ε the problem (1),(2) has the stable stationary solution $u_s(x,\varepsilon)$ satisfying

$$\lim_{\varepsilon\to 0} u_s(x,\varepsilon) = \varphi_1(x), \quad x \in D_0^{(-)}; \quad \lim_{\varepsilon\to 0} u_s(x,\varepsilon) = \varphi_2(x), \quad x \in D_0^{(+)}.$$

(A3). Let $J(x) \neq 0$, $x \in \partial D$. (A4). Let there be only finite number of sets and single points (separated one from another) in $D_0^{(+)}$ where $J(x) \geqslant 0$ and only finite number of sets and single points (separated one from another) in $D_0^{(-)}$ where $J(x) \leqslant 0$. (?5). $u_0(x) \in C^2(\bar{D}), \left.\frac{\partial u_0}{\partial n}\right|_{\partial D} = 0$ and $\bar{u}(x) < u_0(x) < \hat{u}(x)$, $x \in \bar{D}$. (A6). $\exists x^{(-)} \in D_0^{(-)}$ and $\exists x^{(+)} \in D_0^{(+)}$ such that: $u_0(x^{(-)}) < 0$ and $u_0(x) < 0$ in all points $x \in D_0^{(-)} \cup \partial D$ where $J(x) \leqslant 0$; $u_0(x^{(+)}) > 0$ and $u_0(x) > 0$ in all points $x \in D_0^{(+)}$ where $J(x) \geqslant 0$. 'heorem. *If assumptions (A1)-(A5) are satisfied then for sufficiently small ε function $u_0(x)$ belongs to the global domain of attraction $G(u_s)$ of the solution $u_s(x,\varepsilon)$, i.e.* $\lim_{t\to+\infty} \|u(x,t,\varepsilon) - u_s(x,\varepsilon)\|_{C(\bar{D})} = 0$.

Cardone G.
(Second University of Naples)

Compactness results for a class of non standard lagrangians in the case of elasticity

Let us consider the class $\mathcal{A}$ of integrands $f(x,\xi)$ such that

i) $f(x,\xi)$ is a measurable function of $x \in \mathbf{R}^n$;

ii) $f(x,\xi)$ is convex with respect to the $n \times n$ matrix ξ;

iii) $-c_0 + c_1|\xi|^\alpha \leqslant f(x,\xi) \leqslant c_0 + c_1|\xi|^\beta$ with $1 < \alpha \leqslant \beta < +\infty$, $c_0 \geqslant 0, c_1, c_2 > 0$

where Ω is a bounded domain with lipschitz boundary and

$$e(u) = (e_{ij}(u)) = \left(\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right)$$

is the strain tensor. Let us now consider, given a sequence $(f_\varepsilon)_\varepsilon$ of the class $\mathcal{A}$, the following sequence of integral functionals

$$A_\varepsilon^p(u) = \begin{cases} \int_\Omega f_\varepsilon(x, e(u))dx & \text{if } u \in W^{1,p}(\Omega;\mathbf{R}^n), \\ +\infty & \text{if } u \in W^{1,\alpha}(\Omega;\mathbf{R}^n) \setminus W^{1,p}(\Omega;\mathbf{R}^n). \end{cases} \tag{2}$$

We prove compactness results for the sequence of functionals $A_\varepsilon^p(u)$. More precisely if A^p is its Γ-limit with respect to the weak topology of $W^{1,\alpha}(\Omega;\mathbf{R}^n)$, we want to clarify the structure of $A^p(u)$. The first step is to prove the integral representation of $A^p(u)$ on smooth functions.

Theorem 1. *Let Ω be a bounded domain with lipschitz boundary, $\mathcal{A}$ the class of integrands defined in (1), $A^p_\varepsilon(u)$ integral functionals defined in (2). Then there exists an integrand $f(x, e(u))$ in $\mathcal{A}$ such that*

$$A^p(u) = \int_\Omega f(x, e(u))dx, \qquad \text{for every } u \in C^\infty(\overline{\Omega}).$$

The second step is the following theorem.

Theorem 2. *Let Ω be a bounded domain with lipschitz boundary, $\mathcal{A}$ the class of integrands defined in (1), $A^p_\varepsilon(u)$ integral functionals defined in (2) and α, $\beta > 1$ such that the imbedding $W^{1,\alpha}(\Omega;\mathbf{R}^n) \hookrightarrow L^\beta(\Omega;\mathbf{R}^n)$ is continuous. Then there exists an integrand $f(x, e(u))$ in $\mathcal{A}$ such that*

$$A^p(u) \geqslant \int_\Omega f(x, e(u))dx, \qquad \text{for every } u \in W^{1,\alpha}(\Omega;\mathbf{R}^n).$$

We explictly observe that we need in Theorem 2 of the continuity of the imbedding $W^{1,\alpha}(\Omega;\mathbf{R}^n) \hookrightarrow L^\beta(\Omega;\mathbf{R}^n)$ and that the general case is open. These problems of compactness in the scalar case were treated in [2] and [3], while in [1] the authors considered the homogenization in the vectorial case when the integrands are of the type $f_\varepsilon(x, Du(x)) = f\left(\frac{x}{\varepsilon}, Du(x)\right)$, where $f(\cdot, \xi)$ is 1-periodic with respect to x.

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Subsolutions and supersolutions of a boundary-value problem with Emden-Fowler equations

In this work, we are concerned with boundary-value problem (BVP) for the Emden-Fowler equation $y''(x) = \lambda x^p y^q(x)$, with $\lambda > 0$, $p > -1$, $q \in \mathbb{R}$, wich satisfies the boundary conditions $y'(0) = 0$, $y(1) = 1$. This BVP has been considered by Mooney [3] for $p = 0$, $q > 0$. This particular BVP arises in heat-transfer problems.

Mooney presented some numerical results when $q > 1$. For $0 < q < 1$ the iterative schemes based on Newton's and Picard's methods, presented by him, they don't converge. To construct convergents iterative schemes we need a subsolution of the BVP. In the present paper we introduce subsolutions and supersolutions for $0 < q < 1$ and also for $q > 1$ and $q < 0$. With these subsolutions and supersolutions we can conclude that exists a unique non-negative solution for $q > 0$. When $q < 0$ we can have one or two solutions or none at all, according to the value of λ. The supremum of values of λ for wich there are two solutions is obtained. Among the several subsolutions and supersolutions achieved, we determine the highest subsolution and the lowest supersolution when q takes all its real values, for specific values of λ and p. With this last subsolution and supersolution we get efficients iterative schemes to obtain the solution of the BVP in a similar way that was gotten for related BVP [1, 2]. Numerical results, obtained by different methods, are presented.

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Boundary value problem for layer-growth kinetics in ion nitriding of pure iron

The present work is devoted to study a boundary value problem arising in the mathematical modeling of layer growth kinetics in ion nitriding of pure ion. Wellposedness of the boundary value problem is discussed.

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Absolutely continuous invariant measures of interval maps serving as a model in laser physics

The object of study is the real rational function

$$s_{a,b} : x \mapsto b + \frac{ax}{1+x^2},$$

depending on two scalar parameters a and b. The iterations of $s_{a,b}$ model the evolution of a Fabry-Perot cavity in Laser Physics. The values of parameters that correspond to the regular (with periodic sinks) behaviour of $s_{a,b}$, are the most preferable from the applied point of view. Thus it makes sense, in particular, to determine the parameter ranges for chaos in the system. Earlier some partial results in this direction were obtained in the framework of topological dynamics (see [1]). We prove the following result.

Theorem. *There is a set Σ of positive Lebesgue measure in the space of parameters $\{(a,b)\}$, very closely to the point $(\infty, -2)$, such that for any $(a,b) \in \Sigma$ there exists an absolutely continuous invariant probability measure of the map $s_{a,b}$.*

This theorem can be considered as an analogue of Jakobson Theorem (see [2], [3]) for a two-parameter family of one-dimensional maps close to some limit map with a neutral fixed point.

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Macdonald polynomials and algebraic integrability

In late 80-s I.G.Macdonald introduced remarkable families of multivariable orthogonal polynomials [1,2], related to simple complex Lie algebras, or root systems. Each family depends, apart from a root system, on two parameters q, t and specializes to several remarkable families of symmetric functions, such as Schur functions and characters of corresponding Lie groups, Hall–Littlewood functions and Jack polynomials. This makes Macdonald polynomials very interesting from the point of view of the representation theory, combinatorics, special function theory and mathematical physics. We present an alternative approach to Macdonald polynomials which uses some remarkable properties of the difference operators by Macdonald [1] in case $t = q^k$ with integer k. Namely, it turns out that in this case the Macdonald operators act naturally in the coordinate ring of a certain singular affine algebraic variety. This provides an elementary construction of the Bloch eigenfunctions for Macdonald operators. These eigenfunctions, in its turn, are parametrized by the points of another, dual algebraic variety. This is a multidimensional analogue of the phenomenon known as algebraic integrability and well studied in the context of the finite-gap theory in dimension one. In this way we obtain a generalization of earlier results [3-5]. One of the applications is a new proof of several conjectures concerning Macdonald polynomials (norm conjecture, evaluation formula and symmetry identity). These have been suggested by Macdonald and proved (for all root systems) by I.Cherednik [6,7]. Our proof is much simpler since it doesn't use Cherednik's double affine Hecke algebras.

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An analog of Cameron-Johnson theorem for the linear $\mathbb{C}$-analytic equations in Hilbert space.

The well-known Cameron-Johnson's theorem [1,2] affirms that the equation

$$x' = A(t)x \qquad (1)$$

with recurrent (almost periodic by Bhor) matrix may be reduced by the Lyapunov-Perron transformation to the equation $y' = B(t)y$ with skew-symmetric matrix $B(t)$, if all solutions of the equation (1) and all its limit equations are bounded on the whole real axis. The generalization of this result on the linear $\mathbb{C}$-analytic equations in the Hilbertian space is our main scope in the article. Let $\mathbb{R} = (-\infty, +\infty), \mathbb{C}^m$ – m–dimensional complex Euclidean space, G be a domain from $\mathbb{C}^m$, $\mathbb{H}$ be a real or complex Hilbertian space with scalar product $< \cdot, \cdot >$ and with norm $|\cdot|^2 =< \cdot, \cdot >$. Denote by $\mathbb{H}_w$ a space $\mathbb{H}$ equipped with weak topology, and by $[\mathbb{H}]$ (respectively $[\mathbb{H}_w]$) the family of all linear continuous operators acting into $\mathbb{H}$ (resp. $\mathbb{H}_w$) and equipped with operational norm (with weak topology). Assume that $\mathcal{H}(G, [\mathbb{H}])$ (resp. $\mathcal{H}(G, [\mathbb{H}_w]), \mathcal{H}(G, \mathbb{C}^m)$) is the family of all holomorphic functions $h : G \to [\mathbb{H}]$ (resp. $h : G \to [\mathbb{H}_w], h : G \to \mathbb{C}^m$) equipped with compact-open topology and consider the system

$$\begin{cases} x' = \mathcal{A}(z)x \\ z' = \Phi(z) \end{cases} \quad (z \in G), \qquad (2)$$

where $\Phi \in \mathcal{H}(G, \mathbb{C}^m)$ and $\mathcal{A} \in \mathcal{H}(G, [\mathbb{H}])$. We suppose that the second equation of system (2) generates the dynamical system $(G, \mathbb{R}, \sigma)$ on G. Denote by $U(t, z)$ the Cauchy's operator of equation

$$x' = \mathcal{A}(zt)x \quad (z \in G), \qquad (3)$$

where $zt = \sigma(t, z)$. It follows from general property of solutions of differential equations (see, for example, [3-5]) that the family of operators $\{U(t, z) | t \in \mathbb{R}, z \in G\}$ satisfies the following conditions:

1. $U(0, z) = I \quad (\forall z \in G)$, where I is a unit operator on $\mathbb{H}$;
2. $U(t + \tau, z) = U(t, z\tau)U(\tau, z) \quad (\forall t, \tau \in \mathbb{R}$ and $z \in G$);
3. the mapping $U : \mathbb{R} \times G \to [\mathbb{H}] \quad (U : (t, z) \mapsto U(t, z))$ is continuous and for every $t \in \mathbb{R}$ the mapping $U(t, \cdot) : G \to [\mathbb{H}]$ is holomorphic.

The following assertion takes place.

Theorem 1 *Suppose that there exists a positive canstant C such that*

$$\|U(t, z)\| \leqslant C \qquad (4)$$

for all $t \in \mathbb{R}$ *and* $z \in G$, *then there exists* $P \in \mathcal{H}(G, [\mathbb{H}])$ *such that:*

a. P*- is biholomorphic, i.e. the operator* $P(z)$ *is invertible for all* $z \in G$ *and the mapping* $P^{-1} : G \to [\mathbb{H}]$ $(P^{-1} : z \mapsto P^{-1}(z))$ *is holomorphic;*

b. $P_*(z) = P(z)$ *for all* $z \in G$, *where* $P_*(z)$ *is an operator adjoint for* $P(z)$;

c. the operator $P(z)$ *is positive definite for all* $z \in G$;

d. $C^{-1}|x| \leqslant |P(z)x| \leqslant C|x|$ *for all* $z \in G$ *and* $x \in \mathbb{H}$;

e. the change of variables $x = P(zt)y$ *takes equation (3) into*

$$y' = B(zt)y \tag{5}$$

with skew-Hermitian operator $B \in \mathcal{H}(G, [\mathbb{H}])$, *i.e.* $B_*(z) = -B(z)$ *for all* $z \in G$.

Remark 1 *a. If the point* $z \in G$ *is stationary (ω-periodic, quasiperiodic, almost periodic etc), then according to [6] the operator-function* $P(zt)$ *will be also stationary (ω-periodic, quasiperiodic, almost periodic etc). b. The Theorem 1 takes place for the system of difference equations*

$$\begin{cases} x(k+1) = A(zk)x(k) \\ z(k+1) = \Phi(zk) \end{cases} \quad (k \in \mathbb{Z}),$$

where $A \in \mathcal{H}(G, [\mathbb{H}])$ *and* $\Phi \in \mathcal{H}(G, \mathbb{C}^m)$ *and also for the system of differential and difference equations with multidimensional time.*

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What is quantum stochastic differential equation from the point of view of functional analysis

We specify two fundamental objects which determine the well-posedness of a master Markov equation and a quantum stochastic differential equation. The first object is a completely positive contractive map $Q(\cdot)$, which can be expressed in terms of the coefficients of the formal generator of a master Markov equation. We prove that if there exists an eigenoperator $X : Q(X) = X$, then the minimal quantum dynamical semigroup does not preserve the unit operator and the probability of the event {*quantum system prepared in initial state ρ explodes during time t*} differs from null. A nonexplosion criterion is suggested, which generalizes the conditions proved for stochastic processes by Khasminsk'i and Taniguchi.

The analogy between the Carleman criterion for self-adjointness of a symmetric operator and the Gikhman–Skorokhod nonexplosion condition for Markov jump processes becomes clear. The second fundamental object is a symmetric boundary value problem on the Fock building. We prove that if the deficiency index of this problem is $(0,0)$, then the related quantum stochastic differential equation has a unique unitary solution, and the corresponding master Markov equation has a trace preserving solution. Sufficient conditions for the essential self-adjointness of the boundary value problem are proved.

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On behavior of a body randomly perforated along the boundary

We consider randomly perforated domain. Suppose that the diameter of the cavities and the distance from each cavity to the outer boundary of the domain are equal to ε. Also we assume that the set of the cavities is random, statistically homogeneous and nondegenerated. We introduce a definition of nondegeneraty on the base of the imbedding theorem for probability spaces.

For such structures we obtain the effective behavior, i.e. we consider boundary – value problem in such domain and prove the homogenization theorem. Also we estimate the difference between the solution of the initial problem and the solution of the homogenized problem. It appears that the homogenized problem is nonrandom.

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Matching Conditions for Harmonic Fields and for the Solutions of Maxwell Equations (Quasistatics)

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Trajectory and global attractors for a dissipative wave equation

The *trajectory attractor* $\mathfrak{A}_\varepsilon$ is constructed for the following non-linear wave equation with dissipation:

$$\partial_t^2 u + \gamma\left(x, \frac{x}{\varepsilon}\right) \partial_t u = \Delta u - b\left(x, \frac{x}{\varepsilon}\right) f(u) + g\left(x, \frac{x}{\varepsilon}\right), \ u|_{\partial\Omega} = 0. \qquad (0.1)$$

Here $x \in \Omega \Subset \mathbb{R}^n$ and ε is a fixed positive parameter. We assume that $\gamma(x,z)$ and $b(x,z)$ are continuous functions and $0 < \gamma_1 \leqslant \gamma(x,z) \leqslant \gamma_2$, $0 < \beta_1 \leqslant b(x,z) \leqslant \beta_2$ for all $x \in \overline{\Omega} \times \mathbb{R}^n$. The nonlinear term $f(v)$ satisfies the inequalities $|f(v)| \leqslant \gamma_0(|v|^\rho + 1)$, $F(v) \geqslant \gamma_3|v|^{\rho+1} - C_1$, $f(v)v \geqslant \gamma_4 F(v) - C_2$ for all $v \in \mathbb{R}$. Here $F(v) = \int_0^v f(w)dw$. Let also $g(x, x/\varepsilon) \in L_2(\Omega)$. We study a family $\mathcal{K}_\varepsilon^+ = \{(u(x,t), \partial_t u(x,t)), t \geqslant 0\}$ of weak solutions (trajectories) of the hyperbolic equation (0.1) such that $u(\cdot,t) \in L_\infty^{loc}(\mathbb{R}_+; L_{\rho+1}(\Omega) \cap H_0^1(\Omega))$, $\partial_t u(\cdot,t) \in L_\infty^{loc}(\mathbb{R}_+; L_2(\Omega))$, $u(x,t)$ is a solution of the equation in the distribution sense, and $u(x,t)$ satisfies the energy decrease inequality. If $n \geqslant 3$ and $\rho \geqslant \frac{n}{n-2}$, then the uniqueness theorem for the corresponding Cauchy problem is not proved yet. The trajectory attractor $\mathfrak{A}_\varepsilon$ consists of trajectories $\{(u(x,t), \partial_t u(x,t)), t \geqslant 0\} \in \mathcal{K}_\varepsilon^+$ that are bounded in the space $L_\infty\left(\mathbb{R}_+; L_{\rho+1}(\Omega) \cap H_0^1(\Omega)\right) \times L_\infty\left(\mathbb{R}_+; L_2(\Omega)\right)$ and have the bounded prolongation as solutions of the wave equation (0.1) on the entire time axis $\mathbb{R}$. The set $\mathfrak{A}_\varepsilon$ attracts bounded sets of trajectories from $\mathcal{K}_\varepsilon^+$ in the space $C([h, h+T]; H^{1-\delta}(\Omega)) \times C([h, h+T]; H^{-\delta}(\Omega))$ as $h \to +\infty$ for every $T \geqslant 0$. Since any trajectory $\{(u(x,t), \partial_t u(x,t)), t \geqslant 0\}$ from $\mathfrak{A}_\varepsilon$ belongs to the space $C(\mathbb{R}_+; H^{1-\delta}(\Omega)) \times C(\mathbb{R}_+; H^{-\delta}(\Omega))$ the following set is well defined:

$$\mathcal{A}_\varepsilon := \mathfrak{A}_\varepsilon(0) = \{(u(\cdot,0), \partial_t u(\cdot,0)) \mid (u, \partial_t u) \in \mathfrak{A}_\varepsilon\}.$$

The set $\mathcal{A}_\varepsilon$ is called the *global attractor* of the equation (0.1). It is bounded in $H_0^1(\Omega) \times L_2(\Omega)$ and compact in $H^{1-\delta}(\Omega) \times H^{-\delta}(\Omega)$ for $\delta > 0$. Moreover the set $\mathcal{A}$ has attracting properties known for the global attractors of semigroups generating by dissipative evolution equations for which the uniqueness theorems of the Cauchy problems hold. We now assume that the functions $\gamma(x, x/\varepsilon)$, $b(x, x/\varepsilon)$, and $g(x, x/\varepsilon)$ have weak averages $\bar{\gamma}(x)$, $\bar{b}(x)$, and $\bar{g}(x)$ as $\varepsilon \to 0+$, respectively. For example, this is true if the functions $\gamma(x,z)$, $b(x,z)$, and $g(x,z)$ are periodic, quasiperiodic, or almost periodic with respect to $z \in \mathbb{R}^n$. Along with the equation (0.1) we consider the averaged equation

$$\partial_t^2 \bar{u} + \bar{\gamma}(x) \partial_t \bar{u} = \Delta \bar{u} - \bar{b}(x) f(\bar{u}) + \bar{g}(x), \ \bar{u}|_{\partial\Omega} = 0, \qquad (0.2)$$

which also has the trajectory attractor $\overline{\mathfrak{A}}$ and the global attractor $\overline{\mathcal{A}} = \overline{\mathfrak{A}}(0)$ in the corresponding spaces. We have proved the following theorem: the trajectory attractor $\mathfrak{A}_\varepsilon$ and the global attractor $\mathcal{A}_\varepsilon$ of the equation (0.1) converges as $\varepsilon \to 0+$ to the trajectory attractor $\overline{\mathfrak{A}}$ and the global attractor $\overline{\mathcal{A}}$ of the equation (0.2), respectively. All these results were obtained in the collaboration with M.I.Vishik.

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Fridrichs' model and Titchmarsh-Weyl's function

In the space $H = L^2_\rho(0;\infty)$ the operator

$$T = S + V \tag{0.1}$$

is considered as perturbation of the operator $S\varphi(\tau) \equiv \varphi(\tau)$, $\tau > 0$ with maximal domain of definition $D(S)$. The perturbation $V = A^*B$ is given by the operators

$$A\varphi = \int_0^\infty \varphi(\tau)\alpha(\tau)\rho(\tau)d\tau, \quad B\varphi = \int_0^\infty \varphi(\tau)\beta(\tau)\rho(\tau)d\tau, \quad \varphi \in H$$

which act from H in anxiliary Hillbert space G. The conditions on the vector functions α, β and scalar function ρ are such that its permet to reduce some Sturm-Liouville's operators to the form (0.1). The maximal operator $\widetilde{S} : H \to H$ corresponding to the operator S, is given by the relation

$$\begin{cases} D(\widetilde{S}) = \left\{\varphi \in H | \exists c = c(\varphi) : \int\limits_0^\infty |\tau\varphi(\tau) + c(\varphi)|^2 \rho(\tau)d\tau < \infty\right\}, \\ \widetilde{S}\varphi = \tau\varphi(\tau) + c(\varphi),\ \tau > 0. \end{cases}$$

Let $T(\theta) = \widetilde{S} + V$, the domain $D(T(\theta))$ is defined by some condition $\theta(\varphi) = 0$.
Theorem 1 *1) The branchement of the resolvent $T_\zeta(\theta) = (T(\theta) - \zeta)^{-1}$ is generated by the branchement of the eigenvector $h_{\theta,\zeta} \in H$, $\theta(h_{\theta,\zeta}) = -\bar{k}_0$, $\zeta \in \rho(T(\theta))$ of the maximal operator $T(\theta)_{max}$ i.e.*

$$T_\zeta(\theta)\varphi = (a, b(\bar{\zeta}))h_{\theta,\zeta} + \widetilde{T}_\zeta(\theta)\varphi, \qquad \zeta \in \Omega \cap \rho(T(\theta)) \tag{0.2}$$

where the functions $\zeta \to (a, b(\bar{\zeta}))$, $\widetilde{T}_\zeta\varphi$, $\varphi \in \Phi$ are unique analytic in the domain Ω, $b(\bar{\zeta})$ is eigenfunctional of the operator $T(\theta)^*$, corresponding to $\bar{\zeta}$ and

$$\begin{cases} \widetilde{T}_\zeta(\theta)(T(\theta) - \zeta)\varphi = \varphi,\ \varphi \in D(T(\theta)) \cap \Phi, \\ (T(\theta)_{max} - \zeta)\widetilde{T}_\zeta(\theta)\varphi = \varphi,\ \varphi \in \Phi,\ \zeta \in \Omega \end{cases}$$

2) The branchement of the vector $h_{\theta,\zeta}$ in Ω is generated by some function $m_\theta(\zeta)$, analytic in the resolvent set $\rho(T(\theta))$, i.e.

$$(h_{\theta,\zeta}, \psi)_H = (a(\zeta), \psi)m_\theta(\zeta) + (R_{\theta,\zeta}, \psi), \qquad \zeta \in \Omega \cap \rho(T(\theta)) \tag{0.3}$$

where the values $(a(\zeta), \psi)$, $(R_{\theta,\zeta}, \psi)$, $\psi \in \Phi$ represente the eigenfunctionals of the operators $T(\theta)$, $T(\theta)_{max}$ and they are unicue analytic functions in Ω. 3) The *function* $m_\theta(\zeta)$ *is under the form* $m_\theta(\zeta) = (T_\zeta(\theta)\omega, \omega^*)$, $\zeta \in \rho(T(\theta))$ *where* $\omega = 1 + k_1\chi$, $\omega^* = 1 + \bar{k}_1\eta$ *– some singular elements,* $\chi, \eta \in H$ *4) The operator-function* $\zeta \to \widetilde{T}_\zeta(\theta)$ *is pseudoresolvent in* Ω, *bounded in some norm* $\|\cdot\|_\Omega$. One can find the results apropos of the prolongation of the resolvent in [1]. The theorem 1 is published in [2].

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On solvability of problems for parabolic equations with growing (near the boundary) coefficients

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О курсе геометрии в общеобразовательной школе

Состояние школьного математического образования в последние годы вызывало обоснованную тревогу математической общественности. Особенно ярко это выразил В.И. Арнольд, который, в частности, писал: " ... вся геометрия и, следовательно, вся связь математики с реальным миром и другими науками была исключена из математического образования ... Особенно опасна тенденция исчезновения всех доказательств из школьного обучения. Тот, кто не научится искусству доказательства в школе, не способен отличить правильное рассуждение от неправильного. Такими людьми могут легко манипулировать безответственные политики." [1].

Недостатки в постановке школьного математического образования имеют свое наиболее яркое проявление в преподавании курса геометрии, который в конце (теперь уже) прошлого века стал второстепенным по значимости, практически неудобоваримым для восприятия школьниками и фактически полностью обескровленным по содержанию.

Однако в последние годы в нашей стране возникло движение за восстановление должного места геометрии в школьном образовании. То же самое наблюдается и за рубежом; особенно активно этот процесс протекает во

Франции. В этой связи интересно познакомиться с компетентным мнением французской Комиссии по математическому образованию [2, 3].

На основной вопрос — надо ли сегодня продолжать преподавание геометрии в школе? — ответ без колебаний должен быть полложительным. Аргументы в пользу преподавания геометрии многочисленны. Здесь надо вспомнить и о развитии пространственного представления, и об обучении искусству доказательства, что геометрия выполняет лучше, чем любая другая дисциплина, и о значимости геометрии в реальной жизни, в культуре и в эстетике, и о фундаментальной роли в науке и технике геометрического мышления, способности опираться на геометрическую интуицию.

Изучение геометрии можно подразделить на несколько этапов. Например, начальное (элементарное) обучение геометрии проводится в младших классах средней школы, носит наглядный уровень и предусматривает знакомство с геометрическими фигурами и понятиями как планиметрии, так и стереометрии. Преподавание же в колледжах, лицеях, старших классах средней школы рекомендуется сконцентрировать вокруг таких вопросов: геометрия в пространстве (включая полиэдрф и сферическую геометрию); роль инвариантов (длина, угол, площадь, объем); геометрические места и построения; геометрические преобразования и др.

Интересно напомнить в этой связи те соображения и предложения, которые оставил нам в своих публикациях и высказываниях А.Н.Колмогоров. Он полагал, что изучение геометрии в школе должно состоять из четырех этапов: I - III, IV - V, VI - VIII, IX - X (сохранена действовавшая раньше нумерация классов десятилетней школы).

Изучение геометрии в I - III классах носит наглядно-иллюстративный характер. Рассматриваются формы различных предметов, даются пояснения об их свойствах, вводятся первые определения. Учащиеся знакомятся с употреблением линейки, циркуля, угольника и транспортира, выполняют простейшие построения и измерения. В IV - V классах обучение остается наглядным, но расширяется круг изучаемых фигур и начинается целенаправленная работа по развитию дедуктивного мышления. На основе выводов из наблюдений появляются первые дедуктивные умозаключения, первые теоремы и их доказательства. Ученики на начальном уровне знакомятся с отображениями геометрических фигур на примерах параллельного переноса, осевой симметрии.

В VI-VIII классах учащиеся постепенно подводятся к пониманию логического строения геометрии; последовательно проводится идея понимания геометрической фигуры как множества точек; для доказательства теорем и решения задач систематически используются геометрические преобразования. Вектор вводится как понятие, конкретными ориентирами которого являются сила, скорость в физике, параллельный перенос в геометрии. Наконец, в IX - X классах изучается систематический курс сте- реометрии. На двух последних этапах особое внимание уделяется системе основных понятий и обозначений. Следует подчеркнуть, что развитие школьного математического образования требует издания различных вариантов учебников, участия в этой работе ведущих математиков и педагогов страны.

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Chiado' Piat V.
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Homogenization of monotone operators

In this talk we give an account of the main contributions to the study of homogenization of boundary-value problems defined by monotone operators, starting from the linear case studied by De Giorgi and Spagnolo (1973) up to the recent results by Zhikov (2001), concerning non-linear equations involving periodic measures. We try to analyse the evolution of homogenization techniques, entering into details in the case with measures. More precisely, we examine the limit behaviour of solutions u^ε of boundary-value problems of the type

$$\int_\Omega \left(a\left(\frac{x}{\varepsilon}, \nabla u^\varepsilon\right) \cdot \nabla \varphi + |u^\varepsilon|^{p-2} u^\varepsilon \varphi \right) d\mu_\varepsilon = \int_\Omega f \varphi \, d\mu_\varepsilon \qquad \forall \varphi \in C_0^\infty(\Omega),$$

where $\mu_\varepsilon(A) = \varepsilon^N \mu(\varepsilon^{-1} A)$, and μ is a given non-negative Borel measure on $\mathbb{R}^N$, that is 1-periodic with respect to each of the variables $x_1, \ldots, x_N$ and is normalized by the condition $\int_Y d\mu = 1$, with $Y = [0,1[^N$. The function $f \in C_0^\infty(\overline{\Omega})$, $p > 1$, and the function $a(x, \xi)$ is μ-measurable with respect to $x \in \Omega$, it is strongly monotone with respect to $\xi \in \mathbb{R}^N$, for μ-a.e. $x \in \mathbb{R}^N$ and satisfies suitable growth conditions with rspect to $\xi \in \mathbb{R}^N$. We illustrate the main homogenization theorem in the case μ is p-connected, and we examine some examples of p-connected measures. In the end, we try to point out which are the main problems that are still open in this field.

Chizhonkov E.V.
(Moscow State University)

To optimization of one symmetric algorithm for saddle point problem

We consider the abstract linear nonsingular saddle point problem $L_0 z = F$:

$$L_0 z \equiv \begin{pmatrix} A & B \\ B^T & 0 \end{pmatrix} \begin{pmatrix} u \\ p \end{pmatrix} = \begin{pmatrix} f \\ g \end{pmatrix} \equiv F, \tag{0.1}$$

where A is a symmetric, positive definite $N_u \times N_u$ matrix, and B is an $N_u \times N_p$ matrix (in the general case $N_u \geqslant N_p$), f, g ($g \perp \text{kernel}(B^T)$) are given and u, p are the unknows. The problem (0.1) can be reformulated in the following way ([1]):

$$M_0 z \equiv \begin{pmatrix} Q^{-1}(A+\beta B_0) & Q^{-1}B \\ B^T Q^{-1}(\nu A + \nu\beta B_0 - Q) & \nu B^T Q^{-1} B \end{pmatrix} \begin{pmatrix} u \\ p \end{pmatrix} = \begin{pmatrix} \tilde{f} \\ \tilde{g} \end{pmatrix} \equiv \tilde{F}.$$

Here $C = C^T > 0, Q = Q^T > 0, B_0 = BC^{-1}B^T$ are matrices, and $\nu > 0, \beta$ are parameters. For the preconditioners C and Q the following inequalities are assumed: there are positive constants γ, Γ ($\gamma < \Gamma$) and δ, $\Delta(\delta \leqslant \Delta)$, such that

$$\gamma C \leqslant B^T A^{-1} B \leqslant \Gamma C,$$
$$\delta Q \leqslant A \leqslant \Delta Q.$$

Introduce the preconditioner $\bar{M}_0$ with parameter $\alpha > 0$ for the operator M_0 in the form: $\bar{M}_0 = \text{diag}\{I, \alpha C\}$, where I is identity matrix. Denote $H = \text{kernel}\, B^T$, then we expand the Euclidean space of vectors U with dimension N_u into the direct sum $U = H \oplus G$, where $G = H^\perp$. The following theorem holds.

Theorem. *Let subspaces H and G are invariant under $Q^{-1}A$, then the spectrum $\sigma(T)$ of the preconditioned operator $T = \bar{M}_0^{-1} M_0$ belongs to the set Λ:*

$$\Lambda = \{s\} \cup \{\theta \pm \sqrt{\theta^2 - ts/\alpha}\}, \quad \theta = s(1+\beta t + \nu t/\alpha)/2,\ t \in [\gamma, \Gamma],\ s \in [\delta, \Delta].$$

Using the analytic representation of the preconditioned operator spectrum, it is possible to formulate and to solve the asymptotic optimization problem of the method $\bar{M}_0 z_t + M_0 z = \tilde{F}$ (see, e.g. [2]).

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Chronopoulos A.Th.
(The Univ. of Texas at San Antonio)

Block S-step Iterative Methods for Nonsymmetric Linear Systems

(jointly with Kucherov)

S-step and block conjugate gradient type iterative methods have been studied and implemented in the past. In this work we derive a block s-step conjugate gradient type iterative method (the block OSOmin) for nonsymmetric linear systems. We also derive the 'seed' or 'projected' version of OSOmin. Finally, we derive a new averaging algorithm to combine several approximations to the solution of a single linear system using the block method with multiple initial guesses. We implement these new methods with ILU preconditioners on a parallel computer. We make comparisons between the different methods.

Chueshov I.D.
(Kharkov National University)

Order-preserving skew-product flows and nonautonomous parabolic equations

Let X be a nonempty set in a real Banach space V with a closed convex cone $V_+ \subset V$ such that $V_+ \cap (-V_+) = \{0\}$. This cone defines a partial order relation on X via $x \leqslant y$ if $y - x \in V_+$. The cone V_+ is *solid*, if it has nonempty interior $\operatorname{int} V_+$. The cone V_+ is *normal*, if for any a, $b \in V$ such that $a \leqslant b$ the interval $[a, b]$ defined by the formula $[a, b] \equiv \{x \in V \; : \; a \leqslant x \leqslant b\}$ is bounded. The cone V_+ is said to be *minihedral* if for any finite set $B \subset V$ there exist supremum $\sup B$ and infimum $\inf B$. As an example of a normal minihedral solid cone we can point out the standard cone in the space of continuous real functions on a compact set.

A *skew-product flow* (SPF) with time $\mathbb{R}_+$ and state space X is a pair (π, σ) consisting of the following two objects: (i) a *base flow* $\sigma \equiv (\Theta, \{\sigma_t, t \in \mathbb{R}\})$ on Θ, i.e. a family of transformations $\{\sigma_t : \Theta \mapsto \Theta, t \in \mathbb{R}\}$ such that $\sigma_0^s = id$, $\quad \sigma_t \circ \sigma_s = \sigma_{t+s}$ for all $t, s \in \mathbb{R}$ and $(t, \theta) \mapsto \sigma_t\theta$ is a continuous and (ii) a mapping $\pi \; : \; \mathbb{R}_+ \times X \times \Theta \mapsto X \times \Theta$ of the form $\pi(t, x, \theta) = (\varphi(t, \theta)x, \sigma_t\theta)$, where φ is a *cocycle* over σ with the state space X and time $\mathbb{R}_+$, i.e. a continuous mapping $(t, \theta, x) \mapsto \varphi(t, \theta)x$ from $\mathbb{R}_+ \times \Theta \times X$ to X satisfing the cocycle property:

$$\varphi(0, \theta) = id, \quad \varphi(t + s, \theta) = \varphi(t, \sigma_s\theta) \circ \varphi(s, \theta) \quad \text{for all} \quad t, s \in \mathbb{R}_+, \; \theta \in \Theta.$$

A skew-product flow (π, σ) is said to be *order-preserving* if $x \leqslant y$ implies $\varphi(t, \theta)x \leqslant \varphi(t, \theta)y$ for all $t \geqslant 0$ and $\theta \in \Theta$.

The concepts of equilibria, sub- and super-equilibria turn out to be of prime importance for the study of order-preserving SPF (for autonomous and periodic systems these concepts are well-known, see [3] and [5], for instance). The following definitions are motivated by the corresponding concept for random dynamical systems (see [1, 2] and the discussion therein).

A mapping $u : \Theta \mapsto X$ is said to be: (i) an equilibrium of the SPF (π, σ) if it is invariant under φ, i.e. if $\varphi(t,\theta)u(\theta) = u(\sigma_t\theta)$ for all $t \geqslant 0$ and $\theta \in \Theta$; (ii) a sub-equilibrium if $\varphi(t,\theta)u(\theta) \geqslant u(\sigma_t\theta)$ for all $t \geqslant 0$ and $\theta \in \Theta$; (iii) a super-equilibrium if $\varphi(t,\theta)u(\theta) \leqslant u(\sigma_t\theta)$ for all $t \geqslant 0$ and $\theta \in \Theta$.

Our first result is the following theorem.

Theorem 1. *Let (π,σ) be an order-preserving SPF. Assume that there exist a sub-equilibrium a and a super-equilibrium b such that $a(\theta) \leqslant b(\theta)$ and the set $\varphi(t_0,\theta)[a(\theta), b(\theta)]$ is precompact in X for some $t_0 \in \mathbb{R}_+$ and all $\theta \in \Theta$. Then the limits*

$$\underline{u}(\theta) = \lim_{s\to+\infty} \varphi(t,\sigma_{-t}\theta)a_s(\sigma_{-t}\theta) \quad \text{and} \quad \bar{u}(\theta) = \lim_{s\to+\infty} \varphi(t,\sigma_{-t}\theta)b_s(\sigma_{-t}\theta)$$

exist, and they are equilibria such that $a(\theta) \leqslant \underline{u}(\theta) \leqslant \bar{u}(\theta) \leqslant b(\theta)$.

This theorem is well-known for autonomous and periodic systems (see, e.g., [3, 5]). In fact similar assertions can be also found in [4] for ordinary differential equations with almost periodic coefficients. It was also proved in [1] for random dynamical systems.

An order-preserving SPF (π,σ) on $X = V_+$ is said to be *strictly sublinear* if for any $x \in \operatorname{int} V_+$ and for any $\lambda \in (0,1)$ we have $\lambda\varphi(t,\theta,x) < \varphi(t,\theta,\lambda x)$ for all $t > 0$ and $\theta \in \Theta$.

The orbit $\gamma_a(\theta) := \cup_{t\geqslant 0}\varphi(t,\sigma_{-t}\theta)a(\sigma_{-t}\theta)$ of the cocycle φ of SPF (π,σ) in $X = V_+$ emanating from a is said to be *bounded* if there exists a constant $C > 0$ such that $\|\varphi(t,\sigma_{-t}\theta)a(\sigma_{-t}\theta)\| \leqslant C$ for all $t \geqslant 0$ and $\theta \in \Theta$. We will say that the orbit γ_a is unbounded if it is not bounded. We will also say that SPF (π,σ) is *compact* if

$$\Gamma_a^\tau = \bigcup\{(\varphi(t,\theta)a(\theta), \sigma_t\theta) \,:\, t \geqslant \tau, \theta \in \Theta\}$$

is a precompact set in $X \times \Theta$ for every $a(\theta) \in V_+$ with bounded orbit $\gamma_a(\theta)$ and for some $\tau \geqslant 0$.

Our second result describes the possible long-term behavior of a sublinear SPFs.

Theorem 2. *Let (π,σ) be a strictly sublinear compact order-preserving SPF on V_+, where V_+ is a normal minihedral solid cone in the Banach space V. Assume that $\varphi(t,\theta)\operatorname{int} V_+ \subset \operatorname{int} V_+$ for all $t \geqslant 0$ and $\theta \in \Theta$. Then precisely one of the following three cases applies:*

(i) for all $b \in \operatorname{int} V_+$, the pull back orbit γ_b emanating from b is unbounded;

(ii) *for all $b \in \operatorname{int} V_+$, the pull back orbit γ_b emanating from b is bounded, but the closure of γ_b contains elements that do not belong to* $\operatorname{int} V_+$;

(iii) *there exists a unique compact equilibrium $u \in \operatorname{int} V_+$, and for all $b \in \operatorname{int} V_+$ the orbit emanating from b converges to u, i.e. $\lim_{t\to+\infty} \varphi(t, \sigma_{-t}\theta)b = u(\theta)$ for all $\theta \in \Theta$.*

Our main example is the SPF generated by the system of parabolic differential equations

$$\partial_t u_j = \Delta u_j + f_j(t, x, u_1, \dots, u_m), \quad j = 1, \dots, m,$$

in a smooth bounded domain $D \subset \mathbb{R}^d$, $d \leqslant 3$, with the Neumann boundary conditions. Here the function $f = (f_1 \dots, f_m)$ is almost periodic with respect t and it is *cooperative*, i.e. $\partial f_i(t, x, u)/\partial u_j \geqslant 0$, when $i \neq j$ and $u = (u_1, \dots, u_m) \in \mathbb{R}^m_+$.

We note that in contrast with the autonomous or periodic case almost periodic systems can possess nontrivial completely ordered omega-limit sets. This is one of the obstacles which prevent the direct expansion of results available for autonomous or periodic systems to the general nonautonomous case.

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Chung, S.-Y.
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Uniqueness Theorem and reflection principle for the Solutions of Heat Equation via generalized functions

It is well known that solutions of the heat equation in the upper half space is not uniquely determined by its initial value. But many mathematicians, such as Tychonoff, Hayne, and so on, have constructed a uniqueness class of solutions with some constraint on the growth of solutions. Here, we give a much larger uniqueness class which improves the results ever known so far. In addition, a reflection principle for the solutions of heat equation will be improved and used to get a uniqueness theorem for the temperatures in semi-infinite rod. Throughout this talk, it will be seen how theory of generalized functions, such as distributions, ultradistributions, hyperfunctions, work properly in this area.

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The Steiner Ratio

Steiner's Problem is the "Problem of shortest connectivity", that means, given a finite set of points in a metric space (X, ρ), search for a network interconnecting these points with minimal length. This shortest network must be a tree and is called a Steiner Minimal Tree (SMT). It may contain vertices different from the points which are to be connected. Such points are called Steiner points.
If we do not allow Steiner points, that means, we only connect certain pairs of the given points, we get a tree which is called a Minimum Spanning Tree (MST).

Observation I.

In general, methods to solve Steiner's Problem, that means to find an SMT, are still unknown or hard in the sense of computational complexity. In any case, we need a subtle description of the geometry of the space.

On the other hand,

Observation II.

It is easy to find an MST by an algorithm which is simple to realize and fast to run in all metric spaces. The algorithm needs only the mutual distances between the points.

A natural question, derived from these observations, is to ask, what is the performance ratio of an approximation of an SMT by an MST? Consequently, we are interested in the greatest lower bound for the ratio between the lengths of these both trees:

$$m(X,\rho) := \inf\left\{\frac{L(\text{SMT for } N)}{L(\text{MST for } N)} : N \subseteq X \text{ is a finite set}\right\},$$

which is called the Steiner ratio (of the metric space (X,ρ).
We will discuss this quantity for specific metric spaces. Particularly, we will consider the class of all

- two-dimensional Banach spaces;
- finite-dimensional $\mathcal{L}_p$-spaces;
- Riemannian surfaces;
- graphs; and
- sequence spaces.

Cojuhari P.A.
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Hardy type inequalities for abstract operators

Inequalities of Hardy [1,2], due to their importance for many principle problems from various domains, intensively have been discussed by many authors (see, for instance, [3-6] and the references quated there). We propose a general method for studing Hardy type inequalities. Our approach is based on the technique of abstract operator theory as far as possible and then involving concrete operators in order to obtain the classical Hardy inequalities, and others. In this context, the following result can be considered as an abstract version of Hardy type inequalities. For the sake of simplicity let us consider only the case of the Hilbert spaces (see, also, [7]).

Theorem 1 *Let A and B be densely defined linear operators in a Hilbert space $\mathcal{H}$, A is self-adjoint and $Ker A = 0$. Suppose that the operator BA^{-1} is densely defined and there is a definite operator C (so that either $C \geqslant \gamma I$ or $C \leqslant -\gamma I$ for some $\gamma > 0$) such that $Re(BA^{-1}) = C$ on a linear manifold Ω of $\mathcal{H}$. Then, for each complex number λ with $Re\lambda = 0$, the following inequality holds*

$$m\|Au\| \leqslant \|(B - \lambda I)u\|$$

for all $u \in Dom(A) \cap Dom(B)$ such that $Au \in \Omega$, where $m = m(C)$ is the greatest lower bound of the operator $|C|$.

We note that the constant m from above inequality is optimal. This fact follows at once by considering concrete situations which correspond to the case of well known classical Hardy inequalities. In this respect, the connection with the remarkable results of B.Muckenhoupt [8] is shown. Applications to spectral theory is also considered.

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A dynamical model for the interaction among large neural aggregates.

A dynamical model that describes the synaptic interaction among large neural aggregates as a competitive and cooperative process of propagation of waves of excitation and inhibition is described. The derivation of a nonlinear hyperbolic system of coupled equations is made, for which, averaging over the spatial variable, turns it into a system of ordinary differential reaction equations, with nonlinear terms that depend on the unknown function in a sigmoid form. These unknown functions represent the level of stimulation of the neural aggregates, because of the synaptic activity that is performed on them. This sigmoid term represent the ability of the aggregates to answer to a given level of stimulation. Some results about the asymptotic behavior of the solutions in relation to certain small parameters, the existence of periodic solution and travelling waves and the controllability of the model, are obtained. Also, several characterization and identification problems of the parameters that define the sigmoid functions that are able to generate stable periodic solutions, are solved. The results obtained are applicable to the analysis of the rhythms of the brain cortex and to the study of the so-called limbic circuit and its role in the schizophrenia disease.

Corbo E. A.
(University of Cassino)

A problem of approximation of BV functions with values of fixed norm

Danilin A.R.
(IMM UrD RAS, Ekaterinburg)

Asymptotics of solutions to a system of elliptic equations with parameter and small coefficient at higher derivatives.

By the matched asymptotic expansions method [1] the asymptotics of solutions of the following Dirichlet problem for system of singularly perturbed elliptic equations with scalar parameter and additional integral restriction

$$\begin{cases} \mathcal{L}_\varepsilon z_\varepsilon + u_\varepsilon = f, \ \mathcal{L}_\varepsilon^* u_\varepsilon - \lambda_\varepsilon \cdot z_\varepsilon = 0, \ (x,y) \in \Omega = (0;1) \times (0;1), \\ z_\varepsilon(x,y) = u_\varepsilon(x,y) = 0, \ (x,y) \in \partial\Omega, \end{cases}$$

$$\| u_\varepsilon \|^2 = \int\limits_\Omega u_\varepsilon^2(x,y)\,dx\,dy = R^2,$$

is constructed, where $\mathcal{L}_\varepsilon z = \varepsilon^2 \Delta z - b(x)\dfrac{\partial z}{\partial y} - a(x,y)z$, $\mathcal{L}_\varepsilon^* u = \varepsilon^2 \Delta u + b(x)\dfrac{\partial u}{\partial y} - a(x,y)u$,

$$f, a \in C^\infty(\overline{\Omega}),\ a(x,y) \geqslant A > 0 \text{ for } (x,y) \in \overline{\Omega},$$
$$b \in C^\infty[0;1],\ b(x) \geqslant B > 0 \text{ for } x \in [0;1]$$

and

$$b(x)\frac{\partial f(x,y)}{\partial y} - a(x,y) \cdot f(x,y) \not\equiv 0 \text{ in } \Omega.$$

The problem of finding optimal control and corresponding state in problem of optimal control of solutions of the equation $\mathcal{L}_\varepsilon z_\varepsilon + u_\varepsilon = f$ in domain Ω, integral restrictions for control and quadratic quality criterion [2] is reduced to the above problem.

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Danilov V.G.
(MSIEM, Moscow)

Solutions to quasilinear equations describing interaction of nonlinear structures

In my talk I will speak about a new approach to constructing formulas of solutions describing the merging process for shock waves, weak discontiniuties (shock wave generation) and decay of nonstable shocks for scalar conservation laws.

Da Silva A.R.
(Federal University of Rio de Janeiro)

Some results of S.Bernstein on the 2-point problem revisited

The purpose of this talk is to present the geometric viewpoint of the classical 2-point boundary value problem, introduced by M.M.Peixoto. We shall show that this approach allows us to extend some results due S.Bernstein, so as to put them within the framework of the modern theory of Whitney stratifications. Our results, based on previous work by Peixoto and Thom, allow us to handle a large class of equations left out by their treatment. Further, we shall see that these matters are closely related to the escape-time problem for second-order ordinary differential equations.

Davydov A.A.
(Vladimir State University)

Limiting direction metamorphoses of generic implicit ODEs

An implicit differential equation of order n is defined as zero level (=*equation surface*) of a smooth function on $(n+2)$-dimensional manifold S_n endowed by 2-dimensional distribution D_n which are the result of Goursat prolongation procedure from standard contact structure on the space of directions on the plane. A *generic* equation is defined by a function from an open everywhere dense subset in the space of functions in fine C^∞-topology. The surface of a generic equation is either empty or a smooth hypersurface in S_n because the differential of a generic function does not vanish at any point of its zero level. A solution of an equation is an immersed curve which lies in its surface and is tangent to the distribution D_n. A direction on the plane is *admissible* if there exists solution of the equation which passes through this point with the slope supplying by this direction. A boundary point of the closure of set of admissible directions gives the *limiting* direction at the respective point. A point of the plane provides metamorphose of limiting directions if the number of such directions is not constant near this point. Here generic metamorphoses of limiting directions are classified. Ones are closely related to generic singularities of limiting direction fields of generic dynamic inequalities on surfaces [1]. The restriction to the equation surface of natural projection from S_n to the space of the directions on the plane is called *equation 1-folding*.

Theorem 1 *A germ of 1-folding image of a generic equation of order $n \geqslant 2$ at any point of its boundary is the germ at zero of one of the following 9 sets:* 1) 0; 2) $x \leqslant \pm z^2$; 3) $x \leqslant z^3 + yz$; 4) $z \leqslant |x|$; 5) $x \leqslant |z|$; 6) $\{y \leqslant z\} \cup \{x \leqslant \pm z^2\}$; 7) $z \leqslant \max\{|x|, y\}$; 8) $y \leqslant \max\{|x| - z, z\}$; 9) $z \leqslant \max\{-w^4 + xw^2 + yw | w \in R\}$ *in appropriate local smooth coordinate system x, y, z in the space S_1 which has the origin at this point and moves the canonical projection along the direction axis to the form $(x, y, z) \mapsto (x, y)$. Also such image for any equation near the given one is carried to the initial one by C^∞-diffeomorphism of this space being close to the identity and commutating with this projection.*

Metamorphoses can be subdivided in two classes. One of them consists of *point* metamorphoses every of which takes place near some direction at a point of the plane and the other one is formed by *multipoint* metamorphoses when at a point of the plane there are observed at least two different point metamorphoses. Set of all points on the plane which provide point metamorphoses is a *bifurcation set*. Generic point and multipoint metamorphoses are also described and their stability is proved. For $n = 2$ some metamorphoses and their stability were investigated by M. LeMasurier [2].

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Delitsyn A.L.
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The spectral properties of some operators of mathematical physics in deformed cylinder.

The problem of the existence of the discrete spectrum of different operators of mathematical physics in deformed cylinder is the subject of many studies [1-3]. Nevertheless there are no general criterions of determination if discrete spectrum empty or not. We consider two problems. The first spectral problem is:

$$-\Delta u = k^2 u \qquad u|_{\partial Q} = 0 \qquad u \in L_2(Q), \tag{0.1}$$

where Q is locally enlarged cylinder with smooth boundary, $Q_1 = ((x,y) \in \Omega, z \in (-\infty, \infty)) \in Q$. The second problem is

$$rotrotE = k^2 E \qquad E \times n|_{\partial Q} = 0 \tag{0.2}$$

$$divE = 0 \qquad E \in L_2(Q)$$

which considerd in Q. The main result is

Theorem. *The discrete spectrum of the problem (1) and (2) is not empty.*

For the consideration of the problem (1) we consider [4] the nonlinear spectral problem in finite domain bounded by two sections $z = z_1, z = z_2$ and the boundary of cylinder ∂Q

$$-\Delta u = k^2 u \quad u|_{\partial Q} = 0$$

$$\frac{\partial u}{\partial z}|_{z=z_2} = -\sum_n \gamma_n(k)(u, \psi_n)|_{L_2(\Omega_2)}\psi_n$$

$$\frac{\partial u}{\partial z}|_{z=z_1} = \sum_n \gamma_n(k)(u,\psi_n)_{L_2(\Omega_1)}\psi_n$$

where ψ_n are the solutions of the spectral problem in the section of regular cylinder

$$-\Delta_\perp \psi_n = \lambda_n \psi_n \quad \psi_n|_{\partial\Omega} = 0$$

$\gamma_n = \sqrt{\lambda_n - k^2}$ We prove that there are exist the nontrivilal solution of this problem. The analogous method is applied for the (2) problem.

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Demidenko G.V.
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On pseudohyperbolic equations

In the paper we continue the study of equations not solved with respect to the highest derivative

$$L_0(x;D_x)D_t^l u + \sum_{k=0}^{l-1} L_{l-k}(x;D_x)D_t^k u = f(t,x). \tag{1}$$

Probably, for the first time, equations of such type were studied in Poincaré's well-known article [1]. Subsequently, they were considered in some articles by mathematicians and mechanics. The most intense interest to equations of the form (1) arose in connection with investigations of the problem on small oscillations of rotating fluids by S.L.Sobolev [2]. It is well known also that after the appearance of S.L.Sobolev's articles "... I.G.Petrovskii pointed out the necessity of studying the general differential equations and systems not solved for the highest derivative with respect to time (systems not of Kovalevskaya type)" (see [3, p. 27]). At present, there are a great number of works devoted to the study of equations and systems of such type (see, for example, the bibliography in [4]). In the present paper we consider one of three classes of equations of the form (1) introduced in the book [4]. The class is called pseudohyperbolic and contains particularly one class of equations defined by S.A.Gal'pern [5]. The main result of the paper is proof of solvability of the Cauchy problem for pseudohyperbolic equations with variable coefficients. In the case of constant coefficients analogous results were published in [6]. The research was supported by the Russian Foundation for Basic Research (N 99-01-00533, N 01-01-00609).

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Demidov A.S.

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The Stokes-Leibenson problem and non-standard analysis

It is shown why this problem in the case of a source does not admit a classical solution when initial contour is non-analytic. This "peculiarity" can be relaxed by using non-standard analysis. Let $\Omega_0 \subset \mathbb{R}^2$ be a simply connected open domain (symmetric with respect to x-axis) such that the origin belongs to Ω_0 and its boundary Γ_0 is smooth enough. This domain is deformed in the following way: for $t \geqslant 0$, we get a domain Ω_t such that, along its boundary Γ_t, each point $\mathrm{x} = (x, y)$ moves as velocity $\dot{\mathrm{x}} = \nabla u$, where u satisfies: $u_{xx}+u_{yy} = 2\delta$, in Ω_t, and $u = 0$, on Γ_t, where δ is the "Dirac function" concentrated at the origin. Let $v = v(x,y)$ be the function harmonically conjugate to u in $\Omega_t \setminus \{(0,y) : y \leqslant 0\}$ such that $v(x,0) = 0$ for $x > 0$. Let $b(t,\sigma)$ be the angle between ($x > 0$)-axis and the exterior normal to Ω_t at a point $s \in \Gamma_t \cap \{y \geqslant 0\}$ such that $v = \sigma \in [0,1]$ at the point s. Example: the function $(t,\sigma) \mapsto \hat{b}(t,\sigma) = \pi\sigma$ corresponds to a cerle. Let $\mathrm{H}\phi(t,\sigma) = \sum_{k\geqslant 1} \phi_k(t)\cos\pi k\sigma$, where $\phi(t,\sigma) = \sum_{k\geqslant 1}\phi_k(t)\mathrm{e}_k(\sigma)$ and $\mathrm{e}_k : \sigma \mapsto \sin \pi k\sigma$.

Theorem 1. *Function $b(t,\sigma) = \hat{b}(t,\sigma) + \beta(t,\sigma)$, where $\beta(t,\sigma) = \sum_{k\geqslant 1}\beta_k(t)\sin\pi k\sigma$, satisfies:*

$$(t+t_0)(b - \mathrm{K}(b))\dot{\beta}(t,\sigma) = e^{-2\alpha(t,\sigma)}\alpha'(t,\sigma) + \\ + b'(t,\sigma)e^{-\alpha(t,\sigma)} \int_0^\sigma [\pi e^{\alpha(t,\xi)} - b'(t,\xi)e^{-\alpha(t,\xi)}]\,d\xi, \quad (0.1)$$

$$\mathbf{K}(\phi)(t,\sigma) = b'(t,\sigma)e^{-\alpha(t,\sigma)} + \int_0^\sigma e^{\alpha(t,\xi)}\mathbf{H}\phi(t,\xi)\,d\xi,$$

$$\alpha = \mathbf{H}(\beta),\ \sqrt{2\pi t_0}\int_0^1 \exp\alpha(0,\sigma)\,d\sigma = |\Gamma_0|/2,$$

where $\dot g(t,\sigma) = \partial g/\partial t$, $g'(t,\sigma) = \partial g/\partial\sigma$. *The function* $b = b(t,\cdot) \in L^2(0,1)$ *defines* Γ_t *(see [1]).*

Theorem 2. *Vector* $\mathbf{e}_1$ *is transversal to* $\mathcal{M} = \{b \in L^2, \|b - \hat b\| << 1 :< (1-\mathbf{K})(b), \mathbf{e}_1 >_{L^2} = 0\}$. *The projection of the equation (1) on a normal* η *to* $\mathcal{A} \subset \mathcal{M}$ *(codim* $\mathcal{M}$ *= codim* $\mathcal{A} = 1$*) has (a convenient scale) the form:* $-2\eta\dot\eta = 1$, *its solution is defined only for* $t \leqslant \eta(0)^2$. *The solution of the regularizing equation* $-2(\eta + \lambda_\varepsilon(\eta(0),t))\dot\eta = 1$, *is defined for all* $t \geqslant 0$ *and oscillates with frequency* $(8\varepsilon^2)^{-1}$ *and amplitude* ε *for* $t > \eta(0)^2$. *Here* λ_ε *is a function which takes alternately values* ε *and* $-\varepsilon$ *with frequency* $(8\varepsilon^2)^{-1}$ *for* $t > \eta(0)^2$, *where* ε *is an infinitely small in the sense of non-standard analysis.*

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On Solution of Some Interpolation Problems with μ-splines [9]

Denisov V.N.
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On stabilization of the solution of the Cauchy problem with lowest terms

In half space $\{t \geqslant 0\} \equiv \{x; t : x \in E^N, t \geqslant 0\}$ we consider the Cauchy problem

$$\sum_{k,l=1}^N \frac{\partial}{\partial x_k}\left(a_{kl}(x)\frac{\partial u}{\partial x_l}\right) + \sum_{k=1}^N b_k(x,t)\frac{\partial u}{\partial x_k} + c(x,t)u - \frac{\partial u}{\partial t} = 0, \tag{1}$$

$$u(x,0) = u_0(x). \tag{2}$$

We assume that following conditions holds: coefficients $a_{kl}(x) = a_{lk}(x)$ $(k,l = 1,\ldots,N)$ are bounded and measurable, and uniform parabolic condition holds, initial function $u_0(x)$ — bounded and continuous in E^N, $c(x,t) \leqslant 0$ for all $(x,t) \in \{t \geqslant 0\}$, functions $b_k(x,t)$ $(k = 1,\ldots,N)$, $c(x,t)$ are bounded in every strip $\{0 < t \leqslant T\} = \{E^N \times (0,T)\}$. The Cauchy problem (1), (2) we understand in usual weak sence, that is in sence of integral identity [1]. The solutions of the problem (1), (2) we takes from class of

[9]The research is partly supported by RFFI Grant 98-01-00345.

uniqueness, that $u(x,t)$ is solutions bounded in each strip $\{0 < t \leqslant T\}$. We consider the question of sufficient conditions, which guarantee the following zero limit

$$\lim_{t\to\infty} u(x,t) = 0 \tag{3}$$

exists, uniformly on every compact $K \subset E^N$ for all bounded initial function $u_0(x)$.
Theorem 1. *Let $N = 1$ or $N = 2$ and following conditioness holds:*

$$c(x,t) \leqslant -\alpha^2 < 0 \tag{4}$$

for $|x| < k$, for some $k > 0$ and all $t \geqslant 0$, and

$$|b_k(x,t)| \leqslant C_1(1+|x|)^{-1-\varepsilon}, \quad x \in E^N,\ t > 0, \quad \text{and} \quad C_1 > 0,\ \varepsilon > 0. \tag{5}$$

Then the solution of the Cauchy problem (1), (2) stabilizes to zero, as $t \to +\infty$ uniformly on any compact $K \subset E^N$ for any bounded function $u_0(x)$.
Remark. On examples we demonstrated that conditions Theorem 1 are precise and Theorem 1 does not hold in cases $N \geqslant 3$.

Theorem 2. *Let $N \geqslant 3$ and following conditioness holds:*

$$c(x,t) \leqslant \frac{-\alpha^2}{|x|^2} \tag{6}$$

for $|x| > h$, where h — some positive;

$$|b_k(x,t)| \leqslant \frac{C}{(1+|x|)^{1+\epsilon}}, \quad (k = 1,\ldots,N). \tag{7}$$

Then the solution of the Cauchy problem (1), (2) stabilizes to zero, as $t \to +\infty$ uniformly on any compact $K \subset E^N$ for any bounded function $u_0(x)$.
Remark. On examples we demonstrated that conditions Theorem 2 are precise.

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On indefinite generalized interpolation

Denote by S_κ or H_κ^∞ the class of matrix valued functions s which can be represented by $s_0 b^{-1}$, where b is a Blaschke–Potapov product and s_0 is from the Schur class or $s_0 \in H^\infty$, respectively. We consider the following problem $IP_\kappa(w,\theta)$, which is an indefinite variant of the Sarason problem. Given are an inner matrix function θ and $w \in H^\infty$. Find $s \in S_\kappa$ such that $[s(z) - w(z)]\theta(z)^{-1} \in H_\kappa^\infty$. In the case where θ is a Blaschke–Potapov product the problem $IP_\kappa(w,\theta)$ is a tangential interpolation problem. Associated with the problem $IP_\kappa(w,\theta)$ is a functional Pontryagin space $\mathcal{H}$ and an isometric operator V in $\mathcal{H}$. Solutions of the problem $IP_\kappa(w,\theta)$ and unitary extensions of the operator V are shown to be in a one–to–one correspondence. Making use of Kreˇin – Naimark formula for generalized resolvents we obtain a description of all solutions of the problem.

Dezin A.A.
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On the deductive approach to elementary quantum mechanics

The outline of deductive approach to elementary quantum mechanics is given. As a model of physical space a locally compact abelian group is used.

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Characterization of Solutions of Hermite Heat Equation and Schrödinger Equation

We show that smooth solution to the Hermite heat equation $(\partial_t-\Delta+|x|^2)U(x,t)=0$ on $\mathbb{R}^n\times\mathbb{R}^+$ which does not increase faster than t^{-N} for some $N>0$, can be expanded into a Hermite series

$$U(x,t)=\sum_k c_k\, e^{-(2|k|+n)t}\, h_k(x)$$

with the polynomial growth in c_k and vice-versa. Moreover, due to the polynomial growth of c_k, the initial value of the solution turns out to be a tempered distribution. We introduce an integral transform $\tilde{U}(x,s+it):=\langle U(y,t),E(x,y,s)\rangle$ in $\mathbb{R}^n\times\mathbb{R}^+\times\mathbb{R}^1$ associated with the Mehler kernel $E(x,y,s)$ and call it the Mehler transform of $U(\cdot\,,\,t)\in\mathcal{T}'_{\frac{1}{2}}(\mathbb{R}^n)$. As an application of the above result, we show that every solution $U(x,t)\in C^1\left(\mathbb{R}^1_t,\ \mathcal{T}'_{\frac{1}{2}}(\mathbb{R}^n)\right)$ of the Schrödinger equation $(-i\partial_t-\Delta+|x|^2)U(x,t)=0$ subject to $\sup_{x\in\mathbb{R}^n}|\tilde{U}(x,s+i0)|=O\left(s^{-N}\right)$ as $s\to0^+$ for some $N>0$, is a tempered distribution in the form of Hermite series $\sum_k c_k\, e^{-(2|k|+n)it}\, h_k(x)$ with the polynomial growth in c_k and vice versa. Here the space $\mathcal{T}'_{\frac{1}{2}}(\mathbb{R}^n)$ is the dual of $\mathcal{T}_{\frac{1}{2}}(\mathbb{R}^n)$, a variant of the Gel'fand-Shilov space $\mathcal{S}_{\frac{1}{2}}(\mathbb{R}^n)$.

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New universal method of numerical solution of Mathematical Physics Problems

Many important problems of Physics, Technology, Engineering and Science integrally are described Mathematical Physics Problems (MPP). Such problems may be solved exactly in exclusively scarce cases when equations are one-dimensional, linear and coefficient constant. But important practical problems are nonlinear and many-dimensional. Such problems may be solved successfully only with help numerical method. Among numerical methods solving of MPP more universal, effective, flexible, simple are Finite Difference Method (FDM) and Finite Element Method (FEM). Although indicated methods are differed via principle of construction and investigation according discrete scheme a idea besis these methods equally. In present message are accounted absolutely new approach to numerical solving of MPP. Principle of construction and a idea besis of this method are differed from FDM and FEM. New method are demonstreted on example of boundary inverse problems (incorrect) for simplest hyperbolic equation. Results broutht of numerical experiments on PC are shown effectivity and absolutely stability of accounted method

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Absence of poles in integrand of the Green's function for special model of three layers medium

The Green's function $u(z,x)$ satisfies the Helmholtz equation with an ingomogenious term of δ-function type

$$\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial x^2} + k^2(z)u = -2\delta(z-z')\delta(x-x')$$

and the radiation condition at infinity - $u \to 0$ when $r = \sqrt{z^2+x^2} \to \infty$. The function u has the logarithmic singularity in the source point $M'(z',x')$. The equation is considered when $z, z', x, x' \in R$. We suppose that the coefficient $k^2(z)$ depends on one variable z and takes three values $k_l^2 = \varepsilon_l + j\sigma_l$, $\varepsilon_l \in R, \sigma_l > 0$, $l = 0,1,2$, j is the imaginary unit.

We obtain the problem solution expanding the function u as the function of variable x in the Fourier integral.

The integrant of the Green function represents a fraction with a denominator - the Wronskian of linear independent depending of the variable z solutions of the ordinary equation . The Wronskian contains the factor which could be equal to zero

$$w(\alpha) = 1 - q$$

where

$$q = \rho^2\rho^0 e^{-2\eta^1 H},$$

$$\rho^2 = \frac{\eta^1 - \eta^2}{\eta^1 + \eta^2}, \rho^0 = \frac{\eta^1 - \eta^0}{\eta^1 + \eta^0},$$

$$\eta^l = \sqrt{\alpha^2 - k_l^2},\ l = 0,1,2,\ Re\,\eta^l \geqslant 0,$$

H is the thickness of the middle layer.
α is a spectral parameter.

We prove that in the case of a special connection between the parameters of the problem ε_l and σ_l when

$$\frac{(\varepsilon_1 - \varepsilon_0)^2 + (\sigma_1 - \sigma_0)^2}{(\varepsilon_2 - \varepsilon_1)^2 + (\sigma_2 - \sigma_1)^2} = 1 \qquad (1)$$

the intergrant of the Green's function has no poles in the comlex plane α.

For that purpose we consider the integrand in the upper half-plane of a complex plane of the special parameter $\alpha = \alpha_1 + j\alpha_2$. We divide the upper half-plane by hyperbolas $\alpha_2 = \sigma_i/2\alpha_1, i = 0,2$ into three parts.

Taking into account that $Im\,\eta^l < 0, l = 0,1,2$ in the domain on the left of the hyperbola $\alpha_2 = \sigma_i/2\alpha_1$ and $Im\,\eta^l > 0$ on the right of it we prove that on the boundary of the internal domain $1 >| q |$. Using the Rouché theorem we obtain that $1 - q \neq 0$ in the whole domain. It is easy to show that last equality is valid for external domains.

The formula (1) is connected with the condition $1 >| q |$ on the both hyperbolas.

Dobrokhotov S.Yu.
(Institute for Problems in Mechanics, Moscow)

From Hugoniot-Maslov chains for singularities of quasilinear equations to pendulum type behavior of typhoons trajectories

The dynamics of solitary weak point singularities (vortices) is studied for the system of shallow water equations with the variable Coriolis force on the β-plane. According to Maslov's viewpoint, this dynamics can be described by an infinite chain of ordinary differential equations similar to the Hugoniót conditions arising in the theory of shock waves and can be used for the description of the typhoon "eye" trajectories. We show that after some reasonable closure, one obtains the nonlinear system of 17 ODE with very curious properties, like integrability, which allow us to say that in some approximation the vortex in question can be treated as a rigid body whose trajectory is determined by the Hill and Physical Pendulum equations.

Dolgopiat D.I.
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Transversely hyperbolic systems with symmetries

We discuss partially hyperbolic dynamical systems where the central direction is generated by the symmetry group of the system. We relate mixing properties of such systems with accessability properties of stable and unstable foliations. We also investigate the imapct of small non-symmetric perturbation on the system obtaining diffusion approximation for the slow motions and compute weak derivatives of assymptotic measures. In some cases we show that after generic perturbation the system becomes robustly non-uniformly hyperbolic.

Drábek P.
(University of West Bohemia, Check republic)

Resonance Problems for the Models of Suspension Bridges

The boundary value problem

$$\begin{cases} u_{tt} + u_{xxxx} + bu^{+} = h \text{ in } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \times \mathbf{R}, \\ u(\pm\frac{\pi}{2}, t) = u_{xx}(\pm\frac{\pi}{2}, t) = 0, \\ u(x,t) = u(-x,t) = u(x,-t) = u(x, t+T), \end{cases} \tag{0.1}$$

where $u^{+}(x,t) = \max\{u(x,t), 0\}$, serves as a model of suspension bridge. If h is positive, symmetric and time independent forcing term, problem (0.1) has a positive stationary solution. Depending on the magnitude of $T > 0$, different types of nonstationary solutions to (0.1) exist for certain ranges of parameter $b > -1$. The problem (0.1) can be transformed to the usual bifurcation scheme and the existence of nonstationary solutions is proved by using of the bifurcation theory.

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Some new differential equations with adelic solutions

p-Adic analysis has a successful application in mathematical physics (for a review, see [1]). Adelic approach seems to be the most natural in p-adic generalization of classical and quantum mechanics [2]. In this context we usually encounter adelic functions, as sequences of functions which are adelic valued for adelic arguments. These adelic functions are solutions of the Euler-Lagrange equations of motion. Thus it is important to investigate adelic aspects of differential equations in mathematical physics. In this contribution we will present construction and analysis of some new differential equations which are of interest for adelic physical models. There exist p-adic and real solutions, which together make adelic solutions. Some of these results are presented in Ref. [3]. It is worth noting that real and p-adic components of these adelic solutions have different form. In this way we have Lagrangian and equation of motion, which are invariant under change of the number fields R and Q_p but the corresponding solutions are not invariant. It is natural to expect that the fundamental physical laws are invariant under interchange of R and Q_p. So we have here some examples which may serve to study a new kind of spontaneous symmetry breaking related to number fields. This can be used to explain the dominance of real numbers at distances larger than the Planck length.

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Drozhzhinov Yu.N., Zavialov B.I.
(Steklov Mathematical Institute)

Tauberian Theorem for Generalized Functions in Banach Spaces

We disccuss new general tauberian theorem for Banach valued distributions. This theorem includes a lot of interesting particular cases. We give some of its applications. In particular we study the asymptotic properties of solutions of Couchy problem for Heat equation.

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Some Orthogonal Decompositions of Sobolev Spaces and their Applications

We consider the decompositions of the Sobolev and Sobolev–Clifford spaces in the sum of the analytic and coanalytic subspaces. These decompositions are the base for the constructions of the corresponding nonlinear analytic and coanalytic boundary value problems.

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Well-posedness of linear integral equations with Wnon--Fredgolm kernels and natural spaces of their solutions[10]

We consider linear integral equations of the second type with non-Fredgolm kernels. The notion of quasifunctional is introduced, and the resolvent adjoint equation is considered. As a result, we select certain functional spaces that are natural for a given integral equation (or a system of integral equations). The existence and uniqueness theorems in these natural functional spaces are proved. Also, we demonstrate that outside these natural spaces the solution may be, generally speaking, non-unique.

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The new method of a finding of the first eigenvalues of the spectral problem of Orr-Zommerfeld.

This article deals with the elaboration of the new method of a finding of the first eigenvalues of the non-self-conjugate operators on the basis of the theory of the regularized traces of discrete operators. The formulas, which help to calculate the corrections of the theory of perturbations of the necessary order, are also obtained. The technique of numerical experiment, tested on the spectral problem of Orra-Sommerfeida, has been elaborated.

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Oscillation theorem for eigenfunctions of the indefinite Sturm–Liouville problem

We consider the boundary problem of the form

$$-y'' + (\lambda f(x) + q(x))y = 0, \quad a \leqslant x \leqslant b;$$
$$y(a) = y(b) = 0.$$

It is called the indefinite Sturm–Liouville problem if the function f changes sign on $[a,b]$. We assume that the operator $Ay = -y'' + q(x)y$ subject to the given boundary conditions defines a positive definite operator in $L^2(a,b)$. This problem has a countable set of positive eigenvalues $0 < \lambda_0 < \lambda_1 < \dots$ (as well as negative ones). The main results obtained are the following.

Theorem 1. *The eigenfunction y_k corresponding to the eigenvalues $\lambda_k > 0$ has exactly k zeros on the open interval (a,b). If the function f is smooth and has a finite set of turning points then the number of zeros of y_k on a segment $[\alpha,\beta] \subset [a,b]$ can be estimated as follows*

$$\lim_{k\to\infty} \frac{N_k[\alpha,\beta]}{k} = \frac{\int_\alpha^\beta \sqrt{f_-}dx}{\int_a^b \sqrt{f_-}dx}, \qquad f_- = \max\{-f, 0\}.$$

Moreover, if $f(x) \geqslant 0$ at the segment $[\alpha,\beta]$ then y_k has no more than one zero at $[\alpha,\beta]$.

Theorem 2. *Let the operator $Ay = -y'' + q(x)y$ with Dirichlet boundary donditions on intervals (a,c) and (c,b) be positive for any $c \in (a,b)$. Then the zeros of the eigenfunctions possess the interplacing property: exactly one zero of the function y_k lies between two neighbouring zeroes of y_{k+1}.*

The talk is based on the joint work with A.A. Shkalikov.

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Some badly conditioned stationary problems in energy spaces and optimization of numerical methods for such problems

The paper deals, first of all, with special types of stationary boundary and spectral problems in energy spaces like classical Sobolev spaces and their modifications on composed manifolds (they have not only fundamental theoretical importance but are also of special interest for many applied problems like those on multistructures in theory of elasticity) and, secondly, with justification of the famous Bakhvalov—Kolmogorov principle about asymptotically optimal algorithms for classes of correct problems with the solutions in a given compact set M of the energy space (see [1–6] and references therein). This principle states that, *given a prescribed tolerance $\varepsilon > 0$, it is possible to indicate a computational algorithm such that it yields an ε-approximation to the solution $u \in M$ with the computational work*

$$W(\varepsilon) \asymp N(\varepsilon)$$

(see [1,2,4–6]); $N(\varepsilon)$ denotes the minimal value of N such that $\pi(N) < \varepsilon$, where $\pi(N)$ is the classical N-width in the sense of Kolmogorov for compact set M. The main attention in the paper is paid to hard (stiff) stationary problems which, for the simplest linear case, can be formulated as operator equations

$$\Lambda_\alpha u = f \tag{1}$$

in a Hilbert space H_1; they depend on a singular parameter $\alpha \to +0$ which yields very large condition numbers

$$\kappa(\Lambda_\alpha) \equiv \|\Lambda_\alpha\| \|(\Lambda_\alpha)^{-1}\| = O(1/\alpha)$$

of original operators Λ_α. For special types of problems (1) when (1) can be written in the form

$$L_{1,1}u_1 + \frac{1}{\alpha} L_{2,1}^* L_{2,2}^{-1} L_{2,1} u_1 = f_1, \tag{2}$$

($L_{i,j} \in \mathcal{L}(H_j; H_i)$, $i = 1,\ 2, j = 1,\ 2$, $\mathrm{Im}\, L_{2,1} = H_2$) their reduction to ones with strongly saddle operators

$$L_\alpha \equiv \begin{bmatrix} L_{1,1} & L_{2,1}^* \\ L_{2,1} & -\alpha L_{2,2} \end{bmatrix} \tag{3}$$

in a Hilbert space $H \equiv H_1 \times H_2$ leads to the remarkable improvement of correctness and to the estimates $\kappa(L_\alpha) = O(1)$; the indicated regularization leads sometimes even to the construction of asymptotically optimal numerical methods and algorithms with the estimates $W(\varepsilon)$ independent of the singular parameter and under natural conditions on the smoothness of the solution (see [1,2,4–6]). For nonlinear problems with strongly monotone and Lipschitz operators (we call them Vishik's operators), we make use of

$$\|\Lambda_\alpha\| \equiv \sup_{u-v\neq 0} \frac{\|\Lambda_\alpha u - \Lambda_\alpha v\|}{\|u-v\|}, \quad \|(\Lambda_\alpha)^{-1}\| \equiv \sup_{u-v\neq 0} \frac{\|(\Lambda_\alpha)^{-1}u - (\Lambda_\alpha)^{-1}v\|}{\|u-v\|}$$

in the definition of $\kappa(\Lambda_\alpha)$; the number $\log\{\kappa(\Lambda_\alpha)\} \equiv h(\Lambda_\alpha) \geqslant 0$ can serve as a characterizion of hardness of the given invertible operator Λ_α since $h(\Lambda_\alpha$ defines a distance between Λ_α and the class of isometric maps; localized versions of $h(\Lambda_\alpha$ are of help if a priori estimate of the solution of (1) is available;

Theorem 1. *Let, in problem (1), $L_{1,1}$ and $L_{2,2}$ be Vishik's operators, $\alpha > 0$. Then, for nonlinear operator (3), there exists a constant K independent on $\alpha \geqslant 0$ and $L_{2,2}$ and such that $\|(L_\alpha)^{-1}\| \leqslant K$.* We concentrate also on cases when the proof that $L_{2,1} \in \mathcal{L}(H_2; H_1)$ in (2) and (3) is normally invertible requires ruther unusal extension theorems like those in Sobolev space $W_2^1(Q)$ when the boundary of the domain may be of non-Lipschitz type on manyfolds of smaller dimension. The case of several singular parameters is also considered.

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The manifolds of eigenfunctions and Arnold's hypothesis of transversality

Consider the family of eigenvalue problems

$$-u'' + p(x)u = \lambda u, \quad x \in S^1, \quad \oint u^2 dx = 1 \tag{1}$$

where a real valued potential $p \in P = C^r(S^1) \cap \{p : \oint p dx = 0\}$ $(r \geqslant 2)$ serves as a functional parameter. For fixed p, the eigenvalues form nondecreasing sequance $\lambda_0(p) < \lambda_1^-(p) \leqslant \lambda_1^+(p) < \ldots$. If $\lambda_n^-(p) < \lambda_n^+(p)$, then both these eigenvalues are simple; if $\lambda_n^-(p) = \lambda_n^+(p) = \lambda_n^*(p)$, then the eigenvalue is double. The eigenfunction u corresponding to the nth eigenvalue has $2n$ non-degenerate zeros x_i. Denote by U_n^- (U_n^+) the set of all eigenfunctions of family (1) associated with simple eigenvalues λ_n^- (λ_n^+, respectively); U_n^* denote the set of all einenfunctions of family (1) associated with double eigenvalues λ_n^*. It is clear that $U_n^- \cap U_n^+ = \varnothing$, $U_n^\pm \cap U_n^* = \varnothing$. Let $U_n = U_n^- \cup U_n^+ \cup U_n^*$. Denote by $P_n^* \subset P$ the set of all potentials associated with double eigenvalues λ_n^*.

Theorem 1. *The set U_n is a C^{r-1} manifold modeled on P. The subsets $U_n^\pm \subset U_n$ are open submanifolds. The subset $U_n^* \subset U_n$ is the submanifold and $codim U_n^* = 1$. The manifold U_n is trivial fiber space over U_n^*, its fiber is diffeomorphic to $\mathbb{R}$.*

Consider the mapping $\alpha : U_n \to S^1$, where $\alpha(u) = (\sum_{i=1}^{2n} x_i) mod\, 2\pi$. Denote by α^* the restriction of the mapping α onto U_n^*.

Theorem 2. *The mapping α is trivial fiber bundle. The mapping α^* is trivial fiber subbundle. The manifolds $U_n^\pm$, U_n^* and U_n are homotopically equivalent to S^1.*

Theorem 3. *The subset $P_n^* \subset P$ is C^{r-1} submanifold of codimention two. Let $p \in P_n^*$ and u_n, v_n are orthonormal eigenfunctions corrersponding to $\lambda_n^*(p)$; then the tangential space $T_p P^* = \{q \in P : \oint q u_n v_n dx = \oint q(u_n^2 - v_n^2)dx = 0\}$. The space P is trivial fiber bundles over U_n^*, its fiber is diffeomorphic to $\mathbb{R}^2$.*

Theorem 4. *For any integer $n_1, n_2, ..., n_k$ the intersection $\cap_{i=1}^k P_{n_i}^* \subset P$ is C^{r-1} submanifold of codimention $2k$.*

Corollary (Arnold's hypothesis of transversality). *Suppose M is finite-dimensional C^{r-1} manifold. If $r - 1 > dim M - 2k$, then maps transversal to $\cap_{i=1}^k P_{n_i}^*$ are residual in $C^{r-1}(M, P)$.*

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Variational Schrödinger Operators and Feynman Integrals

Classical configurations of neutral massive boson fields on $\mathbb{R}^D$ are real-valued functions $\phi = \phi(x)$, $x \in \mathbb{R}^D$. Their quantum states are (non-linear) complex functionals $\Psi = \Psi(\phi)$.

A symbolic *variational Schrödinger operator*, defined on the quantum states, is

$$\frac{1}{2}\int\left(-\frac{\partial^2}{\partial\phi(x)^2} + (\nabla\phi)^2(x) + \phi(x)^2\right)dx + \int p\left(\phi(x), \frac{\partial}{\partial\phi(x)}\right)dx.$$

The first integral is a free Hamiltonian operator H_0, a continual harmonic oscillator; the second integral is a perturbation Hamiltonian operator P, a continual polynomial of "products with $\phi(x)$" and of "variational derivatives $\partial/\partial\phi(x)$" indexed by $x \in \mathbb{R}^D$.

We present appropriate symbolic operators $H_0 + P$ (in particular, with quartic P) as selfadjoint operators H on the Fock space $\mathcal{F}$ of quantum states and describe the quantum propagators $\exp(-iHt)$ via a rigorous Feyman type integral.

The Wiener-Ito-Segal model of the Fock space $\mathcal{F}$ is $\mathcal{L}^2(\mathrm{Re}\,\mathcal{S}, e^{-\langle\phi|\phi\rangle/2}\mathcal{D}(\phi))$, where $\mathrm{Re}\,\mathcal{S}$ is the real part of the Schwartz space $\mathcal{S} = \mathcal{S}(\mathbb{R}^D)$ and $e^{-\langle\phi|\phi\rangle/2}\mathcal{D}(\phi)$ is the standard Gaussian promeasure on $\mathrm{Re}\,\mathcal{S}$. (Hermitean scalar products are modified to be antilinear on the left.) The costant 1 is the *vacuum quantum state* $\Omega_0 \in \mathcal{F}$.

The derivatives $\partial/\partial\phi$ in the directions of non-zero $\phi \in \mathrm{Re}\,\mathcal{S}$ act in the space of continuous polynomials on $\mathrm{Re}\,\mathcal{S}$. For complex $\psi \in \mathcal{S}$, $\partial/\partial\psi = \partial/\partial(\mathrm{Re}\,\psi) - i\partial/\partial(\mathrm{Im}\,\psi)$. Their Hermitean conjugates $(\partial/\partial\psi)^\dagger$ act in the space of continuous polynomials on $\mathrm{Re}\,\mathcal{S}$ as well.

Let h_D denote the harmonic oscillator $(1/2)(-\nabla^2 + x^2)$ on $\mathcal{L}^2(\mathbf{R}^D)$. Its differential second quantizaton $\widehat{h}_D$ is the Friedrichs selfadjoint extension on $\mathcal{F}$ of the operator

$$\widehat{h}_D \prod_{j=1}^{N} (\partial/\partial\psi_j)^\dagger \Omega_0 = \sum_k [\partial/\partial(h\psi_k)]^\dagger \prod_{j\neq k} (\partial/\partial\psi_j)^\dagger \Omega_0, \ \widehat{h}_D \Omega_0 = 0.$$

Let $\mathcal{F}_\infty$ be the Frechet space of $\Psi \in \mathcal{F}$ such that $\langle\Psi|(1+\widehat{h_D})^r|\Psi\rangle < \infty$ for all $r \geqslant 0$. This is an infinite-dimensional analogue of the Schwartz space $\mathcal{S}$. Its antidual space $\mathcal{F}_{-\infty}$ is an infinite-dimensional analogue of the Schwartz space $\mathcal{S}'$.

The *coherent states*, Ω_0 and $\Omega_\psi = \sum_0^\infty (n!)^{-1}(\partial/\partial\psi)^{\dagger n}\Omega_0$ for non-zero $\psi \in \mathcal{S}$, are quantum states from $\mathcal{F}_\infty$. They form a continual orthonormal basis in $\mathcal{F}$:

$$\langle\Omega_{\psi'}|\Psi\rangle = \int \mathcal{D}(\overline{\psi},\psi) e^{\langle\psi'-\psi|\psi\rangle}\langle\Omega_\psi|\Psi\rangle, \ \langle\Psi''|\Psi'\rangle = \int \mathcal{D}(\overline{\psi},\psi)e^{-\langle\psi|\psi\rangle}\langle\Psi''|\Omega_\psi\rangle\langle\Omega_\psi|\Psi'\rangle.$$

The derivative $\partial/\partial\psi$ is a continuous operator on $\mathcal{F}_\infty$. It is continuous in the parameter $\psi \in \mathcal{S}$ with respect to the topology of $\mathcal{S}'$. By the continuity, $\partial/\partial\psi$ is extended to all $\psi \in \mathcal{S}'$. In particular, we have the *variational derivatives* $\partial/\partial\phi(x) = (\partial/\partial\delta_x)$, $x \in \mathbf{R}^D$, in "the directions of the delta functions δ_x". The Hermitian conjugates $(\partial/\partial\delta_x)^\dagger$ are continuous operators on $\mathcal{F}_{-\infty}$.

A *polynomial operator* $P : \mathcal{F}_\infty \to \mathcal{F}_{-\infty}$ *of order* n with coefficients $c_{kl} \in \mathcal{S}'(\mathbf{R}^{D(k+l)})$ is a finite sum of weak operator integrals

$$P = \sum_{k+l\leqslant n} \int c_{kl}(x_1, \dots, x_{k+l}) \prod_{j=1}^{k} (\partial/\partial\phi(x_j))^\dagger \prod_{j=k+1}^{k} (\partial/\partial\phi(x_j)) \prod_{j\leqslant k+l} dx_j.$$

The *Wick symbol* P^w of the operator P is the continual polynomial on $\mathcal{S}$:

$$P^w(\overline{\psi},\psi) = \sum_{k+l\leqslant n} \int c_{kl}(x_1, \dots, x_{k+l}) \prod_{j=1}^{k} \overline{\psi}(x_j) \prod_{j=k+1}^{k} \psi(x_j) \prod_{j\leqslant k+l} dx_j.$$

The *principal symbol* P_0^w of P is the partial sum in P^w over $k+l = n$.

A polynomial operator P is Re-*elliptic* if its Wick symbol is real, and for a positive constant C its principal symbol $P_0^w(\phi) \geqslant C||\phi||_{-r}^n$ for all $\phi \in \mathrm{Re}\,\mathcal{S}$.

If $\langle c_{kl}|h_{nD}^{-r}|c_{kl}\rangle < \infty$ for some $r > nD$ then the operator P is *tame*. The *anti-Wick symbol* $P^a(\overline{\psi},\psi)$ of a tame operator P is the continual polynomial on $\mathcal{S}$ such that

$$\langle\Omega_{\psi'}|P|\Psi\rangle = \int \mathcal{D}(\overline{\psi},\psi)e^{\langle\psi'-\psi|\psi\rangle} P^a(\overline{\psi},\psi)\langle\Omega_\psi|\Psi\rangle.$$

The Wick symbol of the free Hamiltonian operator H_0 is $\langle\psi|\psi\rangle$, and $\langle \mathrm{Re}\,\psi|\,\mathrm{Re}\,\psi\rangle^2$ is the Wick symbol of a quartic self-interaction Hamiltonian operator. The operators are Re-elliptic but not tame.

Let a self-interaction polynomial operator P be Re-*elliptic. Assume that its Wick symbol P^w is continuous on $\mathcal{L}^2(\mathbf{R}^D)$.* The following theorems improve on [1].

Theorem 1 *The operator H_0+P has a Friedrichs type extension to a selfadjoint operator H on the Fock space $\mathcal{F}$.*

Theorem 2 *There exists a sequence of* Re-*elliptic tame polynomial operators $\{P_N\}$ that strongly converges to P on the coherent states Ω_ψ and such that the matrix element $\langle\Omega_{\psi''}|\exp(-iHt)|\Omega_{\psi'}\rangle$ is the limit at $N=\infty$ of the continual integrals over S^{N-1}*

$$\int\prod_{j=1}^{N-1}\mathcal{D}(\overline{\psi}_j,\psi_j)\exp\sum_{j=0}^{N-1}\left[\langle\psi_{j+1}-\psi_j|\psi_j\rangle-\frac{it}{N}(\langle\psi_{j+1}|\psi_j\rangle+P_N^a(\overline{\psi_j},\psi_j))\right],$$

$$\psi_N=\psi'',\psi_0=\psi'.$$

The limit is a rigorous mathematical version of a Hamiltonian Feynman integral over classical histories $\{\psi_\tau, 0\leqslant\tau\leqslant t\}$ for the quantum amplitude from ψ_0 to ψ_t (cf.[2]):

$$\int_{\psi_0}^{\psi_t}\prod_{s=0}^{t}\mathcal{D}(\overline{\psi_s},\psi_s)\exp i\int_0^t d\tau\left[-i\langle\dot{\psi}_\tau|\psi_\tau\rangle-\langle\psi_\tau|\psi_\tau\rangle-P^w(\overline{\psi_\tau},\psi_\tau)\right].$$

Both theorems, with Berezin continual integrals, hold for *massive fermion fields* as well. (The fermion classical configurations are Grassmann functions on $\mathbf{R}^D$, and the harmonic oscillator h is replaced with the Dirac operator on $\mathbf{R}^D$.) Further extensions to massive supersymmetric fields with elliptic self-perturbations are straightforward.

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Limit Laws of Entrance Times for Circle Homeomorphisms with Singularities

Limit laws of entrance times have been obtained in various contexts such as: hyperbolic automorphisms of the torus and Markov chains, Axiom A diffeomorphisms and shifts of finite type with a Holder potential, and piesewise expanding maps of the circle [1]. Given a orientation-preserving homeomorphism f of the unit circle S^1 with irrational rotation number α. For any subset $A \subset S^1$, we consider a map $N_A : S^1 \to N$, by

$$N_A(x) = \min\left\{j > 0 : f^j(x) \in A\right\}$$

for all $x \in S^1$. We call $N_A(x)$ the first entrance time of x in A. We assume that the expansion of α in a continued fraction of the form $\alpha = [a_1, a_2, \cdots, a_n, \cdots]$. We denote the finite continued fractions by $\frac{p_n}{q_n} = [a_1, a_2, \cdots, a_n]$. Let $J_n, n \geqslant 1$ are a descending chain of renormalization intervals of f. For $A = J_n$ we denote $N_n(x) = N_{J_n}(x)$ and $\bar{N}_n(x) = q_n^{-1} N_n(x)$. Let μ be the unique ergodic invariant probability measure for f and λ be Lebesque measure on S^1. For every $n \geqslant 1$ we define two distribution functions:

$$F_n(t) = \mu\{x \in S^1 : \bar{N}_n(x) \leqslant t\}$$

$$\Phi_n(t) = \lambda\{x \in S^1 : \bar{N}_n(x) \leqslant t\}$$

If subsequence $\{F_{n_i}(t)\}$ converge (pointwise or uniformly), then the limit distribution $F(t)$ is either the uniform distribution on the unit interval, or $F(t)$ is piesewise linear function [2]. This is true for $\Phi_n(t)$ in the case, when f is a C^1 - conjugate to a rotation. An interesting question is the study of limit points of $\{\Phi_n(t); n \geqslant 1\}$ and its properties for circle homeomorphisms with points of singularity(i.e. when normalized invariant measure μ is singular with respect to Lebesque measure [4]). We study homeomorphisms of the circle satisfying the following conditions: a) the rotation number $\alpha(g)$ is "golden mean"; b) either $g(x)$ is nontrivial fixed point of renormalization group transformation (RG) on the space of real -analitic critical circle maps with cubic critical point [3] or $g(x)$ is unique periodic point of period 2 of RG on the space circle homeomorphisms $C^{2+\varepsilon}(S^1/\{x_c\})$ for some $\varepsilon > 0$, with a single break point (at which there is a jump of the first derivative) . We construct a thermodynamic formalism for $g(x)$ and on its basic to study the limit of $\{\Phi_n(t)\}$. It is shown, that for any $t \in [0, 1]$

$$\lim_{n \to \infty} \Phi_n(t) = \Phi(t)$$

exists and the limit $\Phi(t)$ is continuous and singular function. The cases when f is C^1-conjugated to homeomorphism $g(x)$ diffeomorphism are also discussed

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About solvability of the cauchy problem for loaded nonlinear parabolic equations

Let the separable reflected Banach Space $\{V, \|\cdot\|\}$ and Hilbert Space $\{H, |\cdot|, (\cdot,\cdot)\}$ be given and the following dense continuous embeddings $V \subset H \equiv H' \subset V'$ be valid. Let γ be the norm of embedding operator $V \subset H$. We consider the loaded nonlinear differential operator equation [1], [2] in space V

$$L(t)u \equiv u'(t) + A_0(t)u(t) + \sum_{k=1}^{m} A_k(t)u(t_k) = f(t) \text{ на } (0,1), \tag{0.1}$$

with initial condition

$$u(0) = \varphi, \tag{0.2}$$

where $u'(t) = du(t)/dt$, $A_k(t), k = 0,1,...,m,-$ are family of given operators, $A_0(t) : V \to V'$, $A_k(t) : H \to H$, $k = 1,...,m$, the points t_k, $k = 1,...,m$, of interval $[0,1]$ are fixed and given and $t_1 < t_2 < ... < t_m$. Suppose that function $f(t) : (0,1) \to V'$ and element $\varphi \in H$ are given. Let the following conditions be satisfied:

Assumption 1. a). Function $t \to < A_0(t)u, v >$ is measurable for any fixed $u, v \in V$; b). function $\lambda \to < A_0(t)(u + \lambda v), w >$ is continuous in zero for almost all $t \in (0,1)$ and for any fixed $u, v, w \in V$; c). operator $A_0(t) : V \to V'$ is monotone for almost all $t \in (0,1)$; d). there exist such positive numbers α, β, θ, p $(p \geqslant 2)$ that

$$\begin{cases} < A_0(t)v, v > \geqslant \alpha\|v\|^p - \theta, \ \|A_0(t)v\|_{V'} \leqslant \beta(1 + \|v\|^{p-1}), \\ |A_k(t)v| \leqslant a_{0k}|v|, \ k = 1,...,m, \end{cases} \tag{0.3}$$

is valid for any $v \in V$ uniformly with respect to $t \in (0,1)$. We shall say that operator L satisfies to the condition $\Lambda(\varepsilon_1, a)$ designated as $L \in \Lambda(\varepsilon_1, a)$ iff the following inequalities

$$\frac{a_{0k}}{\varepsilon_1} \leqslant \delta_1 ... \delta_{k-1}(1 - \delta_k)\chi_k, \ 0 < \delta_k < 1, \tag{0.4}$$

$$\text{where } \chi_k = \begin{cases} a \cdot \exp(-at_k)/(1 - \exp(-at_k)), & a \neq 0, \\ 1/t_k, & a = 0, \end{cases}$$

$$k = 1, ..., m,$$

are valid.

Theorem 1. *Let the Assumption 1 be satisfied and* $L \in \Lambda(\varepsilon_1, m\varepsilon_1 - \eta(1-\delta_0))$, $\eta = 2\alpha\gamma^{-1}$. *Then the problem (0.1)–(0.2) is solvable for any* $f \in L^q(0,1;V')$, $\varphi \in H$ $(1/p + 1/q = 1)$.

Proof of Theorem 1. We are using Galerkin method. Let $\{w_j\}$ be the basis in V. We are looking for the approximate solution $u^N(t) = \sum_{i=1}^N g_i^N(t) w_i$ using the solutions of Cauchy problem of loaded nonlinear ordinary differential equations

$$(u_t^N(t), w_j) + (A_0(t)u^N(t) + \sum_{k=1}^m A_k(t)u^N(t_k) - f, w_j) = 0, \; 0 \leqslant j \leqslant N, \tag{0.5}$$

$$u^N(0) = \varphi^N \equiv \sum_{i=1}^N f_i^N w_i, \; \lim_{N\to\infty} \varphi^N \to \varphi. \tag{0.6}$$

The following statement is valid for the problem (0.5)–(0.6). **Lemma 1.** *If* $L \in \Lambda(\varepsilon_1, m\varepsilon_1 - \eta(1-\delta_0))$, *then the problem (0.5)– (0.6) is solvable in interval* $(0,1)$. The measurability of function $t \to < A_0(t)u^N(t), w >$ is supported by Assumption 1 about operator $A_0(t)$. The mapping $t \to A_0(t)u^N(t) : R \to V'$ is also measurable by virtue of separability of V. Thus by virtue of condition (0.3) and estimation uniformly with respect to N

$$\|u^N(t)\|_{L^\infty(0,1;H)\cap L^p(0,1;V)} \leqslant \text{const}, \tag{0.7}$$

we have the following estimation

$$\|A_0(t)u^N(t)\|_{L^q(0,1;V')} \leqslant \text{const} \tag{0.8}$$

uniformly with respect to N. Further for Fourier image $\hat{u}^N(\tau)$ (of extension $\bar{u}^N(t)$ from $(0,1)$ into R by zero value) we obtain uniformly with respect to N the following estimation

$$\int_R |\tau|^{2\sigma} |\hat{u}^N(\tau)|^2 d\tau \leqslant C(\sigma), \; \sigma \in (0, 1/4), \tag{0.9}$$

which means that fractional derivative $D_t^\sigma u^N(t)$ is restricted. The estimations (0.7), (0.8), (0.9) and the following equalities

$$A_0(t)u(t) = U(t) \text{ in } L^q(0,1;V'),$$
$$\sum_{k=1}^m A_k(t)u(t_k) = \xi(t) \text{ in } L^q(0,1;H),$$

completes the proof of Theorem 1.

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Egorov Yu.V.
(University Paul Sabatier, Tulouse)

On elliptic boundary value problems in a cone with nonlinear boundary conditions

We are studying the weak solutions of an elliptic linear equation of second order

$$Lu := \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}(a_{ij}(x)\frac{\partial u}{\partial x_j}) - c(x)u = 0 \tag{1}$$

in an unbounded domain $Q \subset \mathbf{R}^n$ such that

$$Q \subset \{x = (x', x_n)\ : |x'| < Ax_n) + B,\ \ 0 < x_n < \infty\},$$

supposing that u satisfies the boundary condition

$$\frac{\partial u}{\partial N} + b(x)|u(x)|^{p-1}u(x) = 0 \tag{2}$$

on the lateral surface $S = \partial\Omega \cap \{x_n > 0\}$, belonging to C^1. Here $p > 0$, $b(x) \geqslant b_0 > 0$, and

$$\frac{\partial u}{\partial N} = \sum_{i=1}^{n} a_{ij}(x)\ \frac{\partial u}{\partial x_j}\ \cos\theta_i,$$

θ_i is the angle between the direction of the x_i-axis and the outer normal direction.

Theorem 1. *A function u, satisfying* (1),(2) *and the inequality* $|u(x)| \leqslant bx_n^a$ *with* $a < a_0$ *in the domain Q, tends to* 0 *as $x_n \to \infty$ uniformly in Q.*

We study also the asymptotic behavior as $x_n \to \infty$ of the solutions of (1) satisfying the nonlinear boundary condition

$$\frac{\partial u}{\partial N} - b(x)|u(x)|^{p-1}u(x) = 0 \tag{3}$$

on S.

Theorem 2. *Let*

$$Q = \{x = (x', x_n)\ : |x'| < Ax_n^{\sigma} + B,\ \ 1 < x_n < \infty\}.$$

Suppose that $v(x)$ satisfies (1), (3) *and* $v(x) \geqslant 0$ *in Q. If*

$$1 < p \leqslant 1 + \frac{2-\sigma}{\sigma(n-2)},\quad \frac{1}{n-1} < \sigma \leqslant 1$$

or

$$p > 1, \quad 0 \leqslant \sigma(n-1) < 1,$$

then $v(x) \equiv 0$.

The talk is based on the joint work with V.A. Kondratiev.

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Eichhorn J.

(Greifswald University, Germany)

Bordism theory for open manifolds

Our classsification approach for open manifolds consists of 4 steps, 1. defini- tion of uniform structures of open metrized manifolds, 2. description of their (arc) components as rough equivalence classes, 3. characterization of them by new invariants like Lipschitz or Gromov-Hausdorff cohomology, 4. charac- terization of the elements inside an equivalence class. For the fourth step we developed several bordism theories adapted to geometric features of the ends. We present independent generators and partial characterization of bordism classes.

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Engquist B.

(KTH, Stockholm)

On existence of solutions to the Prandtl equations

The Prandtl boundary layer equations describe solutions of the Navier Stokes equations close to a boundary in some flow regimes. We shall discuss the lack of well posedness, existence of classical solutions and the convergence to Euler solutions as the Reynolds number increases. We shall also give computational results and present a theorem of finite time blow up.

Exner P.
(Nuclear Physics Institute, Academy of Sciences, Prague, Czech Republic)

Quantum graphs with tunneling

There have been a renewed interest to quantum mechanics on graphs recently - cf. [1] and references therein. In all these studies, however, the graphs are regarded as perfect, which is certainly an idealization from the viewpoint of applications. In this talk we consider quantum graphs which allow tunneling being described by Hamiltonians which can be formally written as

$$H = -\Delta - \alpha\delta(x - \Gamma), \quad \alpha > 0,$$

where Γ is a planar graph. We present several results concerning such operators. In particular, we show that if Γ is a non-straight bent curve which is asymptotically straight in a suitable sense, $\sigma_{\mathrm{disc}}(H)$ is non-empty [2], which is a property analogous to that of bent Dirichlet waveguides [3]. We also demonstrate a similar result for non-straight arrays of point interactions [4]. Furthermore, we show that the eigenvalues of H with Γ being a smooth curve tend for $\alpha \to \infty$ to those of the operator $-\frac{d^2}{ds^2} - \frac{1}{4}\kappa(s)^2$ on $L^2(\Gamma)$, where κ is the curvature of Γ [5]. If Γ is a loop, we get in this way an estimate on the number of bound states with a correct semiclassical behaviour in contrast to the result in [6].

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Faminskii A.V.[11]
(Friendship of Nations University of Russia)

On boundary value problems for some generalizations of the KdV equation

We consider the problems on nonlocal well–posedness of initial boundary value problems for the generalized Korteweg – de Vries equation (KdV)

$$u_t + u_{xxx} + au_x + (g(u))_x = 0 \qquad (0.1)$$

[11] The work was supported by RFFI grant (project 99–01–01139).

in three domains: a right half-strip $\Pi_T^+ = (0,T)\times\mathbb{R}_+$, a left half-strip $\Pi_T^- = (0,T)\times\mathbb{R}_-$ and a bounded rectangle $Q_T = (0,T)\times(0,1)$ ($T>0$ - arbitrary). For each of these problems we set the initial condition

$$u(0,x) = u_0(x) \tag{0.2}$$

and the following boundary conditions: 1) for the problem in Π_T^+

$$u(t,0) = u_1(t), \tag{0.3}$$

2) for the problem in Π_T^-

$$u(t,0) = u_2(t), \qquad u_x(t,0) = u_3(t), \tag{0.4}$$

3) for the problem in Q_T

$$u(t,0) = u_1(t), \qquad u(t,1) = u_2(t), \qquad u_x(t,1) = u_3(t). \tag{0.5}$$

The function g is assumed to be in $C^3(\mathbb{R})$, a is an arbitrary constant.

Theorem 1 *Let $(1+x)^\alpha u_0 \in L_2(\mathbb{R}_+)$ for some $\alpha \geqslant 0$, $u_1 \in (L_{6+\varepsilon}\cap W_1^{1/3}\cap W_2^{1/6})(0,T)$ for some $\varepsilon > 0$ and*

$$|g'(u)| \leqslant c|u| \qquad \forall u \in \mathbb{R}. \tag{0.6}$$

Then the problem (0.1)–(0.3) has a solution $u(t,x)$ such, that

$$(1+x)^\alpha u \in L_\infty(0,T;L_2(\mathbb{R}_+)), \qquad \sup_{m\geqslant 0}\int_0^T\int_m^{m+1} u_x^2\,dxdt < \infty,$$

and if $\alpha > 0$, then $(1+x)^{\alpha-1/2}u_x \in L_2(\Pi_T^+)$. For $\alpha \geqslant 3/8$ the constructed solution is unique.

Theorem 2 *Let $u_0 \in H^1(\mathbb{R}_+)$, $u_1 \in H^1(0,T)$, $u_0(0) = u_1(0)$, and for each $\delta > 0$ there exists a constant $c(\delta) > 0$ such, that*

$$\int_0^u g(v)\,dv \leqslant \delta|u|^{10/3} + c(\delta)u^2 \qquad \forall u \in \mathbb{R}.$$

Then the problem (0.1)–(0.3) has a unique solution $u(t,x)$ such, that

$$u \in C([0,T];H^1(\mathbb{R}_+)), \qquad \sup_{m\geqslant 0}\int_0^T\int_m^{m+1} u_{xx}^2\,dxdt < \infty,$$

$$\int_0^T \sup_{x\geqslant 0} u_x^4\,dt < \infty, \qquad \sum_{m=0}^{+\infty}\sup_{(t,x)\in[0,T]\times[m,m+1]} u^2 < \infty.$$

Theorem 3 *Let $u_0 \in L_2(\mathbb{R}_-)$, $u_2 \in W_1^{5/6+\varepsilon}(0,T)$ for some $\varepsilon > 0$, $u_3 \in L_2(0,T)$, and the function g satisfies the inequality (0.6). Then the problem (1),(2),(4) has a unique solution $u(t,x)$ such, that*

$$u \in L_\infty(0,T;L_2(\mathbb{R}_-)), \qquad \sup_{m\geqslant 0}\int_0^T\int_{-m-1}^{-m} u_x^2\,dxdt < \infty.$$

Theorem 4 *Let $u_0 \in L_2(0,1)$, and the functions u_1, u_2, u_3, g satisfy the same conditions as in the Theorems 1 и 3. Then the problem (1),(2),(5) has a unique solution $u(t,x)$ such, that $u \in C([0,T];L_2(0,1))$, $u_x \in L_2(Q_T)$.*

Corresponding continuous dependence results and a theory of nonlocal well–posedness of these problems in more smooth classes are also established.

Fatibene L., Francaviglia M., Raiteri M.
(Universita di Torino-via C.Alberto)

Conserved quantities and entropy in general relativity

The notion of entropy of exact solutions of General Relativity and, more generally, of gauge covariant field theories, is reviewed. A definition resembling the Clausius formulation of classical gas thermodynamics is considered and analyzed by an extensive use of the geometrical framework for field theories as well as Nöther theorem. This new definition of entropy applies in particular to all covariant theories of gravitation, in any dimension and signature. A complete correspondence with Brown-York original formulation of the first principle of black hole thermodynamics is finally established.

Fiedler B.
(Free University of Berlin, Germany)

Quantitative Homogenization

Diophantine conditions are well-known from Kolmogorov-Arnold-Moser theory. In contrast, we consider semilinear reaction diffusion equations with *spatially* quasiperiodic coefficients in the nonlinearity, rapidly varying on spatial scale ε. Under Diophantine conditions on the spatial frequencies, we derive quantitative homogenization estimates of order ε^γ on Sobolev spaces H^σ in the triangle

$$0 < \gamma < \min(\sigma - n/2, 2 - \sigma).$$

Here n denotes spatial dimension. The estimates measure the distance to a solution of the homogenized equation with the same initial condition, on bounded time intervals. The same estimates hold for C^1-convergence of local stable and unstable manifolds of hyperbolic equilibria. Our results apply to homogenization of the Navier-Stokes equations with spatially rapidly varying quasiperiodic forces in space dimensions 2 and 3. Our

results also extend to quantitative homogenization of global attractors for near-gradient system. All results are joint work with Mark I. Vishik.

Filinovskii A.V.
(Moscow State University)

On the behavior for small values of a parameter for the resolvent of the Dirichlet problem for the Laplace operator in unbounded domains

Let Ω be an unbounded domain in $\mathbb{R}^n$, $n \geqslant 2$, with smooth boundary surface Γ. Consider the self-adjoint operator $L = -\Delta : L_2(\Omega) \to L_2(\Omega)$ with the domain $D(L) = \left\{u : u \in \overset{\circ}{\to} W_2^1(\Omega), \Delta u \in L_2(\Omega)\right\}$. Since the spectrum of L belongs to $[0, +\infty)$, we see that for all $k \in \{\operatorname{Im} k > 0\}$ and for any $f(x) \in L_2(\Omega)$ there exists the function $v(x,k) = (L - k^2)^{-1} f \in \overset{\circ}{\to} W_2^1(\Omega)$. The function $v(x,k)$ is the solution of the boundary-value problem for the Helmholtz equation

$$\Delta v + k^2 v = -f, \quad x \in \Omega, \tag{0.1}$$

$$v|_\Gamma = 0. \tag{0.2}$$

We study the behavior of the function $v(x,k)$ at $k \to 0$ and $k \in K_N = \{\operatorname{Im} k > 0, |k| < N\}$, $N > 0$. Let $\varphi(x) = \sum\limits_{j=1}^{n} \alpha_j x_j^2$ with $0 < \alpha_1 \leqslant \ldots \leqslant \alpha_m < \alpha_{m+1} = \ldots = \alpha_n = 1$, $0 \leqslant m \leqslant n-1$. Suppose the domain $\Omega \neq \mathbb{R}^n$ and the origin does not belongs to $\overline{\Omega}$. The domain Ω is said to be the *domain of class* G_φ ([1], [2]) if $(\nu, \nabla\varphi(x)) \leqslant 0$, $x \in \Gamma$ (ν is the outer unit normal vector to Γ), and exist the positive constant C_1 such that $\sum\limits_{j=1}^{m} x_j^2 \leqslant C_1 \left(\sum\limits_{j=m+1}^{n} x_j^2\right)^{\alpha_m}$, $x \in \Omega$. By definition, put the space $L_{2,\gamma_1,\gamma_2}(\Omega)$, $-\infty < \gamma_1 < +\infty$, $-\infty < \gamma_2 < +\infty$, as a completion of the space of a functions from $C^\infty(\Omega)$ with bounded support by norm

$$\|u\|_{L_{2,\gamma_1,\gamma_2}(\Omega)} = \left(\int\limits_\Omega |u|^2 r^{\gamma_2} e^{r^{\gamma_1}}\, dx\right)^{1/2}.$$

By definition, put the space $H^1_{\gamma_1,\gamma_2,\gamma_3}(\Omega)$, $-\infty < \gamma_1 < +\infty$, $-\infty < \gamma_2 < +\infty$, $-\infty < \gamma_3 < +\infty$, as a completion of the space of a functions from $C^\infty(\Omega)$ with bounded support by norm

$$\|u\|_{H^1_{\gamma_1,\gamma_2,\gamma_3}(\Omega)} = \left(\int\limits_\Omega \left(|\nabla u|^2 r^{\gamma_2} + |u|^2 r^{\gamma_3}\right) e^{r^{\gamma_1}}\, dx\right)^{1/2}.$$

Let $\overset{\circ}{\to} H^1_{\gamma_1,\gamma_2,\gamma_3}(\Omega)$ denote the subspace of functions $u(x) \in H^1_{\gamma_1,\gamma_2,\gamma_3}(\Omega)$ with $u|_\Gamma = 0$.
Theorem. *Let $\alpha_1 < 1$, $0 < \gamma_1 < 1 - \alpha_m$, $-\infty < \gamma_2 < +\infty$, then there exists positive*

constant $C^* = C^*(\gamma_1, \gamma_2, n, m, \alpha_m)$ *such that for all domains* $\Omega \in G_\varphi$ *with* $C_1 < C^*$ *and for* $f \in L_{2,\gamma_1,\gamma_2}(\Omega)$ *the function* $v(x,k)$ *is represented by asymptotic series*

$$v(x,k) \sim \sum_{j=0}^{\infty} v_j(x) k^{2j}, \quad k \to 0, \quad k \in K_N.$$

The functions $v_j \in \overset{\circ}{\to} H^1_{\gamma_1,\gamma_2-4j\alpha_m-2\alpha_m,\gamma_2-4(j+1)\alpha_m}(\Omega)$, $j = 0, 1, \ldots$, *are the solutions of the boundary value problems*

$$\Delta v_j = -v_{j-1}, \quad x \in \Omega, \quad v_{-1} = f(x), \tag{0.3}$$

$$v_j|_\Gamma = 0, \tag{0.4}$$

and satisfies an estimates

$$\|v_j\|_{H^1_{\gamma_1,\gamma_2-4j\alpha_m-2\alpha_m,\gamma_2-4(j+1)\alpha_m}(\Omega)} \leqslant C(n,\gamma_1,\gamma_2,j)\|v_{j-1}\|_{L_{2,\gamma_1,\gamma_2-4j\alpha_m}(\Omega)}.$$

The remainder part of the series is $k^{2l}\tilde{v}$ *where* l *is a number of the first remainder and* $\tilde{v}(x,k)$ *is the solution of the boundary value problem*

$$\Delta \tilde{v} + k^2 \tilde{v} = -v_{l-1}, \quad x \in \Omega, \tag{0.5}$$

$$\tilde{v}|_\Gamma = 0. \tag{0.6}$$

For all $\gamma_3 \in (-1, \min(2\alpha_1 - 1, 1 - 2\alpha_m))$ *we have an estimate*

$$\left\| \frac{(\nabla \tilde{v}, \nabla \varphi)}{|\nabla \varphi|} - i\omega \frac{|\nabla \varphi|}{2\sqrt{\varphi}} \tilde{v} \right\|_{L_{2,\gamma_3}(\Omega)} + \|\nabla_\tau \tilde{v}\|_{L_{2,\gamma_3}(\Omega)}$$

$$+\|\tilde{v}\|_{L_{2,\gamma_3-2\alpha_m}(\Omega)} \leqslant C\|v_{l-1}\|_{L_{2,2}(\Omega)},$$

$$|\nabla_\tau \tilde{v}|^2 = |\nabla \tilde{v}|^2 - \frac{|(\nabla \tilde{v}, \nabla \varphi)|^2}{|\nabla \varphi|^2}, \quad k \in K_N, \quad N > 0.$$

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Filippov V.I.
(Saratov State University)

P.L. Ul'yanov's problen on representation by series on arbitrary function systems in the classes $\varphi(L)$

We consider the function systems in the classes $\varphi(L)$ more general than the Faber-Schauder system, namely systems of the type

$$\{\psi_{n,k}(t)\} = \{\psi(2^n t - k)\}, \quad n = 0, 1, ..., \quad k = 0, 1, ..., 2^n, \tag{1}$$

where $\psi(t) \geqslant 0, \quad t \in (0,1), \quad \psi \in L_\infty(0,1), \quad \psi(t) = 0, \quad t \notin (0,1), \quad \|\psi\|_\infty \neq 0$, in the classes $\varphi(L)$. We can remark that the Faber-Schauder system is a partial case of the system (1). P.L. Ul'yanov [1] considered Faber-Schauder system in the classes $\varphi(L)$ and formulated the following problem. What function systems will be representation systems in the classes $\varphi(L)$? We give some results on this problem. In the paper [2] are considered more general systems but in the spaces L^p. In the talk we will consider more general systems than the systems (1) in the classes $\varphi(L)$ and $L^p, 0 < p < \infty$. Let Φ be the set of even functions φ that are finite and nondecreasing on the halfline $[0, \infty)$, and such that $\lim_{t\to\infty} \varphi(t) = \varphi(\infty) = \infty$. We denote by $\varphi(L)$ the set of measurable functions $f(x)$ on $(0,1)$, for which $\int_0^1 \varphi(f(t))dt < \infty$. If

$$\varphi(t) \in \Phi, \quad \varphi(0) = 0, \quad \varphi(t) > 0, \quad t > 0, \quad \varphi(t) \in C[0, \infty), \tag{2}$$

then the set $\varphi(L)$ is called a generalized Orlicz class. If additionaly $\varphi(t)$ satiesfies the Δ_2-condition (i.e., $\varphi(2t) = O\{\varphi(t)\}, t \to \infty$), then $\varphi(L)$ is a linear class. If $\varphi(t) = |t|^p$, then the class $\varphi(L) = L^p$. Below we will suppose that Δ_2-condition is fulfiled.

Definition 1. A system $\{f_n\}_{n=1}^n$ of the class $\varphi(L)$ is called a representation system in the class $\varphi(L)$, if for any $f \in \varphi(L)$ there exists a series $\sum_{k=1}^\infty c_k f_k$ such that

$$\lim_{n\to\infty} \int_0^1 \varphi(f - \sum_{k=1}^n c_k f_k)dt = 0,$$

Theorem 1. *A subsystem* $\{\psi_{n_l}\}_{l=1}^\infty$ *of the system*

$$\{\psi_{n,k}(t)\} = \{\psi(2^n t - k)\}, \quad n = 0, 1, ..., \quad k = 0, 1, ..., 2^n, \tag{1}$$

where $\psi(t) \geqslant 0, \quad t \in (0,1), \quad \psi \in L_\infty(0,1), \quad \psi(t) = 0, \quad t \notin (0,1), \quad \|\psi\|_\infty \neq 0$, *is a representation system in the class* $\varphi(L)$ *if and onli if*

$$\forall \varepsilon > 0 \quad \forall N \in \mathbf{N} \quad \exists m > N : mes\{t : \sum_{l=N}^m \psi_{n_l}(t) \neq 0\} > 1 - \varepsilon.$$

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Filippov S.S., Tygliyan A.V.
(Keldysh Institute of Applied Mathematics)

On an approach to examining boundary value problems for ODEs with extra boundary conditions

For the steady-state viscous flows of incompressible fluids between parallel planes, there exists a class of so-called 'self-similar' solutions to Navier-Stokes equations. In this case, we come to boundary value problems (BVPs) for ordinary differential equations of 3rd order, which contain one unknown constant, and four boundary conditions. We introduce the notions of equivalent problems and equivalent solutions and prove that all the solutions to relevant BVPs can be found by solving equivalent initial value problems (IVPs). To elucidate the problem, let us consider a simple example. Given an ordinary differential equation, e.g. $y''' - yy'' + y'y' + b = 0$ with an unknown constant b and four boundary conditions, say $y(0) = y'(0) = 0, y(1) = 0, y''(1)+r = 0$, where r is a parameter. Using numerical methods, find all the solutions to this BVP (or state their absence) for r varying in a certain range. The main idea is to embed this one-parameter family of BVPs into an extended two-parameter family of BVPs and to define certain relations of equivalence between the members of this extended family. In the above example, the length H of the interval $[0, H]$ is chosen as the second parameter (the original interval was $[0,1]$). Now, we observe that the transformation $X = x * H; Y = y/H$ changes the values of b and r to B and R, but the form of the equation and boundary conditions is preserved. We define equivalent problems as problems, which have equal products $R \cdot H^3$, and equivalent solutions as solutions, which meet the requirements $B_1 \cdot H_1^4 = B_2 \cdot H_2^4$ and $H_1 \cdot Y_1(H_1 \cdot x) = H_2 \cdot Y_2(H_2 \cdot x)$ for all $x \in [0,1]$. Then we prove that there exists a one-to-one correspondence between equivalent problems and equivalent solutions. Moreover, if we add the missing initial value, say $Y''(0) = C$, then the solutions to all initial value problems with fixed ratio B^3/C^4 constitute a certain class of equivalent solutions to extended BVPs, and one-to-one correspondence between the solutions of IVPs and BVPs can be established. Thus, the task of solving a variety of 'overdetermined' BVPs depending on the parameter r is reduced to solving a one-parameter set of IVPs. In this way, several boundary value problems of similar type were studied. Numerical results are discussed in short. All the examined problems take their origin in the hydrodynamics of incompressible viscid flows and thermocapillary convection.

Filonov N.D.
(SPb State University)

Example of a periodic elliptic operator of second order with an eigenvalue of infinite multiplicity

In the three-dimensionsal case we construct an equation $-div(ggradu) = \lambda u$ having a nontrivial smooth finite solution u. The positively-defined matrix-function g belongs to the Holder class of any order lesser than 1. It is known that for the Lipshitz coefficients the finite solution can not exist due to Hormander theorem of uniqueness of the continuation.

¿From the point of view of the spectral theory our result means in particular that there exists a Schrodinger operator with periodic coefficients such that its spectrum contains an eigenvalue of infinite multiplicity.

Florens V.
(Universite Paul Sabatier UFR MIG, Laboratoire E.Picard, Tulouse)

Generalized Tristram signatures of links and applications to the real algebraic curves

Tupchiev V.A., Fomina N.A.
(Obninsk Institute of Nuclear Power Engineering)

On existence and uniqueness of a global solutions of initial-boundary value problems of chemotaxis

In 1971 Keller and Segel [1] proposed a mathematical model describing the dynamics of populations of so-called chemotactic bacteria, i.e. bacteria which are attracted by the gradient of concentration of some chemical substance. We study a particular case of a general mathematical model, which is the initial-boundary value problem:

$$\frac{\partial u}{\partial t} = \nabla\left(\mu\nabla u - \chi u\nabla v\right), \text{ in } \Omega,\ t > 0, \tag{1}$$

$$\frac{\partial v}{\partial t} = \nu\Delta v - k(v)u, \text{ in } \Omega,\ t > 0, \tag{2}$$

$$\frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0, \text{ on } \partial\Omega,\ t > 0, \tag{3}$$

$$u\left(x,0\right) = u^0\left(x\right), \qquad v\left(x,0\right) = v^0\left(x\right), \text{ on } \Omega, \tag{4}$$

where $u(x,t)$ is the density of bacteria, $v(x,t)$ is the concentration of the chemical substance at the place x and time t and Ω is bounded domain in R^n with boundary $\partial\Omega$. The parameters μ, χ, ν are positive numbers and function $k(v)$ is such as $0 < k(v) < A, k'(v) > 0, k''(v) < 0, k(v) \leqslant k_0 v$. Initial functions are $u^0\left(x\right), v^0\left(x\right) \in C\left(\Omega\right)$ and $0 < m^0 \leqslant u^0\left(x\right), v^0\left(x\right) \leqslant M^0$. For the initial Cauchy problem [2, 3] with $\Omega = R$ and for the initial- boundary value problem in one space dimension the global existence and uniqueness of a classical solution are proved. One of the interesting aspects of the Keller-Segel model is the possibility of blow-up of solutions in finite time. The fact has been studied in [4, 5]. Chemotaxis system in [4] is different from ours since in system from [4] $k(v) < 0$, i.e. the chemical substance is produced by the bacteria, whereas it is consumed in our model. Under definite conditions on parameters and on initial functions the global existence and uniqueness of a classical solution are established for our two-dimensional boundary value problem. The proof is based on obtaining apriori estimates of solution and using the method of continuation of a local classical solution [6].

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Topological solitons for a class of Lorentz Tinvariant equations in three space dimensions

Topological Solitary Waves: Existence and multiplicity results for a class of Lorentz invariant equations in 3 -Space Dimensions

Here we review some results contained in [1, 2]. We consider fields

$$\psi : \mathbf{R}^{3+1} \to M$$

where $\mathbf{R}^{3+1}$ is the space-time and M is a manifold whose 3-homotopy group $\pi_3(M)$ is not trivial. We shall take for definitess $M = \mathbf{R}^4 \setminus \{\bar{\xi}\}$, where $\bar{\xi}$ is a point in $\mathbf{R}^4$. In this case it is well known that $\pi_3(M)$ can be identified with the the integers $\mathbf{Z}$. We study the existence of solitary waves for the Lorentz invariant field equation

$$\frac{\partial}{\partial t}\left[\alpha'(\sigma)\,\frac{\partial \psi}{\partial t}\right] - \nabla\cdot\left[\alpha'(\sigma)\,\nabla\psi\right] + V'(\psi) = 0, \qquad (1)$$

where

$$\sigma = |\nabla\psi|^2 - \left|\frac{\partial\psi}{\partial t}\right|^2$$

$\alpha : \mathbf{R} \to \mathbf{R}$, $V : \mathbf{R}^4 \setminus \{\bar{\xi}\} \to \mathbf{R}$ with $V \geqslant 0$ and ∇, $\frac{\partial}{\partial t}$ denote the appropriate differentiations with respect to the space variable $x \in \mathbf{R}^3$ and the time variable t. Clearly when $\alpha(\sigma) = \sigma$ the above equations (1) reduce to the semilinear wave equation

$$\frac{\partial^2\psi}{\partial t^2} - \Delta\,\psi + V'(\psi) = 0, \qquad (2)$$

It is well known (Derrick's theorem) that the only finite energy static solution of (2) is the trivial one. Now we consider the case in which $\alpha(\sigma)$ is a perturbation of the identity, more precisely we assume that

$$\alpha(\sigma) = \sigma + \varepsilon\sigma^3, \ \varepsilon > 0$$

It can be shown that to any finite energy field ψ there corresponds a homotopy class in $\pi_3(\mathbb{R}^4 \setminus \{\bar{\xi}\})$. So each field ψ can be labelled by an integer q which we call topological charge. If V is sufficiently singular at $\bar{\xi}$, then it is possible to prove that equation (1) has a static finite energy solution whose topological charge is not trivial (see ref. [1]). Moreover, if V has a suitable symmetry, it can be proved that (1) has static finite energy solutions having an assigned topological charge (see ref. [2]).

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Stabilization of parabolic equation and 2D Navier-Stokes system by boundary feedback control

We propose new mathematical formalization for the notion of feedback control adapted for parabolic equations and for Navier-Stokes system. With help of this formalization we solve the problem of stabilization for indicated equations defined in a bounded domain Ω by a control supported in a part Γ of the boundary of this domain. In the case of linear equation stabilization problem is formulated as follows: Given $k > 0$ and initial condition $y_0(x)$, find feedback control supported on Γ such that the solution $y(t,x)$ of the obtained evolutionary boundary value problem satisfies the estimate:

$$\|y(t,\cdot)\|_{H^1(\Omega)} \leqslant ce^{-kt} \quad \text{as} t \to \infty$$

In the nonlinear case (i.e. for Navier-Stokes system) we carry out stabilization near arbitrary steady-state solution. Stabilization can be made for each prescribed rate $k > 0$. Details of formulation and Idea of proof will be given at the talk.

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Synthesis of dynamical systems with given limit cycles

1. A method of synthesis of dynamical systems with given limit cycles (LC) by constructing bounded feedbacks is under consideration. In addition a high quality of transients from the point of view of a chosen cost function is provided to a closed-loop system.

2. A base problem on which an algorithm of synthesis of the limit cycles is justified has the form

$$\begin{gathered} c'x(t^*) \to \max,\ \dot{x} = Ax + bu,\ x(0) = x_0 \\ Hx(t^*) = g,\ |u(t)| \leqslant 1,\ t \in T = [0, t^*]\ (x \in \mathbb{R}^n, u \in \mathbb{R}) \end{gathered} \tag{0.1}$$

Problem (0.1) is solved in the class of discrete positional controls by using fast procedures of calculating optimal open-loop solutions that allow to construct real-time algorithms to the optimal controller [1] generating current values of the optimal feedback for every concrete process.

3. The simplest problem of LC synthesis consists in the following. Let a dynamic control system

$$\dot{x} = Ax + bu, x \in R^n, u \in \mathbb{R}$$

and a closed curve $x = \varphi(t)$, $t \geqslant 0$, a number L be given. Denote by G a given vicinity of $f(t)$, $t \geqslant 0$. LC problem. It is necessary to construct a discrete feedback $u(x)$, $x \in G$, such that the conditions hold: 1) $|u(x)| \leqslant L, x \in G$ 2) the curve $x = \varphi(t)$, $t \geqslant 0$, is a stable LC to the closed system

$$\dot{x} = Ax + bu(x),\ x \in G. \tag{0.2}$$

This problem is solved with the help of problem (0.1).

4. Generalizations of problem (0.1) is considered for a quasi-linear control system

$$\dot{x} = Ax + bu + \mu f(x). \tag{0.3}$$

Asymptotics of switching points of the optimal control is constructed by a small parameter method and is used at solving the LC problem to system (0.3).

5. The second generalization of the LC problem deals with a control system

$$\dot{x} = f(x) + bu, \tag{0.4}$$

where $f(x), x \in R^n$, is a piecewise-linear function. A special procedure of linearization is suggested to be used at fast solving an optimal open-loop control problem and realizing in real time optimal feedbacks that provide the solution of the LC problem.

6. Composition of two procedures (items 4, 5) allows to construct a method of solving the LC problem for general nonlinear systems (0.4) ($f(x)$ is a nonlinear function).

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Optimal'noe vosstanovlenie znacheniy lineynykh i nelineynykh operatorov pri netochno zadannykh iskhodnykh dannykh

Predpolagaetsya rasskazat' o novykh rezul'tatakh, a takzhe nekotorykh bolee rannikh rezul'tatakh, ne voshedshikh v obzornye stat'i V.V. Arestova (Uspekhi matematicheskikh nauk, 1996) i V.V. Arestova i avtora (Izvestiya VUZov. Matematika. 1995), svyazannykh s zadachey optimal'nogo vosstanovleniya (nailuchshego priblizheniya) znacheniy Ux operatora U, esli x opredeleno s oshibkoy, ne prevoskhodyaschey nekotorogo fiksirovannogo chisla, i x prinadlezhit nekotoromu mnozhestvu W iz oblasti opredeleniya operatora U. V svyazi s rassmotreniem zadach v prostranstvakh Lp pri 0¡p¡1 nekotorye obschie rezul'taty avtora rasprostranyayutsya na sluchay lineynykh metricheskikh prostranstv. Krome togo, predpolagaetsya rassmotret' zadachu ob optimal'nom vosstanovlenii operatora differentsirovaniya na nekotorykh nestandartnykh klassakh funktsiy, naprimer, na klassakh differentsiruemykh funktsiy s odnostoronnimi ogranicheniyami na starshuyu proizvodnuyu, a takzhe svyazannye s etoy zadachey neravenstva i sravnit' poluchennye rezul'taty so sluchaem prostranstv tipa Soboleva. Poluchennye ranee neravenstva dlya proizvodnykh resheniy obyknovennykh lineynykh differentsial'nykh uravneniy takzhe primenyayutsya v zadache o vosstanovlenii proizvodnykh etikh resheniy. Drugim operatorom, kotoriy predpolagaetsya rassmotret', yavlyaetsya operator vychisleniya krivizny krivykh, zadannykh vektor-funktsiyami iz klassov, podobnykh klassam Soboleva dlya obychnykh funktsiy, i iz nekotorykh drugikh klassov. V etoy zadache budut ukazany otsenki sverkhu i snizu tochnosti vosstanovleniya, a takzhe ukazany otsenki nekotorykh funktsionalov, svyazannykh s etoy zadachey, inogda neuluchshaemye.

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On analogs of the Helmholtz resonator in homogenization[12]

It is known the scattering problems for E-polarized and H-polarized electromagnetic fields on ideally conductive cylindrical surface whose cross-section is a curve Γ_δ are reduced to the solutions of the following boundary value problems in $\Omega_\delta = \mathbb{R}^2\backslash\overline{\Gamma_\delta}$:

$$\begin{aligned} &(\Delta + k^2)E_\delta = J, \quad x \in \Omega_\delta, \qquad E_\delta = 0, \quad x \in \Gamma_\delta, \\ &E_\delta = O(r^{-1/2}), \qquad \frac{\partial E_\delta}{\partial r} - ikE_\delta = o(r^{-1/2}), \qquad r \to \infty \end{aligned} \tag{1.1}$$

for E-polarized filed and, respectively,

$$\begin{aligned} &(\Delta + k^2)H_\delta = h, \quad x \in \Omega_\delta, \qquad \frac{\partial H_\delta}{\partial \nu} = 0, \quad x \in \Gamma_\delta, \\ &H_\delta = O(r^{-1/2}), \qquad \frac{\partial H_\delta}{\partial r} - ikH_\delta = o(r^{-1/2}), \qquad r \to \infty \end{aligned} \tag{1.2}$$

in the case of H-polarization. Here $x = (x_1, x_2)$, $r = |x|$, k is a positive number, ν is the normal to Γ_δ, J is the component of current vector $\mathbf{j}$ directed parallel to a generatrix of the cylindrical surface, and h is the third component of the vector $-\mathrm{rot}\mathbf{j}$ in the case, when, in the contrary vector $\mathbf{j}$ is perpendicular to a generatrix. The functions J and h are proposed to be from $L_2(\mathbb{R}^2)$ and have bounded supports. Hereafter, $\Gamma_0 \in C^\infty$ is the boundary of the bounded simply connected domain Ω, and for $\delta = \varepsilon > 0$ the curves Γ_ε are obtained from Γ_0 by cutting out a great number of holes, having small size and located almost periodically and closely each to other. Namely, let ω be the unit circle with center at the origin, $\gamma_0 = \partial\omega$, $N \gg 1$ be an integer number, $\varepsilon = 2N^{-1}$, $0 < a(\varepsilon) < \frac{\pi}{2}$, θ be the polar angle, $\gamma_\varepsilon = \{(r,\theta) : r = 1,\ \varepsilon(-a(\varepsilon) + m\pi) < \theta < \varepsilon(a(\varepsilon) + m\pi),\ m = 0, 1, ..., N-1\}$, $\mathcal{P}$ be the diffeomorphism in $\mathbb{R}^2$, $\Omega = \mathcal{P}(\omega)$, $\Gamma_\delta = \mathcal{P}(\gamma_\delta)$. For $\delta = \varepsilon$ we call the boundary value problems (1.1) and (1.2) perturbed problems and consider their solutions in the class of functions which with their first derivatives are square integrable in the neighbourhoods the ends of the curves Γ_ε. Since $\Omega_0 = \Omega \cup (\mathbb{R}^2\backslash\overline{\Omega})$, then for $\delta = 0$ the boundary value problem (1.1) disintegrates in two Dirichlet problems and the problem (1.2) does in two Neumann problems in Ω and $\mathbb{R}^2\backslash\overline{\Omega}$. We call them limit interior and outer problems, respectively. In the case of the boundary value problem (1.2) and the analogue of the problem (1.1) (the join of small two-dimensional domains instead of the join of arcs Γ_ε is considered) it is known if $\mathrm{Im}\, k \geqslant 0$ and k^2 is not an eigenvalue of the limit interior problem, then for sufficient small holes (with respect to distance between them) the solutions of the perturbed problems converge to the solutions of the limit problem in Ω and $\mathbb{R}^2\backslash\overline{\Omega}$. Thus we have the situation which is analogous to Helmholtz resonator, when the perturbed problem (for Helmholtz resonator these are Dirichlet and Neumann problems outside the closed curve in which one small hole is cut out) and the limit outer problem are solvable for all positive k, and the limit interior problem has positive discrete

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spectrum. First part of the work is devoted to proof that, like for Helmholtz resonator, analytic extension of the perturbed problems have poles in the lower half-plane tending to eigen frequencies (to square root of eigenvalues) of limit interior problems. Namely, the following two statements are established.

Theorem 1 *Let $\tau_0 > 0$, τ_0^2 be an eigenvalue of the Dirichlet problem in Ω and $\varepsilon \ln a(\varepsilon) \to 0$ as $\varepsilon \to 0$. Then analytic extension of the Green function $G_\varepsilon^{(1)}$ for boundary value problem (1.1) has the pole τ_ε with small negative imaginary part converging to τ_0 as $\varepsilon \to 0$.*

Theorem 2 *Let $\tau_0 > 0$, τ_0^2 be an eigenvalue of the Neumann problem in Ω and*

$$\frac{\pi}{2} - a(\varepsilon) = \exp\left(-\frac{1}{\varepsilon b(\varepsilon)}\right), \quad b(\varepsilon) > 0, \quad \lim_{\varepsilon\to 0} b(\varepsilon) = 0.$$

Then analytic extension of the Green function $G_\varepsilon^{(2)}$ for boundary value problem (1.2) has the pole τ_ε with small negative imaginary part converging to τ_0 as $\varepsilon \to 0$.

However, the existence of poles with small imaginary parts does not mean that they cause resonance phenomena consisting of, in particular, the fact that for positive frequencies k, close to the eigenvalues of the limit interior problems, the difference between the solution of the perturbed problem outside Ω and the solution of the limit problem is $O(1)$ even in the case when the support of the right hands of the equations is located outside Ω. In the second part of the work for the problem (1.1) in model case when Ω is the unit circle and the holes are situated periodically the rigorous asymptotics of these poles on small parameter ε is constructed and their resonance character is shown. The Bashkir State

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On Global Bifurcations of Limit Cycles

We consider two-dimesional dynamical systems and discuss the modern state of the second part of Hilbert's Sixteenth Problem on the maximum number and relative position of limit cycles in such systems. Unfortunately, this Problem has not been solved completely even for the simplest nonlinear case: the case of quadratic systems. We suggest a new global approach to the complete solution of the Problem in the quadratic case. This approach can be applied also to the study of arbitrary polynomial systems and to the global qualitative analysis of higher-dimensional dynamical systems [1].

1. To use five-parameter canonical systems with field-rotation (dynamic) parameters.

2. To divide the plane of two rest (static) parameters into the domains corresponding to various number and character of finite singularities and to consider the canonical

systems separately in each of such domains, i. e. to reduce the study of limit cycle bifurcations to the analysis of three-parameter domains of dynamic parameters.

3. To prove in every concrete case of finite singularities that the maximal one-parameter family of multiple limit cycles is not cyclic.

4. Using Bautin's result on the cyclicity of a singular point which is equal to three and the Wintner–Perko termination principle stating that the multiplicity of limit cycles cannot be higher than the multiplicity (cyclicity) of the singular point in which they terminate, to prove by contradiction in every case the nonexistence neither of a multiplicity-four limit cycle nor of four limit cycles around a singular point.

5. To control simultaneously bifurcations of limit cycles around different singular points and to prove that the maximum number of limit cycles in a quadratic system is equal to four and the only possible their distribution is (3 : 1).

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Existence and blow-up for higher-order semilinear parabolic equations: majorizing order-preserving operators

As a basic example, we establish that in the Cauchy problem for the $2m$-th order semilinear parabolic equation

$$u_t = -(-\Delta)^m u + |u|^p \quad \text{in } \mathbb{R}^N \times \mathbb{R}_+,$$

where $m > 1$, $p > 1$, with bounded integrable initial data u_0, the critical Fujita exponent is $p_F = 1 + 2m/N$, so that for $p > p_F$ there exists a class of small global solutions and for $p \in (1, p_F]$ blow-up can occur for arbitrarily small initial data. These results are classical for the second-order ($m = 1$) heat equations and were proved by H. Fujita in 60's. The main idea of global and blow-up estimates for higher-order equations are as follows. We show that for any $m > 1$, there exists a majorizing order-preserving equation and a standard comparison applies to describe some key properties of such parabolic flows. In particular, the global solvability follows by comparison with similarity solutions of the majorizing order-preserving equation, which are expressed in terms of eigenfunctions of a Hammerstein's operator with positive kernel. On the other hand, a usual comparison with a spatially flat solutions of the majorizing equation implies a sharp L^∞-bound on any blow-up solutions. Generalizations to differential and pseudodifferential evolution equations and relations to positivity sets for higher-order equations are discussed. A joint work with S.I. Pohozaev.

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Флуктуационно-пассивная теорема

В рамках линейной флуктуационно-диссипативной теории доказана общая флуктуационно-пассивная теорема, обобщающая известную одномерную флуктуационно-диссипативную теорему. При доказательстве нашей теоремы была использована теория пассивных систем, развитая в работах H. Konig и T. Meixner [1], E. Beltrami и M. Wohlers [2], B.C. Владимирова [3], A. Zemanian [4], Р.Х. Галеева [5], и примененная к моделям в физике плазмы, сформулированным в работах R.Kubo [6], М.А.Леонтовича и С.М.Рытова [7], В.П.Силина [8], А.А.Рухадзе и др.[9].

Флуктуационно-пассивная теорема. *Средние значения пассивного оператора -отклика $\hat{Z}_{lk}(\mathbf{r},t)$ и флуктуации плотности тока $\hat{j}_k(\mathbf{r},t)$ связаны следующим соотношением:*

$$< \hat{Z}_{lk} >= (2\pi)^{-4}\hbar^{-2} < \hat{J}_k[\hat{J}_l, \hat{H}] > (\mathbf{r},t),$$

где $[\hat{J}_l, \hat{H}]$ - коммутатор.

Заметим, что оператор -отклика $\hat{Z}_{lk}(\mathbf{r},t)$ связывает, out - флуктуации плотности тока $\hat{\mathbf{J}}(\mathbf{r},t)$, которые вызываются, возникающими внешними in - электромагнитными полями A, с упомянутым полем A следующим образом:

$$\hat{\mathbf{J}} = \hat{Z} * \mathbf{A}.$$

Как следствие нашей теоремы, мы получаем нашу ранее опубликованную флуктуационно-диссипативную теорему:

Антиэрмитова часть оператора отклика и спектральная функция коррелятора тока связаны соотношением:

$$< \hat{Z}_{lk} - \hat{Z}^+_{kl} >= (2\pi)^{-2}\hbar^{-1}\nu < \hat{J}_l\hat{J}_k > \left\{\exp\left(\frac{h\nu}{T}\right) - 1\right\}(\mathbf{k},\nu).$$

Заметим, что одномерный вариант этой теоремы был сформулирован ранее в [6-8].

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Smoluchowskii Equations[13]

The paper is devoted to global genaralized solutions of the Smoluchowskii equations for coagulation processes in nonuniform systems. The Smoluchowskii equation for particles with discrete masses is given by the following relation

$$\frac{\partial f_i}{\partial t} + v_i \frac{\partial f_i}{\partial x} = \frac{1}{2} \sum_{j=1}^{i-1} \Phi_{i-j,j} f_{i-j} f_j - f_i \sum_{j=1}^{\infty} \Phi_{i,j} f_j, \quad i \in \mathbf{N}, \tag{1}$$

where $\Phi_{i,j} = \sigma_{i,j}|v_i - v_j|$ is intensity of particles collisions, $\sigma_{i,j}$ is particles capture cross-section which is simmetric nonnegative function on $\mathbf{N} \times \mathbf{N}$. The solvability of Cauchy problem was established by means of Tartar's compensated compactness method. It is important to emphasize that value of generalized solutions for physical kinetics equations is caused by possibility of nonsmooth space – time singularities for infinitely smooth initial data of Cauchy problem. The example of such singularities we shall propose in the talk for equation (1). The main reason which cause the singularity in this case is noncontinuity of Smoluchowskii collision operator (right - hand side part of the equation (1)) in the norm defined by conservation law of Cauchy problem. The similar problems are considered for spatially uniform Smoluchowskii equations provided rather high rate of particles interaction is supposed and the same we investigated for stationary uniform problem when particles positive sourse is added to Smoluchowskii collision operator. The functional solutions theory was applied to global correctness problem. We proved global existance, uniqueness and stability theorems for nonnegative functional solutions of generalized Boltzmann equation provided free velocity v is Borel locally bounded function and initial data are nonnegative Lebesgue summable functions.

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Inverse Singular Problems of two Spectrums of Sturm-Liouville Equation

Difference notions and methods, generated during the process of investigation of the Sturm-Liouville equation

$$-y''(x) + q(x)y(x) = \mu y(x), \tag{0.1}$$

and associated with this equation the operator of Sturm-Lioville $L = -\frac{d^2}{dx^2} + q(x)$, which also called one-dimensional Schrodinger operator and function $q(x)$ is called potential. The results in the regular case in different statements were enough investigated. The results of regular cases we expand to the some singular cases. **Theorem.** *For sequences of real numbers* $-\infty < \mu_0 < \mu_1^- \leqslant \mu_1^+ < \mu_2^- \leqslant \mu_2^+ < \dots$ *to be spectrums of periodic* $y(0) - y(\pi) = y'(0) - y'(\pi) = 0$ *and antiperiodic* $y(0) + y(\pi) = y'(0) + y'(\pi) = 0$ *value problems, generated by the same equation (1) with the potential*

$$q(x) = \sum_{i=1}^{m} \frac{A_i}{x^{p_i}} + q_0(x), \tag{0.2}$$

A_i *are real constants,* $p_i \in (1; 5/4)$, $q_0(x) \in L_2(0;\pi)$ *and with boundary conditions* $y(0) = y(\pi) = 0$, $y'(0) = y'(\pi) = 0$ *correspondingly, it is necessary and sufficient for that sequences satisfy to asymptotic formula*

$$\mu_k^{\pm} = k^2 + \frac{2}{\pi} \sum_{i=1}^{m} A_i C_{p_i} k^{p_i - 1} - 2A + \varepsilon_k^{\pm},$$

where $\mu_{2k}^{\pm}$ *are eigenvalues of periodic and* $\mu_{2k+1}^{\pm}$ *eigenvalues of antiperiodic boundary value problems,* $p_i \in (1; 5/4)$, A_i, A *are arbitrary real numbers,* $C_{p_i} = \int\limits_0^\infty \frac{\sin^2 \xi}{\xi^{p_i}} d\xi$ *and* $\sum\limits_{k=1}^{\infty} |\varepsilon_k^-|^2 + |\varepsilon_k^+|^2 < \infty$.

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Gasymov M.G., Orudzhev E.G.
(Baku State University)

On Expansion Of Function By Solition Of Spectral Problem For One Differential Equation

A boundary value problem on interval $[0,1]$ was considered

$$y^{(2n)}(x)+P_1(x,\lambda)y^{(2n-1)}(x)+\cdots+P_{2n}(x,\lambda)y(x)=0 \tag{0.1}$$

$$U_i(y)\equiv y^{(j-1)}(0)-y^{(j-1)}(1)=0,\ j=\overline{1,2n} \tag{0.2}$$

where $P_i(x,\lambda)=\sum\limits_{l=0}^{i}P_{il}(x)\lambda^l$, $i=\overline{1,2n}$, $P_{il}(x)\in C^{2n-i+l}[0,1]$, $l<i$, P_i are complex numbers, λ is spectrum parameter with supposition, that corresponding characteristic equation have $k=2m$ different roots $\theta_1,\dots,\theta_k$, each of which are of $n_1,\dots,n_k$ multiplicity ; moreover, these roots are different from zero and such, that numeration divides plane λ to the sectors R_s, $s<4m$, at each of which inequality of Tamarkin type holds. At present paper some conditional algebraic equations relative to coefficients. $P_{i,i-\nu}(x)$, $\nu=\overline{1,n_s-1}$ was found, which are sufficient conditions for existence of partial solutions of equations (1) at each sector R_s, which allow asymptotic representation for $|\lambda|\to\infty$

$$y_i(x,\lambda)=\left[g_{i_0}^0(x)+\frac{1}{\lambda}g_{i_0}^1(x)+\ldots+\frac{1}{\lambda^{n_s-1}}g_{i_0}^{n_s-1}(x)+O\left(\frac{1}{\lambda^{n_s-1}}\right)\right]e^{\theta_s\lambda x},\ i\overset{s=\overline{1,k}}{=}\overline{1,2n}$$

where $g_{i_0}^0(x)$ and $g_{i_0}^j(x)$, $j=\overline{1,n_s-1}$ are fundamental solutions of homogenous and partial solutions of non-homogenous differential equations of n_s-th order correspondingly, which expressed by $P_{il}^{(\mu)}(x)$, $\mu<n_s$. For functions, which have continuous derivatives up to the order $2n+\mu$, $\mu=\max\{n_1,\dots,n_k\}-\min\{n_1,\dots,n_k\}$ and vanishes with derivatives up to the order $2n+\mu-1$ at points "0" and "1", the expansion by eigenfunctions of problem (1)-(2) was obtained.

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(Italy)

Homogenization of Unbounded Functionals and Nonlinear Elastomers

The homogenization process for some energies of integral type arising in the modelling of rubber-like elastomers is carried out. The main feature of the variational problems taken into account is the presence of pointwise oscillating constraints on the gradients of the admissible deformations. The classical homogenization result is estabilished also in this framework, both for Dirichlet with affine boundary data, Neumann, and mixed problems, by proving that the limit energy is again of the integral type, gradient constrained. An explicit computation for the homogenized integrand relative to energy density in a particular relevant case is derived.

Gelfreich V.G.
(St. Petersbourg)

Splitting of Separatrices: perturbation theory and exponential smallness

This talk is a survey of main results related to the separatrices splitting for area-preserving maps and Hamiltonian systems with one and a half degrees of freedom. The special attention is paid to the problems, where the separatrices splitting is exponentially small with respect to a perturbation parameter.

Georgievskii D.V.
(Moscow State University)

The generalized Orr–Sommerfeld problem for viscoplastic flows

The spectral problem [1]

$$\varphi^{IV} - 2s^2\varphi'' + s^4\varphi - 4\kappa s^2\left(\frac{\varphi'}{|v^{\circ\prime}|}\right)' = is\left[\left(v^\circ - \frac{i\alpha}{s}\right)(\varphi'' - s^2\varphi) - v^{\circ\prime\prime}\varphi\right]Re \qquad (0.1)$$

$$x_2 = 0, \quad x_2 = 1 \ : \quad \varphi = \varphi' = 0 \qquad (0.2)$$

describes stability of shear flow of viscoplastic material inside the layer $\{\Omega: 0 < x_2 < 1\}$ in case of absence of rigid zones in this layer. In (0.1) Re, κ are the Reynolds' number and the Il'yushin number respectively; $\varphi(x_2)$ is an amplitude of stream function disturbance ψ: $\psi(x_1, x_2, t) = \varphi(x_2)\exp(isx_1 + \alpha t)$; α is complex frequency being spectral parameter (its real part sign is in connection with stability of flow which is defined by the section $v^\circ(x_2)$); $s > 0$ is wave number of disturbance along the axis x_1; a prime means derivation with respect to x_2. The last term in left hand of (0.1) takes into account an influence of plastic properties of medium in comparison with viscous ones. Without this term (0.1) is the same as the classical Orr—Sommerfeld equation. The function v° in the problem (0.1), (0.2) is supposed to be monotone continuously differentiable inside Ω and such that $\int_0^1 (\chi/|v^{\circ\prime}|)\, dx_2 < \infty$ for any function $\chi(x_2)$ vanishing by $x_2 = 0$ and $x_2 = 1$. The section of the Couette' viscoplastic flow may be example of v° here. If the points $\{\xi \in \Omega\colon v^{\circ\prime}(\xi) = 0\}$ exist then they are boundaries of rigid interlayers $\Omega_r \in \Omega$. The latter appear, for example, either in the Poiseuille' viscoplastic flow or in motion of heavy layer along inclined plane. Then the following conditions

$$x_2 = \xi \;:\quad \varphi' = 0, \quad \varphi'' + s\left(s - \frac{iv^{\circ\prime\prime}}{\alpha + isv^\circ}\right)\varphi = 0, \tag{0.3}$$

take place instead of the sticking conditions (0.2). The conditions (0.3) both contain the spectral parameter and take into account a change of rigid interlayer thickness in disturbed motion. It should be noted that the problem (0.1), (0.3) is considered only inside subdomains $\Omega_f = \Omega \setminus \Omega_r$. If change of thickness of domains Ω_r can be neglected then the conditions (0.3) are simplified and no longer contain α:

$$x_2 = \xi \;:\quad \varphi' = 0, \quad \varphi'' + s^2\varphi = 0 \tag{0.4}$$

In the present work the integral methods for estimation of critical parameters on plane (Re, κ) are developed for spectral problem (0.1), (0.2) called as the generalized Orr—Sommerfeld problem as well as for the problems (0.1), (0.3) and (0.1), (0.4). These estimates are based on variations inequalities in $H_2(\Omega_f)$ and have sufficient nature. They generalize the Joseph—Yih inequalities [2, 3] obtained on the basis of analysis of the Orr—Sommerfeld problem. It is shown that the presence of plasticity parameter κ stabilizes corresponding Newtonian flow.

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Gindikin S. G.

(Center Sofus Lie (Moscow); Rutgers University)

Integral geometry on symmetric spaces and nonlinear differential equations

Gladkov A.L.
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Cauchy problem for nonlinear parabolic equations with any growing at infinity initial data

We study the Cauchy problem for the semilinear parabolic equations of higher order

$$u_t + \sum_{i=1}^{m}(-1)^i a_i D^{2i} u + \sum_{j=1}^{m} b_j D^{2j-1} u + cu + d|u|^\alpha u = f, \quad (x,t) \in S_T, \tag{1}$$

$$u(x,0) = u_0(x), \quad x \in \mathbb{R}, \tag{2}$$

where $m \geqslant 1$, $D = \partial/\partial x$, $S_T = \mathbb{R} \times (0,T)$, $T > 0$, a_i, b_i, $i = 1,\dots,m$, c, d and α are constants, $a_i > 0$ $(i = 1,\dots,m)$, $d > 0$, $\alpha > 0$, $f(x,t) \in L_{loc}^{(\alpha+2)/(\alpha+1)}(S_T)$, $u_0(x) \in L_{loc}^2(\mathbb{R})$. No assumption has to be made on the behaviour of $u_0(x)$ as $|x| \to \infty$. We can consider the equation (1) as a generalization of classical Fisher-Kolmogorov equation ($b_i = 0$, $i = 1,\dots,m$, $a_i = 0$, $i = 2,\dots,m$, $a_1 = d = 1$, $c = -1$, $\alpha = 2$). Special case of (1)

$$u_t = -\gamma D^4 u + D^2 u + u - u^3 \tag{3}$$

has been investigated in connection with studying of phase transitions of critical points in [1], [2] and as a model equation of higher order for bistable systems in [3]. The existence and the uniqueness of a generalized solution of the Cauchy problem (3), (2) in S_T with initial data which satisfies the inequality

$$\int_{\mathbb{R}} u_0^2(x) \exp(-\beta |x|^{4/3}) dx < \infty$$

for some positive constant β and any $T < 9/[256\gamma\beta^3]$ have been proved in [4]. We shall define the notion of the generalized solution of problem (1), (2). Let Ω be any bounded domain in $\mathbb{R}$ and $Q_T = \Omega \times (0,T)$, where $T > 0$. By $W(\Omega,T)$ we shall denote the space of real functions v with properties

$$v \in L^2(0,T; H^m(\Omega) \cap L^{\alpha+2}(\Omega)) \cap L^\infty(0,T; L^2(\Omega)),$$
$$v_t \in L^2(0,T; H^{-m}(\Omega) \oplus L^{(\alpha+2)/(\alpha+1)}(\Omega)).$$

Definition. *A function $u(x,t)$ is said to be a generalized solution of (1), (2) in S_T if $u(x,t) \in W(\Omega,T)$ for any domain Ω and satisfies initial data (2) in the following sense*

$$\int_\Omega \{u(x,t) - u_0(x)\} h(x) dx \to 0 \quad \text{as} \quad t \to 0$$

for any function $h(x) \in H_0^m(\Omega) \cap L^{\alpha+2}(\Omega)$ and integral identity

$$\int\int_{Q_T} [u_t\varphi + \sum_{i=1}^{m} a_i D^i u D^i \varphi + \sum_{j=1}^{m} (-1)^{j-1} b_j D^j u D^{j-1} \varphi +$$

$$+cu\varphi + d|u|^\alpha u\varphi]\,dxdt = \int\int_{Q_T} f\varphi\,dxdt$$

for any function $\varphi(x,t) \in L^2(0,T; H_0^m(\Omega) \cap L^{\alpha+2}(\Omega))$.

We prove the existence and the uniqueness of the generalized solution of (1), (2) in S_T for any $T > 0$ without any growth restriction at an infinity on the initial data. The results can be extended to a number equations with more general nonlinearities. Maybe the simplest example is equation (1) with nonlinearity $g(u)$ instead of $|u|^\alpha u$, where

$$\liminf_{|u|\to\infty} |g(u)|/|u|^{\alpha+1} > 0 \quad \text{for some} \quad \alpha > 0,$$

$$g(u)\operatorname{sign} u > 0 \quad \text{for} \quad u \neq 0,$$

$$|g(u_1) - g(u_2)| \geqslant c_1|g(u_1 - u_2)|, \quad \text{for} \quad u_1, u_2 \in \mathbb{R} \quad \text{and some} \quad c_1 > 0.$$

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Glaskova M.Yu.
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On the uniform cone operators

This talk is based on a joint work with T.Ya. Azizov, A. Dijksma, and K.-H. Förster.

In the following $\mathbf{E} = \mathbf{C}$ or $\mathbf{E} = \mathbf{R}$, $\mathbf{E}^m$ is the m-dimensional Hilbert space with the Euclidean inner product, and $\mathbf{E}^m_+$ is the cone of the nonnegative vectors $u = \{\xi_1, \dots, \xi_m\} \in \mathbf{E}^m$ with $\xi_i \geqslant 0$, $i = \overline{1,m}$. We say that an $m \times m$ matrix $\mathcal{A}$ is uniformly cone positive in $\mathbf{E}^m$ if there is a positive number k_A such that $(\mathcal{A}u, u) \geqslant k_A(u,u)$, $u \in \mathbf{E}^m_+$. We prove some new and reprove some old results related to uniform cone positivity and the cosine.

Theorem. *Let $\mathcal{A}$ be a selfadjoint $m \times m$ matrix and let*

$$P_A^0 = \{u \in \mathbf{E}^m : (\mathcal{A}u, u) = 0\}.$$

Then $\mathcal{A}$ is uniformly cone positive if and only if $P_A^0 \cap \mathbf{E}^m_+ = \{0\}$ and there is a vector $u_0 \in \mathbf{E}^m_+ = \{0\}$ such that $(\mathcal{A}u, u) > 0$.

Corollary 1. *Assume that A is a nonnegative matrix. Then A is uniformly cone positive if and only if* kernel $A \cap E^m_+ = \{0\}$.

Corollary 2. *Let $A = (\alpha_{ij})^m_{i,j=1}$ be a selfadjoint matrix with real entries such that*(a) $\alpha_{ii} > 0$, $i = \overline{1,m}$, *and*(b) $\alpha_{ii} + \sum_{j\in\gamma(i)} \alpha_{ij} > 0$ *for every i with $\gamma(i) \neq \varnothing$, where*

$$\gamma(i) = \{j | (i,j) \in \gamma\}, \quad \gamma = \{(i,j) | i \neq j, \ \alpha_{ij} < 0\}.$$

Then A is uniformly cone positive.

Consider an operator pencil $A_0 + \lambda_1 A_1 + \cdots + \lambda_n A_n$ in which, for example (other cases are also considered), A_0 is a maximal accretive operator, $A_1, \ldots, A_n$ are closed accretive operators, and dom $A_0 \subset$ *rmdom* A_j, $j = \overline{1,n}$. We give a sufficient condition under which it is closed for all $\lambda_j \geqslant 0$, $j = \overline{1,n}$. In case $n = 1$, dom A_0 = dom A_1, and A_0, A_1 are maximal uniformly accretive, this condition is also necessary. The condition is that the matrix $(\cos(A_i, A_j))^n_{i,j=0}$ is uniformly cone positive.

Finally, we study the closedness of some 2 × 2 matrices with operator entries.

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Gliklikh Yu.E.
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Viscous hydrodynamics in terms of stochastic processes on groups of diffeomorphisms

We construct special stochastic perturbations of the curves on the groups of diffeomorphisms, describing the motion of perfect fluids, whose expectations describe the motion of viscous fluids. The processes satisfy a certain stochastic analogue of equation of geodesics constructed in previous works by the author (see, e.g., [1] and [2]), i.e., this generalizes the approach to perfect fluids by V. Arnold, D. Ebin and J. Marsden. For the flat n-dimensional torus $\mathcal{T}^n$ denote by $\mathcal{D}^s_\mu(\mathcal{T}^n)$ the Hilbert manifold of Sobolev H^s-diffeomorphisms of $\mathcal{T}^n$ ($s > \frac{n}{2} + 1$), preserving the volume, with group structure relative to the composition $\circ$. Let $g(t)$, $g(0) = e$, $\dot{g}(0) = u_0$ be a curve in $\mathcal{D}^s_\mu(\mathcal{T}^n)$ describing the motion of perfect incompressible fluid without external force. It exists for $t \in [0, \varepsilon)$ where $\varepsilon > 0$ depends on u_0. Consider a Wiener process $w(t)$ in R^n. The process $w_*(t) = -\int_0^t \frac{w(s)}{s} ds + w(t)$ is well-posed for $t \in [0,\infty)$. Denote by $w^*(t)$ the conditional expectation of $w_*(t)$ with respect to the σ-algebra generated by w at t. Introduce the random diffeomorphism $\sigma W^*(t) \in \mathcal{D}^s_\mu(\mathcal{T}^n)$ that sends the point $m \in \mathcal{T}^n$ to $m + \sigma w^*(t)$ where $\sigma > 0$ is a constant. Construct the process $\eta(t) = \sigma W^*(t) \circ g(t)$. Obviously it exists for $t \in [0, \varepsilon)$.

Theorem 1. *The process $\eta(t)$ satisfies the relation $D_* D_* \eta(t) = 0$ (a stochastic analog of equation of geodesics) where D_* is Nelson's (covariant) backward mean derivative.*

Construct the vector $u(t) = E(TR^{-1}_{\eta(t)} \tilde{D}_* \eta(t)) \in T_e \mathcal{D}^s_\mu(\mathcal{T}^n)$ where $R_{\eta(t)}$ is the right shift on $\mathcal{D}^s_\mu(\mathcal{T}^n)$ and E is the expectation; $u(t)$ is a time-dependent divergence-free H^s-vector field on $\mathcal{T}^n$ with $u(0) = u_0$.

Theorem 2. *If the vector field $u(t,m)$ on T^n is C^1 in t and C^2 in $m \in T^n$, it satisfies the Navier-Stokes (NS) equation with viscosity coefficient $\frac{\sigma^2}{2}$ and zero external force.*

The process $\eta(t)$ and so the vector field $u(t,m)$ are well-posed for $s > \frac{n}{2}+1$ but $u(t,m)$ may not be smooth enough to satisfy NS equation in classical sense. In this case we call $u(t,m)$ a generalized solution.

Corollary. *(i) For $s > \frac{1}{2}n + 1$, an H^s divergence free vector field u_0 on T^n and a real number $\sigma > 0$ there exists a generalized solution $u(t)$ of the above-mentioned NS equation with initial value u_0, well-posed on the same time interval (depending on u_0) as the solution $\kappa(t)$ of Euler equation without external force with initial value u_0;*

(ii) $u(t)$ tends to $\kappa(t)$ as $\sigma \to 0$ (i.e., the viscosity coefficient tends to zero);

(iii) For $n = 2$ the generalized solution of Navier-Stokes equation exists for all $t \in [0, +\infty)$ since solutions of Euler equation exist for those t;

(iv) For $s > \frac{1}{2}n + 2$ the above-mentioned generalized solution of NS equation is a classical solution.

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Glutsyuk A.A.

(École Normale Supérieure de Lyon)

An estimate of the number of zeroes of abelian integral for special hamiltonian of arbitrary degree.

This is a joint work with Yu.S.Ilyashenko. We consider a real polynomial $H(x,y) = p(x)+q(y)$, where p and q are monic polynomials of degree $n+1 > 2$ with complex critical values contained in unit disc, such that the complex critical values of the polynomial H are distinct and are "not too close to each other": the distance between any two of them is at least $\frac{1}{n^2}$. Let H have a continuous family γ_t of ovals, i.e., closed curves in real level lines $H(x,y) = t$. For a deformation of the hamiltonian vector field with the hamiltonian H in the class of real polynomial vector fields of the same degree, we give an explicit upper bound (depending on n) of the number of the ovals γ_t that can generate limit cycles of the perturbed field. More precisely, we show that an abelian integral over γ_t of a real polynomial 1- form of degree not greater than n can have at most e^{cn^4} zeroes, as a function in t. The constant c is independent on n.

Godunov S.K., Gordienko V.M.
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Galilei Invariant and Thermodynamically Compatible Equations

Golo V. L.
(Moscow State University)

Solitonnye resheniya nelokalnogo uravneniya Shredingera

Gomilko A.M.
(Institute of Hydromechanics, National Academy of Science, Ukraine)

Factorization of the operator-functions in the Hilbert space

Gorbachuk V.M.
(National Technical University of Ukraine "KPI")

Locally integrated semigroups of normal operators in a Hilbert space

We consider the problem

$$y'(t) = Ay(t) + \frac{t^k}{k!}x, \quad t \in [0,b], \quad b < \infty, \tag{0.1}$$

$$y(0) = 0, \tag{0.2}$$

where A is a normal operator on a Hilbert space $\mathfrak{H}$. It is assumed also that the resolvent set of the operator A is nonempty. We say that problem (1)-(2) is well-posed if for every vector $x \in \mathfrak{H}$, there exists a unique vector-function $y(t) \in C([0,b], \mathcal{D}(A)) \cap C^1([0,b], \mathfrak{H})$ satisfying (1) and (2). Here $\mathcal{D}(A)$ denotes the domain of A. If problem (1)-(2) is well-posed, then there exists a strongly continuous operator-function $S(t)$ on $[0,b]$, such that for any $x \in \mathfrak{H}$, any $t \in [0,b]$,

$$\int_0^t S(s)x\,ds \in \mathcal{D}(A), \quad A\int_0^t S(s)x\,ds = S(t)x - \frac{t^k}{k!}x.$$

The function $S(t)$ is called k-times integrated semigroup generated by the operator A. We give the conditions on the operator A, necessary and sufficient for problem (1)-(2) to be well-posed. The conditions are formulated in terms of location of the spectrum of A. In the case, where the operator A is selfadjoint, these conditions are equivalent to upper semiboundedness of A. It is given also a necessary and sufficient condition on the vector x in order that the solution of problem (1)-(2) be an entire vector-function of finite order and finite type.

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On solvability in classes of entire vector-functions of the Cauchy problem for differential equations in a Banach space

We consider the Cauchy problem

$$y'(t) = Ay(t), \quad t \in [0,\infty), \tag{0.1}$$

$$y(0) = y_0, \tag{0.2}$$

where A is a closed operator with dense domain $\mathcal{D}(A)$ in a Banach space $\mathfrak{B}$, $y_0 \in \mathcal{D}(A)$. If $\mathcal{D}(A) = \mathfrak{B}$, then problem (1)-(2) is solvable for any $y_0 \in \mathfrak{B}$, and its solution is an entire vector-function of exponential type. This is not the case when A is unbounded. A vector $x \in \bigcap\limits_{n\in\mathbb{N}} \mathcal{D}(A^n)$ is entire for the operator A if the series $\sum\limits_{k=0}^{\infty} \frac{t^k A^k x}{k!}$ converges in the whole complex plane. We say that an entire vector for the operator A has a finite order if there exists a number $\beta \in (-\infty, 1)$ such that $\|A^n x\| \leqslant n^{n\beta}$ for safficiently large $n \in \mathbb{N}$. The infimum $r = r(x)$ of such β is called an order of x. Define the type $s = s(x)$ of an r-order vector x as $s = \inf\{\alpha > 0 : \|A^n x\| \leqslant \alpha^n n^{nr}$ for sufficiently large $n \in \mathbb{N}\}$. We show that problem (1)-(2) is solvable in the class of ρ-order and σ-type entire vector-functions ($\rho < \infty$, $\sigma < \infty$) if and only if y_0 is an entire vector of finite order r and finite type s, related to ρ and σ by the relations $\rho = (1-r)^{-1}$, $\sigma = (\rho e)^{-1}(se)^{\rho}$. In the case where $R(\lambda, A) = (A - \lambda I)^{-1}$ is a meromorphic operator-function, then the set of initial data y_0 for which problem (1)-(2) has a solution in the class of entire vector-functions of exponential type is dense in $\mathfrak{B}$ if and only if the set of root vectors for the operator A is complete in $\mathfrak{B}$. In order that the latter be hold, it is necessary and sufficient that there exist an integer $m > 0$, a total set $M \subset \mathfrak{B}$ and a number sequence $r_n \to \infty$ as $n \to \infty$, such that

$$\forall x \in M \ \sup_n \int_0^{2\pi} \ln \left\| \frac{R(r_n e^{i\varphi}, A)x}{r_n^m} \right\| d\varphi < \infty.$$

If A is the generator of C_0-group of bounded linear operators $U(t)$, $t \in \mathbb{R}$, in $\mathfrak{B}$, then the set of initial data y_0, for which problem (1)-(2) has an entire solution of finite order $\rho > 1$ and type $\sigma < \infty$, is dense in $\mathfrak{B}$. In the case where $\rho = 1$, $\sigma < \infty$, the density holds under the condition

$$\int_{-\infty}^{\infty} \frac{\ln\|U(t)\|}{1+t^2}\,dt < \infty.$$

Gorin E.A., Karaulova L.V.
(Moscow State Pedagogical University)

Convolution equations on infinite-dimensional Abelian groups

Let X be a complete separable metric Abelian group. In what follows we fix a continuous function $u : X \to \mathbb{R}_+$ such that $u(0) = 0$ and $\{x \mid u(x) < \varepsilon\}, \varepsilon > 0$, form a neighborhood base at $0 \in X$. The typical example gives a Banach space X and $u(x) = \|x\|^p$.

We consider $\mathbb{R}_+$ as an Abelian semi-group with respect to an operation $\oplus$ and under the usual topology. The main examples are given by $s + t$ and $\max\{s, t\}$.

It will always be assumed that the following (balance) condition holds:

if $t \geqslant 0$ then there exists a sequence $e_n = e_n(t) \in X$
such that $u(x + e_n) \to u(x) \oplus t$ for all $x \in X$.

If the above condition takes place then X *is not* locally compact. At the same time the condition holds (get $s \oplus t = s + t$) if X is an infinite-dimensional L^p-space and $u(x) = \|x\|^p$ as well as (get $s \oplus t = \max\{s, t\}$) if $X = C_0(\Omega)$, where Ω is a locally-compact but not compact metric space, and $u(x) = \sup_\Omega |x(\omega)|$.

Let f be a Borel function on $\mathbb{R}_+$ and let μ be a regular Borel measure on X of finite total variation. Put

$$(f \circ u) * \mu(a) \stackrel{\text{def}}{=} \int_X (f \circ u)(a - x)\,\mu(dx). \tag{0.1}$$

We should like to present sufficiently wide classes of f's such that the triviality of the potential (1) leads to $\mu = 0$. It can be shown in the usual way that $(f \circ u) * \mu(a) = 0$ leads to the relation

$$\int_0^\infty f(s \oplus t)\,dg_a(s) = 0 \quad \text{for all} \quad t \in \mathbb{R}_+, \tag{0.2}$$

where

$$g_a(t) = \mu\{x \mid u(x - a) < t\}.$$

There exist cases such that the equation (2) arises however f is bounded on the segments $0 < \alpha \leqslant t \leqslant \beta < \infty$ only. In particular, the growth of f in zero does not play a role at all (we use a variant of the Cartan covering lemma).

If $s \oplus t = s + t$ then the equation (2) belongs to the classical harmonic analysis. In this case many non-trivial results was fined.

If $s \oplus t = \max\{s, t\}$ then harmonic analysis does not work for a non-trivial continuous character on $(\mathbb{R}_+, \oplus)$ does not exist. Fortunately, the business is not so poorly. In this case if (2) takes place and if variation $V_t^{t\pm\varepsilon}(f) > 0$ for all $\varepsilon > 0$ then $g_a(t) = 0$.

In particulary, if Ω is a locally compact but not compact separable metric space if Ω does not contain isolated points if $X = C_0(\Omega)$ if $u(x) = \sup_\Omega |x(\omega)|$, and if $(f \circ u) * \mu = 0$ then either $f = \text{const}$ or $\mu = 0$.

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Regularity of central leaves of partially hyperbolic sets

The following theorem was announced by Yu.Ilyashenko and the author in [1]. **Theorem A.** *Given an open finite interval $I \subset \mathbb{R}, 0 \in I$ and a closed manifold M, $\dim M \geqslant 3$, there exists an open set $U \subset Diff^2(M)$ such that any $f \in U$ has a locally maximal invariant set $\Delta \subset M$ with the following properties: (i) There exist two numbers l_1 and $l_2 = l_1 + 1$ such that the hyperbolic periodic orbits with stable manifolds of dimension l_i are dense in Δ, (ii) For any $\lambda \in I$ there exists an orbit dense in Δ with one of the intermediate Lyapunov exponents equal to λ.* **Addendum.** *The set Δ in Theorem A may for $\dim M \geqslant 4$ be taken to be a partially hyperbolic attractor.* The proof of Theorem A require the folowing technical results which is also of its own interest. Let the map $S : U(\Lambda) \to U(\Lambda)$ be hyperbolic with locally maximal hyperbolic set Λ, $\Lambda = \cap_{n \in \mathbb{Z}} S^n(U(\Lambda))$. Let M be a closed manifold. **Theorem B.** *Let the map $\mathfrak{F} : U(\Lambda) \times M \to U(\Lambda) \times M$, $\mathfrak{F} = S \times id_M$ be of class $C^{r+1}, 0 \leqslant r < \infty$. Then any C^{r+1}-diffeomorphism $\mathfrak{G}$, which is C^{r+1}-close to $\mathfrak{F}$, has an invariant subset Δ homeomorphic to $\Lambda \times M$, the projection $\Phi : (\Delta, \mathfrak{G}) \to (\Lambda, S)$ is a semiconjugacy, the leaves $\Phi^{-1}(x)$ are C^{r+1}-smooth and depend Hölder continuously on a point $x \in \Lambda$ in C^r-norm. The Hölder exponent and the Hölder constant are uniform on a small C^{r+1}-neighborhood of the map $\mathfrak{F}$.* For the case when S is an Anosov diffeomorphism of $\mathbb{T}^n$ and $M = \mathbb{T}^k$ this result (by different method) was recieved by V.Nitica and A.Török [3].

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Unitary reflection groups and automorphisms of isolated hypersurface singularities

A classical result by Arnold states that simple hypersurface singularities are classified by the ADE Weyl groups. Arnold also showed that simple hypersurfaces with a reflectional Z_2-symmetry correspond to the groups B_k, C_k and F_4. I shall speak about the first appearance of Shephard-Todd unitary reflection groups in singularity theory. This is provided by consideration of arbitrary finite order symmetries of simple hypersurfaces.

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About the applications of KAM — theory in new problems of the cosmic dynamics

In the articles [1, 2] some new models of cosmic dynamics were suggested, that gained the name of "bounded problems of $n > 3$ bodies". We determined all of the equilibrium points in the dynamic models in cases $n = 4, 5, 6, 7$ and investigated their Lyapunov stability [3, 4, 5] using the well-known theorem by Arnold – Mozer [6, 7, 8]. We investigated a universal analytical algorithm of normalization of the Hamiltonians of the models under consideration, perforemed by System of Symbol Computations "Mathematica 4" [9].

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Invariant subspaces of families of Krein space binoncontractions

This talk is based on a joint work with T.Ya. Azizov and A. Dijksma.

The problem of research of invariant subspaces of sets of operators has been paid a lot of attention in the scientific literature. L.S. Pontryagin and S.L. Sobolev were first who considered invariant subspaces for selfadjoint operators with regard to problems of hydrodynamics. It was examined in another ways in works of M.G. Krein, I.S. Iohvidov, G. Langer, P.S. Phillips, etc.

The problem of the existence of maximal semidefinite invariant subspaces for a set of operators which act in Krein spaces and, in particular, in Pontryagin spaces, is one of central problems in the operator theory in spaces with an indefinite metric. It was started in works of M.G. Krein. His statement of the problem was formulated as follows: let $\mathcal{U} = \{V\}$ be a commutative set of J- noncontractive operators and let $\mathcal{L}_+$ be their common invariant nonnegative subspace. Is there a maximal nonnegative subspace $\widetilde{\mathcal{L}}_+$ such, that $\mathcal{L}_+ \subset \widetilde{\mathcal{L}}_+$ and $U\widetilde{\mathcal{L}}_+ \subset \widetilde{\mathcal{L}}_+$? The solution of this problem is not found if to assume that $\mathcal{L}_+$ is a completely invariant concerning U subspace and the set U is arbitrary. It is solved for unitary operators in J-space (if an appropriate group is bounded), for π-noncontractive operator in Π_κ, $\kappa = \dim \Pi_\kappa^+$, and for some other special cases.

The main result of our work is a proof of the following theorem.

Theorem. *Let $\mathcal{U} = \{V\}$ be a commutative set of J- binoncontractions, let $\mathcal{L}_+$ and $\mathcal{L}_-$ be a maximal completely invariant nonnegative and a maximal invariant nonpositive subspaces of $\mathcal{U}$. If either* $\operatorname{def}\mathcal{L}_+ < \infty$ *or* $\operatorname{def}\mathcal{L}_- < \infty$, *then $\mathcal{L}_+$ and $\mathcal{L}_-$ are a maximal nonnegative and a maximal nonpositive subspaces, respectively.*

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Higher eigenvalues for elliptic operators on Riemannian manifolds

Upper and lower estimates are given for heigher eigenvalues of certain second order elliptic operators on Riemannian manifolds, including Schrodinger operators. Some applications are shown, in particular, estimates of the stability index of minimal surfaces.

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On topological classification of Morse-Smale diffeomorphisms on three-manifolds

This report contains the results obtained by the author in cooperation with C. Bonatti, V. Medvedev E. Pécou (see [1]-[3]). Let $f\colon M \to M$ be a Morse-Smale diffeomorphism on a smooth closed orientable 3-dimensional manifold M. By S. Smale, a point $x \in M$ is called heteroclinic point if it belongs to intersection $W^u(p) \cap W^s(q)$ where p, q some saddle different periodic points of diffeomorphism f such that $dim\ W^u(p) = dim\ W^u(q)$. If p, q different saddle periodic points of diffeomorphism f such that $W^u(p) \cap W^s(q) \neq \varnothing$ and $dim\ W^u(p) \neq dim\ W^u(q)$. then a component of the intersection $W^u(p) \cap W^s(q)$ we call a *heteroclinic curve*.

Theorem. *Let M be a three-dimensional closed, connected, orientable manifold. There exists a Morse-Smale diffeomorphism without heteroclinic curve on M admitting k saddle periodic points and l sinks and sources if and only if M is the sphere if $k = l - 2$, or M is the connected sum of $(k - l + 2)/2$ copies of $S^2 \times S^1$.*

Next we solve the problem of topological conjugacy of Morse-Smale diffeomorphisms on M which do not admit any intersections of stable and unstable manifold of saddle periodic points (neither heteroclinic points, nor heteroclinic curves).

To each diffeomorphism f we associate an enriched graph of Smale $G(f)$ and for each sink ω, we define the *scheme* $S(\omega)$ which is a link of tori, Klein bottle and curves embedded in $S^2 \times S^1$. This scheme describes the topological position of the invariant unstable separatrices of saddle points which are contained in the basin of ω.

We show that two diffeomorphisms f_1 and f_2 are topologically conjugate if, and only if: 1) The corresponding graphs are equivalent. The equivalence is a conjugacy between the natural permutations induced by the dynamics f_1 and f_2 on the vertices and edges of the graphs. 2) Two corresponding sinks have the same schemes (up to diffeomorphism). 3) For any two corresponding saddles with one-dimensional unstable manifolds the corresponding pairs of curves associated to the separatrices are *concordantly embedded*. The research was partially supported by the INTAS grant 97-1843 and RFBR grant 99-01-00230

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Some imbedding theorems for weighted Sobolev spaces

Let $R^n_{++} = \{x : x \in R^n, x_i > 0, i = 1, \dots, n\}$, $\rho(x) = \sum\limits_{i=1}^{n} |x_i|^{1/a_i}$, $a_i > 0$, $i = 1, \dots, n$, $|a| = \sum\limits_{i=1}^{n} a_i$. Let ω be a positive measurable function defined on a domain R^n_{++}. Denote by $L_{p,\omega}(R^n_{++})$ the space of measurable functions on R^n_{++} with finite norm $\|f\|^p_{L_{p,\omega}(R^n_{++})} = \int_{R^n_{++}} |f(x)|^p \omega(x)dx$, $1 \leqslant p < \infty$. We define the *anisotropic Sobolev space* $W^{l_1,\dots,l_n}_{p,\omega_0,\omega_1,\dots,\omega_n}(R^n_{++})$, $l = (l_1, \dots, l_n) \geqslant 0$, $l_i \geqslant 0; i = 1, \dots, n$ the nonnegative integers, as the set of functions $f(x)$, $x \in R^n_{++}$, that have generalized derivatives $D^{l_j}_j f$, with and finite norm

$$\left\| f; W^{l_1,\dots,l_n}_{p,\omega_0,\omega_1,\dots,\omega_n}(R^n_{++}) \right\| = \|f\|_{L_{p,\omega_0}(R^n_{++})} + \sum_{j=1}^{n} \|D^{l_j} f\|_{L_{p,\omega_j}(R^n_{++})}.$$

Theorem. *Let* $k = (k_1, \dots, k_n)$, $l = (l_1, \dots, l_n) > 0$, $æ = (k, 1/l) \leqslant 1$, $(k + 1/p - 1/q, 1/l) = 1$, $1 < p \leqslant q < \infty$, $a = (a_1, \dots, a_n)$, $a_i = 1/l_i$, $i = 1, \dots, n$, *and let weight functions* $\omega, \omega_0, \omega_1, \dots, \omega_n$ *depend only on* $\rho(x)$. *Also, let the weight pairs* (ω_j, ω), $j = 0, 1, \dots, n$, *of monotone functions satisfies the following condition 1) or 2):*
1) ω *and* ω_j *are increasing functions on* $(0, \infty)$, *and*

$$\exists C > 0 : \ \forall \rho \in (0, \infty), \ \omega(\rho)^{p/q} \leqslant C \omega_j(\rho);$$

2) ω *and* ω_j *are decreasing functions on* $(0, \infty)$, *and*

$$\sup_{t>0} \left(\int_0^{t/2} \omega(\rho) d\rho \right)^{p/q} \left(\int_t^{\infty} \omega_j(\rho)^{1-p'} \rho^{-1-|a|p'/q} d\rho \right)^{p-1} < \infty.$$

Then for $æ \leqslant 1$ *the continuous imbedding*

$$D^k W^{l_1,\dots,l_n}_{p,\omega_0,\omega_1,\dots,\omega_n}(R^n_{++}) \hookrightarrow L_{q,\omega}(R^n_{++}). \tag{0.1}$$

is valid. Theorem 1 was proved by Yu.S.Nikol'skii [1] in the case $\omega_i(\rho) = \rho^{\alpha_i}(x)$, $i = 0, 1, \dots, n$.

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On the Green formula for nonlocal ellitic problems

While considering nonlocal elliptic problems in bounded n-dimensional ($n \geqslant 3$) domains in the case when support of nonlocal terms intersects with boundary of a domain, we must study some model nonlocal problems in plane angles (see [1, 2]). As an example let us consider the following model problem:

$$-\Delta u + u = f(y) \quad (y \in K), \tag{1}$$

$$u(y)|_{\gamma_1} + \alpha u(\mathcal{G}y)|_{\gamma_1} = g_1(y)\ (y \in \gamma_1), \quad u(y)|_{\gamma_2} = g_2(y)\ (y \in \gamma_2). \tag{2}$$

Here $K = \{y \in \mathbb{R}^2 :\ b_1 < \varphi < b_2\}$, $\gamma_i = \{y \in \mathbb{R}^2 :\ \varphi = b_i\}$ $(i = 1,\ 2)$; $-\pi < b_1 < 0 < b_2 < \pi$; $\mathcal{G}$ is the map given by $(\varphi,\ r) \mapsto (\varphi + |b_1|,\ \chi_1 r)$, where $(\varphi,\ r)$ are polar coordinates in $\mathbb{R}^2$, $\chi_1 > 0$. Let us introduce functional spaces, in which problem (1)–(2) is well defined. Denote by $E_a^l(K)$ the completion of the set $C_0^\infty(\bar{K}\backslash\{0\})$ with respect to the norm $\|u\|_{E_a^l(G)} = \big(\sum\limits_{|\beta|\leqslant l}\int\limits_K |y|^{2a}(|y|^{2(|\beta|-l)}+1)|D^\beta u|^2\big)^{1/2}$. Denote by $E_a^{l-1/2}(\gamma)$ the space of traces on a ray $\gamma \subset \bar{K}$ with the norm $\|\psi\|_{E_a^{l-1/2}(\gamma)} = \inf \|u\|_{E_a^l(K)}$ $(u \in E_a^l(K) :\ u|_\gamma = \psi)$. We study solutions $u \in E_a^2(K)$ for problem (1)–(2) supposing that $(f,\ g_1,\ g_2) \in E_a^0(K) \times E_a^{3/2}(\gamma_1) \times E_a^{3/2}(\gamma_2)$. In order to establish necessary and sufficient conditions of the Fredholm solvability for problem (1)–(2), we consider an auxiliary nonlocal problem with the parameter λ for an ordinary differential equation

$$\tilde{u}_{\varphi\varphi} - \lambda^2\tilde{u} = 0 \quad (\varphi \in (b_1,\ b_2)), \tag{3}$$

$$\tilde{u}(\varphi)|_{\varphi=b_1} + \alpha e^{i\lambda \ln \chi_1}\tilde{u}(\varphi + |b_1|)|_{\varphi=b_1} = 0, \quad \tilde{u}(\varphi)|_{\varphi=b_2} = 0. \tag{4}$$

Theorem. *Model nonlocal problem (1)–(2) is Fredholm iff the line* $\operatorname{Im}\lambda = a - 1$ *contains no eigenvalues of auxiliary problem (3)–(4).* The proof of this result is based on studying the following nonlocal transmission problem:

$$-\Delta v_i + v_i = f_i(y) \quad (y \in K_i;\ i = 1,\ 2), \tag{5}$$

$$\begin{gathered} v_1(y)|_{\gamma_1} = g_1(y)\ (y \in \gamma_1),\ v_2(y)|_{\gamma_2} = g_2(y) \quad (y \in \gamma_2), \\ v_2(y)|_\gamma - v_1(y)|_\gamma = h_1(y), \quad \frac{\partial v_2}{\partial n}(y)|_\gamma - \frac{\partial v_1}{\partial n}(y)|_\gamma - \alpha\chi_1^{-1}\frac{\partial v_1}{\partial n_1}(\mathcal{G}^{-1}y)|_\gamma = h_2(y), \end{gathered} \tag{6}$$

where $y \in \gamma$. Here $K_1 = \{y \in \mathbb{R}^2 :\ b_1 < \varphi < 0\}$, $K_2 = \{y \in \mathbb{R}^2 :\ 0 < \varphi < b_2\}$; $\gamma = \{y \in \mathbb{R}^2 :\ \varphi = 0\}$; n_1 and n are the normals to γ_1 and γ directed inside K_1 and K_2 correspondingly. Problems (1)–(2) and (5)–(6) are formally adjoint. This means that for any $u \in E_a^2(K)$ satisfying homogeneous conditions (2) and $v_i \in E_{-a+2}^2(K_i)$ $(i = 1,\ 2)$ satisfying homogeneous conditions (6) we have $\sum\limits_{i=1}^{2}\int\limits_{K_i}(-\Delta u + u)\bar{v}_i\,dy = \sum\limits_{i=1}^{2}\int\limits_{K_i} u(-\Delta\bar{v}_i + \bar{v}_i)\,dy$. Further, it can be shown that 1) kernals of problems (1)–(2) and (5)–(6) are of finite dimension and 2) dimension of cokernal for problem (1)–(2) is equal to dimension of kernal for problem (5)–(6). This implies the Theorem. All the results are obtained for arbitrary elliptic differential operators of order $2m$ and general boundary conditions with a finite number of nonlocal terms, which may have quite a complicated form.

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Boundary properties of the solution of the Dirichlet problem for an elliptic equation and its applications.

This research is a natural continuation of works [1,2]. Let u is a solution of the following Dirichlet problem

$$\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{i,j}(x)\frac{\partial u}{\partial x_j}\right) = 0, \qquad x \in Q, \tag{1}$$

$$u\,|_{\partial Q} = u_0, \qquad u_0 \in L_2(\partial Q),$$

for uniformly elliptic equation in bounded domain $Q \subset \mathbf{R}_n$ with a smooth boundary ∂Q. We suppose, that the normal (outward and unit) to ∂Q is Dini-continuous, the coefficients $a_{ij} = a_{ji}$ are measurable and bounded in Q and they are Dini-continuous on the boundary (see [1]). We consider the case, in which index number (exponent of smoothness) $\alpha > 0$ is sufficiently small for simplicity.

Theorem. *Let measure ϕ supported in $\bar{Q} \times \bar{Q}$ satisfies the condition there exists constant C_0 such that*

$$\iint_{\{(x,y):|x-x^0|\leqslant r,\, y\in Q\}\cup\{(x,y):x\in Q,\, |y-x^0|\leqslant r\}} [\max\{|x-y|, r(x), r(y)\}]^{-2\alpha}\phi(x,y) \leqslant C_0 r^{n-1},$$

for any $x_0 \in \partial Q$ and $r > 0$;$r(x) = \mathrm{dist}\{x, \partial Q\}$. Then the estimate

$$\iint_{\bar{Q}\times\bar{Q}} \frac{|u(x)-u(y)|^2}{|x-y|^{2\alpha}}\,d\phi(x,y) \leqslant \mathrm{const}\|u_0\|^2_{L_2(\partial Q)},$$

holds for an arbitrary $u_0 \in L_2(\partial Q)$ with a constant independent on u_0 and ϕ.

This property is applied to investigation of the solvability of non-local problems for an elliptic equation.

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The KdV equation on a half-line

An initial boundary value problem for the Korteweg-de Vries equation is considered on a half-line with zero boundary conditions at the origin and with arbitrary smooth initial values vanishing rapidly enough. The problem is effectively integrated by means of the inverse scattering method when the associated linear problem has no discrete spectrum. In this case the global solvability theorem is proved. A kind of large time asymptotics of the solution is discussed. For instance, at the end $x = 0$ the first derivative decays as

$$u_x(0,t) = 1/t(1+o(1)) \quad \text{for} \quad t \to \infty.$$

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Wedderburn type theorems in traditional and 'quantized' functional analysis

From the time of von Neumann there was an interest to conditions on a given operator algebra that are equivalent (or closely related) to the existance of what could be reasonably called the Wedderburn structure, on the lines of the classical Wedderburn theorem for semisimple finite-dimentional algebras. The homological approach to this circlė of questions, inherited from algebra, lead us to the study of the condition of the so-called spatial projectivity of a given algebra. It was shown (1994) that spatially projective von Neumenn algebras form a class similar to that of Wedderburn algebras, but containing a little bit less algebras. Afterwards, the methods of 'quantized' functional analysis provided the respective 'quantized' version of the mentioned condition. It turned out (1999) that quantum spatially projective von Neumenn algebras are exactly those with the Wedderburn structure. Quantum spatially projective operator C^*-algebras and quantum projective Hilbert modules over C^*-algebras can be also completely described.

Hryniv R. O
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Sturm-Liouville operators with singular potentials

In this talk, we discuss some properties of Sturm–Liouville operators given by the formal differential expression

$$lf := -\frac{d^2 f}{dx^2} + qf$$

with complex-valued singular potentials $q \in W_2^{-1}(0,1)$. In general, there are many operators that can be associated to l and, say, the Dirichlet boundary conditions. However, it was shown in [1] that among them there exists a distinguished operator T that is the uniform resovent limit of Sturm-Liouville opeartors with regular potentials. Some properties of the operator T were thoroughly studied in [1] and [2]; in particular, it was proved that T has a compact resolvent and asymptotics of its eigenvalues and eigenfunctions were found. Our main objective is to study further properties of the operator T. We show that T is similar to a rank two perturbation of the potential-free Sturm-Liouville operator.

Theorem. *The operator T with $q \in W_2^{-1}(0,1)$ is similar to an operator M_p defined by $(M_p f)(x) = -f'' + p_1(x)f'(1)$ on the domain $\mathfrak{D}(M_p) = \{f \in H^2(0,1) \mid y(0) + \int_0^1 p_2(t)y(t)\,dt = 0,\ y(1) = 0\}$ with $p_1, p_2 \in L_2(0,1)$. The similarity is performed by the operator $I + K$, where*

$$(Kf)(x) := \int_x^1 k(x,t)f(t)\,dt$$

is an integral Volterra operator. Moreover, either p_1 or p_2 can be made identically zero.

As one of numerous corollaries of this result we get that for the operator T with $q \in W_2^{-1}(0,1)$ all but finitely many eigenvalues are semisimple and the corresponding set of normalized eigen- and associated functions forms a Bari basis of $L_2(0,1)$ (i.e., a basis that is quadratically close to an orthonormal one).

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I.G.Petrovskii: from a student to the rector

The entire Ivan Georgievich Petrjvsky's path "From the student up to the rector" can conditionally be divided into three parts. During each of them he got acquainted with certain particulars of life of the Moscow university and of the country, as a whole, he acquired experience and personal qualities, which subsequently became a basis of his "rector's credo".

The years of 1922-1930. These are student's and post-graduate's years of I.V.Petrovsky. During the twentieth years the ideology did not interfere with natural sciences. Authorities aspired to attract all experienced experts - those who descended from old intelligentsia to the side of the Soviet state (this policy discontinued in 1928 in connection with so called "Shakhty's Case"). The situation at Moscow university and at its physico-mathematical faculty was rather quiet. All this has in the end allowed I.V.Petrovsky to successfully complete his student's years obtaining high-quality professional mathematical education and to take firm decision on his further carrier of a scientific worker and a higher school teacher. Out of numerous facts and events of the 1920-th, which have rendered or could render perceptible influence on I.V.Petrovsky's formation as a person and as the future rector, it is worth to pay special attention to activities of the former Moscow university rectors V.P.Volgin, A.Ya.Vishinsky, I.D.Udaltsov; to organization of the scientific-research institute of mathematics and mechanics under the 1-st MSU and D.F.Egorov's fate.

The years of 1930-1940. During these years I.V.Petrovsky develops as an original, distinguished mathematician and a university teacher. In the second edition of the book "Universities and scientific establishments" printed in 1935 it was stated: "As one of the significant phenomenon of last years we should rather recognize extremely great works of A.N.Kolmogorov and I.V.Petrovsky on analytical methods of the theory of probabilities related to partial differential equations of the second order, in particular a deep analysis of the thermal conductivity equation carried out by I.V.Petrovsky who has brought a number final results to this classical task". The process of I.V.Petrovsky's personal making coincided with the realization of scale and radical changes in the structure of scientific and educational establishments of the country. At the same time, the beginning of the 30-th was the period of heavy struggle for preservation of the Moscow university, which had being led by the rectors V.N.Kassatkin and A.S.Boutyagin. I.V.Petrovsky had an opportunity to witness and to analyze relations between the university, on the one hand, and all the higher authorities, on the other. It was the time when almost ten years' period of fast quantitative growth of higher school had been completed. A large technical and engineering sector had been created in it that had caused fast and wide growth of education in the fields of natural sciences and especially of mathematics. The task of training national technical and engineering staff for the country had been solved. The function was defined for universities to become centers of training scientific personnel. A system of their state certification had been introduced. A number of important events has occurred in the life of the Moscow university during this very decade including final approval of its principle structure based on faculties, creation and consequent dismissal of Moscow institute of a history and philosophy (MIPHLI); assignment of a name of M.V.Lomonosov to the university, approval of the Charter MSU in 1939. I.V.Petrovsky - was a witness and a direct participant of these processes. At the same time a unified system of the USSR Academy of Sciences was made out. In second half of the 30-th years I.V.Petrovsky runs into such a phenomenon, as a rise of scientific and political opposition between various groups of soviet scientists, including opposition between scientists of the Academy of Sciences and those of the Moscow university. Then came sharp political and ideological confrontations that accompanied conflicts arisen here (the most notorious of them was so called "The Case of the academician N.N.Luzin" and annihilation "of a historical school of M.N.Pokrovsky"). I.V.Petrovsky confronted directly with these phenomena later, when in 1953 a group of employees of the physical faculty accused him of discrediting "scientists-communists".

The years of 1940-1950. This interval of time is especially important. It was during Great Patriotic war and immediately on its ending that "an administrative self-determination" of I.V.Petrovsky took place. In the system of higher school he was the dean of the MSU faculty of mechanics and mathematics; in the system of the USSR Academy of Sciences he was the deputy director the leading academic scientific establishment- the V.A.Steklov's Mathematical institute of the USSR Academy of Sciences, and the Academician-secretary of the Branch of physico-mathematical sciences of the Academy. The accumulation of administrative and political experience by I.V.Petrovsky took place in the most extreme conditions - the war was going on. His first managing administrative work was connected to evacuation (begun on October 14, 1941) and re-evacuation (completed by June 10, 1943) of the Moscow university. At the end of the 1940-th I.V.Petrovsky becomes already known, authoritative and independent administrator. He closely cooperates with leaders of the USSR Academy of Sciences S.I.Vavilov, A.V.Topchiev, rectors of Moscow university I.S.Galkin and especially with A.N.Nesmeyanov. The post of the rector of Moscow university was included into the Politburo of CPSU's nomenclature. After A.N.Nesmeyanov had been elected as the President of the USSR Academy of Sciences in 1951 a consent between the higher party, state and academic hierarchies was reached to assign I.V.Petrovsky to the post of the MSU rector.

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On the nonlocal investigation of bifurcations for a class of nonlinear elliptic equations

Let F_λ be a real functional on Banach space W. We discuss the problem of the calculation of *sufficient and necessary interval* $(\lambda_j, \lambda_{j+1}) \subseteq \mathbb{R}$ for the existence of the solution for the equation $F_\lambda(u) = 0$ on W when $\lambda \in (\lambda_j, \lambda_{j+1})$ and nonexistence one when $\lambda \in \mathbb{R} \setminus (\lambda_j, \lambda_{j+1})$. Generally, the calculation of the points λ_j in linear cases is a subject of Spectral Theory. We present a method for constructive calculation of sufficient and necessary intervals in nonlinear cases through the following example of a class of nonlinear elliptic equations

$$(1) \qquad -\Delta_p u = \lambda |u|^{p-2}u + f(x)|u|^{\gamma-2}u, \;\; u > 0 \text{ in } \Omega,$$

$$(2) \qquad u = 0 \text{ on } \partial\Omega,$$

where Ω is bounded and smooth domain in $\mathbb{R}^n$, Δ_p denotes the p-Laplacian, $1 < p < \gamma \leqslant p^*$, where $p^* = np/(n-p)$ if $p < n$ and $p^* = +\infty$ if $p \geqslant n$. We suppose that the problem (1)-(2) has indefinite nonlinearity, i.e. $f(x)$ may changes sign. Define $C_0^1 = \{w \in C^1(\overline{\Omega}) | w > 0 \text{ in } \Omega \text{ and } \partial w/\partial\nu < 0, w = 0 \text{ on } \partial\Omega\}$, where ν is the outward normal of $\partial\Omega$.

Let λ_1 be a first eigenvalue of $-\Delta_p$ in $W_0^{1,p}$ and ϕ_1 eigenfunction associated with λ_1. Define

$$\lambda^* = \sup_{u\in C_0^1} \{\inf_{\phi}\{\frac{\int |\nabla u|^{p-2}(\nabla u, \nabla\phi)}{\int |u|^{p-2}u\phi} | \int f(x)|u|^{\gamma-2}u\phi \geqslant 0, \phi \geqslant 0, \phi \in C_0^\infty(\Omega)\}.$$

Our main result on an least upper bound for the sufficient and necessary interval of the existence of the positive solution for (1)-(2) is the following:

Theorem. *Let $1 < p < \gamma \leqslant p^*$, $f \in C(\Omega)$. Then*
(i) $\lambda^* < +\infty$ iff $\Omega_0 = \{x \in \Omega | f(x) \geqslant 0\} \neq \varnothing$. Moreover the problem (1)-(2) has no positive solution if $\lambda > \lambda^*$.
(ii) $\lambda_1 < \lambda^*$ iff $F(\phi_1) < 0$. Moreover $\forall\lambda \in (\lambda_1, \lambda^*)$ there exists a positive solution u_λ of the problem (1)-(2).

Remark. It is well known that in subcritical cases $1 < p < \gamma < \gamma^*$ there exists a positive solution of (1)-(2) for all $\lambda \in (-\infty, \lambda_1]$.

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Justification of the asymptotics by the separation of variables method at large times.

Let us consider the system:

$$\begin{cases} \dot{I} = \varepsilon g_1(I, \varphi, \varepsilon) \\ \dot{\varphi} = \omega(I) + \varepsilon g_2(I, \varphi, \varepsilon), \end{cases}$$

where g_1, g_2 are indefinitely differentiable, periodic in φ functions. To get an asympthotic solution of the system, we apply the separation of variables method [see, for example, [1]] which consists in elimination of the fast variable from the first equation. In the lecture it is proved that in the case when the limiting problem is stable the approached solution received by this method, approximates the required solution of an initial poblem for any large values of time The system $\begin{cases} \dot{I} = \varepsilon F(I, \varphi) \\ \dot{\varphi} = \omega(I) + \varepsilon G(I, \varphi) \end{cases}$ $I(0) = 1; \varphi(0) = 0$, is considered. Functions F, G, ω are indefinitely differentiable, periodic in φ functions, bounded on I. For F the condition of stability is fulfilled: $\int\limits_0^{2\pi} \frac{\partial F(J,\psi)}{\partial J} d\psi = -\gamma < 0$

Theorem. Under the given conditions for any given, $k, p \in N$ it is possible to choose number $n \in N$, such that partial sum of constructed asymptotic series received by the separation of variables method will differ from the exact solution by $O(\varepsilon^{p+1})$ for any $t \in [0, \varepsilon^{-k}]$.

The proof consists in check of boundedness of the solution for all t, construction asymptotic solution on phase plane. Further, estimation is carried on a plane (t, x).

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Граничное управление процессом колебаний, описываемым уравнением $K(x)[K(x)u_x(x,t)]_x - u_{tt} = 0$

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On treatment of quantum chaos by chaos degree

There exist several approaches in the study of chaotic behavior of dynamical systems. But these concepts are rather independently used in each field. In 1991, M.Ohya proposed Information Dynamics (ID for short) to treat such chaotic behavior of systems altogether. A chaos degree to measure the chaos in classical dynamical systems is defined by mea ns of a complexity in ID. In particular, among several chaos degrees, the en tropic chaos degree is applied to some dynamical systems [1]. In this talk, we study the chaotic behaviour and the quantum-classical correspondence for the baker's transformation. The chaos degree is computed and it is demonstra ted that it describes the chaotic features of the model. The correspondence between classical and quantum chaos degrees is considered. This talk is main ly based on the joint paper [2].

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On operators collinear to the uniformly J-nonexpansive ones

We consider linear operators acting in Krein space H :

$$H = H_+ \oplus H_-, \quad H_\pm = P_\pm H, \quad P_\pm^2 = P_\pm^* = P_\pm, \quad P_+ + P_- = I,$$

with an indefinite metric $[x,y]=(Jx,y)$, $J=P_+-P_-$, $x,y\in H$. An operators are not supposed to be bounded, and they may be defined on arbitrary linear manifold of Krein space H.

Definition 1. *A linear operator V is called uniformly J-nonexpansive (with the constant $\delta>0$)), if*

$$[Vx,Vx]\leqslant[x,x]-\delta\|x\|^2\quad\forall x\in D_V$$

(D_V denotes the domain of the operator V).

Definition 2. *The symbol K_α ($\alpha\geqslant 0$) will denote the following set of vectors of Krein space H :*

$$K_\alpha=\{x\in H\mid\quad \alpha\|P_+x\|^2\geqslant\|P_-\|^2\}.$$

It is easy to see, that $K_0=H_+$, and that for every $\alpha<1$ the set K_α is uniformly positive, i.e. there exists the number $\gamma=\gamma(\alpha)>0$ such that

$$[x,x]\geqslant\gamma\|x\|^2\quad\forall x\in K_\alpha.$$

Definition 3. *For an arbitrary linear operator T, $D_T\supset L$, where $L\neq\{0\}$ is an arbitrary linear manifold of the space H, we introduce the following numbers:*

$$\omega_-(T|L)=\inf_{x\in L,x\neq 0}\frac{[Tx,Tx]}{\|x\|^2}\quad и\quad \omega_+(T|L)=\sup_{x\in L,x\neq 0}\frac{[Tx,Tx]}{\|x\|^2}.$$

Theorem 1. *Let*

$$T|L=\lambda\cdot(V|L),$$

where $\lambda\in C$, $\lambda\neq 0$, the operator $(V|L)$ is uniformly J-nonexpancive with the constant $\delta>0$, and let the condition

$$\omega_-(T|L)>-|\lambda|^2\cdot(1+\delta). \tag{0.1}$$

fulfils. Then

$$L\subset K_\alpha,\quad where\quad \alpha=\frac{|\lambda|^2(1-\delta)-\omega_-(T|L)}{|\lambda|^2(1+\delta)+\omega_-(T|L)}.$$

Theorem 2. *If under the conditions of Theorem 1 instead of inequality (0.1) the inequality*

$$\omega_-(T|L)>-|\lambda|^2\delta, \tag{0.2}$$

holds, then the linear manifold L is uniformly positive with the constant

$$\gamma=\frac{1}{|\lambda|^2}[\omega_-(T|L)+|\lambda|^2\delta](>0).$$

Theorem 3. *Let $L\neq\{0\}$ be a uniformly positive linear manifold, $L\subset D_T$, where T is a linear operator, and let the condition*

$$\omega_+(T|L)<+\infty$$

is realized. Then the operator $(T|L)$ is collinear to some uniformly J-nonexpansive operator. In particular, under the additional condition

$$\omega_+(T|L) < \epsilon_-(L), \tag{0.3}$$

where

$$\epsilon_-(L) = \inf_{x\in L, x\neq 0} \frac{[x,x]}{\| x \|^2},$$

$(T|L)$ is itself uniformly J-nonexpansive operator with the constant

$$\delta = \epsilon_-(L) - \omega_+(T|L)(> 0).$$

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Numerical modeling of derivative nonlinear Schrödinger equation

The problem of our interest is given by the derivative nonlinear Schrödinger equation

$$\frac{\partial u}{\partial t} = i\frac{\partial^2 u}{\partial x^2} + ic_3|u|^2u + ic_5|u|^4u + \alpha|u|^2\frac{\partial u}{\partial x} + \beta u\frac{\partial |u|^2}{\partial x}, \tag{0.1}$$

where $u = u(x,t)$ is unknown complex function, $(x,t) \subset Q = (0,1) \times (0,T]$, and c_j, α, β are real constants. The information on the applications of the model (0.1) can be found in [1]. In computer simulations, the initial boundary-value problem is dealt with. We consider boundary conditions of two different types, both coming from physical models. Namely, Dirichlet boundary conditions

$$u(0,t) = u(1,t) = 0, \qquad 0 \leqslant t \leqslant T, \tag{0.2}$$

or periodic ones

$$u(0,t) = \theta u(1,t), \qquad \left.\frac{\partial u}{\partial x}\right|_{x=0} = \theta\left.\frac{\partial u}{\partial x}\right|_{x=1} \qquad 0 \leqslant t \leqslant T. \tag{0.3}$$

are assumed. The complex parameter (phase shift) θ is such that $|\theta| = 1$. In this note we provide with the main results (convergence and stability), proved in [2], about two-step algorithm, proposed to solve the above models. We also confirm these results by numerical experiment. In the first step, we employ some invertible transformation [2], which can be computed to the required precision numerically, to obtain the system of two equations which, redenoting the solution u, appears as

$$\frac{\partial u}{\partial t} = i\frac{\partial^2 u}{\partial x^2} + f_1(u, u^*, v, v^*),$$
$$\frac{\partial v}{\partial t} = i\frac{\partial^2 v}{\partial x^2} + f_2(u, u^*, v, v^*), \tag{0.4}$$

here f_1 and f_2 are polynomials. Note that, by the applied transformation, we have the following relation between u and v:

$$v = \frac{\partial u}{\partial x} + i\frac{\alpha}{2}|u|^2 u. \tag{0.5}$$

The boundary conditions (0.2) transform to

$$u(0,t) = u(1,t) = 0, \qquad \left.\frac{\partial v}{\partial x}\right|_{x=0} = \left.\frac{\partial v}{\partial x}\right|_{x=1} = 0. \tag{0.6}$$

and the transformation of (0.3) boundary conditions is

$$u(0,t) = \tilde{\theta}u(1,t), \qquad \left.\frac{\partial u}{\partial x}\right|_{x=0} = \tilde{\theta}\left.\frac{\partial u}{\partial x}\right|_{x=1},$$
$$v(0,t) = \tilde{\theta}v(1,t), \qquad \left.\frac{\partial v}{\partial x}\right|_{x=0} = \tilde{\theta}\left.\frac{\partial v}{\partial x}\right|_{x=1}, \tag{0.7}$$

here $|\tilde{\theta}| = 1$. Then, as a second step, we apply Crank–Nicolson finite difference scheme to solve (0.4), (0.6) or (0.4), (0.7) numerically. To linearize obtained difference problem we employ iterative method and prove its convergence [2]. Denote p, q to be the solutions of difference scheme and $\varepsilon = u - p$ and $\delta = v - q$ to be the error functions.

Theorem 1. (Convergence, Dirichlet problem). *Suppose that there exist the unique solutions $u, v \in C^{4,3}$ of (0.4), (0.6). Then there exist constants $\tau_0, h_0 > 0$ such that, if $\tau \leqslant \tau_0$, $h \leqslant h_0$, then there exists the unique solution p, q of finite difference scheme and*

$$\|\varepsilon\|_{C,h} + \|\delta\|_{C,h} = O(\tau + h), \qquad \tau, h \to 0. \tag{0.8}$$

Theorem 2. (Convergence, periodic problem). *Suppose that there exist the unique solutions $\partial u/\partial x$, $\partial v/\partial x \in C^{4,3}$ of (0.4), (0.7). Then there exist constants $\tau_0, h_0 > 0$ such that, if $\tau \leqslant \tau_0$, $h \leqslant h_0$, then there exists the unique solution p, q of finite difference scheme and*

$$\|\varepsilon\|_{H^1,h} + \|\delta\|_{H^1,h} = O(\tau^2 + h^2), \quad \tau, h \to 0. \tag{0.9}$$

Theorem 3. (Stability). *For any of the considered initial boundary-value problems, Let p_1, q_1 be the solution of the difference scheme with initial functions $u_1^{(0)}(x)$, $v_1^{(0)}(x)$. Let also p_2, q_2 be the solution of the same difference problem with another initial functions $u_2^{(0)}(x)$, $v_2^{(0)}(x)$. Then there exist constants $\tau_0, h_0 > 0$ such that, if $\tau \leqslant \tau_0$, $h \leqslant h_0$, then the following estimate hold:*

$$\|p_1 - p_2\|_{H^1,h} + \|q_1 - q_2\|_{H^1,h} \leqslant c_S\left(\|u_1^{(0)} - u_2^{(0)}\|_{H^1,h} + \|v_1^{(0)} - v_2^{(0)}\|_{H^1,h}\right). \tag{0.10}$$

Constant c_S does not depend on the grid steps τ and h.

Remark 1. Due to the imbedding $H^1 \to C$, by Theorem 2, the convergence in C norm follows with the same rate as in (0.9). Also, by Theorem 3, the stability in C norm follows.

Remark 2. Due to (0.5), the proposed two-step numerical algorithm converges and is stable in C^1 norm.

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Extreme networks in normalised spaces

Criteria for networks exremality with respect to the length functional in a normalised space are presented. Considered both cases of deformations preserving and changing the topolofy of the network.

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Control Systems governed by PDE and exponential families[14]

Let us consider the control system

$$\dot{y}(t) = Ay(t) + Bu(t),\ y(0) = 0,$$

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where A is a (differential) operator in H with eigenfunctions φ_n and eigenvalues λ_n; u is a control

$$u(t) \in \mathcal{U}, \quad u(\cdot) \in U = L^2(0,T;\mathcal{U}).$$

One of the possible of statements of control problems connected with this system is to describe the reachibility set $R(T)$ — the set of all final states $y(T) = y^u(T)$ when u runs the whole control space. Using the Fourier method we reduce the control problem to a moment one relative to the exponential family $\mathcal{E} := \{B^*\varphi_n e^{\lambda_n t}\}$. This approach reveals the deep connections between properties of the control system and properties of $\mathcal{E}$ [1]. In particular, under natural conditions, $R(T) = H$ *iff* $\mathcal{E}$ *forms a Riesz basis in its span in* $L^2(0,T;\mathcal{U})$; $R(T)$ *contains all eigenmodes* φ_n *iff* $\mathcal{E}$ *is minimal in* $L^2(0,T;\mathcal{U})$; These connections allow us to involve the well developed theory of exponential families in order to find $R(T)$. Let us present three examples, which use this connection in both directions.
(i) Let we have the heat equation

$$y_t = \Delta y + \sum_1^N b_j(x)u_j(t), x \in \Omega,$$

where b_j are fixed functions, u_j — controls. In this case $\mathcal{E} = \{\eta_n e^{i\lambda_n t}\}$, $\eta_n \in \mathbb{C}^N$. It is known [1] that $\mathcal{E}$ is minimal in $L^2(0,T;\mathbb{C}^N)$ only if the spectrum $\{\lambda_n\}$ is separated: $\inf_{m\neq n}|\lambda_n - \lambda_m| > 0$. In view of the Weil asymptotics of λ_n, *for* $dim\Omega > 1$ *the reachibility set of the system does not contain all eigenmodes* φ_n (ii) Let we have the wave equation

$$y_{tt} = \Delta y, x \in \Omega, y|_{t=0} = y_0, \ y_t|_{t=0} = y_1, \quad y|_{\partial\Omega} = u.$$

It is well known that the system can be steered to the rest in time $T > 2diam\,\Omega$ for any initial state $\{y_0, y_1\} \in W_2^1 \oplus L^2$ by the control u^T. This implies that the family $\mathcal{E} = \{\frac{\partial}{\partial n}e^{\pm i\sqrt{\lambda_n}t}\}$ forms a Riesz basis in its span in $L^{(}0,T;\partial\Omega)$. Using this fact it was proved that the L^2–norm of u^T decreases as $1/\sqrt{T}$ while the $L^1([0,T];\partial\Omega))$–norm of u^T is approximately constant [2]. (iii) Let we have a gibrid system (coupled beam and string)

$$\frac{\partial^2}{\partial t^2}u_1 - \frac{\partial^2}{\partial x^2}u_1 + Au_1 + Bu_2 = 0 \qquad in(0,\pi)\times\mathbb{R},$$

$$\frac{\partial^2}{\partial t^2}u_2 + \frac{\partial^4}{\partial x^4}u_2 + Cu_1 + Du_2 = 0 \qquad in(0,\pi)\times\mathbb{R},$$

$$u_1 = u_2 = \frac{\partial^2}{\partial x^2}u_2 = 0 \qquad for x = 0,\pi,$$

$$u_1 = y_1 \in H_0^1(0,\pi), \ \frac{\partial}{\partial t}u_1 = y_2 \in L^2(0,\pi), \qquad for t = 0,$$

$$u_2 = \frac{\partial}{\partial t}u_2 = 0 \qquad for t = 0$$

(A, B, C, D are constants). The problem is to prove the partial observability

$$\left\|\frac{\partial}{\partial x}u_1(0,t)\right\|^2_{L^2(0,T)} \succ \|y_0\|^2_{H^1} + \|y_1\|^2_{L^2}. \tag{0.1}$$

It means that we can recover the initial state via the observation $\|\frac{\partial}{\partial x}u_1(0,t)\|^2_{L^2(0,T)}$ during the time T. Also using the Fourier method, C. Baiocchi, V. Komornik, and P. Loreti [3] have proved (0.1) for $T > 4\pi$ (for almost all 4-tuples (A,B,C,D)). The authors conjectured that the system is probably partially observed even for $T > 2\pi$. In [4] this conjectured has been proved. The point is that the spectrum is not separated, what complicates the study of the system, and it was suggested to use so called divided differences of exponentials (functions of the form $e^{\mu t}$, $(e^{i\mu t} - e^{i\lambda t})/(\mu - \lambda)$ etc.).

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A local version of Riemann-Roch theorem on a compact complex manifold and symbolic calculus

Let M be a n dimensional compact complex manifold, and let $L_q = \bar\partial_{q-1}\bar\partial^*_{q-1} + \bar\partial^*_q\bar\partial_q$ be the Laplacian actiong on differential $(0,q)$-forms $A^{(0,q)}(M) = \Gamma(\wedge^{(0,q)}T^*(M))$. Then Riemann-Roch theorem states as follows:

$$\sum_{q=0}^{n}(-1)^q \dim H_q = \int_M (2\pi i)^{-n}[Td(TM)]_{2n} \tag{1}$$

where H_q is the set of all harmoniq $(0,q)$-forms for L_q and $Td(TM)$ is the Todd class defined as

$$Td(TM) = \det(\frac{\Omega}{e^{\Omega}-1}) \tag{2}$$

with the curvature form Ω. Analytical prooves of the above thorem is based on

$$\sum_{q=0}^{n}(-1)^q \dim H_q = \int_M \sum_{q=0}^{n}(-1)^q \operatorname{tr} e_q(t,x,x)dv_x, \tag{3}$$

where $tre_q(t,x,x)$ is the trace of the kernel $e_q(t,x,x)$ of the fundamental solution $E_q(t)$ of the following heat equation;

$$\begin{cases} (\frac{\partial}{\partial t} + L_q)E_q(t) & = 0 \text{ in } (0,T) \times M, \\ E_q(0) & = I \text{ in } M. \end{cases}$$

So, an analytical proof for Riemann-Roch theorem is complete, if the following equation holds;

$$\int_M \sum_{q=0}^{n} (-1)^q \operatorname{tre}_q(t,x,x)dv = \int_M (2\pi i)^{-n}[Td(TM)]_{2n}$$

We call the following equation "a local version of Riemann-Roch theorem "

$$(4) \qquad \sum_{q=0}^{n} (-1)^q \operatorname{tre}_q(t,x,x)dv_x = (2\pi i)^{-n}[Td(TM)]_{2n} + O(t^{\frac{1}{2}}).$$

We shall give a rough sketch of a proof of the above formula (4), constructing the fundamental solution according to the method of symbolic caluculus for a degenerate parabolic operator instead of that of a parabolic operator. Our point is that we can prove the above formula by only calculating the main term of the fundamental solution, introducing a new weight of symbols of pseudodifferential operators. In this paper, we study also "a local version of Riemann-Roch theorem " under the condition that M is a compct complex manifold. In this situation, does "a local version of Riemann-Roch theorem " hold? If this answer is negative, the next problem is to characterize manifolds where "a local version of Riemann-Roch theorem " holds. Our results about this problem is following. Let Φ be the Kaehler form of M. We remark that a complex manifold M is a Kaehler if and only if $d\Phi = 0$. **Theorem**

(1) If $\partial\bar{\partial}\Phi \neq 0$ and n is even, then we have

$$\sum_{q=0}^{n} (-1)^q \operatorname{tre}_q(t,x,x)dv_x = (2\pi)^{-n}(-1)^{\frac{n(n-1)}{2}} \frac{(i\partial\bar{\partial}\Phi)^{\frac{n}{2}}}{(\frac{n}{2})!} t^{-\frac{n}{2}} + O(t^{-\frac{n}{2}+\frac{1}{2}}).$$

(2)If $\partial\bar{\partial}\Phi = 0$, then we have

$$\sum_{q=0}^{n} (-1)^q \operatorname{tre}_q(t,x,x)dv_x = O(1).$$

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Estimating measure of dynamical systems with Sinai-Ruelle-Bowen measures

We consider one parameter families of non-hyperbolic dynamical systems and estimate measure of parameter values such that respective systems have attractors with Sinai-Ruelle-Bowen measures.

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Gas–Solid Phase Transition Problem

The classical Stefan problem is concerned with phase transitions in an immovable substance. Meanwhile, a lot of phase changes occurs in the presence of hydrodynamic flow in a liquid phase. The interest in the study of these processes is motivated by numerous technological applications. Phase transitions with convection have been considered for incompressible fluids. However, the role of compressibility phenomenon still requires a basic study. In [1] we propose a new approach to liquid–solid phase transitions within the frame of complete Navier-Stokes model for a liquid (gas) phase, which takes into account such properties of liquid as compressibility, viscosity and heat conductivity. Concerning a solid phase, we suppose that
(i) the solid phase is immovable;
(ii) the density of the solid phase depends only on space variables;
(iii) the heat equation governs the solid phase. The accepted assumptions allow us to formulate the initial-boundary value problem which describes a phase transition between viscous gas (or liquid) and solid including all conservation laws on the phase boundary. The classical Stefan problem is the particular case of this new problem when a liquid phase is at rest and the densities of both liquid and solid phases are the same constants. The local existence and uniqueness of a smooth solution to the related one-dimensional problem are proved. In [2] we neglect the terms with derivative of the velocity with respect to the space variable in the heat equation for the liquid phase. Global existence and uniqueness of one-dimensional smooth solutions are proved for some class of boundary and initial data providing a monotony of a free boundary. The work was supported by Russian Fund for Basic Researches (grant code 00-01-00911).

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On equality to unit of weight norms of Riesz projectors

Let T is the operator with the symbol $(\zeta_1/|\zeta|, \dots \zeta_n/|\zeta|)$ – vector Riesz transform on the functions on R^n, and T' is its adjoint. Let $T^{t,s} = T^t(T')^s$, $s+t$ is even, degrees are in tensor sense. We consider the action of $T^{t,s}$ on the space $L_{2,a} = L_2(R^n; |x|^a)$ (rigorously speaking, from $L_{2,a}^{n^s}$ to $L_{2,a}^{n^t}$). We establish for $n > 3$, $s \neq t$ that the norm of $T^{t,s}$ equals unit not only for $a = 0$ (obviously by Parseval's equality), but on some segment $[a_*, 0]$ for $t > s$ or $[0, a^*]$ for $t < s$. Moreover, the estimates stronger in lower terms are established inside the segment. We mention that the problem arises from the degenerated nonlinear elliptic equations of high order and has the direct application to existence and uniqueness of solutions.

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One criteria of root vectors' completeness of Tricomi problem mixed type equations

Next result was proved in the work.

Theorem. *The root vectors' completeness of Tricomi problem is equivalent to their traces' completeness in free characteristics.*

The main idea of the truth of this theorem is the study on traces properties of the Tricomi problem for the mixed type equations.

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Best extension operators for sobolev spaces on the halfline

Let $W_2^m(I)$ be the Sobolev space of all functions $f(x)$ defined on the interval $(\alpha,\beta) \subset R^1$, having absolutely continuous derivative $f^{(m-1)}(x)$ and such that the norm

$$\|f\|_{W_2^m(I)} := \left(\int_I (|f(x)|^2 + |f^{(m)}(x)|^2))\, dx\right)^{1/2} < \infty$$

In [1] the extension operators $T_m : W_2^m(R^1_-) \to W_2^m(R^1)$ have been constructed whose norms do not exceed 8^m. On the other hand in [2] it was shown that $W_2^m(R^1_-)$ contains such function $f_m(x)$ that any its extension onto the whole line has a norm in $W_2^m(R^1)$ greater than $0.08m^{-1/4}2^m\|f\|_{W_2^m(R^1_-)}$. Our goal here is to establish the following (sharp in logarithmic scale) asymptotic formula.

Theorem. *As* $m \to \infty$ $\min \ln \|T_m\|_{W_2^m(R^1_-)\to W_2^m(R^1)} \approx K_0 m$ *where*

$$K_0 := \frac{4}{\pi}\int_0^{\pi/4} \ln(\mathrm{ctg}\, x)\, dx = 1.166... = \ln 3.21...$$

Denote by $y(x) := y(x;f), x > 0$ the solution of the equation $(-1)^m y^{(2m)} + y = 0$ tending to 0 as $x \to +\infty$ and satisfying the initial conditions $y^{(s)}(+0) = a_s = f^{(s)}(-0), s \in$

$\{0, 1, \ldots, m-1\}$. Then the extension of f onto the positive halfline by y (which is linear operator) provides the minimal possible norm in $W_2^m(R^1)$. The quantity $\|y\|^2_{W_2^m(R^1_+)}$ is positively determined quadratic form $(G_m a, a)$ of the initial values vector $a := (a_0, a_1, \ldots, a_m)$. The matrices G_m can be expressed by means of certain Vandermonde matrices and those inverse to them. The study of G_m (in particular their maximal and minimal eigenvalues and corresponding eigenvectors) is the central part of the Theorem's proof. Author would like to express his gratitude to participants of the Seminar on Function Theory at the Steklov Institute Mathematics headed by S.M.Nikol'skii, L.D.Kudryavtzev and O.V.Besov. The work was supported by the grant of RFBR-99-01-000868.

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Asymptotic solution of a problem on autoresonance

Autoresonance is a phase locking phenomenon occurring in nonlinear oscillatory system, which is forced by oscillating perturbation. Many physical applications of the autoresonance are known in nonlinear physics. The essence of the phenomenon is that the nonlinear oscillator selfadjusts to the varying external conditions so that one remains in resonance with the driver over long time. This long time resonance leads to a strong increase of the response amplitude under weak driving perturbation. We consider a simple mathematical model of forcing oscillations given by the nonlinear ordinary differential equation

$$u'' + (1 + \gamma u^2)u = 2\alpha f(t)\cos(\varphi(t)),\ t > 0;\quad 0 < \alpha \ll 1$$

where the right hand side represents a small external force. The zero initial condition is here added $(u, u')|_{t=0} = (0, 0)$, so the system is starting from stable equilibrium. A nonlinearity may have a different signs: $\gamma = \pm 1$. The driving amplitude f is a slow varying function in contrast to the phase function so that $f'/\varphi'(t) = o(1)$, $\alpha \to 0$. In the report we find a condition under which the system's energy grows up to the order of $\mathcal{O}(1)$ as $t \to \infty$ while the driver is being small: $0 < \alpha \ll 1$, $f(t) = \mathcal{O}(1)$. The WKB type anzatz is taken as an asymptotic solution of the problem:

$$u = \alpha^{1/3}\Big[\mathrm{A}(\varepsilon t, \varepsilon)\exp(i\varphi(t, \varepsilon)) + \text{c.c.} + \alpha^{2/3} u_1(t, \varepsilon) + \mathcal{O}(\alpha^{4/3})\Big],\quad \alpha \to 0$$

The nonlinear equation under the zero initial data is derived for the leading order amplitude

$$2i\mathrm{A}' - 2\Phi'\mathrm{A} + 3\gamma|\mathrm{A}|^2\mathrm{A} = f,\quad \mathrm{A}(\tau)|_{\tau=0} = 0;\quad (\tau = \alpha^{2/3} t).$$

Our main discovery is a class of data Φ', f, under which a solution $A(\tau)$ is infinitely increasing as $\tau \to \infty$. This slow increasing is interpreted as the initial stage of the autoresonance. Acknowledgments. This research has been supported by the RFFR under Grants 00-01-00663, 00-15-96038 and by INTAS under Grant 99-1068

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On the initial value problem for mixed linear difference-differential equations

There is considered an initial value problem for the linear mixed difference-differential equation in a nonrectangular domain

$$a_0(s)y^{(m)}(t,s) + a_1(s)y^{(m-1)}(y,s) + \cdots + a_m(s)y(t,s) + \\ +b_0(s)y^{(n)}(t,s-h) + b_1(s)y^{(n-1)}(t,s-h) + \cdots + b_n(s)y(t,s-h) = f(t,s), \qquad (0.1)$$

where $y^{(j)}(t,s) := \frac{\partial^j y}{\partial t^j}$, $h > 0, S > 0$. Let $\gamma : [-h, S] \to \mathbb{R}$, $\gamma \in C^1 \gamma(0) = 0$ and $\gamma'(s) < 0$. On the set $E_0 = E_0^1 \bigcup E_0^2, E_0^1 = \{(t,s) | -h \leqslant s \leqslant 0, \gamma(s+h) \leqslant t < \infty\}, E_0^2 = \{(t,s) | \gamma(s+h) \leqslant t \leqslant \gamma(s)\}$ there is given an initial function $\varphi(t,s)$ and the solution of equation (0.1) must satisfy the initial value condition: $y(t,s) = \varphi(t,s)$ for $(t,s) \in E_0$. About theory and applications of mixed difference-differential equations see [1] and [2]. It is developed here a variant of operational calculus which permits to reduce the solution of a linear mixed difference-differential on a nonrectangular domain to solution of a linear difference equation. γ-original is called such a function $y(t,s), y : \mathbb{R} \to \mathbb{C}$, which is defined on $D\{\gamma(s) \leqslant t \leqslant \infty\}$, and satisfies following conditions: There is given on E_0 an initial function $\varphi(t,s)$ and it is supposed that $y(t,s) = \varphi(t,s)$ for $(t,s) \in E_0$, $y(t,s) = 0$ for $t < \gamma(s+h)$. and there exist such constants $M > 0$ and q that $|y(t,s)| < Me^{qt}$ for $(t,s) \in D$. γ-transform corresponding to the γ-original $y(t,s)$ is called the function
$Y(p,s) = \int_{\gamma(s)}^{\infty} y(t,s)e^{-pt}\,dt$, $p \in \mathbb{C}$. By application of γ-transformation to problem (0.1) it is reduced to the difference equation

$$Y(p,s)(a_0p^m + a_1p^{m-1} + \cdots + a_m) + Y(p,s-1)(b_0p^n + b_1p^{m-1} + \cdots + b_n) = \\ = F(p,s) + \Gamma(p,s) - I(p,s), \qquad (0.2)$$

where F, Γ and I are some functions determined by initial and right-hand functions in (0.1).

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Positivity and hierarchy of Green's functions for boundary value problems for deflection of a beam

We consider simple boundary value problems for deflection of a beam on an elastic foundation under tension. Proved positivity of Green's functions and showed hierarchical structure between Green's functions with different boundary conditions.

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Hamilton - Jacobi functional differential equations with unbounded delay

Let H denote the Haar pyramid

$$H = \{(t,x) = (t,x_1,\ldots,x_n) \in R^{1+n}: \ t \in [0,a], \ -b + h(t) \leqslant x \leqslant b - h(t)\}$$

and $E = (-\infty, 0] \times [-b, b] \subset R^{1+n}$ where $b = (b_1, \ldots, b_n) \in R_+^n$ $R_+ = [0, +\infty)$, and $h = (h_1, \ldots h_n) \in C([0,a], R_+^n)$, $a > 0$. We assume that h in nondecreasing, $h(0) = 0$ and $b > h(a)$. (We use vectorial inequalities with the understanding that the same inequalities hold between their corresponding components.) Let Y be the space of initial functions $w : E \to R$. We assume that Y is a linear space with the norm $\|\cdot\|_Y$ and that $(Y, \|\cdot\|_Y)$ is a Banach space. For $0 < t \leqslant a$ we put $H_t = H \cup ([0,t] \times R^n)$. For each t, $0 < t \leqslant a$, we consider the space X_t consisting of functions $z : E \cap H_t \to R$. We assume that X_t is a linear space with the norm $\|\cdot\|_{X_t}$. Write $X = X_a$ and $\|\cdot\|_X = \|\cdot\|_{X_a}$ and assume that $V : X \to X(H, R)$ is a given operator. Let $\Omega = H \times R \times R^n$ and assume that $f : \Omega \to R$ and $\varphi : E \to R$ are given functions. We consider the functional differential equation

$$\partial_t z(t,x) = f(t, x, (Vz)(t,x), \partial_x z(t,x)) \tag{0.1}$$

with the initial condition

$$z(t,x) = \varphi(t,x) \ \text{ on } \ E \tag{0.2}$$

where $\partial_x z = (\partial_{x_1} z, \ldots, \partial_{x_n} z)$. We assume that the operator V satisfies the following Voterra condition: if z, $\bar{z} \in X$ and $z(\tau, y) = \bar{z}(\tau, y)$ for $(\tau, y) \in H_t$ then $(Vz)(t,x) = (V\bar{z})(t,x)$. In this time numerous papers were published concerning equation (0.1) with initial condition given on a bounded domain. The following questions were considered: functional differential inequalities, uniqueness of solutions, existence of classical or generalized solutions, numerical methods. We start the investigation of first order partial functional differential equations on the Haar pyramid with unbounded delay. We give sufficient conditions for the existence of solutions of problem (0.1), (0.2). The phase space for the above problem is constructed. The Cauchy problem is transformed into a system of integral functional equations. The unknown functions are z and $\partial_x z$. The method of bicharacteristics and integral functional inequalities are used. Examples of phase spaces are given.

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Rayleigh type surface waves in elastic wedge

The problem of vibration of isotropic elastic wedge with a free boundary is considered. The existence of the waves, which are oscillating along the edge and exponentially decreasing at distance from it, was predicted in [1,2] on a base of numerical calculations. The forthcoming papers mostly deals with the methods of theory of perturbations, asymptotic analysis and functional equations of Malyuzhinets type while trying to construct such wave. We present a proof of existence of surface wave and give estimates demonstrating its localization. The method we use is close to [3,4] and can be briefly outlined as following: we associate with the problem a self-adjoint operator, deduce some estimate of quadratic form which gives us the explicit information about location spectrum and by means of variational approach (by choosing of test function) indicate the point spectrum, which corresponds to Rayleigh type surface waves.

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Inverse problems for parabolic equations with final overdetermination

We prove the unique solvability of the inverse problem of identification of the pair $\{u(t,x), f(x)\}$ satisfying the equation

$$\rho(t,x)u_t - u_{xx} = f(x)g(t,x) + h(t,x), \ (t,x) \in Q_T \equiv [0,T] \times [0,l],$$

the initial condition

$$u(0,x) = u_0(x), \ x \in [0,l],$$

the boundary conditions

$u(t,0) = u(t,l) = 0,\ t \in [0,T],$

and the condition of final overdetermination

$u(T,x) = \varphi(x),\ x \in [0,l].$

Let us note that earlier one had obtained the results on unique solvability of the inverse problem for parabolic equations with the condition of final overdetermination for the equations with the coefficients independent on t [1]. On the other hand, if the coefficients of the equation depend also on time one had obtained only the Fredholm solvability of the corresponding inverse problem [2]. THEOREM. *Let*

$$\Lambda_1 \leqslant \rho(t,x) \leqslant \Lambda_2,\ 0 \leqslant \rho_t(t,x) \leqslant K_\rho,$$

$$|g(t,x)| \leqslant K_g,\ |g_t(t,x)| \leqslant K_g^*,\ |g(T,x)| \geqslant g_0 > 0,$$
$$h(t,x), h_t(t,x) \in L_2(Q),\ u_0(x), \varphi(x) \in W_2^2([0,l]),$$
$$u_0(0) = u_0(l) = \varphi(0) = \varphi(l) = 0.$$

Suppose also that the following inequality holds:

$$\frac{\Lambda_2^2}{g_0^2}\left[\left(\frac{K_g^2}{\Lambda_1^2} - \frac{(\theta^2 K_g^*)^2\Lambda_2}{\Lambda_1}\right)e^{-T/(\Lambda_2\theta^2)} + \frac{(\theta^2 K_g^*)^2\Lambda_2}{\Lambda_1}\right] < 1. \tag{1}$$

(Here θ is the constant from Poincare – Steklov inequality:

$$\|u\|_{L_2([0,l])} \leqslant \theta \|u_x\|_{L_2([0,l])},$$

valid for any function from the space $\overset{0}{W}{}_2^1([0,l])$. Then the concerned inverse problem has a solution which is unique.

REMARK 1. If $g(t,x) = const$, $\rho(t,x) = const$, then the condition (1) is always true. This fact corresponds to the results of the papers [1,2].

REMARK 2. If $g(t,x) = const$, $\rho(t,x) \neq const$, then the condition (1) is valid for large $T > 0$.

REMARK 3. The obtained results are easily extended to the case of many-dimensional inverse problem

$\rho(t,x)u_t - \Delta u = f(x)g(t,x) + h(t,x),\ (t,x) \in P_T \equiv [0,T] \times \Omega,$
$u(0,x) = u_0(x),\ x \in \Omega,$
$u(t,x) = 0,\ t \in [0,T],\ x \in \partial\Omega,$
$u(T,x) = \varphi(x),\ x \in \Omega,$

where Ω is a smooth bounded domain in $\mathbb{R}^n$. This researh is fulfilled under RFBR support, Grant N 00 – 01 – 00638.

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On prolongation of solutions to the first order nonautonomous differential equations on singular set

We study properties of solutions to ordinary differential equation

$$a(t,x)\frac{dx}{dt} = 1, \qquad (1)$$

in neighborhoods of points where function $a(t,x)$ is equel to zero. The same problem had been discussed by Petrovsky in [1]. The function $a(t,x)$ is supposed to be defined and continuously differentiable on $\mathbb{R}^2$. The set $S = \{(t,x) \in \mathbb{R}^2 : a(t,x) = 0\}$ is called [2] *a singular set* of the equation (1). A problem of solution prolongations to the equation (1) on (via) the singular set S is studied. Similar problem was studied in [2] where, in particular, an approach of solution prolongation by help of the first integral was developed. We have proved the following theorems. **Theorem 1.** *Let the singular set S define a closed Jordan curve, $Int\ S$ be a domain bounded by the curve S, $Ext\ S = \mathbb{R}^2 \setminus \overline{Int\ S}$, the set S contain none connected subset $\{(t,x) : t = \tilde{t}, x \in [a,b]\}$, where $b > a$, $\tilde{t}$ are real. Then any solution to the equation (1) with initial data belonging to domain $Int\ S$, defined on maximal interval of its existence, reaches the singular set S. Moreover, a solution to the equation (1) with initial data belonging to domain $Int\ S$ is prolongated via the singular set S iff function $a(t,x)$ has different signs in both domains $Int\ S$ and $Ext\ S$.* **Theorem 2.** *Let the singular set S contain none connected subset $\{(t,x) : t = \tilde{t}, x \in [a,b]\}$, where $b > a$, $\tilde{t}$ are some real. Then for every point $(t_*, x_*) \in S$ there are two solutions $x_k = x_k(t)$, $t \in (\alpha_k, \beta_k)$, $k = 1, 2$, such as*

$$\lim_{t \to t_*} x_k(t) = x_*, \quad \text{where} \quad t \in (\alpha_k, \beta_k), k = 1, 2.$$

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Singular Problems for Parabolic Equations

Let us consider the problem

$$\varepsilon^2 u_{xx} - A(t,x)u_x - C(t,x)u - u_t = F(t,x), \quad (t,x) \in D = \{(0,T] \times (0,\infty)\},$$
$$u(0,x) = \phi(x), \qquad u(t,0) = h(t), \tag{1}$$

ε is a positive small parameter. We assume that the functions $A(t,x)$, $C(t,x)$, $F(t,x)$, $h(t) \in C^\infty(\overline{D})$ and are uniformely bounded; $\phi(x)$ is a bounded function with a finit discontinuity at the point $a > 0$, and $\phi(x)$ is infinitely differentiable when $x \neq a$; $\phi(0) = h(0)$, $A(t,x) \leqslant A < 0$, $C(t,x) > 0$. The solution of the problem (1) $u(t,x,\varepsilon)$ is infinitely differentiable when $t > 0$; at the same time the solution of the corresponding degenerated problem (when $\varepsilon = 0$) is discontinuous at the characteristic line $x = x_0(t)$, defined by the equation $x_0'(t) = A(t, x_0(t))$, $x_0(0) = a$. We assume that the characteristic line $x = x_0(t)$ intersects the boundary $x = 0$. Our aim is to construct the continuous asymptotic solution of the problem (1), posessing the continuous first order derivative with respect to variable x in D. **Theorem.** *The solution of the problem (1) has the asymptotic representation*

$$U_N(t,x,\varepsilon) = \sum_{i=0}^{N} \varepsilon^i \Big\{u_i(t,x) + v_i(t,\xi) + p_i(\tau,\xi) + w_i(t,\eta) + z_i(t,\xi,\varepsilon)\Big\},$$

$\xi = x/\varepsilon^2$, $\tau = t/\varepsilon^2$, $\eta = (x - x_0(t))/\varepsilon$. Here $u_i(t,x)$ are regular functions [1]; the rest ones are boundary layer functions [1],[2] describing the solution near the boundary $x = 0$ and characteristic line $x = x_0(t)$. The following estimate holds for all $(t,x) \in \overline{D}$

$$|u(t,x,\varepsilon) - U_N(t,x,\varepsilon)| \leqslant M\varepsilon^{N+1},$$

where constant M does not depend on ε.

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Mixed finite element methods for nonlinear differential equations of high order

The mixed finite element methods for quasilinear partial differential equations of forth order

$$\sum_{|j|=2} D^j A_j(x,u,u_{xx}) - \sum_{|j|=1} D^j A_j(x,u,u_x) + A_0(x,u,u_x) = f, \quad x \in \Omega,$$

with Dirichlet boundary conditions

$$u = 0, \quad \partial u/\partial \nu = 0, \qquad x \in \partial\Omega,$$

are considered. The approximate solution y is determined as element of the space H_l of Lagrangian splines vanish on the boundary and the integral identity

$$\int_\Omega \Big(\sum_{|j|=2} A_j(x, y, w^h(y)) w_j^h(\eta) + \sum_{|j|\leqslant 1} A_j(x, y, y_x)\, D^j\eta - f\eta \Big) dx = 0 \ \forall \eta \in H_l$$

satisfies. The approximation of second derivatives $w^h(y)$ is constructed by similarly of Herman-Johnson or Herman-Miyoshi metods. The solvability of discrete problems and convergence of approximate solutions by condition of coerciveness of differential operator are obtained. The error estimations by conditions on the functions A_j that ensure the strong monotonicity of the differential operator are established. The analogous methods for partial differential equations systems with fourth order derivatives that arise in nonlinear thin elastic shell theory and are formulated as minimization problems for functionals in the form

$$F(u) = \int_\Omega \varphi(\varepsilon, \kappa) d\Omega - \int_\Omega u \cdot f d\Omega$$

are constructed and investigated. The tangential and bending components of deformation (ε, κ) are computed by the displacement vector u of the shell middle surface on the base of nonlinear theory of middle bending of the shells.

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Combinatorial Morse theory and minimal networks

Consider a set $\mathcal{K}$ and a function $f : \mathcal{K} \to \mathbb{R}$. Let $\Sigma = \{\mathcal{K}_i\}$ be a given finite covering of the set $\mathcal{K}$ by it's subsets $\mathcal{K}_i \subset \mathcal{K}$: $\mathcal{K} = \cup \mathcal{K}_i$. Like the classical Morse theory we study the bifurcations of the set $\mathcal{K}_{\leqslant c} = \{x \in \mathcal{K} : \ f(x) \leqslant c\}$ under the changing c from $-\infty$ to $+\infty$. Actually the bifurcations of the homotopy type of the $\mathcal{K}_{\leqslant c}$ is studied in the classical Morse theory, but in our case we consider a nerve $\mathcal{N}(\Sigma_{\leqslant c})$ of the covering $\Sigma_{\leqslant c} = \{\mathcal{K}_{\leqslant c} \cap \mathcal{K}_i\}$ of the set $\mathcal{K}_{\leqslant c}$. The number $\tilde{c}$ is called a *critical value* if the combinatorial structure of the complex $\mathcal{N}_{\leqslant c}$ is changed passing through $\tilde{c}$. The set $f^{-1}(\tilde{c})$ is the *critical set* corresponding to the critical value $\tilde{c}$. Any point x belonging to a critical set is called a *critical point*. The changing of the nerve $\mathcal{N}(\Sigma_{\leqslant c})$ at the critical point $\tilde{c}$ is characterized by the Morse pair $(\mathcal{N}(\Sigma_{\leqslant \tilde{c}+\varepsilon}), \mathcal{N}(\Sigma_{\leqslant \tilde{c}-\varepsilon}))$, where ε is a sufficiently small positive number. We define the *index* of the critical value $\tilde{c}$ by the formula: $\operatorname{ind}_f \tilde{c} := \chi(\mathcal{N}(\Sigma_{\leqslant \tilde{c}+\varepsilon})) - \chi(\mathcal{N}(\Sigma_{\leqslant \tilde{c}-\varepsilon}))$. Then the following theorem holds.

Theorem. *The sum of the indices over all critical values of function f is equal to the Euler characteristic of the nerve $\mathcal{N}(\Sigma)$.*

Now we apply the combinatorial Morse theory to some problems from the minimal network theory [1]. Let $\mathcal{K}$ be the space of all linear network-traces spanning a given boundary set $\mathcal{A}$ and a function f be the length of a network. This space has a natural covering $\Sigma = \{\mathcal{K}_i\}$. The elements $\mathcal{K}_i$ of Σ correspond to the spaces of all fixed type binary trees spanning a given boundary set $\mathcal{A}$. It turns out [2] that critical points (networks), which has a binary tree as a canonical representative, are the local minimal networks spanning the set $\mathcal{A}$ and are the local minima of the function f at the same time. Estimating indices of the critical points and using the previous theorem we prove the following assertions.

Assertion. *The number of all local minimal networks spanning a given boundary set of 4 points in general position in the Euclidean or Lobachevskian plane is equal to either 1 or 2.*

Assertion. *The number of all local minimal networks spanning a given boundary set of 5 points in general position in the Euclidean or Lobachevskian plane is not exceed 8.*

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Kasymov K.A.
(Kazakh State University)

On a method of solving multipoint boundary value problems for linear differential equations

Katanaev M.O.
(Steklov Mathematical Institute)

Tho-dimensional Gravity

Two-dimensional gravity model based on Riemann–Cartan geometry is considered. A general Lagrangian quadratic in torsion and with arbitrary dependence on the scalar curvature yields a system of nonlinear partial differential equations of motion. A general solution to this system of equations is found. It describes (i) surfaces of constant curvature and zero torsion and (ii) surfaces of nonconstant curvature and nontrivial torsion. The last class of solutions contains physically interesting solutions: black-hole solutions and solutions describing changing topology of space in time.

Kersner R.
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On equations of the type $u_t = u_{xx} - 1/u$

When studying the general reaction-diffusion PDE

$$u_t = (D(u)u_x)_x + F(u) \tag{0.1}$$

or the corresponding ODE for travelling-wave solutions $u = f(x - ct)$

$$(D(f)f')' + cf' + F(f) = 0$$

with different initial and boundary conditions, one always supposes — explicitly or implicitly — the integrability of DF. The variable

$$Q(s) = \left|2\int_0^s D(r)F(r)\,dr\right|^{1/2}$$

plays a definitive role (we suppose that u and $D(u)$ are nonnegative, and, $F(0) = 0$; so $-1/u$ in the title means $-1/u$ for $u > 0$ and 0 for $u = 0$). If $Q(s) < \infty$ for $s > 0$, we are usually able to build up a satisfactory theory of continuous (often only weak) solutions: we have existence, uniqueness, monotone dependence on the data; and, we can deal with different inner properties, special solutions, etc. We do not know very much (in fact, almost nothing) about PDEs and ODEs with $Q(s) = \infty$ for $s > 0$. In my talk I try to shed some light on "what can we expect"-type questions. This will be done by explaining some facts for the simplest (0.1)-type equation with $Q(s) = \infty$ for $s > 0$:

$$u_t = u_{xx} - \begin{cases} 1/u & \text{if } u > 0 \\ 0 & \text{if } u = 0 \end{cases}, \quad x \in \mathbb{R}, \quad t > 0. \tag{0.2}$$

One may ask what kind of phenomena correspond to (0.2) or, more generally, to (0.1) with $Q(s) = \infty$ for $s > 0$? What will be clear from the talk (joint results with Brian H. Gilding of the University of Twente, NL) is that the mechanisms behind the phenomena are rather brutal ones. Here are some examples.

1. Consider first the Cauchy Problem with

   $$u(x, 0) = u_0(x), \quad x \in \mathbb{R},$$

   where u_0 is nonnegative and continuous. Suppose that $M > u_0 > m > 0$. We know: there exists a first moment $t_q > 0$ and a point x_q such that $u(x_q, t_q) = 0$. We show: $u(\cdot, t) \equiv 0$ for $t > t_q$. Thus, in general, $u(x, \cdot)$ is discontinuous at t_q and looks like a precipice.

2. If u_0 is compactly supported ($u_0(x) = 0$ for $|x| \geqslant l > 0$), then $u(\cdot, t)$ is identically zero for $t > 0$. This may also happen when $u_0(x)$ is zero only for $x \leqslant l_1$ or $x \geqslant l_2$ or even at a single point. We see that any concept of solution cannot be applied here: $u(x, t) \to 0$ as $t \downarrow 0$ at any point.

3. The classical Kawarada Problem is the following (see the vast literature on the so-called "Quenching Problem"). Consider (0.2) in $(-a,a) \times (0,\infty)$ with initial-boundary conditions

$$u(x,0) = 1, \quad x \in (-a,a), \qquad u(\pm a,t) = 1, \quad t > 0.$$

It is well known that if $a > 0.8$ (so we could take $a = 1$), then there is a first moment $t_q > 0$ (the "quenching time") such that $u(0,t_q) = 0$ and that the profile as $t \uparrow t_q$ is parabola-like. ¿From our results it follows that $u(x,t) = 0$ for $x \in (-a,a)$ and $t > t_q$. Thus one can see once again that the global solution is discontinuous (at $t = t_q$ and at $x = \pm a$ for $t > t_q$).

Khanin K.M.

(Isaac Newton Institute, Cambridge and Landau Institute)

Burgers Turbulence and Random Lagrangian Systems

The talk is based on a joint paper with Renato Iturriaga (CIMAT, Mexico) ([1]). We discuss two closely related problems: construction of a stationary distribution for solutions for d-dimensional spatially periodic inviscid random forced Burgers equation and properties of minimizing trajectories for random time-dependent Lagrangian systems related to it. The random forced Burgers equation was a subject of intensive study in the physical literature in the last 5 years. Although it arises naturally in many different physical problems recent interest was mostly motivated by the hydrodynamics applications and the theory of turbulence. The mathematical theory in the one-dimensional case was developed in [2]. It is important to mention that the results proved in [2] allows not only to analyse the qualitative properties of a stationary distribution for solutions, but also lead to quantitative predictions for universal scaling exponents related to the pdf (probability distribution function) for the velocity gradients ([3]). One of the main aims of the present work was to study what happens in the d-dimensional case. The methods which were used in [2] were purely one-dimensional and in order to achieve our goals we use in a more systematic way the Lagrangian formalism and connection with the random Hamilton-Jacobi equation. Our results lead to a construction of a unique stationary distribution for "viscosity" solutions of the random Burgers equation. We also show that with probability 1 there exists a unique minimizing trajectory for the random Lagrangian system which generates a non-trivial ergodic invariant measure for the non-random skew-product extension of the Lagrangian flow. Finally we introduce the notion of topological shocks and study their properties.

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Khe K. Ch.
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On Mixed Boundary Value Problems for the Degenerating Elliptic Equations with the Bessel Operators

It is known in general the classical boundary value problems [1] are becoming ill-posed [2-4]. On the whole in papers of many authors concerned with mixed boundary value problems for degenerating elliptic equations there is Dirichlet condition on the elliptic part boundary and there is Neyman condition or its analog or the degenerating part of the boundary curve is becoming free of boundary conditions [2-4]. At the talk mixed boundary value problems will be investigated for two degenerating elliptic equations with differential Bessel operators $x\frac{\partial^2}{\partial x^2}+\alpha\frac{\partial}{\partial x}$ and $y\frac{\partial^2}{\partial y^2}+\beta\frac{\partial}{\partial y}$ when Dirichlet condition is given on the elliptic part and a one-half degenerating part of the boundary curve and an other one-half degenerating part of the boundary curve is given a analog of Neyman condition with a weight function. In this case the order of the boundary differential operator and the weight function are depend essentially on the coefficient of the differential equations. There will be posed two mixed boundary value problems for two degenerating elliptic equations and there will be formulated uniqueness and existence theorems. Proof of uniqueness theorem is carried out by the maximum principle of solutions for degenerating elliptic equations [3] using properties of solutions of differential equations with Bessel operators. Existence of solutions of the boundary value problems is proved by methods of the theory of singular integral equations. The details will be given at the talk.

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The theory of p-adic dynamical systems

Investgations in $p-$adic quantum physics [1]-[3] stimulated an increasing interest in studying $p-$adic dynamical systems, see for example [3]-[5]. Some steps in this direction [3] demostrated that even the simplest (monomial) discrete dynamical systems over the fields of $p-$adic numbers Q_p have quite complex behavior. This behavior depends crucially on the prime number $p > 1$ (which determines Q_p). By varying p we can transform attractors into centers of Siegel discs and vice versa. The number of cycles and their lengths also depend crucially on p [3]. Some applications of discrete p–adic dynamical systems to cognitive sciences and neural networks were considered in [1]. Some of these cognitive models are described by random dynamical systems in the fields of p–adic numbers [6].

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Khruslov E.Ya.

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Soliton asymptotics of rear part of non-localized solutions of the Kadomtsev-Petviashvili equation

We consider the KP-1 equation

$$\frac{\partial}{\partial x}\left(u_t + \frac{3}{2}uu_x + \frac{1}{4}u_{xxx}\right) - \frac{3}{4}u_{yy} = 0$$

and construct a special class of non-localized solutions $u(x, y, t)$ – these solutions tend to zero as $x \to -\infty$ but they do not decrease as $x \to +\infty$. We study the long-time asymptotic behaviour of the rear part of the solutions. We prove that in the domains

$$D_N = \left\{x, y: \ x < C(Y)t + \frac{1}{2a(Y)}\ln t^N, \ -\infty < y < \infty\right\}$$

the solutions are represented as follows

$$u(x,y,t) = \sum_{n=1}^{N-1} \frac{a^2(Y)}{\cosh^2\left[x - C(Y)t - \frac{1}{2a(Y)}\ln t^{n+1/2} + \alpha_n(Y)\right]} + O(\frac{1}{t^{1/4}}),$$

where N is an arbitrary integer, $a(Y), C(Y), \alpha_n(Y)$ are functions of argument $Y = \frac{y}{t}$. It means that curved asymptotic solitons are graduately edjected at the rear part of the solution. Analogous phenomenon of edjection of asymptotic solitons at the front of the solution are described in [1].

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On Boundary and Periodic Solutions of Nonautonomous Ordinary Differential Equations

Problems of existence and uniqueness of periodic and bounded solutions of the equation

$$u^{(n)} = f(t, u, \dots, u^{(n-1)}) \tag{0.1}$$

are investigated, where $f : R \times R^n \to R$ is a continuous function. In particular, in the case where f is periodic in the first argument with the period $\omega > 0$, the following theorems are proved.

Theorem 1. *Let there exist $\sigma \in \{-1, 1\}$, continuous ω-periodic functions $h_0 : R \to R$, $h : R \to [0, +\infty[$ and $h_k : R \to [0, +\infty[$ $(k = 1, \dots, n)$ such that on $R \times R^n$ the inequalities $|f(t, x_1, \dots, x_n)| \leqslant \sum_{k=1}^{n} h_k(t)|x_k| + h(t)$ and*

$$f(t, x_1, \dots, x_n)\mathrm{sgn}(\sigma x_1) \geqslant h_0(t)|x_1| - \sum_{k=2}^{n} h_k(t)|x_k|$$

hold. Let, moreover, $\int_0^\omega h_0(t)\,dt > 0$ and

$$\eta_0 \ell_1 \int_0^\omega h_1(t)\,dt + (1 + \eta_1) \sum_{k=2}^{n} \ell_k \int_0^\omega h_k(t)\,dt < 1,$$

where $\eta_i = \int_0^\omega |h_i(t)|\,dt / \int_0^\omega h_0(t)\,dt$ $(i = 0, 1)$, $\ell_k = \frac{\omega}{2}(\frac{\omega}{2\pi})^{n-k-1}$ $(k = 1, \dots, n-1)$, $\ell_n = 1$. Then the equation (1) has at least one ω-periodic solution.

Theorem 2. *Let there exist numbers $\sigma \in \{-1, 1\}$, $\lambda_k \in]0, 1[$ $(k = 1, \dots, n)$ and ω-periodic continuous functions $h_0 : R \to R$, $h : R \to [0, +\infty[$ and $h_k : R \to [0, +\infty[$ $(k = 1, \dots, n)$ such that $\int_0^\omega h_0(t)\,dt > 0$ and on $R \times R^n$ the inequalities*

$$f(t, x_1, \dots, x_n)\mathrm{sgn}(\sigma x_1) \geqslant h_0(t)|x_1|^{\lambda_1} - \sum_{k=2}^{n} h_k(t)|x_k|^{\lambda_k} - h(t),$$

$$|f(t, x_1, \dots, x_n)| \leqslant \sum_{k=1}^{n} h_k(t)|x_k|^{\lambda_k} + h(t)$$

hold. Then the equation (1) *has at least one* ω*-periodic solution.*

Kilbas A.A.
(Belarusian State University)

Differential equations of fractional order in the spaces of summable and integrable functions

We consider the Caushy-type problem for the model nonlinear differential equation of fractional order $\alpha n\mathbf{C}$, $\mathrm{Re}(\alpha) > 0$,

$$(D^{\alpha}_{a+}y)(x) = f[x, y(x)]\ (n-1 < \alpha \leqslant n,\ n = -[-\mathrm{Re}(\alpha)],$$

$$(D^{\alpha-k}_{a+}y)(x) = b_k,\ b_k \in \mathbf{C}\ (k = 1, 2, \cdots, n),$$

on a finite interval $[a, b]$ of the real axis. Here $D^{\alpha}_{a+}y$ is the the Riemann-Liouville fractional derivative defined for $\alpha \in \mathbf{C}$, $\mathrm{Re}(\alpha) > 0$, by

$$(D^{\alpha}_{a+}y)(x) = \left(\frac{d}{dx}\right)^{n} \frac{1}{\Gamma(n-\alpha)} \int_{a}^{x} \frac{y(t)dt}{(x-t)^{\alpha-n+1}},\ n = [\mathrm{Re}(\alpha)] + 1,$$

where $\Gamma(n-\alpha)$ is the Gamma-function and $[\mathrm{Re}(\alpha]$ is the integer part of $\mathrm{Re}(\alpha)$, see Section 2 in [1] and [2]. The above problem is studied in the space $L(a, b)$ of summable functions on $[a, b]$ and in the weighted space $C^{n-\alpha}[a, b]$ of continuous functions $y(x)$ such that $(x-a)^{n-\alpha}f(x) \in C[a, b]$. The equaivalence of the problem considered and the nonlinear Volterra integral equation is established. The existence and uniqueness of the solution $y(x)$ of the above problem is proved by the method of successive approximation. The corresponding assertions for the linear differential equations are given. The explicit solutions of some types of such linear equations are presented in terms of special functions generalizing the classical Mittag-Leffler functions.

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Kim D.
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Periodic and almost periodic hyperfunctions

Every periodic hyperfunction is a bounded hyperfunctionand can be represented as an infinite sumof derivatives of bounded continuous periodic functions. Also, Fouriercoefficients c_α of periodic hyperfunctions are of infra-exponentialgrowth in $\mathbb{R}^n$, i.e., $c_\alpha < C_\epsilon e^{\epsilon|\alpha|}$ for every $\epsilon > 0$ and every $\alpha \in \mathbb{Z}^n$. This is a natural generalization of the polynomial growth of the Fouriercoefficients of distributions. To show these we introduce these $B_{\mathcal{L}\checkmark}$ of hyperfunctions ofL^p growth which generalizes the space $\mathcal{D}'_{\mathcal{L}\checkmark}$ of distributions of L^p growth and represent generalized functions as the initial values ofsmooth solutions of the heat equation. Also, we show some related results on almost periodic hyperfunctions.

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Bifurcation of eigenvalues of nonselfadjoint differential operators and stability of nonconservative systems

Behavior of eigenvalues of nonselfadjoint operator due to changing of parameters (the load parameter, for example) is of interest when deal with stability problems for multiparameter nonconservative systems. In the generic case the spectrum of a family of nonconservative systems contains multiple eigenvalues. Such eigenvalues can be critical from the point of view of stability theory because of their splitting due to variation of parameters can lead to qualitative changes in behavior of mechancal system. In the present paper eigenvalue problems for nonselfadjoint linear differential operators smoothly dependent on a vector of real parameters are considered. Bifurcation of eigenvalues along smooth curves in the parameter space is investigated. The case of multiple eigenvalue with Keldysh chain of arbitrary length is studied. Explicit expressions describing bifurcation of eigenvalues are found. The obtained formulae use eigenfunctions and associated functions of the adjoint eigenvalue problems as well as the derivatives of the differential operator taken at the initial point of the parameter space. As an application of the developed theory the extended Beck's problem of stability of an elastic column under action of potential force and tangential follower force is considered and discussed in detail.

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Relatively undistorted progressing waves

In the classical monograph [1], much attention is paid to solutions of the wave equation,

$$u_{xx} + u_{yy} + u_{zz} - c^{-2} u_{tt} = 0, \quad c = const,$$

of the specific form

$$u = g(x, y, z, t) f(\theta),$$

where $f(\theta)$ is an arbitrary function of one variable, and the phase, $\theta = \theta(x, y, z, t)$, and the amplitude, or distortion factor, g, are fixed functions. In [1] such solutions were termed *relatively undistorted progressing waves.* The best known examples are plane waves with $\theta = z \pm ct$ and $g = 1$, and spherical waves, in which case, $\theta = R \pm ct$, $R = \sqrt{x^2 + y^2 + z^2}$, and $g = 1/R$. Recently Bateman's solution with $\theta = z - ct + (x^2 + y^2)/(z - ct)$, and $g = 1/(z + ct)$ (to be precise, its complexification) attracted much attention in the course of seeking simple highly localised solutions (e.g. [2] and references therein). We present new solutions of this form. One of several examples is

$$\theta = (x \pm iy)(z \pm ct), \quad g = A + B(x^2 + y^2 - z^2 + c^2 t^2),$$

where A and B are constants. A support from the RFFI Grant 00-01-00485 is acknowledged.

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Asymptotic solution of primary resonance equation in bifurcation layer[15]

We study a primary resonance equation with external fast oscillating force:

$$\varepsilon i \partial_t \psi + |\psi|^2 \psi = \exp\left(it^2/(2\varepsilon)\right), \quad 0 < \varepsilon \ll 1 \tag{0.1}$$

The studied asymptotic solution has fast oscillating behaviour with frequency of external force when $t < t_*$ and has two phase oscillation when $t > t_*$. Our goal is to investgate a bifurcation layer in a neighborhood of t_* and to construct a matching asymptotic solution before and after the bifurcation. Formulate the results of our study.

When $(t - t_*)\varepsilon^{-4/5} \gg 1$ The asymptotic solution has a form:

$$\psi(t, \varepsilon) = \exp\left(it^2/(2\varepsilon)\right) \sum_{n=0}^{\infty} \varepsilon^n \overset{n}{U}(t) \tag{0.2}$$

The main term is equal to a middle root of roots of algebraic equation $|u|^2 u - tu = 1$. Other terms are algebraic functions of t.

When $|t - t_*| \ll 1$ the asymptotic is defined by four expansions of different types. First of them is:

$$\psi = \left(U_* + \varepsilon^{2/5} \sum_{n=0}^{\infty} \varepsilon^{2n/5} \overset{n}{\alpha}(\tau) + i\varepsilon^{3/5} \sum_{n=0}^{\infty} \varepsilon^{2n/5} \overset{n}{\beta}(\tau)\right) \exp\left(it^2/(2\varepsilon)\right), \tag{0.3}$$

[15]This work was supported by INTAS grant No99-1068, RFBR grant No 01-00-00663 and 00-15-96038

here U_* is a double root of the equation $|u|^2u - t_*u = 1$. The coefficients of the expansion are dependent on new scaling variable $\tau = (t - t_*)\varepsilon^{-4/5}$. The main term and corrections are defined by their asymptotic as $\tau \to -\infty$, when they are matched with (0:2). For example the main term is a special solution of the Painlevé-1 equation: $\overset{0}{\alpha}{}'' - 3\overset{0}{\alpha}{}^2 + \tau = 0$ with asymptotic behaviour as $\tau \to -\infty$:

$$\overset{0}{\alpha}(\tau) = \sum_{n\geqslant 0} \alpha_n \tau^{-\frac{(5n-1)}{2}}, \quad \text{and } \alpha_0 = \frac{1}{\sqrt{3}},\ \alpha_1 = \frac{1}{24}.$$

If τ is bounded then $\overset{0}{\alpha}$ has poles on the real axis. Let's denote the least of them by τ_0. The expansion (0.3) is suitable as $(\tau - \tau_0)\varepsilon^{-1/5} \gg 1$.

In the neighborhood of τ_0 the coefficients of the asymptotic expansion are dependent on one more fast scale $\theta = (\tau - \tau_0)\varepsilon^{-1/5}$. Denote by $\theta_0 = \theta + \sum_{n=1}^{\infty} \varepsilon^{n/5} \overset{n}{\theta}_0$. Then as $-\varepsilon^{-1/5} \ll \theta_0 \ll \varepsilon^{-1/10}$ the asymptotic solution has the form:

$$\psi(t,\varepsilon) = \left(U_* + \overset{0}{w}(\theta_0) + \varepsilon^{4/5} \sum_{n=1}^{\infty} \varepsilon^{(n-1)/5} \overset{n}{w}(\theta_0)\right) \exp\left(it^2/(2\varepsilon)\right) \qquad (0.4)$$

The constants $\overset{n}{\theta}_0$ may be defined by matching of (0.3) and (0.4). The leading term of the asymptotic expansion is separatrix solution of the equation:

$$i\,\overset{0}{w}{}' + U_*\left(2|\overset{0}{w}|^2 + \overset{0}{w}{}^2\right) + U_*^2\left(\overset{0}{w}{}^* - \overset{0}{w}\right) + |\overset{0}{w}|^2 \overset{0}{w} = 0,$$

namely: $\overset{0}{w}(\theta_0) = -2(\theta_0 - iU_*)^{-2}$.

As $-\theta_0 \gg 1$ the asymptotic solution is defined by a sequence of two asymptotic expansions which are called intermediate and separatrix asymptotic expansions. Denote one more scaling variable $T_k = \theta_k \varepsilon^{1/6}, k = 1, 2, \ldots$. The intermediate asymptotic expansion has the form:

$$\psi(t,\varepsilon) = \left(U_* + \varepsilon^{1/3} \sum_{n=0}^{\infty} \varepsilon^{n/6} \overset{n}{A}_k(T_k) + i\varepsilon^{1/2} \sum_{n=0}^{\infty} \varepsilon^{n/6} \overset{n}{B}_k(T_k)\right) \exp\left(it^2/(2\varepsilon)\right). \quad (0.5)$$

The leading term may be represented by using of the Weierstrass $\wp$-function:

$$\overset{0}{A}_k(T_k) = -2\wp(T_k, \lambda_k/2, g_3(k,\varepsilon)), \quad g_3(k,\varepsilon) = \overset{0}{g}_3(k) + \sum_{n=1}^{\infty} \varepsilon^{n/30} \overset{n}{g}_3(k).$$

Here $\lambda_k(\varepsilon) = \varepsilon^{1/6}\left(\sum_{j=1}^{k-1} \Omega_j + \sum_{n=1}^{\infty} \varepsilon^{(n-1)/30} \sum_{j=1}^{k-1} \overset{n}{x}{}_j^+\right)$. The Ω_j is a real period of the function $\overset{0}{A}_j(T_j)$. The constants $\overset{n}{x}{}_j^+$ and $\overset{n}{g}_3(k)$ may be defined by matching of the nearest-neighbor intermediate (0.5) and separatrix expansions (0.6).

This intermediate expansion is suitable in the intervals between the poles of the Weierstrass function as $-\varepsilon^{-1/6}T_k \gg 1$, $\varepsilon^{-2/15}(T_k + \Omega_k) \gg 1$.

The separatrix expansions are suitable in small neighborhoods of the poles of the Weierstrass function. Denote by $\theta_k = (T_k + \Omega_k - \sum_{n=1}^{\infty} \varepsilon^{n/30} \overset{n}{x}_k{}^{+})\varepsilon^{-1/6}$, $k = 1, 2, \dots$. Then as $|\theta_k|\varepsilon^{1/6} \ll 1$ the asymptotic solution of (0.1) has a form:

$$\psi = \left(U_* + \overset{0}{W}(\theta_k) + \varepsilon^{4/5} \sum_{n=1}^{\infty} \varepsilon^{(n-1)/30} \overset{n}{W}(\theta_k)\right) \exp\left(it^2/(2\varepsilon)\right). \qquad (0.6)$$

The leading term of the asymptotic expansion $\overset{0}{W}(\theta_k) = -2(\theta_k - iU_*)^{-2}$.

The sequence of the intermediate and separatrix asymptotic expansions are suitable when $t > t_*$ and $\varepsilon^{-1/6}(t_* - t) \ll 1$.

As $(t_* - t)\varepsilon^{-2/3} \gg 1$ the asymptotic solution of (0.1) became two phase behaviour:

$$\psi = \left(\overset{0}{U}(t_1, t, \varepsilon) + \varepsilon \overset{1}{U}(t_1, t, \varepsilon) + \varepsilon^2 \overset{2}{U}(t_1, t, \varepsilon)\right) \exp\left(it^2/(2\varepsilon)\right),$$

Here t_1 is new scaling variable $t_1 = S(t)/\varepsilon + \phi(t)$. The leading term of the asymptotic expansion is situated on the curvarepsilon $\Gamma(t)$: $\frac{1}{2}|U|^4 - t|U|^2 - (U + U^*) = E(t)$, and satisfy the equation

$$iS'\partial_{t_1} \overset{0}{U} + |\overset{0}{U}|^2 \overset{0}{U} - t\overset{0}{U} = 1.$$

The function $S(t)$ is solution of the Cauchy problem:

$$iS' \int_{\Gamma(t)} dy(3y^3 + (2E + t^2)y^2 + 2ty + 1)^{-1/2} = T, \quad S|_{t=0} = 0,$$

where $T = \text{const} > 0$. The function $E(t)$ is a solution of a transcedental equation $i\int_{\Gamma(t)} u^* du = \pi$. The phase shift ϕ is defined by a Cauchy problem:

$$\frac{\phi'}{\partial_E S}\partial_E I = \phi_1, \quad \phi(t_*) = \phi_0 \quad \text{where} \quad I = i\int_{\Gamma(t)} u^* du.$$

In our work the values of the constants ϕ_0 and ϕ_1 remain unknown.

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Recent developments in nonlinear wave equations

The subject of nonlinear wave equations has been transformed in the last twenty years by three fundamental developments.

1. The emergence of sophisticated geometric techniques, i.e. techniques wich involve only geometric constructions in the physical space.

2. Systematic application of Littlewood-Paley decompositions and introduction of paradifferential calculus.
3. The development of Fourier spacetime techniques such as Strichartz type inequalities and Bilinear estimates.

The goal of my lectures is to discuss some of these new ideas and techniques and illustrate them through the main new results which they led to. In particular I will discuss the recent result of T. Tao on wave maps and the new result I have obtained in collaboration with I. Rodnianski on the Einstein field equations.

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Cloud of points: singular invariant measures in skew products

Consider a random dynamical system on a circle: we are given a finite number of homeomorphisms, and while iterating them on each step we choose one of them randomly and independently on different steps. In computer modeling of random dynamical systems on a circle Maxim Nalskiy discovered the following strange effect, later called as "cloud of points": distances between iterations of different points tended to 0 as number of iterations increased. In the talk a rigorous proof of this effect will be presented (joint result of Maxim Nalskiy and the speaker). Namely, we will prove that under sertain conditions on the homeomorphisms for any two points distances between their iterations tend to 0 almost surely. In [1] is obtained the following analogical result: we iterate one diffeomorphism, and after each iteration the circle is rotated by random angle. Then, under sertain conditions, distances between iterations of any two points tend to 0 almost surely.

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Lyapunov Functions and Solutions of the Lyapunov Matrix Equation for Marginally Stable Systems

We consider linear systems of differential equations $I\ddot{x} + B\dot{x} + Cx = 0$ where I is the identity matrix and B and C are general complex n x n matrices. Our main interest is to determine conditions for complete marginal stability of these systems. To this end we find solutions of the Lyapunov matrix equation and characterize the set of matrices

(B, C) which guarantees marginal stability. The theory is applied to gyroscopic systems, to indefinite damped systems, and to circulat ory systems, showing how to choose certain parameter matrices to get sufficient conditions for marginal stability. Comparison is made with some known results for equations with real system matrices. Moreover more general cases are investigated and several examples are given.

Knejevich-Miljanovich J.
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Asymptotic properties of differential equation

Asymptotic properties of solutions have been considered for some nonlinear differential equations. The paper deals with investigation of bounded solutions, of prolongation of solutions, oscillatory solutions and another asymptotic properties. The example s have been stated which illustrate the given methods and have got physical interest. The paper is divided in two parts and each of them investigating some of asymptotic properties for certain differential equation. For general information is referred a short reference. The part one deals with asymptotic behavior of the solution of the differential equation (1) on [0,?), particularly with respect to oscillation. The part two deals with asymptotic behavior of solutions. They are also related to oscillation theory, to study the nonoscillatory solutions of the nonlinear equation (2) where r(x) is positive and continuous on with and f(x,y) is continuous on with f(x,y)¿0 if , and for equation (3) - 1991 Mathematics Subject Classification: 34-02, 34C10, 34D35 Key words: oscillation solutions, zeroes, growth, boundary solution Affiliation of author(s): Faculty of mathematics Studentski trg 16 Belgrade 11000, Yugoslavia knezevic@matf.bg.ac.yu Kobelkov Georgy Mikhajlovitch

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Differential equations over fields of positive characteristic

It is well known that any non-discrete, locally compact field of characteristic p is isomorphic to the field K of formal Laurent series with coefficients from the Galois field $\mathbf{F}_q$, $q = p^k$, $k \in \mathbf{Z}_+$. The foundations of analysis over K were laid by Carlitz, Wagner, Goss, and Thakur. An interesting property of many functions introduced by them as analogues of classical elementary and special functions, is their $\mathbf{F}_q$-linearity. Tools of classical calculus are not sufficient to study behavior of $\mathbf{F}_q$-linear functions. For example, if such a function f is differentiable then $f'(t) \equiv \text{const}$, and all the higher derivatives vanish irrespective of possible properties of f. In particular, one cannot reconstruct the Taylor coefficients of a holomorphic function – the classical formula contains the expression $\frac{f^{(n)}(t)}{n!}$ where both the numerator and denominator vanish. The correct analogue of the factorial has been found by Carlitz. In this work we give a counterpart of a higher derivative. This results in a formula for Taylor coefficients of a $\mathbf{F}_q$-linear holomorphic function, a definition of classes of $\mathbf{F}_q$-linear smooth functions which are characterized in terms of coefficients of their Fourier-Carlitz expansions. A Volkenborn-type integration theory for $\mathbf{F}_q$-linear functions is developed; in particular, an integral representation of the Carlitz logarithm is obtained. We study certain classes of equations for $\mathbf{F}_q$-linear functions, which are the natural function field counterparts of linear ordinary differential equations. It is shown that, in contrast to both classical and p-adic cases, formal power series solutions have positive radii of convergence near a singular point of an equation. Our approach is based on a function field analogue of the Schrödinger and Bargmann-Fock representations of the canonical commutation relations of quantum mechanics [1, 2, 3].

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Homogenization of Optimal Control Problems in Banach Spaces with Nonscalar Criterions

We study optimal control problems with nonscalar criterion. All components of their mathematical description may depend on a small parameter ε. Since numerical research of such problems is impossible with respect to small values of ε, the limit analysis of these problems is considered as $\varepsilon \to 0$. As an example let us consider the following optimal control problem. Let $U = V^*$ be a control space, which is dual of separable Banach space V, U_∂^ε be an admissible class of control from U, K_ε be a weakly closed subset in a separable Banach space Y, $Y \subset X$ with continuous and dense injection, where X is a reflexive Banach space, Z_1 be a Banach space which is semiordered by reproducing cone L. Let (Z_2, μ) be a topological vector space semiordered by reproducing cone Λ. Denote by $\Lambda-\operatorname{Inf}(\Omega)$ the set of all $\overset{(\Lambda)}{\leqslant}$-infimum elements for $\Omega \subset Z_2$. Now we have the following problem

$$\Lambda-\operatorname{Inf}(I_\varepsilon(u,y))$$

$$A_\varepsilon(u,y) = f_\varepsilon, \quad F_\varepsilon(u,y) \geqslant 0, \quad u \in U_\partial^\varepsilon, \quad y \in K_\varepsilon,$$

where f_ε is a fixed element from Y^*, $A_\varepsilon : U_\partial \times (D(A_\varepsilon) \subset X) \to Y^*$, $F_\varepsilon : U_\partial^\varepsilon \times Y \to Z_1$ are nonlinear operators, which may arbitrarily depend on ε, $I_\varepsilon : U_\partial \times X \to Z_2$ is a cost map. Thus we have the following problems: how to pass to the limit in such optimal control problem as $\varepsilon \to 0$; what can be expected as the result; what form (the structure) can it take? Obviously we can rewrite the previous problem in another form

$$\left\{\left\langle \Lambda - \operatorname*{Inf}_{(u,y)\in\Xi_\varepsilon} I_\varepsilon(u,y)\right\rangle, \varepsilon \to 0\right\},$$

where Ξ_ε is the set of all admissible pairs for the fixed ε. In this connection the abstract sequence is considered, elements of which are the nonscalar optimization problems. For this sequence the notion of variational V-limit is introduced (see [1,2,3]). The variational V-limit (V -homogenization problem) is a result of the passage to the limit when $\varepsilon \to 0$. This limit is some problem of nonscalar optimization which has a certain structure. The sufficient conditions under which there exists a V -homogenized optimal control problem for the above mentioned family of problems have been obtained.

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The Phenomenon of Bufferness In Nonlinear Hyperbolic Systems[16]

One says, that in some system of the differential equations with partial derivatives of a parabolic or hyperbolic type the phenomenon of bufferness is watched, if it is possible to provide the existence it of any fixed finite number of its stable cycles with an appropriate choice of parameters of this system (solutions, periodic on time).

It appears, that bufferness is a property of a wide class of mathematical models that adequately describe physical processes in the terms of the hyperbolic equations. For example, one can consider a nonlinear boundary value problem for system of a hyperbolic type consisting of the linear system of the telegraphic equations (considered on the finite interval with respect to the space variavle) and nonlinear boundary conditions (at the ends of the interval). Such problems are mathematical models of different self-oscillators containing a cut of a long line and nonlinear devices (say, tds) at the ends of a line.

[16]in Ruassian

Let's assume, that in a problem on the stability of a zero position of equilibrium of some hyperbolic system from a class described above the critical case of dountable number of purely imaginary eigenvalues is implemented, and with the change of some parameters of this system there is a bias of a part of a spectrum of stability in the right complex half-plane. Then the natural fashion a problem on existence and stability of self-oscillations of such system, bifurcating from zero, arises.

Without of particular resonance relations between eigenfrequencies of system in the indicated case there is a number of quasiharminic (i.e. close to harmonic with respect to time) stable cycles, and this number can be made as much as major by the appropriate choice of parameters. Besides there are still unstable quasiharmonic invariant toruses of different dimensions dividing the domains of attraction of different inconvertible cycles.

In case of a resonance spectrum of eigenfrequencies the study of self-oscillations in systems of the indicated type leads to two model nonlinear boundary value problems carrying, thus, generalpurpose character.

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Simple states in gas dynamics

A geometrical approach for solving hyperbolic quasilinear systems PDEs in three-dimensional space-time with coefficients depending on both dependent and independent variables is developed. A system describing propagation of short waves in gas dynamics is considered. The approach applied here is based on a detailed investigation of the respective overdetermined system and through a special construction, involving the Pfaff systems we reduce the system under consideration to system of ODEs.

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Boundary value problems for Hamiltonian systems with degenerate Hamiltonians, small time asymptotics for degenerate parabolic equations, and Young schemes

As is known, for Hamiltonian function of the form

$$H(x,p) = \frac{1}{2}(G(x)p,p) - (A(x),p) - V(x) \tag{1}$$

with invertible symmetric matrix G and uniformly bounded (together with their derivatives) G, G^{-1}, A and V, the boundary value problem for the Hamiltonian system

$$\dot{x} = \frac{\partial H}{\partial p}, \dot{p} = \frac{\partial H}{\partial x}, \quad x(0) = x_0, x(t) = x, \tag{2}$$

is always solvable. Moreover, there exists a solution which provides the absolute minimum $S(t, x, x_0)$ for the corresponding problem of the calculus of variations (with the Lagrangian $L(x, v)$ being the Legendre transform of H with respect to the second variable) with fixed boundary points x, x_0. At last, for small t and $x - x_0$ such a solution is unique and the function $tS(t, x, x_0)$ has a regular asymptotic expansion

$$tS(t, x, x_0) \sim \sum_{j=2}^{\infty} P_j(x_0)(t, x - x_0),$$

where $P_j(x_0)$ are homogeneous polynomials of degree j in variables t and $x - x_0$.

Here we describe the whole class of Hamiltonians (which we call regular) of form (1) but with possibly degenerate matrices $G(x)$ such that the global existence of the boundary value problem (2) giving global minimum for the corresponding problem of calculus of variations (with now singular Lagrangian) still holds and the function $t^{\alpha} S(t, x, x_0)$ has again a regular expansion in integer positive powers of t and $x - x_0$ with some constant $\alpha > 0$. In particular, this class of Hamiltonians contains a large subclass, which corresponds to the problems of calculus of variations with Lagrangians depending on higher derivatives of minimised curves. In general, the Hamiltonians of this class are classified by means of Young schemes which are well known in the theory of group representation.

To each Hamiltonian of form (1) there corresponds naturally a parabolic equation

$$\frac{\partial u}{\partial t} = \frac{h}{2}(G(x)\frac{\partial}{\partial x}, \frac{\partial}{\partial})u + (A(x), \frac{\partial u}{\partial x}) - \frac{1}{h}V(x)u,$$

where h is a positive parameter. It turns out that the same class of regular Hamiltonians describes the class of parabolic equations of this form (with possibly degenerate G) such that its Green function (or fundamental solution to the Cauchy problem, or heat kernel) has the small time (and small h) asymptotic expansion of the form

$$u_G(t, x, x_0) = (2\pi h)\alpha_1 t^{\alpha_2} Reg_1(t, x - x_0, h) \exp\{-Reg_2(t, x - x_0)/ht^{\alpha_3}\},$$

where Reg_1 and Reg_2 have asymptotic expansions in positive integer powers of t, $x - x_0$ and h, α_j are constants.

In particular, one can describe the corresonding class of invariant degenerate parabolic equations on manifolds with such a nice behavior of the heat kernel. It contains, for example, the following equation on the cotangent bundle T^*M to a Riemannian manifold M ($g(x) = G^{-1}(x)$ is a Riemannian metric on M)

$$\frac{\partial u}{\partial t} = Lu = \frac{h}{2} tr \left(g(x)\frac{\partial^2 u}{\partial y_i \partial y_j}\right) + \left(G(x)y, \frac{\partial u}{\partial x}\right) - \frac{1}{2}\left(\frac{\partial}{\partial x}(G(x)y, y), \frac{\partial u}{\partial y}\right).$$

which defines a semigroup that corresponds to the stochastic geodesic flow on T^*M. Using the above results, one can prove that the corresponding heat kernel has a trace, and the coefficients of its asymptotic expansion in small times are geometric invariants of M.

These results can be also extended to a certain class of nonlinear parabolic equations (reaction-diffusion equations) that recently became very popular in connection with the theory of superprocesses.

The core of the results described above (and some of their generalisations to non-local parabolic equations) is published in the recent author's monograph "Semiclassical Analysis for Diffusions and Stochastic Processes", Springer Lecture Notes Math., v. 1724, 2000.

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Finite-dimensional model for conductivity of a system of the dense packing particles

We consider Laplas equation in the domain with the large number of small absolutely conducting fillers. The solution takes the constant (not known) values on the fillers. Earlier proposed methods [1,2] deal with the equations with the continuous coefficients cannot be adopted to the such kind media. This paper demonstrates that in the particles filled high-contrast media the energy fluxes are concentrated in the necks between the neighbour particles and the original continuum problem has a finite dimensional approximation. *Formulation of the problem.* Consider the domain $P = [-1,1]\times[-L,L]$ in which the disks $D_i, i = 1,...,N$ of the radius R are distributed in a random way. Denote the remaining part of the domain by $Q = P\setminus \cup Q_i$. Consider the problem

$$\Delta u = 0, x \in Q_P,$$

$$u(x) = t_i, x \in D_i$$

$$\int_{\partial D_i} undx = 0, i = 1,...,N,$$

$$u(x,\pm 1) = \pm 1, \partial u/\partial n(\pm L, x) = 0$$

The effective conductivity of the filled medium is defined as

$$A = (1/2L)\int_{x=\pm 1} undx$$

. *The "net" (finite-dimensional) model.* The flux between the pair of disks (i-th and j-th) is equal to $g_{ij}(t_i - t_j)$, where $g_{ij} = \sqrt{R/\delta_{ij}}$, R is the radius of the disks, δ_{ij} is the distance between the disks. Introduce the net $x_i, t_i, g_{ij}; i,j = 1,...,N$, where x_i are the nodes and t_i are the potentials, which satisfy the equations

$$\sum g_{ij}(t_i - t_j) = 0, i \in I; t_i = \pm 1, i \in S^{\pm 1}$$

I are the interior and $S^{\pm 1}$ are the boundary nodes, corresponding to the disks belonging to the boundaries $x = \pm 1$. *Theorem.* The effective conductivity A has the order of $\sqrt{R/\delta}$ as $\delta \to 0$, where $\delta = max\delta_{ij}$ and max is taken for the neighbour disks. The leading term (of the order of $\sqrt{R/\delta}$) as $\delta \to 0$ is expressed through t_i - solution of the "net" problem in the form $A = 1/4 \sum g_{ij}(t_i - t_j)^2$.

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On attractor of a nonlinear U(1)-invariant 1D Klein-Gordon equation

An attractor is studied for all finite energy solutions to a model nonlinear U(1)-invariant 1D Klein-Gordon equation. The attractor is a union of the solitary waves psi(x)exp(i omega t).

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$C^{0,\alpha}$ solutions of the Dirichlet problem for parabolic equations

In a bounded domain $\Omega \subset D$, where $D = R^n \times (0,T)$, $T < \infty$, $n \geqslant 1$, we consider the parabolic operator of the second order:

$$Lu = u_t - a_{ij}(x,t)\partial_{ij}u - b_i(x,t)\partial_i u - c(x,t)u.$$

We assume that the real-valued coefficients satisfy the following conditions:

$$(\exists \delta > 0)\ (\forall P \in \bar{D},\ \forall \xi \in R^n) \quad a_{ij}(P)\xi_i\xi_j \geqslant \delta|\xi|^2; \tag{0.1}$$

$$a_{ij} \in H_\alpha(\Omega), \quad b_i \in H_\alpha^{(1-\alpha)}(\Omega), \quad c \in H_\alpha^{(2-\alpha)}(\Omega), \quad \alpha \in (0,1). \tag{0.2}$$

No assumption is made on the sign of the coefficient c. Here H_α and H_α^b denote anisotropic Hölder spaces and weighted anisotropic Hölder spaces respectively, see [1]. The latter spaces allow functions (or their derivatives) to blow up in a certain way near the parabolical boundary $\partial\Omega = \Sigma \cup B_0$, where Σ is the "lateral" boundary of Ω and the domain $B_0 \subset R^n$ lies in the plane $\{t = 0\}$. Denote $\Sigma_\tau = \Sigma \cap \{t = \tau\}$ and

$$\Lambda = \max\left\{\|a_{ij},\Omega\|_\alpha, \|b_i,\Omega\|_\alpha^{(1-\alpha)},\ \|c,\Omega\|_\alpha^{(2-\alpha)}\right\}.$$

We consider the Dirichlet problem:

$$\begin{cases} Lu = f \in H_\alpha^{(2-\alpha)}(\Omega), \\ u|_{B_0} = \phi \in H_\alpha(B_0), \\ u|_\Sigma = \psi \in H_\alpha(\Sigma). \end{cases} \tag{0.3}$$

The following result is established.

Theorem. *Suppose $\Sigma \in H_{1+\alpha}$ and the coefficients of the operator L satisfy conditions (0.1), (0.2). Let $u \in C(\bar{\Omega}) \cap C^{2,1}(\Omega)$ be the solution of (0.3) with $\phi|_{\Sigma_0} = \psi|_{\Sigma_0}$. Then $u \in H_{2+\alpha}^{(-\alpha)}(\Omega)$. Furthermore, there exists a positive constant C depending on n, α, δ, Ω, Λ, such that*

$$\|u, \Omega\|_{2+\alpha}^{(-\alpha)} \leqslant C\left[\|\phi, B_0\|_\alpha + \|\psi, \Sigma\|_\alpha + \|f, \Omega\|_\alpha^{(2-\alpha)}\right].$$

The special case of a cylinder $\Omega = B_R \times (0, T)$, where B_R is a ball, and Hölder in the closure of Ω coefficients of the operator L was considered by G. Lieberman [1]. The corresponding statement for elliptic equations was obtained by D. Gilbarg and L. Hörmander [2] under less restrictive assumptions on the coefficients a_{ij}. The present theorem is proved by the barrier method. We avoid local flattening of the boundary with the help of a "global" barrier. The latter is constructed using a solution of an auxiliary parabolical problem and results of E. Baderko [3] on smoothness and potential representations of such solutions.

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On solutions of nonlinear Emden-Fowler elliptic systems

We study elliptic systems of the form

$$\begin{cases} \Delta u_0 = p_0(x) u_1^{\lambda_0} \\ \dots \\ \Delta u_i = p_i(x) u_{i+1}^{\lambda_i} \\ \dots \\ \Delta u_{m-1} = p_{m-1}(x) u_0^{\lambda_{m-1}} \end{cases} \tag{1}$$

where $m \geqslant 2$, $x = (x_1, \dots, x_n)$, $n \geqslant 2$, $\lambda_0 \dots \lambda_{m-1} > 1$, $\lambda_i \geqslant 1$, and p_i are nonnegative functions, $i = 0, \dots, m-1$. In particular, we strengthen the results of papers [1] and [2].

Put: $Q_r = \{x : |x| < r\}$. For a real number $\sigma > 1$ and a measurable function $\varphi : \mathbf{R}^n \to \mathbf{R}$, we define $\varphi(r;\sigma) = \operatorname{ess\,inf}_{r/\sigma<|x|<r\sigma} \varphi(x)$. Suppose that $j \in \{0, \dots, m-1\}$. The following notation is used: $\alpha_j = 2+2\sum_{i=1}^{m-1} l(i,j)$, $\beta_j = \lambda_j(n-2)-n+\sum_{i=1}^{m-1} l(i,j)(\lambda_{i+j}(n-2)-n)$, and $F_{\sigma,j}(r) = p_j(r;\sigma)\prod_{i=1}^{m-1} p_{i+j}^{l(i,j)}(r;\sigma)$, where $l(k,j) = \prod_{i=0}^{k-1} \lambda_{i+j}$, $k = 1, \dots, m$. We note that the sum of subscripts is taken modulo m. We also assume that either $p_i \in L_{loc}(\mathbf{R}^n)$ (for solutions of (1) in exterior domains) or $p_i \in L_{loc}(Q_{R_0} \setminus \{0\})$ (for solutions of (1) in $Q_{R_0} \setminus \{0\}$), $R_0 > 0$, $i = 0, \dots, m-1$.

Theorem 1. *Let $u = (u_1, \dots, u_{m-1})$ be a nonnegative solution of system (1) in $\mathbf{R}^n \setminus \overline{Q}_{R_0}$, $R_0 > 0$, and, moreover,*

$$\int^{\infty} r^{\alpha_j - 1} F_{\sigma,j}(r)\, dr = \infty \tag{2}$$

for some $\sigma \in (1,\infty)$ and $j \in \{0, \dots, m-1\}$. Then

$$u_j(x) \leqslant C \left(\int_{R_0}^{|x|} r^{\alpha_j - 1} F_{\sigma,j}(r)\, dr \right)^{-\frac{1}{\lambda - 1}}$$

for all x in a neighborhood of infinity, where the constant $C > 0$ depends only on σ, n, m, $\lambda_0, \dots, \lambda_{m-1}$.

Theorem 2. *Let $u = (u_1, \dots, u_{m-1})$ be a nonnegative solution of system (1) in $\mathbf{R}^n \setminus \overline{Q}_{R_0}$, $R_0 > 0$, and, moreover,*

$$\int^{\infty} r^{\alpha_j - 1} F_{\sigma,j}(r)\, dr < \infty$$

for some $\sigma \in (1,\infty)$ and $j \in \{0, \dots, m-1\}$. Then

$$u_j(x) \leqslant C \left(\int_{|x|}^{\infty} r^{\alpha_j - 1} F_{\sigma,j}(r)\, dr \right)^{-\frac{1}{\lambda - 1}}$$

for all x in a neighborhood of infinity, where the constant $C > 0$ depends only on σ, n, m, $\lambda_0, \dots, \lambda_{m-1}$.

Theorem 3. *Suppose that $u = (u_1, \dots, u_{m-1})$ is a nonnegative solution of system (1) in $\mathbf{R}^n$ and relation (2) is valid for some $\sigma \in (1,\infty)$ and $j \in \{0, \dots, m-1\}$. Then $u \equiv 0$.*

Example 1. Assume that $m = 2$, $\lambda_0\lambda_1 > 1$, and $p_i(x) \sim |x|^{s_i}$ as $x \to \infty$ for some $s_i \in \mathbf{R}$, $i = 0, 1$. If one of the conditions $s_0 + \lambda_0 s_1 + 2\lambda_0 + 2 \geqslant 0$ (the case of $j = 0$) or $s_1 + \lambda_1 s_0 + 2\lambda_1 + 2 \geqslant 0$ (the case of $j = 1$) holds, then, by Theorem 1, each nonnegative solution of (1) identically equals zero.

Example 2. As before, assume that $m = 2$, $\lambda_0\lambda_1 > 1$, but now $p_i(x) \sim |x|^{s_i} \ln^{\mu_i} |x|$ as $x \to \infty$, $i = 0, 1$, where $s_0 + \lambda_0 s_1 + 2\lambda_0 + 2 = 0$ and $s_1 + \lambda_1 s_0 + 2\lambda_1 + 2 < 0$. Then the relation $\mu_0 + \lambda_0\mu_1 + 1 \geqslant 0$ implies that each nonnegative solution of system (1) identically equals zero.

Theorem 4. *Let $u = (u_1, \dots, u_{m-1})$ be a nonnegative solution of system (1) in $Q_{R_0} \setminus \{0\}$, $R_0 > 0$, and, moreover,*

$$\int_0 r^{-1-\beta_j} F_{\sigma,j}(r)\, dr = \infty \tag{3}$$

for some $\sigma \in (1,\infty)$ and $j \in \{0,\dots,m-1\}$. Then there exists $\varepsilon > 0$ such that

$$u_j(x) \leqslant C|x|^{2-n}\left(\int_{|x|}^{R_0} r^{-1-\beta_j} F_{\sigma,j}(r)\,dr\right)^{-\frac{1}{\lambda-1}}$$

for all $x \in Q_\varepsilon \setminus \{0\}$, where the constant $C > 0$ depends only on σ, n, m, $\lambda_0,\dots,\lambda_{m-1}$.

Theorem 5. *Let $u = (u_1,\dots,u_{m-1})$ be a nonnegative solution of system (1) in $Q_{R_0}\setminus\{0\}$, $R_0 > 0$, and, moreover,*

$$\int_0 r^{-1-\beta_j} F_{\sigma,j}(r)\,dr < \infty$$

for some $\sigma \in (1,\infty)$ and $j \in \{0,\dots,m-1\}$. Then there exists $\varepsilon > 0$ such that

$$u_j(x) \leqslant C|x|^{2-n}\left(\int_0^{|x|} r^{-1-\beta_j} F_{\sigma,j}(r)\,dr\right)^{-\frac{1}{\lambda-1}}$$

for all $x \in Q_\varepsilon \setminus \{0\}$, where the constant $C > 0$ depends only on σ, n, m, $\lambda_0,\dots,\lambda_{m-1}$.

Corollary 1. *Suppose that $u = (u_1,\dots,u_{m-1})$ is a nonnegative solution of system (1) in $Q_{R_0} \setminus \{0\}$, $R_0 > 0$, and relation (3) holds for some $\sigma \in (1,\infty)$ and $j \in \{0,\dots,m-1\}$. Then a singularity at zero of u_j is removable, i.e., $u_j \in W_2^1(Q_{R_0}) \cap L_\infty(Q_{R_0})$, $p_j u_{j+1}^{\lambda_j} \in L(Q_{R_0})$, and the j-th equation of system (1) is valid in the entire domain Q_{R_0} (in the integral identity sense).* This work was supported by the Russian Foundation for Basic Research, grant № 99-01-00225.

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On the essential spectrum of a non-elliptic boundary value problem: an extension of the paper of H. Langer and M. Möller

In 1996 H. Langer and M. Möller [1] applied the abstract approach of [2] (see also [3], [4]) to determine the essential spectrum of matrix partial differential operators of the form

$$L_0 := \begin{pmatrix} -\rho^{-1}\partial_1\rho a\partial_1 + b & -\rho^{-1}\partial_1\rho a\partial_2 & -\rho^{-1}i\partial_1\rho c_1 \\ -\rho^{-1}\partial_2\rho a\partial_1 & -\rho^{-1}\partial_2\rho a\partial_2 + b & -\rho^{-1}i\partial_2\rho c_1 \\ -ic_2\partial_1 & -ic_2\partial_2 & d \end{pmatrix},$$

$$\mathcal{D}(L_0) := \{f = (f_1, f_2, f_3) \in (W_2^1(\Omega))^3 \mid \partial_1 f_1 + \partial_2 f_2 \in W_2^1(\Omega),\ \nu_1 f_1 + \nu_2 f_2|_{\partial\Omega} = 0\}.$$

Here $\partial_1 = \frac{\partial}{\partial x_1}$, $\partial_2 = \frac{\partial}{\partial x_2}$, $\Omega \subset \mathbb{R}^2$ is a bounded domain with boundary $\partial\Omega$ which is Lipschitz continuous, piecewise C^2 and whose angles are all convex, $\nu = (\nu_1, \nu_2)$ is the outer normal of $\partial\Omega$, a, c_1, c_2, ρ are Lipschitz continuous functions on $\overline{\Omega}$, and b, d are continuous functions on $\overline{\Omega}$. The functions a and ρ are supposed to be positive on $\overline{\Omega}$. The operator L_0 is closable in $(L^2(\Omega, \rho\, dx))^3$. It was proved in [1] that the essential spectrum of its closure L is given by the formula

$$\sigma_{ess}(L) = b(\overline{\Omega}) \cup (d - \frac{c_1 c_2}{a})(\overline{\Omega})$$

Analogous results were obtained earlier by Raikov [5] for more special case of a linearized magnetohydrodynamic model. The main purpose of this talk is to generalize the results of [1] on matrix partial differential operators of the form

$$K_0 := \begin{pmatrix} -\rho^{-1}\partial_1 \rho a \partial_1 + b_{11} & -\rho^{-1}\partial_1 \rho a \partial_2 + b_{12} & -\rho^{-1} i \partial_1 \rho c_1 \\ -\rho^{-1}\partial_2 \rho a \partial_1 + b_{21} & -\rho^{-1}\partial_2 \rho a \partial_2 + b_{22} & -\rho^{-1} i \partial_2 \rho c_1 \\ -ic_2\partial_1 & -ic_2\partial_2 & d \end{pmatrix}, \quad \mathcal{D}(K_0) := \mathcal{D}(L_0).$$

Here b_{ij}, $i, j = 1, 2$, are continuous functions on $\overline{\Omega}$ such that for any $x \in \Omega$ the 2×2 matrix $(b_{ij}(x))_{i,j=1}^2$ is Hermitian. Coefficients a, c_1, c_2, d, ρ are supposed to be as above. Denote by $\lambda_1(x)$, $\lambda_2(x)$ ($\lambda_1(x) \leqslant \lambda_2(x)$) the eigenvalues of the matrix $(\operatorname{Re} b_{ij}(x))_{i,j=1}^2$ and set

$$m := \min\{\lambda_1(x) : x \in \overline{\Omega}\}, \quad M := \max\{\lambda_2(x) : x \in \overline{\Omega}\}.$$

Denote by K the closure of the operator K_0 in $(L_2(\Omega, \rho\, dx))^3$.

Theorem. *The essential spectrum of the operator K is given by the formula*

$$\sigma_{ess}(K) = [m, M] \cup (d - \frac{c_1 c_2}{a})(\overline{\Omega}).$$

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On Singularly Perturbed Problems with Singularities

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Complete second order linear differential equations in Hilbert space and hydrodinamical applications

There were investigated Cauchy problem for linear complete second order differential operator equations in Hilbert space $\mathcal{H}$

$$\frac{d^2u}{dt^2} + (F + iK)\frac{du}{dt} + Bu = f(t), \quad u(0) = u^0, \quad u'(0) = u^1,$$

where F, K and B are self-adjoint operator coefficients acting in $\mathcal{H}$. Theorem on strong solvability is proved, applications to the famous S. Krein problem on small movements of a viscous fluid in an open vessel and to other problems are studied.

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On a D'Alembert type formula on finite one dimensional networks

We want to suggest a new presentation of the solution $u(x,t)$ of the Cauchy problem for the wave equation on a finite weighted graph Γ

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2} & t \geqslant 0 \\ u(x,0) = \varphi(x) & \\ \dfrac{\partial u}{\partial t}(x,0) = \psi(x) & \end{cases} \tag{0.1}$$

in the form

$$u(x,t) = f(x,t) + \sum_{i=1}^{q} f_i(x,t)\cos\omega_i t + g_i(x,t)\sin\omega_i t$$

where $f(x,t)$ is a 2-periodic in time function, $f_i(x,t)$ and $g_i(x,t)$ are 1-periodic in time. $\cos\omega_i \in \sigma(S) \cap (-1,1)$ $(1 \leqslant i \leqslant q)$, where $\sigma(S)$ denotes the spectrum of a dispersion matrix S. S is a $2|E| \times 2|E|$ matrix depending on the geometrical structure of Γ. The functions $f(x,t)$, $f_i(x,t)$ and $g_i(x,t)$ are difined like $f(x,t) = u_0(x,t)$,

$$f_i(x,t) = \frac{1}{\sin\omega_i}(u_i(x,t)\sin\omega_i(t+1) - u_i(x,t+1)\sin\omega_i t),$$

$$g_i(x,t) = \frac{1}{\sin\omega_i}(u_i(x,t+1)\cos\omega_i t - u_i(x,t)\cos\omega_i(t+1)),$$

where $u_i(x,t)$ $(0 \leqslant i \leqslant q)$ is the solution of (0.1) with the initial data

$$u_i(x,0) = \varphi_i(x), \ \frac{\partial u_i}{\partial t}(x,0) = \psi_i(x).$$

The functions $\varphi_i(x)$ and $\psi_i(x)$ $(0 \leqslant i \leqslant q)$ can be obtained by the ortogonal projecting on the proper subspaces of S. The details will be given in the talk.

Korallov L.B.

Random Perturbations of 2-D Periodic Hamiltonian Flows

We consider the motion of a particle in a periodic two dimensional flow perturbed by small (molecular) diffusion. The flow is generated by a divergence free zero mean vector field. The long time behavior corresponds to the behavior of the homogenized process - that is diffusion process with the constant diffusion matrix (effective diffusivity). We obtain the asymptotics of the effective diffusivity when the molecular diffusion tends to zero. In the case of cellular flows the effective diffusivity has the order of the square root of molecular diffusion.

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On globally stable approximation schemes

For a semigroup $\{S_{\lambda_0}(t,\cdot)\}$ in a Banach space X corresponding, for example, to a evolution equation and having global attractor $\mathcal{M}^{\lambda_0}$ the globally stable approximation (GSA) schemes are studied. GSA scheme has attractor $\mathcal{M}^{\lambda}$ and in addition $\mathcal{M}^{\lambda} \to \mathcal{M}^{\lambda_0}$ when the approximation parameters λ tend to a limit λ_0. An approach to approximate a global attractor of a semidynamical system with error estimates in Hausdorff metric is presented. This approach is based on the properties of a function of rate of attraction to an attractor and on some new results for an unstable manifold in a neighborhood of an essential nonhyperbolical point. For some classes of the semidynamical system we construct an unstable manifold in the neighborhood of a fixed isolated point, prove that each trajectory is attracted to the manifold and find the function of attraction.

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Inverse problem and estimates for Schrödinger operator with periodic potentials

We solve the inverse problem for the Schrödinger operator $Ty = -y'' + q(x)y$ acting on $L^2(R)$, where $q \in L^2(0,1)$ is a 1-periodic real potential and $\int_0^1 q(x)dx = 0$. The spectrum of T is absolutely continuous and consists of intervals separated by gaps $\gamma_n = (a_n^-, a_n^+), n \geqslant 1$. Let $\mu_n, n \geqslant 1$, be the Dirichlet eigenvalue of the equation $-y'' + qy = \mu_n y$ on the interval $[0,1]$. Introduce the vector $g_n = (g_{cn}, g_{sn}) \in R^2$, with components $g_{cn} = \frac{1}{2}(a_n^+ + a_n^+) - \mu_n$, and $g_{sn} = |\frac{1}{4}|\gamma_n|^2 - g_{cn}^2|s_n$, where the sign $s_n = +$

or $s_n = -$ for all $n \geqslant 1$. Using nonlinear functional analysis in Hilbert spaces (the direct method) we show, that the mapping $g : q \to g(q) = \{g_n\}_1^\infty \in \ell^2 \oplus \ell^2$ is a real analytic isomorphism [3]. In particula, this implies a corresponding trace formula. Moreover, we give the new short proof [2] of the well known Marchenko and Ostrovski result. In our approach we do not use the equation of Gelfand-Levitan-Marchenko or a trace formula. Using our method we solve the inverse problems for Zakharov-Shabat systems and for weighted operators [7-8]. We formulate the key result of the direct method, proved in [1].
Theorem A. *Let H, H_1 be real separable Hilbert spaces equipped with the norms $\|\cdot\|, \|\cdot\|_1$. Suppose the map $f : H \to H_1$ satisfies the following conditions: i) f is real analytic and for each $q \in H$ the operator df/dq has an inverse, ii) there exists a nondecreasing function $F : [0,\infty) \to [0,\infty), F(0) = 0$, such that $\|q\| \leqslant F(\|f(q)\|_1)$ for all $q \in H$, iii) there exists a linear isomorphism $J : H \to H_1$ such that the mapping $f - J : H \to H_1$ is compact. Then f is a real analytic isomorphism between H and H_1.* In the second part we prove estimates. For example: $\|q\| \leqslant 2\|\gamma\|(1 + \|\gamma\|^{1/3})$, where $\|\gamma\|^2 = \sum_{n\geqslant 1} |\gamma_n|^2$ and $|\gamma_n| \geqslant 0, n \geqslant 1$, is the gap length of T [4-6].

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Orbital equivalence of some classical integrable cases in rigid body dynamics

Two smooth dynamical systems on manifolds M_1, M_2 are *orbitally equivalent* if there exists a homeomorphism from M_1 onto M_2 which transforms the trajectories of the first system to the trajectories of the second system and preserves orientation (so the trajectories are the same up to the change of time). In this work the pair of integrable cases of the rigid body dynamics is regared. It's formed by so-called Lagrange and Euler cases. As for the Euler case which describes the rotation of the rigid body about its center of mass, the special case is taken, when two of three principal moments of inertia are equal, so that the body is axisymmetric. It's proved, that under some conditions these two cases are orbitally equivalent on their rather high isoenergy levels. These systems can be determined by hamiltonians $H_L = \frac{1}{2}\left(s_1^2 + s_2^2 + \frac{s_3^2}{\beta}\right) + V(r_3)$ and $H_E = \frac{1}{2}\left(s_1^2 + s_2^2 + \frac{s_3^2}{\gamma}\right)$ respectively. Here $V(x)$ is a convex smooth function (potential). It caracterizes the force field involved. The additional integral in both cases is $F = s_3$. The classical Lagrange case describes the rotation of an axisymmetrical heavy rigid body about a fixed point on the symmetry axis, and it corresponds to the potential $V(x) = ax$. It's well known that on the high-energy levels these two problems are *Liouville equivalent* (see for example [1]). This means that their Liouville foliations are diffeomorphic. Orbital equvalence is more delicate. It turns out that the systems $v_E = \operatorname{sgrad} H_E$ and $v_L = \operatorname{sgrad} H_L$ for the specially selected parameters are orbitally equvalent on the constant energy surfaces for enough great values of energy. In particular it means that from the qualitative point of view the motion of the axisymmetric rigid body about a fixed point situated on the symmetry axis in gravity force field would be the same as the motion of another axisymmetric rigid body about its center of mass (and without any potential). The precise result can be formulated in the following way:

Theorem. Let the value of area integral be $g = 0$. Then

1. The Lagrange system with parameter β and potential $V(x)$ restricted to the constant-energy surface $Q^3 = \{H_L = h\}$ for enough great h is orbitally equivalent to the Euler system with some parameter $\gamma = \gamma(h)$.

2. For any γ, β and h there exists a family of potentials $V(x)$ so that Lagrange system with parameter β and potential $V(x)$ on the energy level $Q^3 = \{H_L = h\}$ is orbitally equivalent to the Euler system with parameter γ.

In [2] Lagrange systems were regarded from the point of view of the orbital classification, but this was done by means of computer analysis. But there was formulated a hypotesis which states that Lagrange systems, corresponding to the same parameters β, $V(x)$ but to the different isoenergy levels are not equivalent (orbitally). So the results obtained do not contradict to the hypotesis mentioned from [2].

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Граничные задачи для гиперболических уравнений второго и третьего порядка

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Regularity of solutions for nonlinear problems of continuum media

The main question of regularity for weak solutions of some boundary values problems concerning the elliptic and parabolic systems describing the continuum media are discussed. For example the regular (Hälder continuous) solutions for the nonlinear elastic system,the system of Navier-Stokes and some other ones. Since thise problems are described by, more or less deneral second order partial differential system it is impossible to use in general the maximum principle. So, the main results follov from some coercive explicit estimales with sharp constants. The functions which gives the optimal situations for these inegualities help to obtain sometimes exact conditions for the loss of regularity for weak solutions of the problem. For example in some problems of nonliniar elasticity with these results we come to sharp conditions for the appearance of unbounded solutions. For the Navier-Stokes system the chaotic solutions can appear in the case of sufficiently big Reynolds numbers. The details will be given at the talk.

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The comparison theorems to solutions of Neuman problem for elliptic equations

Let $D \subset R^n$ be a bounded domain with C^1 boundary Γ. Let $u(x)$ be a solution to the following problem

$$Lu \equiv \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}(a_{ij}\frac{\partial u}{\partial x_j}) = ku \quad \text{in } D, \qquad \left.\frac{\partial u}{\partial \nu}\right|_{\Gamma} = R > 0, \tag{1}$$

where $\partial/\partial\nu$ is conormal derivative and L is uniformly elliptic operator. We note by A and p the corresponding Lebesgue measures of domain D and boundary Γ. The solution of problem (2) can be represented as

$$u = \frac{Rp}{kA} + z + \omega, \tag{2}$$

where ω satisfies the inequality $\max|\omega| < C|k|$ in D and z does not depend on k. This result follows from [1] and [2]. Let u_i be the solutions to (1) in the domains D_i $(i = 0, 1)$. From (2) we immediately obtain:

Theorem. *Let domains D_0 and D_1 be such that inequality*

$$\frac{p_0}{A_0} > \frac{p_1}{A_1}$$

holds. Then there exists a number $k_0 > 0$ such that for any positive number $k < k_0$ the inequality

$$\inf_{D_0} u_0 > \sup_{D_1} u_1$$

holds.

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Nonlinear Schrödinger equation on the half- line

The present talk is concerned with the direct and inverse scattering problems for compatible differential equations connected with the nonlinear Schrödinger equation on the half-line. The corresponding initial boundary value problem $(x, t \in \mathbb{R}_+)$ was studied recently by A.S.Fokas and A.R.Its. They found that the key to this problem is to linearize the initial boundary value problem using a Riemann-Hilbert problem. The main goal of this talk is to obtain characteristic properties of the scattering data for compatible differential equations. Our approach uses the transformation operators for both x- and t-equations. For the Schwartz type initial and boundary functions we get the characteristic properties of the scattering data and derive the so-called xt- and t-integral equations of the Marchenko type. The xt-integral equations guarantee the existence of the solution of the nonlinear Schrödiger equation as well an expression of the solution with given scattering data. In turn, the t-integral equations guarantee that one can recover from the scattering data boundary Dirichlet data $v(t)$ and corresponding Neumann data $w(t)$ consistent with the given initial function $u(x)$.

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Energy criteria for separatrix crossing in near-Hamiltonian stochastic systems

The separatrix-crossing processes in near-Hamiltonian systems have until recently been studied for deterministic systems. In this work we examine the effect of small random perturbations on these processes. The phase space of a near-Hamiltonian system can be divided into domain corresponding to different types of motions: librations (oscillations) within the domains inside the separatrix and rotation in the domains beyond the separatrix. Unbounded rotations in the outer domains are associated with failure of the system, domains inside the separatrix are considered as safe regions. Small perturbations may induce passage through separatrix into a safe region (i.e., capture) or out of a safe region (i.e., escape), or passage from one safe region to another (i.e., jump). From the physical point of view, captured motions occurs if the full energy of the system decreases due to dissipation in the near-separatrix domain, and escape occurs if the energy increases due to effect of nonconservative forces in this domain. This study develops the perturbation method for computing the difference of energy associated with the crossing of the separatrix in stochastic systems. It is shown that the difference of the energy can be approximated by the Melnikov integral. The perturbation method gives a procedure to calculate the probabilities of escape out of the safe region and capture into the safe region. In addition, we demonstrate that the Melnikov method and the perturbation method exhibit equivalent criteria for escape and capture in stochastic systems.

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Nonlinear evolution equations on noncylindrical domains

Kozyrev S.V.

Wavelet analysis as a p-adic spectral analysis

New orthonormal basis of eigenfunctions for the Vladimirov operator of p-adic fractional derivation is constructed. The map of p-adic numbers onto real numbers (p-adic change of variables) is considered. This map (for $p = 2$) provides an equivalence between the constructed basis of eigenfunctions of the Vladimirov operator and the wavelet basis in $L^2(R)$ generated from the Haar wavelet. This means that the wavelet analysis can be considered as a p-adic spectral analysis.

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Deformations and integrable systems

Geometrical approach to differential equations [1] gives rise to important cohomological theories providing invariants of equations. A theory of $\mathcal{C}$-cohomology introduced in [2] relates to deformations of equation structures.

1. Deformations. Let M be a smooth manifold and $\pi: E \to M$ be a locally trivial bundle, $J^k(\pi)$ be the manifold of k-jets, and $\mathcal{E}^\infty \subset \mathcal{J}^\infty(\pi)$ an infinitely prolonged equation, assumed to be formally integrable. Then $\pi_\infty: \mathcal{E}^\infty \to M$ possesses a flat connection $\mathcal{C}$: $\mathrm{D}(M) \to \mathrm{D}(\mathcal{E}^\infty)$, where $\mathrm{D}(\cdot)$ is the module of vector fields (if N is a manifold and P is a $C^\infty(N)$-module, then $\mathrm{D}(P)$ is the module of P-valued derivations $C^\infty(N) \to P$). The connection form $U_\mathcal{C}(\mathcal{E}) \in \mathrm{D}(\Lambda^1(\mathcal{E}^\infty))$ is called the structural element of $\mathcal{E}$. Consider the operator $\partial_\mathcal{C} = [\![U_\mathcal{C}, \cdot]\!]$, where $[\![\cdot, \cdot]\!]$ is the Frölicher–Nijenhuis bracket. $(\mathrm{D}(\Lambda^i(\mathcal{E}^\infty)), \partial_\mathcal{C})$ is a complex. Its cohomology $H^i(\partial_\mathcal{C})$ is called $\mathcal{C}$-cohomology. $H^1(\partial_\mathcal{C})$ consists of equivalence classes of deformations of the equation. $H^2(\partial_\mathcal{C})$ contains obstructions for continuation of infinitesimal deformations up to formal ones.

2. $\mathcal{C}$-cohomology and recursion operators. $X \in \mathrm{D}(\Lambda^i(\mathcal{E}^\infty))$ is vertical, if it vanishes on $C^\infty(M)$. Denote the module of such derivations by $\mathrm{D}^v(\Lambda^i(\mathcal{E}^\infty))$. Restriction of $\partial_\mathcal{C}$ to $\mathrm{D}^v(\Lambda^i(\mathcal{E}^\infty))$ gives the vertical complex $(\mathrm{D}^v(\Lambda^i(\mathcal{E}^\infty)), \partial_\mathcal{C})$. Its cohomology is denoted by $H^i_\mathcal{C}(\mathcal{E})$. $H^0_\mathcal{C}(\mathcal{E})$ is the Lie algebra of higher symmetries. Modules $H^i_\mathcal{C}(\mathcal{E})$ inherit the inner product. $H^1_\mathcal{C}(\mathcal{E})$ is an associative algebra with respect to $\rfloor$ and $H^0_\mathcal{C}(\mathcal{E})\rfloor H^1_\mathcal{C}(\mathcal{E}) \to H^0_\mathcal{C}(\mathcal{E})$ is a representation of this algebra. Elements of $H^1_\mathcal{C}(\mathcal{E})$ act on symmetries and are recursion operators. Let $\mathcal{C}\Lambda^1(\mathcal{E}^\infty) \subset \Lambda^1(\mathcal{E}^\infty)$ be the submodule of 1-forms vanishing on the horizontal distribution $\mathcal{H}$ of $\mathcal{C}$.

Theorem. *Nontrivial recursion operators are in* 1-1 *correspondence with solutions of the equation* $\ell_\mathcal{E}\Omega = 0$, $\Omega \in \mathcal{C}\Lambda^1(\mathcal{E}^\infty) \otimes \Gamma(\pi)$, *where* $\ell_\mathcal{E}$ *is the linearization operator of* $\mathcal{E}$, $\Gamma(\pi)$ *is the module of section of* π.

3. $\mathcal{C}$-cohomology and Bäcklund transformations. Let W be a manifold with a $\dim M$-dimensional integrable distribution $\tilde{\mathcal{H}}$. A bundle $\tau\colon W \to \mathcal{E}^\infty$ is a covering over $\mathcal{E}$, if $d\tau(\tilde{\mathcal{H}}_y) = \tilde{\mathcal{H}}_{\tau(y)}$, $y \in W$. Then $\mathcal{C}$ is extended to a connection $\tilde{\mathcal{C}}$ in $\pi_\infty \circ \tau$ and the previous theory can be constructed in the same way for this bundle. $\varphi \in C^\infty(W) \otimes \Gamma(\pi)$ is a shadow of a nonlocal τ-symmetry, if $\tilde{\ell}_\mathcal{E}\varphi = 0$, where $\tilde{\ell}_\mathcal{E}$ is a natural lifting of the linearization operator to W. A BT is a pair of coverings $\mathcal{E}^\infty_1 \xleftarrow{\tau_1} W \xrightarrow{\tau_2} \mathcal{E}^\infty_2$. A standard situation where nonlinear superposition effects arise is as follows. Consider family of nonequivalent coverings $\tau_\lambda : W \to \mathcal{E}^\infty$, $\lambda \in \mathbb{R}$, and a diffeomorphism $A\colon W \to W$ preserving $\tilde{\mathcal{H}}$. Equivalently, one can consider a covering $\tau = \tau_0$ and a family of diffeomorphisms $A_\lambda : W \to W$.

Theorem. *Let* $\tau\colon W \to \mathcal{E}^\infty$ *be a covering and* $A_\lambda : W \to W$ *be a smooth family of diffeomorphisms such that* $A_0 = \mathrm{id}$ *and* $\tau_\lambda = \tau \circ A_\lambda$ *is a covering for any* λ. *Then* U_{τ_λ} *is of the form* $U_{\tau_\lambda} = U_\tau + \lambda[\![U_\tau, X]\!] + O(\lambda^2)$, *where* X *is a* τ*-shadow.*

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Vector fibrations and Lax equations on algebraic curves

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Helmholts equation in 2-d domains bounded by closed and opened curves

The boundary value problems for elliptic equations in plane domains bounded by closed and open curves were not studied before. Solvability of problems outside open curves in the plane [2,3,6] and solvability of problems in domains bounded by closed curves [1,6] were treated separately because different methods were used in their analysis. The present note is attempt to join these problems together and to consider external domains bounded by closed and open curves. From practical stand-point such domains have great significance, because open curves can model cracks, screens or wings in some physical problems. In the present note we study the Dirichlet problem for the 2-D Helmholtz equation in an external domain bounded by closed and open curves [4]. The existence of classical solution is proved by potential theory. The problem is reduced to the Fredholm equation of the second kind, which is uniquely solvable.

In the plane $x = (x_1, x_2) \in R^2$ we consider the external multiply connected domain bounded by simple open curves $\Gamma_1^1, ..., \Gamma_{N_1}^1 \in C^{2,\lambda}$ and simple closed curves $\Gamma_1^2, ..., \Gamma_{N_2}^2 \in C^{2,\lambda}$, $\lambda \in (0,1]$, so that the curves do not have points in common. We put $\Gamma^1 = \bigcup_{n=1}^{N_1} \Gamma_n^1$, $\Gamma^2 = \bigcup_{n=1}^{N_2} \Gamma_n^2$, $\Gamma = \Gamma^1 \cup \Gamma^2$. The external connected domain bounded by Γ^2 will be called $\mathcal{D}$. We assume that each curve Γ_n^k is parametricized by the arc length $s : \Gamma_n^k = \{x : x = x(s) = (x_1(s), x_2(s)),\ s \in [a_n^k, b_n^k]\}$, $n = 1, ..., N_k$, $k = 1, 2$, so that $a_1^1 < b_1^1 < ... < a_{N_1}^1 < b_{N_1}^1 < a_1^2 < b_1^2 < ... < a_{N_2}^2 < b_{N_2}^2$ and the domain $\mathcal{D}$ is to the right when the parameter s increases on Γ_n^2. Therefore points $x \in \Gamma$ and values of the parameter s are in one-to-one correspondence except a_n^2, b_n^2, which correspond to the same point x for $n = 1, ..., N_2$.

We put $C^{k,r}(\Gamma_n^2) = \{\mathcal{F}(s) : \mathcal{F}(s) \in C^{k,r}[a_n^2, b_n^2],\ \mathcal{F}^{(m)}(a_n^2) = \mathcal{F}^{(m)}(b_n^2),$ $m = \overline{0,k}\ \}$, $k = 0, 1$, $r \in [0,1]$ and $C^{k,r}(\Gamma^2) = \bigcap_{n=1}^{N_2} C^{k,r}(\Gamma_n^2)$.

We say, that the function $w(x)$ belongs to the smoothness class **K** if

1) $w \in C^0(\overline{\mathcal{D}}) \cap C^2(\mathcal{D} \backslash \Gamma^1)$,

2) $\nabla w \in C^0(\overline{\mathcal{D} \backslash \Gamma^1} \backslash X)$, where X is a point-set, consisting of the end-points of Γ^1, so that $X = \bigcup_{n=1}^{N_1} \left(x(a_n^1) \cup x(b_n^1) \right)$,

3) in the neighbourhood of any point $x(d) \in X$ for some constants $C > 0$, $\epsilon > -1$ the inequality holds

$$|\nabla w| \leqslant C \, |x - x(d)|^{\epsilon}, \tag{1}$$

where $x \to x(d)$ and $d = a_n^1$ or $d = b_n^1$, $n = 1, ..., N_1$.

Let us formulate the Dirichlet problem for the Helmholtz equation in the domain $\mathcal{D} \backslash \Gamma^1$.

Problem **U**. To find a function $w(x)$ of the class **K** which satisfies the Helmholtz equation

$$w_{x_1 x_1}(x) + w_{x_2 x_2}(x) + \beta^2 w(x) = 0, \quad x \in \mathcal{D} \backslash \Gamma^1, \quad \beta = \text{const} > 0, \tag{2}$$

the boundary condition

$$w(x(s))|_{\Gamma} = f(s), \tag{3}$$

and the conditions at infinity

$$w = O(|x|^{-1/2}), \quad \frac{\partial w}{\partial |x|} - i\beta w = o(|x|^{-1/2}), \quad |x| = \sqrt{x_1^2 + x_2^2} \to \infty.$$

All conditions of the problem **U** must be satisfied in the classical sense.

On the basis of the Rellich lemma and energy equalities we can easily prove the following assertion.

Theorem 1. *If $\Gamma \in C^{2,\lambda}$, $\lambda \in (0,1]$, then the problem* **U** *has at most one solution.*

To construct the solution of the problem we assume that $f(s)$ from (3) is an arbitrary function from the Banach space $C^{1,\lambda}(\Gamma)$, $\lambda \in (0,1]$. With the help of the single layer potential and the double layer potential we reduce the problem to the boundary integral equation. By means of some transformations [2], [3], we reduce this equation to the Fredholm equation of the second kind, which is uniquely solvable in the appropriate Banach space. The solution of the Fredholm equation and therefore the solution of the problem can be easily computed by standard codes. The theorem holds.

Theorem 2. *If $\Gamma \in C^{2,\lambda}$, $f(s) \in C^{1,\lambda}(\Gamma)$, $\lambda \in (0,1]$, then the solution of the problem* **U** *exists and can be represented in the form of single and double layer potentials for the equation (2). The density in potentials can be found by solving the Fredholm equation of the second kind, which is uniquely solvable.*

It can be checked directly that the solution of the problem U satisfies condition (1) with $\epsilon = -1/2$. Explicit expressions for singularities of the solution gradient at the end-points of the open curves can be easily obtained with the help of formulas presented in [2].

The Dirichlet and Neumann problems for the dissipative Helmholtz equation in both interior and exterior domains, bounded by closed and open curves has been studied in [7], [8]. Similar problems for harmonic functions in exterior domains were treated in [9], [10]. A nonlinear problem on stratified fluid flow over obstacles and wings has been studied in [5].

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Semiclassic for the N particles Schrodinger equation with binary potential

We want to suggest a new representation of the spectral problem for the $N \gg 1$ particles Schrodinger equation at the torus $M = (R^3/Z^3)^N$

$$\sum_{i=1}^{N} -\Delta_{x_i} f + \sum_{1 \leqslant i < j \leqslant N} V(|x_i - x_j|) f = \lambda f$$

in the symmetric (boson) subspace $H_B(M)$ of $L_2(M)$. When N tends to infinity that representation allow us to find asymptotic of some low-energy series by semiclassical method. Let us suppose that the binary potential $V(|x-y|)$ is a sum of finite numbers of Fourier harmonics with Fourier coefficients $v_k = v_{-k} > 0; 1 \leqslant |k| \leqslant q$. We consider an algebra of functions A generated by the countable set of symmetrical functions $u_k = N^{-1/2} \sum_{j=1}^{N} \exp\{i2\pi(k, x_j)\}, k \in Z^3$. That algebra A is dense in $H_B(M)$; therefore we can consider functions from $H_B(M)$ as the functions in $u_k; \overline{u_k}$ where $k \in P$. Here P is such subset of Z^3 that $P \cap (-P) = \emptyset; P \cup (-P) = Z^3 \setminus 0$. By direct calculations we obtain for the sum of binary potential the formula: $\sum_{1 \leqslant i < j \leqslant N} V(|x_i - x_j|) = N(N-1)2^{-1} v_0 - (N/2) \sum_{|k|=1} q v_k + 2^{-1} N \sum_{|k|=1}^{q} v_k u_k \overline{u_k}$ and for the eigenfunctions $f(\dots u_k, \overline{u_k} \dots)$ of the N particles Schrodinger equation we obtain exact equation:

$$-8\pi^2 \sum_{m \in P} m^2 \frac{\partial^2 f}{\partial u_m \partial \overline{u}_m} + 4\pi^2 \sum_{m \in P} m^2 \left(u_m \frac{\partial f}{\partial u_m} + \overline{u}_m \frac{\partial f}{\partial \overline{u}_m}\right) + \tag{0.1}$$

$$\frac{N}{2}\left(\sum_{|k|=1}^{q} v_k u_k \overline{u}_k\right) f + \frac{1}{\sqrt{N}} L_2\left(\dots \frac{\partial}{\partial u_k} \dots, u\right) f = \mu f,$$

$$L_2 \stackrel{def}{=} -2 \sum_{k \neq m; k, m \in P} 4\pi^2 (k, m) \frac{\partial^2 f}{\partial u_k \partial \overline{u}_m} u_{k-m} + \sum_{k, m \in P} 4\pi^2 (k, m) u_{k+m} \frac{\partial^2 f}{\partial u_k \partial u_m} +$$

$$\sum_{k, m \in P} 4\pi^2 (k, m) \overline{u}_{k+m} \frac{\partial^2 f}{\partial \overline{u}_k \overline{u}_m};$$

here $\mu = \lambda - 2^{-1} N(N-1) v_0 + 2^{-1} N \sum_{|k|=1}^{q} v_k$. Dividing equation (1) by N we obtain the equation with the little parameter $N^{-1/2}$ near derivatives. Therefore, the WKB method can be applied, and the main term of low energy spectral asymptotic can be obtained from the system of noninteracting quadratic oscillators, which is obtained from equation (1) by neglecting the term $N^{-1/2}L_2f$.The details will be given at the talk.

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Periodic solutions of the N-body problem and applications to planet systems with satellites

The periodic solutions of the N-body problem were investigated in partial cases by Hill (Sun and a planet with a satellite), Poincaré (Sun and two planets) and later by Krassinskii (Sun and planets), Tkhai (Sun and a planet with satellites). We consider the *planar N-body problem* for the Sun and the arbitrary number of planets and the arbitrary number of satellites. Here the mass of the Sun is supposed to be 1, the masses of planets have orders $O(\mu)$, and the masses of satellites have orders $O(\mu\nu)$ where μ and ν are "small parameters". We investigate the periodic solutions of the N-body problem under consideration applying technics of the perturbation theory. The solution of the N-body problem is called *relative periodic* or *periodic* if there exists a pair of real numbers (T, α), called *relative period*, such that $T > 0$, $-\pi < \alpha \leqslant \pi$ and, for any $t \in rR$, the configuration of the mass points at time $t + T$ is obtained by rotation of the configuration at time t of the angle α around the barycenter. The solution is called *symmetric* if there exist time at which all the mass points are on the same line (i.e. one can watch all the planets and the satellites of the Solar system on "parade") and their velocities are orthogonal to this line. As the "unperturbed problem" one considers the collection of $N - 1$ Kepler problems for independent motions of planets around the Sun and satellites around planets. As the "generating periodic solutions" one considers the "circular" solutions of the Kepler problems with the common relative period (T, α). That is, planets uniformly rotate around the Sun along different circular orbits with constant angle velocities ω_i, and satellites uniformly rotate around their planets along different circular orbits with constant angle velocities ω_{ij}. We prove: The N-body problem under consideration has (T, α)-periodic solutions close to the generating solutions, provided that the nondegeneracy condition $|\alpha| > \left|\frac{\omega_i}{\omega_{ij}}\right|^2 T$ holds, that the fractions $\left|\frac{\omega_i}{\omega_{ij}}\right|$ of the "months" to the "years" are sufficiently small, and that the parameters μ and ν are small enough. Moreover, exactely 2^{N-3} of these solutions are symmetric, and hence, every half-period $\frac{T}{2}$ one can watch a parade for them. In the case $\alpha = 0$ there does not exist, in general, such a solution. For the case that all planets rotate in the same (positive) direction, sufficient conditions are given for the stability of some periodic solutions in linear approximation.

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The critical exponent of the weighted Sobolev embedding

It is well known, that the number $p^* = \frac{Np}{N-p}$, $1 < p < N$, is the critical exponent for the embedding of the (classical) Sobolev space $W^{1,p}(\Omega)$ into $L^q(\Omega)$: this embedding is continuous for $1 < q \leqslant p^*$ and compact for $1 < q < p^*$. In the talk, an analogous question for the case of *weighted* spaces is investigated. We deal with the embedding $W_0^{1,p}(\Omega; P) \hookrightarrow L^q(\Omega; Q)$ with $p > 1$ fixed and P, Q weight functions, i.e. with the inequality

$$\Big(\int_\Omega |u(x)|^q Q(x)dx\Big)^{1/q} \leqslant C\Big(\int_\Omega |\nabla u(x)|^p P(x)dx\Big)^{1/p},$$

and derive for some special cases a formula for the critical value $\tilde{p}$ such that the embedding mentioned is continuous and even compact for $q < \tilde{p}$ and does not take place for $q > \tilde{p}$. For the particular case $P = Q \equiv 1$, $\tilde{p}$ coincides with p^*. For the one-dimensional case, where $\Omega = (0, R)$, $W_0^{1,p}(0, R; P) = \{u = u(t),\ u(R) = 0,\ \int_0^R |u'(t)|^p P(t)dt < \infty\}$, this formula reads as

$$\tilde{p} = p' = \liminf_{r \to 0} \frac{|\log \int_0^r Q(t)dt|}{\log \int_r^R P^{1-p'}(t)dt}$$

with $p' = \frac{p}{p-1}$. For $q = \tilde{p}$, the embedding mentioned can be either continuous, or compact, or invalid, according to the choice of the weight functions P, Q.

Kuksin S.B.
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The coupling techniques and the turbulent-limit Tfor randomly forced dissipative PDEs

I shall discuss a coupling-approach to study ergodic properties of dissipative partial differential equations, forced by a random force. In particular, the approach applies to the 2D Navier-Stokes equations under periodic boundary conditions with a random right-hand side, with arbitrary viscosity δ. For any δ the equation has a unique stationary measure μ_δ and the problem of two-dimensional turbulence can be stated as studying of limiting properties of this measure as $\delta \to 0$. Most of the results of my talk are obtained jointly with Armen Shirikyan. Corresponding publications can be found on the web-page *http : //www.ma.hw.ac.uk/ ~ kuksin/*.

Kulikov V.S.
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Generic coverings of the plane

It is well-known that the restriction f of a generic linear projection $\mathbb{P}^r \to \mathbb{P}^2$ to a smooth surface $X \subset \mathbb{P}^r$ of $\deg X \geqslant 3$ has the following properties: (i) f is a finite morphism unramified over $\mathbb{P}^2 \setminus B$, where B is an irreducible plane cuspidal curve whose singular points are ordinary cusps and nodes only; (ii) $f^*(B) = 2R + C$, where R is irreducible and C is reduced; (iii) $f_{|R} : R \to B$ is one-to-one over a generic point of B.
A morphism $f : X \to \mathbb{P}^2$ of a normal surface X is called a *generic covering* if it satisfies these three conditions. Two generic coverings (X_1, f_1), (X_2, f_2) with the same branch curve B are said to be equivalent if there exists an isomorphism $h : X_1 \to X_2$ such that $f_1 = f_2 \circ h$. The set of generic coverings of degree m with the same branch curve B is in one-to-one correspondence with the set of epimorphisms $\varphi : \pi_1(\mathbb{P}^2 \setminus B) \to S_m$ (up to inner automorphisms of the symmetric group S_m) transforming a geometric generator of $\pi_1(\mathbb{P}^2 \setminus B)$ in a transposition. There are three natural problems.

Existence Problem. *For given numbers of cusps and nodes, does there exist a plane caspidal curve B having these invariants and such that B is the branch curve of a generic covering of the plane of given degree?*

Uniqueness Problem. *How many non-equivalent generic coverings do exist for given B?*

Problem of the Existence of Invariants. *Do there exist invariants which define a branch curve and, respectively, a generic covering of the plane uniquely up to symplectic isotopy?* In the talk, a survey of recent results concerning these three problems will be given.

Курина Г.А.
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Приводимость J-аккретивных оператор-функций к блочно-диагональной форме[17]

В докладе приводятся теоремы, обобщающие результаты работ [1,2], посвященных условной приводимости неотрицательно гамильтоновых оператор-функций к блочно-диагональному виду. Пусть G — вещественное или комплексное гильбертово пространство и

$$H = G \oplus G. \tag{0.1}$$

Заданная на отрезке [0,1] со значениями в $L(H)$ функция $\mathcal{H}(t)$ называется гамильтоновой, если она имеет относительно разложения (0.1) матричное представление

$$\mathcal{H}(t) = \left[\begin{array}{cc} A(t) & B(t) \\ C(t) & -A^*(t) \end{array} \right], \tag{0.2}$$

где $B(t)$ и $C(t)$ — самосопряженные операторы при каждом $t \in [0,1]$. $\mathcal{H}(t)$ называется неотрицательно гамильтоновой, если она гамильтонова и $B(t)$, $C(t)$ — неотрицательные операторы при каждом $t \in [0,1]$.

Оператор-функция $\mathcal{H}(t)$ называется э-дихотомической, если при каждом t спектр $\sigma(\mathcal{H}(t))$ оператора $\mathcal{H}(t)$ не пересекается с мнимой осью, т.е. $\sigma(\mathcal{H}(t)) \subset \mathbb{C}_r \cup \mathbb{C}_l$, где через $\mathbb{C}_r$ и $\mathbb{C}_l$ обозначены правая и левая открытые полуплоскости соответственно.

Непрерывная на $[0,1]$ э-дихотомическая оператор-функция $\mathcal{H}(t)$ со значениями в $L(H)$ называется условно (H_1, H_2)-приводимой, если (i) $H = H_1 \oplus H_2$, (ii) существует такая непрерывная и обратимая при каждом $t \in [0,1]$ оператор-функция $V(t)$ со значениями в $L(H)$, что $H_{1,2}$ инвариантны относительно

$$\mathcal{H}_1(t) := V^{-1}(t)\mathcal{H}(t)V(t) : \mathcal{H}_1(t)H_{1,2} \subset H_{1,2}, \quad \text{и} \quad \sigma(\mathcal{H}_1(t)|_{H_{1,2}}) \subset \mathbb{C}_{r,l}.$$

Если $\mathcal{H}(t)$ — 1-периодическая функция, т.е. $\mathcal{H}(0) = \mathcal{H}(1)$, то такое же требование 1-периодичности предъявляется и к функции $V(t)$.

Линейный ограниченный оператор T называется J-аккретивным, если его вещественная часть $\mathrm{Re}T := \frac{1}{2}(T + T^c)$, где T^c —J-сопряженный к T оператор, является J-неотрицательным оператором (определения см. в [3]).

Theorem 1. *Непрерывная на* $[0,1]$ *функция со значениями во множестве J-аккретивных э-дихотомических операторов из* $L(H)$ *является условно* (H^+, H^-)*-приводимой, где*

$$H^+ := \ker(J - I),\ H^- := \ker(J + I).$$

[17]Работа поддержана РФФИ(гранты 99-01-00391, 99-01-00968)

Приведем условия, достаточные для э-дихотомичности J-аккретивного оператора.

Theorem 2. *Если для J-аккретивного оператора T выполнено хотя бы одно из условий: (a) оператор $\mathrm{Re}T$ имеет ограниченный обратный или (b) оператор $i\mathrm{Im}T := \frac{1}{2}(T - T^c)$ является э-дихотомическим, тогда T — э-дихотомический оператор.*

Theorem 3. *Непрерывная на $[0,1]$ неотрицательно гамильтонова э-дихотомическая оператор-функция $\mathcal{H}(t)$ является условно (G,G)-приводимой. Более того, $V(t)$ можно выбрать так, что $\mathcal{H}_1(t)$ — гамильтонов оператор. Если $\mathcal{H}(t)$ — периодическая функция с периодом 1, то функция $V(t)$, а значит, и $\mathcal{H}_1(t)$ могут быть выбраны периодическими с тем же периодом.*

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V-экстремальные решения проблемы моментов.

Рассматривается задача о плотности множества многочленов в пространстве $L^1_\sigma(-\infty,\infty)$, где σ — произвольная мера с конечными моментами. Подход к этой задаче, предложенный М.А. Наймарком (см. [1]), основан на использовании методов теории проблемы моментов. Он позволяет сформулировать достаточные условия плотности многочленов в терминах функций Неванлинны представления решений неопределенной проблемы моментов (см. [2]). Полученный автором результат является продолжением его исследования данной проблемы (см. [3]).

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A General Criterion for the Eckhaus Instability in Gradient/Skew-Gradient Systems

A stablity problem is considered for a family of spatially periodic stationary solutions in 1-D gradient/skew-gradient systems. We found that the equation for stationary solutions have a first integral. Regard this integral as a functional on the set of stationary solutions, we show that a stability-instability transition occurs at extremal points of the functional. The result gives a general but simple criterion for the Eckhaus instability in various pattern formation equations such as the Ginzburg-Landau equation, the Swift-Hohenberg equation, and activator-inhibitor reaction-diffusion systems. This is a joint work with Professor Eiji Yanagida of Tohoku University, Japan.
keywords: gradient/skew-gradient systems, Eckhaus instability

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On dependence of Lax–Phillips scattering matrix on choice of incoming and outgoing subspaces

The possibility of applying the Lax–Phillips scheme [1] for studying the scattering of evolution system described by an operator-differential equation

$$u_{tt} = -Lu, \tag{0.1}$$

where L is a positive self-adjoint operator acting in an abstract Hilbert space $\mathfrak{H}$, depends on the existence of incoming D_- and outgoing D_+ subspaces for the group $W_L(t)$ of solutions of the Cauchy problem for Eq. (1). Assume that the following condition is satisfied:

Condition 1. *there exists a simple maximal symmetric operator B acting in a subspace $\mathfrak{H}_0$ of the Hilbert space $\mathfrak{H}$ such that the operator L is a positive self-adjoint extension (with exit in the space $\mathfrak{H}$) of the symmetric operator B^2.*

Then subspaces $D_\pm$ exist and are defined explicitly by the operator B ([2]). Choosing the operator B and subspace $\mathfrak{H}_0$ in different ways, we obtain different subspaces $D_\pm$. A simple algorithm which allows us to construct various examples of such subspaces for the group $W_L(t)$ is following: Let B be a fixed simple maximal symmetric operator in $\mathfrak{H}_0$, for which Condition 1 is true. Consider an isometric operator A acting in the Hilbert space $\mathfrak{H}_0$ and such that $AB = BA$. Then the operator $B_A = ABA^*$, $D(B_A) = AD(B)$ is simple maximal symmetric in the space $\mathfrak{H}_A = A\mathfrak{H}_0$ and, for B_A, Condition 1 is also true. We denote by $D_\pm$ and $D^A_\pm$ incoming and outgoing subspaces for $W_L(t)$ defined by operators B and B_A, respectively. Let $S(\delta)$ and $S_A(\delta)$ be the Lax–Phillips scattering matrices for the group $W_L(t)$ that are associated with the subspaces $D_\pm$ and $D^A_\pm$, respectively. We investigate the relation between functions $S(\delta)$ and $S_A(\delta)$ in the case where the isometric operator A is a function of the operator B of the following form:

$$A = g(B) = \int_{-\infty}^{\infty} g(\xi) dE_\xi, \tag{0.2}$$

where E_ξ is the spectral (nonorthogonal) function of the operator B and $g(\lambda)$ is a function from the Hardy class H^∞ in the upper half-plane such that $|g(\delta)| = 1$ ($\delta \in \mathbb{R}$). We note that any isometric operator A of form (2) commutes with B.

Theorem. *If an isometric operator A has form (2), then the Lax–Phillips scattering matrices $S(\delta)$ and $S_A(\delta)$ are related as follows:*

$$S_A(\delta) = \frac{g(-\delta)}{g(\delta)} S(\delta) \quad (\delta \in \mathbb{R}).$$

The theorem shows that the Lax-Phillips scattering matrices $S(\delta)$ and $S_A(\delta)$ can have different singularities. In particular, the appearance of 'false' zeroes of the scattering matrix in the lower half-plane, which are not related with evolution system (1) and are caused only by the choice of incoming and outgoing subspaces $D^A_\pm$, is possible.

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Prandtl boundary layer continuation problem in nonsnandard situations

Considered is problem on continuation of boundary layer of viscous incompressible liquid near a rigid wall in the case of downstream pressure increase. It is known [1] that classical solution of this problem can't be continued to the domain $x > x_*$ (x is the longitudinal coordinate), where x_* is the stagnation point of the external liquid flow refering to boundary layer. Also it is known [2] that there exists some class of prescribed initial profiles of longitudinal velocity for which the solution of this problem doesn't exists even for $x > x_o$, where $x_o \in (0, x_*)$. Built here is the other class of initial velocity profiles for which boundary layer can be continued up to the stagnation point of the external flow. Let the salient point of rigid boundary of the motion domain to exist. There was extracted the asymptotic form of the Navier-Stokes equations for the motion of a fluid at large Reynolds number in the angle sector, which is an analog of the system of Prandtl equations. It is shown that the equations are reduced to the classical Von Mises boundary-layer equation by change of variables. Here the problem statement exibit some specific features: discontinuity point of longitudinal pressure gradient; moreover this point is the stagnation point of the external flow. The existence conditions for generalized solution of this problem are found. It is shown, that the solution is regular in the whole motion domain with the exception of (may by) one line.

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Iterative solution of variational inequalities with several M-matrices and maximal monotone operators

Iterative solution for the problem

$$Au + B\gamma + \delta = f, \quad \gamma \in Cu, \quad \delta \in Du \tag{0.1}$$

is studied, where A, B are M - matricies, while C, D are diagonal maximal monotone (multivalued) operators. Finite element and finite difference schemes for the free boundary problems with several constraints and/or with several convex non-differentiable functionals lead to the problem under consideration. We can cite among others Stefan problem with prescribed convection (including continuous casting problem) and the problems of fluid flows in porous medium under the gravity forces. Existence of the unique solution for the problem (0.1) and convergence of iterative methods, including alternating Schwarz methods, are studied.
Theorem 1. *Let*

$$A, B \text{ be } M- \text{ matrices,} \tag{0.2}$$

$$C, D \text{ be diagonal maximal monotone operators in } R^N. \tag{0.3}$$

Let also there exist the subsolution $(\underline{u}, \underline{\gamma}, \underline{\delta})$, $\underline{\gamma} \in C\underline{u}$, $\underline{\delta} \in D\underline{u}$ *and the supersolution* $(\overline{u}, \overline{\gamma}, \overline{\delta})$, $\overline{\gamma} \in C\overline{u}$, $\overline{\delta} \in D\overline{u}$ *of the problem (0.1):*

$$(\underline{u}, \underline{\gamma}, \underline{\delta}) \ll (\overline{u}, \overline{\gamma}, \overline{\delta}), \quad A\underline{u} + B\underline{\gamma} + \underline{\delta} \ll f \ll A\overline{u} + B\overline{\gamma} + \overline{\delta}, \tag{0.4}$$

where $u \gg 0$ *means the componentwise inequality. Then the equation (0.1) has a solution* (u, γ, δ) *for any* $f \in R^N$.

Theorem 2. *Let the assumptions (0.2) - (0.4) be fulfilled,*

$$A, B \text{ be weakly diagonally dominant in columns matrices} \tag{0.5}$$

and one of the following properties holds: (a) either A or B is strictly diagonally dominant in columns or (b) C is either continuous monotone or strictly maximal monotone operator. If $(u^1, \gamma^1, \delta^1)$ *and* $(u^2, \gamma^2, \delta^2)$ *are the solutions of (0.1) with right-hand sides* f^1 *and* f^2*, correspondingly, then the inequality* $f^1 \gg f^2$ *implies the inequalities* $u^1 \gg u^2$, $\gamma^1 \gg \gamma^2, \delta^1 \gg \delta^2$. We look for a variant of the multiplicative method (which includes, e.g., the additive Schwarz alternating method).

Let for $l = 1, 2, ..., p$ $A = A_0^l - A_1^l$, $B = B_0^l - B_1^l$ be the splittings of matrices A, B such that A_0^l, B_0^l are M - matrices and $A_1^l \gg 0$, $B_1^l \gg 0$. Let also $E_l \gg 0$ be the diagonal matrices and $\sum_{l=1}^p E_l = Id$, where by Id the unit matrix is denoted. We consider the iterative method:

$$A_0^l v_l^{k+1} + B_0^l \eta_l^{k+1} + \delta_l^{k+1} = A_1^l u^k + B_1^l \gamma^k + f,$$

$$\eta_l^{k+1} \in C v_l^{k+1},\ \delta_l^{k+1} \in D v_l^{k+1} \quad l = 1, 2, ..., p;$$

$$u^{k+1} = \sum_{l=1}^{p} E_l v_l^{k+1};\ \gamma^{k+1} = \sum_{l=1}^{p} E_l \eta_l^{k+1};\ \delta^{k+1} = \sum_{l=1}^{p} E_l \delta_l^{k+1} \qquad (0.6)$$

for $k = 0, 1, 2, ...$ with initial guess (u^0, γ^0). **Theorem 3.** *Let the assumptions of Theorem 2 for the problem (0.1) be fulfilled. Then iterative method (0.6) converges for any initial guess (u^0, γ^0) from the ordered interval $< (\underline{u}, \underline{\gamma}), (\overline{u}, \overline{\gamma}) >$, and if $(u^0, \gamma^0) = (\overline{u}, \overline{\gamma})$ ($(u^0, \gamma^0) = (\underline{u}, \underline{\gamma})$) then the sequence $\{(u^k, \gamma^k, \delta^k)\}$ converges monotonically decreasing (increasing) to the unique solution $(u^*, \gamma^*, \delta^*)$ of the problem (0.1).* Linear rate of convergence of the iterative methods is also proved under some additional assumptions to input data.

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Some nonexistence results for higher–order evolution inequalities in cone–like domains[18]

Let S^{N-1} be the unit sphere in $\mathbb{R}^N$, $N \geqslant 3$ and (r, ω) be the polar coordinates in $\mathbb{R}^N$. Let K_ω be a domain of S^{N-1} with smooth boundary ∂K_ω. We shall denote by K the cone

$$K = \{(r, \omega) : 0 < r < +\infty, \omega \in K_\omega\}.$$

The lateral surface of the cone K is ∂K. "Cone-like domain" K_R, $R > 0$ denotes the set $\{x \in K : |x| > R\}$ with full surface ∂K_R. Recall that the Laplace operator Δ in polar coordinates (r, ω) has the form

$$\Delta = \frac{1}{r^{N-1}} \frac{\partial}{\partial r}\left(r^{N-1}\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\Delta_\omega = \frac{\partial^2}{\partial r^2} + \frac{N-1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\Delta_\omega,$$

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where Δ_ω denotes the Laplace–Beltrami operator on the unit sphere $S^{N-1} \subset \mathbb{R}^N$. We shall use the first Helmholtz eigenvalue $\lambda_\omega \equiv \lambda_1(K_\omega) > 0$ and corresponding eigenfunction $\Phi(\omega)$ for the Dirichlet problem of Δ_ω in K_ω

$$\begin{cases} \Delta_\omega \Phi + \lambda\Phi = 0 & \text{in } K_\omega, \\ \Phi|_{\partial K_\omega} = 0. \end{cases} \tag{0.1}$$

It is well-known that $\Phi(\omega) > 0$ for $\omega \in K_\omega$. We assume $\Phi(\omega) \leqslant 1$. Let $k \in \mathbb{N}$. Our model problem has the form

$$\begin{cases} \frac{\partial^k u}{\partial t^k} - \Delta u \geqslant |u|^q & \text{in } K \times (0,\infty), \\ u|_{\partial K \times [0,\infty)} \geqslant 0, & \\ \frac{\partial^{k-1} u}{\partial t^{k-1}}|_{t=0} \geqslant 0, & u \not\equiv 0. \end{cases} \tag{0.2}$$

Let us introduce the parameters

$$s^* = \frac{N-2}{2} + \sqrt{\left(\frac{N-2}{2}\right)^2 + \lambda_\omega}, \qquad s_* = -\frac{N-2}{2} + \sqrt{\left(\frac{N-2}{2}\right)^2 + \lambda_\omega}. \tag{0.3}$$

Theorem 1 *Let*

$$1 < q \leqslant q_k^* = \frac{s^* + 2/k + 2}{s^* + 2/k} = 1 + \frac{2}{s^* + 2/k},$$

where s^* is defined in (0.3). Then the problem (0.2) has no nontrivial global solution.

This theorem includes the sharp results for parabolic equation and inequality (i.e. $k = 1$):

Theorem 2 *Let*

$$1 < q \leqslant q_1^* = 1 + \frac{2}{s^* + 2},$$

where s^* is defined in (0.3). Then the problem

$$\begin{cases} \frac{\partial u}{\partial t} - \Delta u \geqslant |u|^q & \text{in } K \times (0,\infty), \\ u|_{\partial K \times [0,\infty)} \geqslant 0, & \\ u|_{t=0} \geqslant 0, & u \not\equiv 0. \end{cases}$$

has no nontrivial global solution.

Let us consider the system

$$\begin{cases} \frac{\partial^k u}{\partial t^k} - \Delta u \geqslant |v|^{q_1} & \text{in } K \times (0,\infty), \\ \frac{\partial^k v}{\partial t^k} - \Delta v \geqslant |u|^{q_2} & \text{in } K \times (0,\infty), \\ u|_{\partial K \times [0,\infty)} \geqslant 0, \quad v|_{\partial K \times [0,\infty)} \geqslant 0, & \\ \frac{\partial^{k-1} u}{\partial t^{k-1}}|_{t=0} \geqslant 0, \quad \frac{\partial^{k-1} v}{\partial t^{k-1}}|_{t=0} \geqslant 0, & u \not\equiv 0, \quad v \not\equiv 0. \end{cases} \tag{0.4}$$

Theorem 3 *Let* $q_1 > 1$, $q_2 > 1$ *and*

$$\max\{\gamma_1, \gamma_2\} \geqslant \frac{s^* + 2/k}{2}, \quad \textit{where} \quad \gamma_1 = \frac{q_1 + 1}{q_1 q_2 - 1}, \quad \gamma_2 = \frac{q_2 + 1}{q_1 q_2 - 1}.$$

Then (0.4) *has no nontrivial global solution.*

Let us consider the inequality of porous medium type

$$\begin{cases} \frac{\partial^k u}{\partial t^k} - \Delta u^m \geqslant |u|^q \quad \text{in } K \times (0,\infty), & m \geqslant 1, \quad q > m, \\ u|_{\partial K \times [0,\infty)} \geqslant 0, & \\ \frac{\partial^{k-1} u}{\partial t^{k-1}}|_{t=0} \geqslant 0, \quad u \not\equiv 0. & \end{cases} \tag{0.5}$$

Theorem 4 *Let*

$$1 \leqslant m < q \leqslant q^* = \frac{m(s^* + 2) + 2/k}{s^* + 2/k} = m + 2\frac{m - m/k + 1/k}{s^* + 2/k},$$

where s^* *is defined in* (0.3). *Then the problem* (0.5) *has no nontrivial global solution.*

In the "parabolic" case $k = 1$ we obtain

Theorem 5 *Let*

$$1 \leqslant m < q \leqslant q^* = m + \frac{2}{s^* + 2},$$

where s^* *is defined in* (0.3). *Then the problem*

$$\begin{cases} \frac{\partial u}{\partial t} - \Delta u^m \geqslant |u|^q \quad \text{in } K \times (0,\infty), & m \geqslant 1, \quad q > m, \\ u|_{\partial K \times [0,\infty)} \geqslant 0, & \\ u|_{t=0} \geqslant 0, \quad u \not\equiv 0. & \end{cases}$$

has no nontrivial global solution.

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Unbounded weak nonlinear pertubations of monotone operators

Let X, Y be separable reflexive Banach spaces, which are continuously embedding in a local convex space V. Suppose that interception $Z = X \cap Y$ is dense in X and Y. It is well known that Z is a separable reflexive Banach space. Let X', Y', Z' be conjugate spaces. Let $A : X \to X'$ be a radial continuous bounded coercive operator with (M)-property and $B : Y \to Y'$ be a radial continuous bounded coercive operator. The domain Y of the operator B has a dense interception with the domain X of the operator A and in this sense the operator B is unbounded with respect to A. The sum $Su = (A+B)u$ is well defined on the space Z and $S : Z \to Z'$ is a radial continuous bounded coercive operator. Let for the operator $B : Z \to Z'$ with the new domain Z the following two conditions hold: 1B. $B : Z \to Z'$ is a weakly compact operator; 2B. The functional (Bu, u) defined for $u \in Z$ is weakly lower semicontinuous.

Theorem 1 *Let the conditions above are satisfied. Then the equation $Au+Bu = f$ has at least one solution $u \in Z$ for any element $f \in Z'$.*

Example 1 *Let us consider the equation in a bounded domain $\Omega \subset R^n$ wuth a regular boundary $\partial\Omega$:*

$$(-1)^m \sum_{|\alpha|=m} D^\alpha(|D^\alpha u|^{p-2} D^\alpha u) + (-\Delta)^l u = f(x),$$
$$D^\beta u|_{\partial\Omega} = 0, \quad |\beta| + 1 \leqslant \max(m,l). \quad (0.1)$$

Here $p > 1$ and $m, l \geqslant 1$ are natural numbers which are arbitrary so it is possible to take $m \leqslant l$ or $m > l$. Let us introduce the spaces $X = \overset{\circ}{W}{}^m_p(\Omega)$ and $Y = \overset{\circ}{W}{}^l_2(\Omega)$. The operator A is defined by the first sum in (0.1) under the boundary conditions $D^\beta u|_{\partial\Omega} = 0$, $|\beta| \leqslant m-1$. The operator $B = (-\Delta)^l$, $D^\beta u|_{\partial\Omega} = 0$, $|\beta| \leqslant l-1$. Using the Theorem 1 we conclude that the problem (0.1) has a solution $u \in \overset{\circ}{W}{}^m_p(\Omega) \cap \overset{\circ}{W}{}^l_2(\Omega)$ for any given function $f \in W^{-m}_{p'}(\Omega) + W^{-l}_2(\Omega)$

Now we consider the evolution equation

$$Au + \frac{d}{dt}Bu = f(t), \qquad u(0) = u_0. \qquad (0.2)$$

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In addition to the previous conditions on the spaces X, Y let us suppose that Y is a Hilbert space. For a fixed number $p > 1$ we introduce the spaces of abctract functions $\mathcal{X} = L^p(0,T;X)$ and $\mathcal{Y} = L^2(0,T;Y)$. Let $A : \mathcal{X} \to \mathcal{X}'$ be a radial continuous bounded coercive operator with (M)-property on the set $W = \{u \in \mathcal{X}; Bu' \in \mathcal{X}' + \mathcal{Y}'\}$. And let $B : Y \to Y'$ be a linear bounded selfconjugate operator with the coercive form $(Bu, u) \geqslant c\|u\|_Y^2$, $c > 0$. Considering the left hand side of the equation (0.2) as an unbounded perturbation of the operator A one can prove the folowing assertion.

Theorem 2 *Let the conditions above are satisfied. Then the equation* (0.2) *has at least one solution* $u \in \mathcal{X} \cap \mathcal{Y}$ *for any* $u_0 \in Y$ *and* $f \in \mathcal{X}' + \mathcal{Y}'$. *The equation* (0.2) *is considered as an equality of elements in the space* $\mathcal{X}' + \mathcal{Y}'$. *The function* $Bu(t) \in C(0,T; w - Z')$ *and as a continuous function takes the value* $Bu(0) = Bu_0$. *In this sense the initial condition is satisfied.*

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Spectral mapping theorems for semigroups and nonlinear Schrödinger and Euler equations

There are many examples of strongly continuous semigroups on Banach spaces for which the spectral mapping theorem $\sigma(e^{tA}) \setminus \{0\} = \exp t\sigma(A)$ does not hold. In particular, the position of the spectrum $\sigma(A)$ does not, generally, determine uniform stability of the semigroup. It is well-known that on Hilbert spaces the semigroup is uniformly stable if and only if the resolvent of the generator is *bounded* in the right-hand half-plane. However, even this result does not, generally, hold on Banach spaces. In the talk we will use operator-valued L^p–Fourier multipliers to give a generalization of this result for Banach spaces, and to present its applications for stability in control theory. We also prove the spectral mapping theorem for the group generated by the linearization around a bound state of the nonlinear Schrödinger equation with space-dependent nonlinearity. As a consequence, we derive the existence of locally invariant manifolds around the bound state. Also, we prove the spectral mapping theorem for linearizations of two dimensional Euler equations. Moreover, using Friedlander–Vishik bicharacteristic amplitude equations, we give explicit construction for approximate eigenfunctions of the linearized Euler equations in dimensions two and three. As a consequence, we give a rigorous poof of the fact that a steady state of the Euler equations is hydrodynamically stable if and only if the linearization has pure imaginary spectrum.

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A nonlinear ultraparabolic type equation with space–periodic solutions

A nonlinear, Fokker–Planck–type, parabolic integro–differential equation, is studied. It arises from the statistical description of the dynamical behavior of populations of infinitely many nonlinearly coupled random oscillators, subject to so–called "mean–field" interaction (the space–integral term in the equation accounts for this). Such a model, proposed in [1], generalizes and improves the celebrated Kuramoto model [2], which describes a variety of phenomena, in particular self–synchronization, in fields ranging from Biology and Medicine, to Physics and Neural Networks. Statement of the problem is as follows. For $(\theta,\omega,t,\Omega) \in Q_T = [0,2\pi] \times \mathbf{R} \times [0,T] \times [-G,G]$, find a function $\rho(\theta,\omega,t,\Omega)$ which solves the problem:

$$\frac{\partial\rho}{\partial t} = \frac{\partial^2\rho}{\partial\omega^2} - \omega\,\frac{\partial\rho}{\partial\theta} + \frac{\partial}{\partial\omega}(\omega\rho) - \Omega\,\frac{\partial\rho}{\partial\omega} - K_s(\theta,t)\,\frac{\partial\rho}{\partial\omega}, \tag{1}$$

$$\rho|_{\theta=0} = \rho|_{\theta=2\pi}, \tag{2}$$

$$\rho|_{t=0} = \rho_0(\theta,\omega,\Omega), \tag{3}$$

where

$$K_s(\theta,t) := K \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{0}^{2\pi} g(\Omega)\,\sin(\varphi-\theta)\,\rho(\varphi,\omega,t,\Omega)\,d\varphi\,d\omega\,d\Omega. \tag{4}$$

The problem (1)–(4) has been studied under the following assumptions: A) The initial profile, $\rho_0(\theta,\omega,\Omega)$, is supposed to be: ($\mathbf{a}_1$) a continuous function in all variables $(\theta,\omega,\Omega) \in Q = \mathbf{R}\times\mathbf{R}\times[-G,G]$, belonging to the Hölder space $C^{2+\alpha_0}(Q)$, $\alpha_0 \in (0,1)$ being a real constant; ($\mathbf{a}_2$) 2π–periodic in angle, θ; ($\mathbf{a}_3$) positive; ($\mathbf{a}_4$) normalized for all $\Omega \in [-G,G]$,

$$\int_{0}^{2\pi}\int_{-\infty}^{+\infty} \rho_0(\theta,\omega,\Omega)\,d\omega\,d\theta = 1;$$

and ($\mathbf{a}_5$) possesses an exponential decay property, in ω, at infinity, namely:

$$\sup_{\theta\in\mathbf{R},\,\Omega\in[-G,G]} \left|D^{l_1,l_2,l_3}_{\theta,\omega,\Omega}\,\rho_0(\theta,\omega,\Omega)\right| \leqslant C_0\,\mathrm{e}^{-M_0\omega^2}$$

for $\omega \in \mathbf{R}$ and $l_1 + l_2 + l_3 \leqslant 2$, where $C_0, M_0 > 0$ are constants and $l_i \geqslant 0$, $i = 1, 2, 3$, are integers. **B**) The frequency distribution density, $g(\Omega)$, is assumed to be: (**b$_1$**) belonging to the space $L^1(\mathbf{R})$; (**b$_2$**) compactly supported on $[-G, G]$; and (**b$_3$**) bounded. **C**) The coupling strength, K, is a constant. The problem should be considered nonstandard by several reasons, namely:

1) The governing equation (1) is of the *second order* with respect to ω, but only of the *first order* with respect to θ and t. Therefore, even besides of the integral term, equation is neither of the parabolic nor of the hyperbolic type.

2) The governing equation (1) is considered in the *slab* Q_T, that is in a domain *unbounded* in ω, which serves as a coefficient of the equation. This fact gives rise to *singularity* phenomena typical for equations with *unbounded coefficients.*

3) The governing equation (1) contains an integral term taking on *unbounded* domain.

4) There is an additional variable, the natural frequency of the oscillators, Ω, with respect to which no derivatives appear, but the integral is also made with respect to it.

5) We are interested *only* in solutions, *periodic* in θ, while the governing equation contains the first time–derivative with respect to θ (cf. "time–periodic solutions to parabolic equations").

Therefore, results available in the literature concerning nonlinear parabolic or even integroparabolic equations cannot be applied in our case under study. Space–degenerate diffusion suggests to consider a *regularized* equation, where a small spatial diffusion is incorporated in the model equation. Moreover, *to avoid* the *unbounded coefficients* of equation, ω, we also replace ω by bounding function $F_N(\omega)$, which is supposed to be smooth, bounded, and $F_N = \omega$ for $|\omega| < N$. So, instead of equation (1), we first study its *parabolic regularization*, that is a family of equations satisfied by $\rho^{\varepsilon,N}(\theta, \omega, t, \Omega)$,

$$\frac{\partial \rho^{\varepsilon,N}}{\partial t} = \frac{\partial^2 \rho^{\varepsilon,N}}{\partial \omega^2} + \varepsilon \frac{\partial^2 \rho^{\varepsilon,N}}{\partial \theta^2} - F_N \frac{\partial \rho^{\varepsilon,N}}{\partial \theta} + \frac{\partial}{\partial \omega}\left(F_N \rho^{\varepsilon,N}\right) - \Omega \frac{\partial \rho^{\varepsilon,N}}{\partial \omega} - \mathcal{K}_s^{\varepsilon,N} \frac{\partial \rho^{\varepsilon,N}}{\partial \omega} \tag{5}$$

in $Q_T \cap \{t > 0\}$, for any given $\varepsilon > 0$ and $N > 0$. Here $\varepsilon > 0$ and $N > 0$ are regularization parameters. The term $\mathcal{K}_s^{\varepsilon,N}$ is defined by formula (4) with the function $\rho^{\varepsilon,N}$ replacing ρ. Adding the second derivative term in θ, we modify the periodic boundary conditions (2) to

$$(\rho, \rho_\theta)|_{\theta=0} = (\rho, \rho_\theta)|_{\theta=2\pi} \tag{6}$$

for $\omega \in \mathbf{R}$, $t \in (0, T]$, and $\Omega \in [-G, G]$. Estimates uniform with respect to the regularization parameters have been obtained. These estimates allow taking limits which identify the solution to the original problem in certain Sobolev and Hölder spaces. *Existence of strong solutions* is thus established. Precise estimates for the decay of convolutions with the fundamental solutions to linear parabolic equations on unbounded domains are used intensively as an essential tool for general linear parabolic equations in $\mathbf{R}^n$.

Theorem. *Suppose that data of the problem* (1)–(3) *satisfy all assumptions* **A**, **B**, *and* **C**. *Then there exists a strong solution,* $\rho(\theta,\omega,t,\Omega)$, *to the problem* (1)–(3). *Moreover, this solution* ρ *is nonnegative in* Q_T *and is normalized,*

$$\int_0^{2\pi}\int_{-\infty}^{+\infty} \rho(\theta,\omega,t,\Omega)\,d\omega\,d\theta = 1$$

for all $t \in [0,T]$ *and* $\Omega \in [-G,G]$.

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Stochastic Wess-Zumino-Novikov-Witten model on the sphere

We define a measure over the space of applications from the sphere into a Lie Group. We show that the spheres are almost surely Hoelder. We do a stochastic cohomology theory of the space of random spheres, which allow to define a general stochastic topological Wess-Zumino term. We show that the Wess-Zumino term is related to stochastic integrals on the sphere

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Ljapunov dimension for Hennon and Lorenz attractors

Consider the Hennon mapping $F : \mathbf{R}^2 \to \mathbf{R}^2$

$$\begin{aligned} x &\to a + by - x^2, \\ y &\to x, \end{aligned}$$

where $a > 0$, $0 < b < 1$ are the parameters of the mapping.

Theorem 1. *Let $K \subset \mathbf{R}^2$ be an invariant with respect to F set, $(x_-, x_-) \in K$. Then*

$$\dim_L K = 1 + \frac{1}{1 - \ln b / \ln \alpha_1(x_-)}$$

where $\alpha_1 = \sqrt{x_-^2 + b} - x_-$, $x_- = \frac{1}{2}\left(b - 1 - \sqrt{(1-b)^2 + 4a}\right)$. Let us consider the Lorenz system

$$\begin{aligned} \dot{x} &= -\sigma x + \sigma y, \\ \dot{y} &= rx - y - xz, \\ \dot{z} &= -bz + xy, \end{aligned} \tag{1}$$

where r, b, σ are positive. Suppose that the following inequalities are true

$$\begin{aligned} &b \geqslant 2, \; r \geqslant 1, \; \sigma + 1 - 2b \geqslant 0, \\ &r\sigma^2(4 - b) + 2\sigma(b-1)(2\sigma - 3b) - b(b-1)^2 \geqslant 0. \end{aligned} \tag{2}$$

Note that the most often considered in numerical experiments values of parameters $r = 28$, $b = 8/3$, $\sigma = 10$ and $r = 40$ $b = 4$, $\sigma = 16$ are included in the set (2).

Theorem 2. *Suppose the inequalities (2) are true and the set K is invariant with respect to the shift operator along the trajectories of the system (1), $0 \in K$. Then the equality*

$$\dim_L K = 3 - \frac{2(\sigma + b + 1)}{\sigma + 1 + \sqrt{(\sigma - 1)^2 + 4r\sigma}}$$

is true.

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Amost invariant elliptic manifolds in a singularly perturbed hamiltonian system

We study a singularly perturbed Hamiltonian system of the type

$$\begin{aligned} \epsilon\dot{x} &= \frac{\partial H}{\partial y}, & \dot{u}_i &= \frac{\partial H}{\partial v_i}, \\ \epsilon\dot{y} &= -\frac{\partial H}{\partial x}, & \dot{v}_i &= -\frac{\partial H}{\partial u_i}, \end{aligned} \qquad i = 1,\ldots,n. \tag{0.1}$$

Here x and y are scalar variables, $u, v \in \mathbf{R}^n$. To this form many equations from applications can be transformed. Our main assumptions concerning this system are the following. 1. The system $\partial H/\partial y = 0$, $\partial H/\partial x = 0$ can be solved with respect to x, y, $x = f(x,y), y = g(x,y)$ in some domain D_0 of variables (u, v); 2. There is a neighborhood of the slow manifold (i.e., the graph of $x = f(x,y), y = g(x,y)$) where determinant

$$\Delta(u,v) = \det \begin{pmatrix} \partial^2_{xx}H & \partial^2_{xy}H \\ \partial^2_{yx}H & \partial^2_{yy}H \end{pmatrix}\Bigg|_{x=f(u,v),y=g(u,v),\epsilon=0}.$$

is positive, $\Delta(u,v) \geqslant C > 0$. It means that fast variables x, y are elliptic. 3. The Hamiltonian is analytic in all its variables in some complex δ-neighborhood of the slow manifold. Our main result is

Theorem *Then there is a canonical change of coordinates $\Phi : (x,y,u,v) \mapsto (X,Y,U,V)$ and constants $\tilde{\delta}, c > 0$ such that in the new coordinates the Hamiltonian H takes the form*

$$H \circ \Phi^{-1} = H_0(I, U, V; \epsilon) + R(X, Y, U, V; \epsilon), \tag{0.2}$$

where $I = \frac{1}{2}(X^2 + Y^2)$ and

$$R = O\left(e^{-\frac{c}{\epsilon}}\right). \tag{0.3}$$

The change of coordinates and the new Hamiltonian are analytic with respect to (X,Y,U,V) in $D_1(\delta/2)$, and bounded in ϵ. The theorem implies that I is almost an integral of the system: $I' = O\left(e^{-\frac{c}{\epsilon}}\right)$. The equation $X = Y = 0$ defines an almost invariant (normally) elliptic slow manifold. In particular, in the case $n = 1$, the system (0.1) has two degrees of freedom and in some neighborhood of this manifold it is exponentially close to an integrable system. This allows one to use powerful methods of Hamiltonian perturbation theory to a further study of the system. For instance, suppose that the slow system (it is derived from (0.1) if one plugges into $\dot{u}, \dot{v}$-equations the functions f, g, this system is Hamiltonian with 1 degree of freedom) has a saddle equilibrium with a homoclinic orbit to it (common situation). Then for $\epsilon > 0$ the full system has a saddle-center equilibrium whose one-dimensional separatrices are splitted and this splitting is exponentially small. If the initial system is, in addition, reversible with respect to some involution, and the homoclinic orbit and slow manifolds are symmetric with respect to this involution, then, under some generic conditions, one can prove that the system possesses multi-round homoclinic orbits as $\epsilon \to 0$. Several important applications will be discussed. This talk is based on the joint paper with V. Gelfreich. The author is thankful for a partial support to the Russian Foundation of Basic Research (grant 01-01-00905)

Levenshtam V.B.

Exponential dichotomy criterion of parabolic operators with almost-periodic coefficients

1. We'll consider parabolic equations like

$$Pu \equiv \frac{\partial u}{\partial t} + \sum_{|\alpha| \leqslant 2m} a_\alpha(x,t) D^\alpha u = f(x,t), \quad (x,t) \in \mathbb{R}^{n+1} \tag{1}$$

where $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$ is multi-index, $|\alpha| = \alpha_1 + \alpha_2 + \dots + \alpha_n$, $D^\alpha = \frac{\partial^{|\alpha|}}{\partial x_1^{\alpha_1} \partial x_2^{\alpha_2} \dots \partial x_n^{\alpha_n}}$. Let $C^{\gamma,\delta}$, $\gamma, \delta \in [0,1]$ designates Banach space of read functions $u(x,t)$, which satisfy a condition

$$\begin{aligned} \|u\|_{C^{\gamma,\delta}} = & \sup_{(x,t)\in\mathbb{R}^{n+1}} |u(x,t)| + b_\gamma \sup_{x' \neq x''} \frac{|u(x'',t) - u(x',t)|}{|x'' - x'|^\gamma} + \\ & + b_\delta \sup_{t' \neq t''} \frac{|u(x,t'') - u(x,t')|}{|t'' - t'|^\delta} < \infty, \quad b_0 = 0, \quad b_s = 1 \text{ for } s \neq 0. \end{aligned}$$

Let a_α, $f \in C^{\gamma,\gamma/2m}$, $\gamma \in (0,1)$ and condition of parabolicity is fulfilled

$$(-1)^m \sum_{|\alpha|=2m} a_\alpha(x,t) \xi^\alpha \geqslant \lambda_0 |\xi|^{2m}, \quad \forall \xi \in \mathbb{R}^n,$$

where $\lambda_0 = \text{const} > 0$. All solutions are considered in classic sense.

2. It is supposed in this point that coefficients a_α are 2π-periodic up to (x,t) functions.

Theorem 1. *If equation (1) with $f = 0$ has no unzero solutions, which belong to $C^{0,0}$, then equation (1) has an only in $C^{0,0}$ solution u with any right part $f \in C^{\gamma,\gamma/2m}$, being $\partial u/\partial t$, $D^\alpha u \in C^{\gamma,\gamma/2m}$, $|\alpha| \leqslant 2m$.*

Theorem 2. *Operator P possesses an exponential dichotomy on time axis $t \in \mathbb{R}$ if and only if uniform equation $Pu = 0$ has no unzero solutions in $C^{0,0}$.*

3. Let coefficients (1) be (almost-) periodic according to Bohr. Let $H(P)$ designates a set of parabolic operators

$$Q = \frac{\partial}{\partial t} + \sum_{|\alpha| \leqslant 2m} b_\alpha(x,t) D^\alpha,$$

coefficients $b_\alpha(x,t)$ ate the uniform (i.e. in $C^{0,0}$) limits of all possible sequence $a_\alpha(x + \xi_k, t + \tau_k)$, where $(\xi_k, \tau_k) \in \mathbb{R}^{n+1}$. Parallel with equation (1) we'll consider the class of uniform equations

$$Qv = 0, \quad Q \in H(P). \tag{2}$$

Theorem 3. *Equation (1) with any right part $f \in C^{\gamma,\gamma/2m}$ has the only $C^{0,0}$ solution u if and only if equation (2) have no unzero solutions in $C^{0,0}$. In this case $\partial u/\partial t$, $D^\alpha u \in C^{\gamma,\gamma/2m}$, $|\alpha| \leqslant 2m$. If f — is almost-periodic function, then u is almost-periodic solution.*

Theorem 4. *Operator P possesses exponential dichotomy on time axis if and only if equations (2) have no unzero solutions, which belong to $C^{0,0}$.*

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Commutators, Spectral Trace Identities, and Universal Inequalities for Eigenvalues

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KZ equations as isomonodromic deformations and solutions of the Schlesinger and Garnier systems

Under n-pointed KZ equation we mean a integrable meromorphic pfaffian system on $\mathbb{C}^n$ the following form

$$d\Psi_n(z_1,\ldots,z_n)=\Omega_n\Psi_n(z_1,\ldots,z_n),\quad \Omega_n=\sum_{1\leqslant i<j\leqslant n}\frac{t_{ij}d(z_i-z_j)}{z_i-z_j}. \tag{1}$$

Here t_{ij} are constant matrices from $sl(p,\mathbb{C})$ that satisfy relations

$$[t_{ij},t_{kl}]=0,\ \{i,j\}\cap\{k,l\}=\varnothing, i,j=1,\ldots,n$$
$$[t_{ij},t_{ik}+t_{jk}]=[t_{jk},t_{ij}+t_{ik}]=[t_{ik},t_{ij}+t_{jk}]=0,\ 1\leqslant i<j<k\leqslant n. \tag{2}$$

The relations (2) are equivalent to the Frobenius conditions of the integrability of the (1). Let $\Psi_{n-1}(z_1,\ldots,z_{n-1})$ and $\Psi_n(z_1,\ldots,z_n)$ be denote a fundamental matrices of solutions of the n and (n-1)-pointed KZ equations. If we consider $(z_1,z_2,\ldots,z_{n-1})\in\hat{\mathbb{C}}^{n-1}=\mathbb{C}\setminus\cup_{i<j}\{z_i=z_j\}$ as deformation parameters then the KZ equation (1) gives us the *unnormalized KZ isomonodromic deformation* of the Fuchsian system

$$\frac{dY}{dz_n}=(\sum_{i=1}^{n-1}\frac{t_{in}}{z_n-z_i^0})Y,\ z^0=(z_1^0,\ldots,z_{n-1}^0)\in\hat{\mathbb{C}}^{n-1} \tag{3}$$

on Riemann sphere $\bar{\mathbb{C}}$. The transformation $\tilde{\Psi}_n=\Psi_{n-1}^{-1}\Psi_n$ in some small neighbourhood U of the $z^0\in\hat{\mathbb{C}}^{n-1}$ reduces unnormalized the KZ deformation (1) of the (3) to the *Schlesinger deformation* (see,[1])

$$dY=\left(\sum_{i=1}^{n-1}\frac{B_i(z_1,\ldots,z_{n-1})d(z_n-z_i)}{z_n-z_i}\right)Y. \tag{4}$$

The matrices

$$B_j(z_1,\ldots,z_{n-1})=\Psi^{-1}(z_1,\ldots,z_{n-1})t_{jn}\Psi(z_1,\ldots,z_{n-1}),\ j=1,\ldots,n-1 \tag{5}$$

in U are some *hypergeometric type solution* of *the Schlesinger equations*

$$dB_i=-\sum_{j=1,\ j\neq i}^{n-1}[B_i,B_j]\frac{d(z_i-z_j)}{z_i-z_j},\ i=1,\ldots,n-1. \tag{6}$$

The solutions (5) of the (6) can be extended to the whole universal covering Z of the space $\hat{\mathbb{C}}^{n-1}$ as holomorphic functions. So, this solutions don't have movable poles as singular points (see,[2]) For $p = 2$ the Schlesinger equations in suitable coordinates on the space formed with $(B_1, \ldots, B_{n-1})$, $B_i \in sl(2, \mathbb{C})$ are reduced to Garnier system (for n=4, it is reduced to well-known Painlevé VI equation) (see,[3]). Hypergeometric type solutions give us some solutions of Garnier system. For $p = 2$ we solve of the algebraic system (2).General solution is parametrized with four parameters. For n=4,5 we give explicit form of hypergeometric type solutions of Schlesinger equations and also Painlevé equation and Garnier system. We study the action of the pure braid groups on these solutions. In terms this action and some asymptotics we formulate sufficient conditions when a holomorphic solutions on Z of the Sclesinger equations are hypergeometric type solutions.

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Asymptotic and numerical approximation of a nonlinear singular boundary-value problem

In this work, we consider a singular boundary-value problem for a nonlinear second-order differential equation of the form

$$g''(u) = ug(u)^q/q, \tag{0.1}$$

where $0 < u < 1$ and q is a known parameter, $q < 0$. We search for a positive solution of (0.1) which satisfies the boundary conditions

$$g'(0) = 0, \tag{0.2}$$

$$\lim_{u \to 1-0} g(u) = \lim_{u \to 1-0} (1-u) g'(u) = 0. \tag{0.3}$$

This problem arises in the study of boundary layer equations for the stationary flow of an incompressible fluid over an impermeable, semi-infinite plane. We assume that the fluid satisfies a power law, that is, a relation of the type

$$\tau_{xy} = k \left(\frac{\partial u}{\partial y} \right)^n, \tag{0.4}$$

where τ_{xy} is the shear stress, u is the velocity, x, y are the coordinates on the plane. Particular cases of such fluids are the newtonian fluids ($n = 1$), the pseudoplastic fluids ($n < 1$) and dilatant fluids ($n > 1$). Under condition (0.3), the equation of one of the components of the shear stress may be reduced to the form (0.1),with $q = -1/n$ (see [4], [5] and [2]). The existence of solution to problem (0.1-0.2-0.3) was proved in [5]. In [4], an iterative method for the computation of the solution was introduced and numerical results were obtained for some particular values of q. In [2] and [3] a numerical method was introduced, based on the use of lower and upper solutions, which enabled to obtain accurate numerical results for a wide range of values of q. In this work we analyse the asymptotic properties of the solution near the singularity, depending on the value of q. We show the existence of a one-parameter family of solutions of equation (0.1) which satisfy the boundary condition (0.3). By means of variable substitutions, asymptotic expansions are obtained for this family of solutions. Three different cases are considered: $q < -1$, $-1 < q < 0$ and $q = -1$. In each case,the asymptotic expansion of the solutions near the singularity has a different form. If $q < -1$ and $q = -3$, for example, we obtained the following expansion:

$$\begin{aligned} g(u,a) &= \left[(1-q)^2[2q(1+q))]\right]^{\frac{1}{(1-q)}} (1-u)^{\frac{2}{(1-q)}} \cdot \\ &\cdot \left\{ 1 - [(1+q)/[(1-q)(3+q))]] (1-u) + a(1-u)^{\frac{-2(1+q)}{(1-q)}} + O\left((1-u)^{1+\mu}\right) \right\}, \end{aligned}$$

as $u \to 1 - 0$, where $\mu = \min(1, \frac{-2(1+q)}{(1-q)}$ and a is the parameter of the considered family of solutions. Using this asymptotic expansions we can choose the value of a by the shooting method in order to satisfy the boundary condition (0.2). The same approach was used in [1], where this and other nonlinear singular boundary-value problems were analysed. Numerical results are obtained and compared with the ones presented in other papers. The authors were supported in this work by the project NATO PST/CLG.976878. The first author was also supported by the RFBR project 99-01-00331.

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Estimates for Fundamental Solutions for some classes of Second-order Parabolic Equations with singular coefficients

We study the second-order parabolic equation

$$\partial_t u(t,x) = \nabla \cdot a(t,x) \cdot \nabla u(t,x) - b(t,x) \cdot \nabla u(t,x) \\ + \nabla \hat{b}(t,x)u(t,x) + V(t,x)u(t,x),$$

in a domain $[0,T] \times \mathbb{R}^d \subset \mathbb{R}^{d+1}$, where $a = (a_{ij})_{i,j=1}^d$ is matrix of bounded measurable coefficients, $b = (b_j)_{j=1}^d$, $\hat{b} = (\hat{b}_j)_{j=1}^d$ are measurable (in general, singular) vector fields, V is a measurable potential, T is a fixed positive number. We introduce a new class of coefficients in the lower order terms for which we prove the existence and the uniqueness of the weak fundamental solution and for this we derive Gaussian upper and lower bounds. Our condition on the potential V is the non-autonomous extension of the Kato class introduced by Qi Zhang and J. Voigt. The condition we impose on the drift coefficients b and $\hat{b}$ is slightly more restrictive than the non-autonomous extension of the class K_{d+1}. This is caused by the lack of regularity of the coefficients a_{ij} in the main part. We also discuss the case when the Gaussian estimates are no longer valid. We obtain pointwise two-sided estimates for the integral kernel of the semigroup associated with second order elliptic differential operators $-\nabla\cdot(a\nabla)+b\cdot\nabla+\nabla\cdot\hat{b}+V$ with real measurable (singular) coefficients, on an open set $\Omega\subset\mathbb{R}^N$. The assumptions we impose on the lower order terms allow for the case when the semigroup exists on $L^p(\Omega)$ for p only from an interval in $[1,\infty)$, and neither enjoys a standard Gaussian estimate nor is ultracontractive in the scale $L^p(\Omega)$. We show however, that the semigroup *is* ultracontractive in the scale of weighted spaces $L^p(\Omega,\varphi^2dx)$ with a suitable weight φ and derive an upper and lower bound on its integral kernel.

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On the convergence rate of biorthogonal function expansions

We consider a restriction an arbitrary differential operator L generated by the differential operation

$$L=\frac{d^{2n}}{dx^{2n}}+\sum_{l=1}^{2n}p_l(x)\frac{d^{2n-l}}{dx^{2n-l}},\quad x\in G=(0,1),n\geqslant 1; \tag{0.1}$$

$$p_1(x)\in\mathcal{L}^f(\mathcal{G},\mathcal{C}),f>\infty,\ \ddagger(\S)\in\mathcal{L}(\mathcal{G},\mathcal{C}),\ddagger=\in,\dots,\in\backslash, \tag{0.2}$$

on the class D_{2n} of functions absolutly continuous on $\overline{G}$ together with their (2n-1)-order derivatives, which system of root functions possess some useful for applications properties: generalized Bessel inequality, generalization of Riesz (Hausdorff-Young) theorem, convergence of biorthogonal expansions on the entire interval $\overline{G}$ in L_p. We obtain estimates for the equiconvergence rate in $L_p(G)$ indicated expansions with the expansion of the same function in the ordinary Fourier trigonometric series. We detect, that the equiconvergence rate of these expansions in certain cases substantially depends on s - integrability degree of function $p_1(x)$. We define the root functions of the operator L in generalized (Il'in) sense by considering them as regular solutions of differential equations without any particular boundary conditions. In this case restrictions are imposed on the properties of the spectrum and the root functions of the operator. This allows one to study function systems like systems of exponentials. To obtain the above-mentioned estimate, we do not use the adjoint L^* of the operator L, becouse the existence of L^* is an additional restrictin on the smoothness of the coefficients $p_l(x)$. The case of substantially nonself-adjoint operator (that is, operatoers whose systems of root functions contain infinitely many associated eigenfunctions) is also admissible. The restriction of the operator is determined by the three *Il'in conditions* 1)-3), described below. We choose an arbitrary numbers $r \in [1, \infty), \gamma > 0$, an arbitrary eigenvalue system $\{\lambda_k\}_{k=1}^{\infty}$ and an arbitrary system $\{u_k(x)\}$ of root functions of the operator L corresponding to these eigenvalues and satisfying the following three conditions: 1) $\lambda_k \in \{\lambda \in \mathcal{C} : \mathcal{R}]\lambda \geqslant \prime, |\mathcal{I}\Updownarrow\lambda| \leqslant \gamma\}, \qquad \sum_{\prime \leqslant |\lambda_{||}| - \lambda \leqslant \infty} \infty \leqslant \rfloor, \forall\lambda \geqslant \prime$; 2) $\{u_k\}$ is a closed and minimal system in $\mathcal{L}^{\triangledown}(\mathcal{G}), \|\sqcap_{||}\|_{\triangledown} \|\sqsubseteq_{||}\|_{\triangledown'} \leqslant \rfloor, \forall ||$, $\{v_k\}$ is the system biorthgonal to $\{u_k\}$; 3) so-called anti- a priori estimate is valid for $\{u_k\}$. Let us impose an additional condition 4) $\|u_k\|_\infty \leqslant c\|u_k\|_r, \forall k$. For an arbitrary function $f(x) \in \mathcal{L}^{\triangledown}(\mathcal{G})$, consider the partial sums $\sigma_\lambda(x; f) = \sum_{|\lambda_k| \leqslant \lambda} f_k u_k(x), \lambda > 0, f_k \equiv (f, v_k)$, of the biorthogonal expansion. By $S_\lambda(x, f)$ we denote the partial sum of the trigonometric Fourier series of the function $f(x)$, viewed as the orthogonal expansion of f for the operator $L_0 : L_0 u = u'', u \in D_2, u(0) = u(1), u'(0) = u'(1)$. Suppose that 5) $\exists \nu = const > 0 : \quad \alpha_k f_k = O(\lambda_k^{-\nu}), \quad |\lambda_k| \geqslant 1, \alpha_k = \|v_k\|_{r'}^{-1}$, in the following we suppose that $\nu_0 \geqslant \nu$ in the same condition for the operator L_0 (if $f \in V(G)$, then $\nu_0 = 1$, and $\nu : \alpha_k(1, v_k) = O(\lambda_k^{-\nu})$).

Theorem 1 *Let conditions 1)-5) be satisfied for the operator L, $p \in [1, \infty)$. Then the estimate $\|\sigma_\lambda(x, f) - S_\lambda(x, f)\|_p = O(\max(\lambda^{-1/p}, \lambda^{-(\nu - 1/\delta)})), \delta = \min(2, q, s)$, $q = \frac{p}{p-1}$ is valid for all sufficiently large $\lambda > 0$.*

Note that the theorem conditions do not guarantee basis property of $\{u_k\}$ in $\mathcal{L}\checkmark$ even for $n = 1$.

Corollary 1 *Let $p \in (1, \infty), f \in \mathcal{L}^{\Updownarrow\dashv\S(\checkmark, \triangledown)}(\mathcal{G})$ and conditions 1)-5), $\nu > 1/\delta$ are fulfilled. Then $\|f - \sigma_\lambda(x, f)\|_p \to 0, \lambda \to \infty$.*

Note that if $\nu = 1/\delta = 1/q, p \geqslant 2$, then the statement of corollary is not valid even for $s = \infty$. The work was supported by the Russian Foundation for Basic Research (project N 99-01-01260).

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Spatial heterogeneities in evolution problems

We analyze the effect of varying coefficients in a general class of semilinear elliptic boundary value problems of sublinear and superlinear type. As a consequence of our analysis it follows the necesity of introducing a new class of generalized solutions, which are not distributional, to describe the asymptotics of the positive solutions of a general class of sublinear problems with vanishing coefficients. Those solutions are referred to as METASOLUTIONS in the specialized literature. Then, we analyze a general class of indefinite superlinear problems characterizing whether or not the problem possesses a stable positive solution and showing the uniqueness of the stable solution when it exists. The uniqueness result is very striking as there are models exhibiting an arbitrarily large number of positive solutions, as an effect of the spatial inhomogeneities of the problem.

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(Moscow State University)

Geometry of boundaries of stability domains for periodic systems

We consider a linear periodic system governed by the equation

$$\dot{x} = G(t)x, \qquad (*)$$

where x is a real vector of dimension m; $G(t)$ is a real matrix of dimension $m \times m$ continuously depending on time t with a period T, $G(t) = G(t+T)$; dot represents the derivative with respect to t. It is assumed that the matrix $G(t,p)$ and the period $T(p)$ smoothly depend on a vector two or three parameters p. Stability condition for the system $(*)$ determines the stability and instability domains in the parameter space. It is well know that the boundary of the stability domain can have singularities, the nonsmooth points, which affect physical properties of the system and lead to computational difficulties. The classification of all generic (typical) singularities of stability boundaries for linear periodic systems of general form is given, and the local form of the stability domain is determined up to a smooth change of parameters. The classification is carried out with the use of methods of versal deformation theory. Explicit formulae for perturbations of simple and multiple multiplies, which determine regular and singular points of the stability boundary, are derived using perturbation theory for eigenvalues and formulae for derivatives of the monodromy matrix with respect to parameters. The formulae describing sensitivities of multipliers to the perturbations of the parameter vector p allow determining the stability domain in the vicinity of a boundary point. Explicit first order approximations of the stability domain near regular and all types of typical singular boundary points are given. The formulae for the approximations use only information on the system at the boundary point under consideration, which is usually available from the stability analysis at this point. This property is especially useful for numerical analysis, since together with calculation of the stability boundary point we get the local information on the stability domain both in the regular and singular cases. The suggested approach is general and can be useful for the analysis of nongeneric singularities appearing in systems with symmetries as well as in the case of four or more parameters. As an example, vibrations of a tube consisting of two equal parts and conveying pulsating fluid are studied. The parts of the tube are connected by an elastic hinge with the stiffness coefficient c. The left side of the tube is attached to the wall by an elastic hinge; the right end of the tube is free. Local analysis of the stability boundary of the straight equilibrium of the tube is performed in the space of three parameters $p = (\delta, w, V)$, where δ and w are dimensionless amplitude frequency of pulsations, V is the mean velocity of the fluid. Using the obtained formulae we found a linear approximation of the stability domain near the smooth point of the boundary. Then it is shown that the stability boundary has a singularity "dihedral angle". Linear approximation of the stability domain in the vicinity of the singular boundary point is found. Numerical calculations confirm the obtained results and show that the computational time spent for finding the approximation is considerably smaller than the time needed for the numerical stability analysis using Floquet method. This work was done together with Alexander P. Seyranian and was supported by the Russian Foundation for Basic Research, grant RFFI 99-01-39129.

Malamud M. M.
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Deficiency indices and selfadjointness of Hamiltonian systems

The purpose of this talk is to investigate the formal deficiency indices $\mathcal{N}_{\pm}(I)$ of a symmetric first order system

$$Jf' + Bf = \mathcal{H}f \tag{1}$$

on an interval I, where $I = \mathbb{R}$ or $I = \mathbb{R}_{\pm}$. Here $J, B, \mathcal{H}$ are $n \times n$ matrix valued functions and the Hamiltonian $\mathcal{H} \geqslant 0$ may be singular even everywhere. We obtain two results for such a system to have minimal numbers $\mathcal{N}_\pm(\mathbb{R}) = 0$ (resp. $\mathcal{N}_\pm(\mathbb{R}_\pm) = n$) and a criterion for their maximality $\mathcal{N}_\pm(\mathbb{R}_+) = 2n$. Some conditions for a canonical system to have intermediate numbers $\mathcal{N}_\pm(\mathbb{R}_+)$ are presented, too. We also obtain a generalization of the well-known Titchmarsh–Sears theorem for second order $n \times n$ matrix Sturm–Liouville type equations

$$(2) \qquad Py := -\frac{d}{dx}\left(A(x)^{-1}\frac{dy}{dx} + Q(x)y\right) + Q^*(x)\frac{dy}{dx} + R(x)y = \mathcal{H}(x)y,$$

where $A, Q, R, \mathcal{H} \in L^1_{loc}(\mathbb{R})$ and $A(x)$ is positive definite for all $x \in \mathbb{R}$ and $\mathcal{H}(x) \geqslant 0$. In order to present the corresponding statement we put

$$(3) \qquad c(x) := \begin{cases} \max(1, \|A(x)^{-1/2}\mathcal{H}(x)^{-1/2}\|), & \det(A(x)\mathcal{H}(x)) \neq 0, \\ \infty, & \text{otherwise.} \end{cases}$$

Theorem. *Let $P_+y = \mathcal{H}y$ $(P_-y = \lambda\mathcal{H}y)$ be the equation of the form (2) considered on $\mathbb{R}_+(\mathbb{R}_-)$ with $A(x)$ being positive definite for $x \in \mathbb{R}_+(\mathbb{R}_-)$, $\mathcal{H} \geqslant 0$ and $c(x)$ be defined by (3). Suppose also that $V := R - Q^*AQ \geqslant -q\mathcal{H}$ where $q \geqslant \delta > 0$ and*

$$\int_0^\infty \frac{1}{c(x)q^{1/2}(x)}dx = \infty \qquad \left(\int_{-\infty}^0 \frac{1}{c(x)q^{1/2}(x)}dx = \infty\right).$$

Moreover, assume that one of the following two conditions is satisfied: (1) $q^{-1/2}$ is absolutely continuous and $\left|\frac{d}{dx}q^{-1/2}(x)\right|c(x) \leqslant C_1$ for $x \in \mathbb{R}_+(\mathbb{R}_-)$; (2) $q(x)$ is monotone increasing (monotone decreasing). Then $\mathcal{N}_\pm(P_+) = n$ $(\mathcal{N}_\pm(P_-) = n)$. Moreover, $\mathcal{N}_\pm(P) = 0$ if $P_\pm$ satisfy the above assumptions. Our criterion generalizes results due to Lidskii [1] (and coincides with it for $A = \mathcal{H} = I$ and $Q = 0$) and Krein [2] ($n = 1$, $Q = 0, A = I$ and $R \geqslant 0$) and may be considered as an essential (in our opinion) generalization of the well-known Titchmarsh–Sears theorem. Results on selfadjointness of second order elliptic differential equations are presented too. The results are obtained jointly with M. Lesch (Koeln)(see [3]).

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The averaging of non-local Hamiltonian structures in Whitham's method

We consider the m-phase Whitham's averaging method and propose the procedure of "averaging" of "weakly-nonlocal" Hamiltonian structures having the form

$$J^{ij} = \sum_{k\geqslant 0} B_k^{ij}(\varphi, \varphi_x, \ldots)\partial_x^k + \sum_{k,l\geqslant 0} e_{kl} S^i_{(k)}(\varphi, \varphi_x, \ldots)\partial^{-1} S^j_{(l)}(\varphi, \varphi_x, \ldots)$$

The procedure is based on the existence of a sufficient number of local commuting integrals of the system and gives the Poisson bracket of Ferapontov type

$$J^{\nu\mu} = g^{\nu\mu}(U)\partial_X + b^{\nu\mu}_\lambda(U)U^\lambda_X + \sum_{k\geqslant 0} e_{kl} S^\nu_{(k)\lambda}(U)U^\lambda_X \partial^{-1} S^\mu_{(l)\delta}(U)U^\delta_X$$

for the Whitham's system. The method can be considered as the generalization of the Dubrovin-Novikov procedure for the local field-theoretical brackets.

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Stokes problem in a three-faced corner

The flow in a trihedral corner induced by a non-zero velocity distribution over one of the corner's sides is considered in the Stokes approximation. A similar problem in a slightly different geometry was studied in [1]. An algorithm of solution developed in the present study is based on the method of superposition. The velocity and pressure fields are presented as sums of three vector and scalar fields, respectively,

$$\mathbf{U} = r^n \sum_{i=1}^{3} \mathbf{u}^{(i)}(\theta^{(i)}, \phi^{(i)}), \qquad P = r^{n-1} \sum_{i=1}^{3} p^{(i)}(\theta^{(i)}, \phi^{(i)}), \tag{0.1}$$

where $(r, \theta^{(i)}, \phi^{(i)})$, $i = 1, 2, 3$ are three spherical coordinate systems with a common origin at the corner's vertex. These coordinate systems are chosen in such a way that i-th corner's wall occupies the domain $0 \leqslant r < \infty$, $\theta^{(i)} = \pi/2$, $0 \leqslant \phi^{(i)} \leqslant \alpha_i$, $i = 1, 2, 3$ in a corresponding coordinate system. It is not necessary for the corner to be a canonical domain. Hence the developed approach may be applied to a more general class of the trihedral corners. However in what follows we restrict our consideration to the corner of a cubic cavity. The intermediate mathematical treatments and the final representation of the solution are considerably simplified in this case. The functions $p^{(i)}$ are surface spherical harmonics, whereas $\mathbf{u}^{(i)}$ are expressed via three surface spherical harmonics by the Lamb's general solution. Choosing the spherical harmonics in the form of Fourier series with respect to $\phi^{(i)}$, one can present the velocity as follows

$$\begin{aligned} u_r^{(i)} &= \sum_{m=1}^{\infty} q_m^{(i)}(\theta^{(i)}) \sin(2m\phi^{(i)}), \quad u_{\theta^{(i)}}^{(i)} = \sum_{m=1}^{\infty} s_m^{(i)}(\theta^{(i)}) \sin(2m\phi^{(i)}), \\ u_{\phi^{(i)}}^{(i)} &= C^{(i)} P_n^{-1}(\cos\theta^{(i)}) + \sum_{m=1}^{\infty} t_m^{(i)}(\theta^{(i)}) \cos(2m\phi^{(i)}), \end{aligned} \tag{0.2}$$

where $q_m^{(i)}$, $s_m^{(i)}$, $t_m^{(i)}$ are expressed via Legendre associated functions of the first kind. Satisfaction of the boundary conditions leads to a triple infinite system of linear algebraic equations for the unknown coefficients of the solution. Asymptotic behaviour of the unknowns was studied by means of the Mellin transformation [2]. It allows one to show that the local behaviour of the velocity field near the corner's edges, where a discontinuity of the boundary conditions is assumed, coincides with the Goodier-Taylor solution for a two-dimensional wedge, while near the quiet edge the Moffatt-type eddies exist. The first integral of motion is found for the flows presented by (0.1) that reduces the number of independent coordinates. If the velocity field is a linear combination of (0.1) for various n, the flow becomes essentially three-dimensional that results in a complicated behaviour of the streamlines. As an example, rotation of a corner's side about a centre displaced from the vertex is considered.

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Malyutin K.G.

Star Function of Type Baernstein for the Half-Plane

Let $u(z)$ be δ-subharmonic (δ.s.h.) in the closed disk $C(R): |z| \leqslant R$. Baernstein [1] defined a function $u^*(z)$ s.h. in the half-disk $C^+(R)$: $|z| < R$, $\operatorname{Im} z > 0$,

$$u^*(z) = \sup_E \int_E u\left(re^{i\omega}\right) d\omega + 2\pi N^-(r),$$

where the supremum is taken over Borel sets E in $(-\pi, \pi)$ whose measure is 2θ. Then Baernstein's Fundamental Theorem asserts that $u^*(z)$ is s.h. in $C^+(R)$ and continuous in $\overline{C^+(R)}\backslash 0$. We shall consider δ.s.h. functions in the half-disk. Suppose that $u(z)$ is δ.s.h. in the half-disk $C^+(R): |z| < R, \operatorname{Im} z > 0$. We associate with $u(z)$ a function $u^*(z)$ s.h. in the fourth-disk $D^+(R): |z| \leqslant R$, $\pi/4 \leqslant \arg z \leqslant 3\pi/4$. For $z \in D^+(R)$, $z = re^{i\theta}$, we define

$$u^*(z) = \sup_E \frac{1}{r} \int_E u\left(re^{i\omega}\right) \sin\omega d\omega + N^-(r), \tag{1}$$

where the supremum is taken over Borel sets E in $(0, \pi)$ such that

$$\int_E \sin\phi \, d\phi = \frac{4}{\pi}\left(\theta - \frac{\pi}{4}\right).$$

Theorem 1 (The theorem on the star function). *Suppose that $u(z)$ is the function of class $J\delta(R)$ and that $u^*(z)$ is defined by (1). Then $|z|u^*(z)$ is continuous in $D^+(R)\backslash 0$ and s.h. in the interior of $D^+(R)$.* Let $g(x)$ be a real-valued measurable function on $[0, \pi]$. The distribution function of g is the function $m(t)$ defined to be the measure $|E(t)|$ of the subset $E(t)$ of $[0, \pi]$, where $g(x)\sin x > t$. We now define for $\pi/4 \leqslant \theta \leqslant 3\pi/4$

$$g^*(\theta) = \sup_E \int_E g(x) \sin x \, dx, \tag{2}$$

where the supremum is taken over Borel sets E in $(0, \pi)$ also as in (1). We denote by $\Phi(y)$ a convex non-decreasing function of y. The relation between convex means and the star function arises from the following theorem. **Theorem 2.** *If $g, h \in L^1[0,\pi]$ then the following three statements are equivalent: (a) for every Φ we have*

$$\int_0^\pi \Phi\left(g(x)\sin x\right)dx \leqslant \int_0^\pi \Phi\left(h(x)\sin x\right);$$

(b) for every $t \in (-\infty, \infty)$

$$\int_0^\pi \left[g(x)\sin x - t\right]^+ dx \leqslant \int_0^\pi \left[h(x)\sin x - t\right]^+ dx;$$

(c)

$$g^*(\theta) \leqslant h^*(\theta) \qquad \left(\frac{\pi}{4} \leqslant \theta \leqslant \frac{3\pi}{4}\right).$$

The details will be given at the talk.

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Mamedov Kh.R.
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Obratnay zadacha teorii rasseyania dlya uravneniya Shturma-Liuvillya so spectralniym parametrom v granichniyx usloviyax[20]

In this work we develop the right and the inverse problem of the theory of dispersion on half an axis for the Shturm-Liouville equation with the spectral parameter in the boundary conditions. We have learned about the spectrum. We have derived the formula for eigenfunction expansion and the Levinson formula. We have also derived the basic equation and solved the inverse problem using the dispersion data.

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On Sturm-Liouville problem for the equation with the complex-valued density

Let's consider the problem on eigenvalues (e.v.)

$$y'' - \lambda^2 a^2(x) y = 0, \quad 0 \leqslant x \leqslant 1, \tag{1}$$

$$y(0) = 0, \quad y(1) = 0, \tag{2}$$

where $a(x) = a_1(x) + ia_2(x)$, $a_2(x) \neq 0$. It must be noted, that we don't know at least one result (ex.[1]) on distribution of e.v. of such problem (1), (2), when $\arg a(x) \neq const$. Namely this case is considered here. We suppose the fulfillment of conditions:

1^0. Function $a(x)$ is contraction of integer function on the segment $[0, 1]$;

2^0. $a_2(x) > 0$, $a_1'(x)a_2(x) - a_1(x)a_2'(x) > 0$, when $x \in [0, 1]$. We introduce the set of complex-valued x and λ determined by equalities

$$M = \left\{ x = x_1 + ix_2 : Im \overline{\int_0^1 a(\xi)d\xi} \cdot \int_0^x a(\xi)d\xi = 0 \right\},$$

$$S_j = \left\{\lambda : (-1)^{j-1} Re\lambda a(0) > 0, (-1)^j Re\lambda a(1) > 0\right\} \quad (j = 1, 2).$$

It is easy to prove that $M \cap [0, 1] = \{0; 1\}$. We suppose the fulfillment of the condition:

3^0. Let the set M contain connected component-the line l with the ends at the points $x = 0$ and $x = 1$ such that on it and on domain which is bounded by curve $l \cup [0; 1]$ there is no turn points of equation (1) (i.e. the points on which $a(x)$ is tern to zero). Let's denote that the example of function $a(x)$, satisfied conditions $1^0 - 3^0$, is

$$a(x) = x + b, \; Im\, b > 0, \; |2b + 1| > 1,$$

moreover for instance when $Re\, b = -1/2$ the equation of line l has the following explicit form:

$$x_2 = -Im\, b + \sqrt{x_1^2 - x_1 + (Im\, b)^2}, \; 0 \leqslant x \leqslant 1.$$

We has to prove

Theorem. *Let conditions $1^0 - 3^0$ hold. Then the problem (1), (2) have countable set e.v. λ_ν^2, where numbers λ_ν which except of may be finite number of which lies* *in sectors S_j $(j = 1, 2)$ and allows asymptotic representation*

$$\lambda_\nu = \left[\int_0^1 a(x)dx\right]^{-1} \nu\pi i \left[1 + O\left(\frac{1}{\nu}\right)\right] \quad (\nu \to \pm\infty).$$

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Key words: Sturm-Liouville, eigenvalues, asymptotic, complex-valued.

Manturov V.O.

On Cohomologies of Homogeneous Spaces and Relevant Integer Equations

It is known that $su(n)$ can be embedded in $su(N)$ by using some representation R, where $N = dimR$. Thus the most important question for the study of the quotient homogeneous space homology ring is whether such an embedding is entirely homologic to zero or not. For the case of symmetric power of $su(n)$ these embeddings are entirely non–homologic to zero, thus the Poicaré polynomial of the homogeneous space is just the fraction of the two polynomials of $su(N)$ and $su(n)$. The same question for the case of exterior powers of $su(n)$ representation was first studied by O.V.Manturov [1, 2]. It is proved in [1], that p–th exterior power for $su(n)$ gives non–trivial homological map in the generator of degree $2s - 1$ iff the following equation holds:

$$p^{s-1} - n(p-1)^{s-1} + C_n^2(p-2)^{s-1} + \cdots = 0, \tag{1}$$

In [2] this equation is solved for $p = 3$. It is easy to see, that the case $p = n$ (dimensional 1) and $p = \frac{n}{2}$ (symplectic or orthogonal representation) gives us solutions for this equation for many s. The main result of the talk is the following

Theorem 1 *For $p = 2^l$ the number of solutions for (1) is finite.*

In particular, for $p = 4$ there exists only one non–trivial (not belonging to the two series described above) solution: $s = 6, n = 12$.

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On operator homotopies

Let $h \in A$ is a selfadjoint, $u \in A$ is a unitary in some nice C^*-algebra A (e.g. unitalized compacts $\mathcal{K}^+$) and let the norm of their commutator $[u, h]$ is small enough. We show that one can then connect u with the unity by a unitary path $u(t)$ so that the norm of $[u(t), h]$ would be also small enough along this path. The idea of the proof uses a canonical form for a pair of almost commuting matrices. Let $H(A)$ denote the space of selfadjoint elements of A and let $G(A)$ denote the quotient metric space of classes of approximate unitary equivalence of $H(A)$. The above described homotopy makes it possible to lift continuous loops in $G(A)$ to continuous loops in $H(A)$. This result, being trivial for $A = \mathcal{K}^+$, is less trivial for more complicated C^*-algebras, e.g. for type II_1 factors. Similar technique helps to give a construction to solve the following problem. Let $u, v \in \mathcal{K}^+$ be two unitary matrices that almost commute. Then there exist two paths $u(t), v(t)$ in the unitary group of $\mathcal{K}^*$, $t \in [0, \infty)$, starting at u and v and such that the commutator $[u(t), v(t)]$ is small enough for all t and vanishes at infinity. This construction shows that any matrix almost representation of a finitely presented abelian group can be extended to an asymptotic representation

Marchuk N.G.

Dirac equation in Riemannian space

Markushevich D.G.

Some agebro-geometric integrable systems versus classical ones

Abstract. Several classical integrable systems are linearized on families of Jacobians of genus-2 curves: Kowalevski top, Jacobi problem, Neumann system, periodic Toda with three particles. It turns out that all these systems are related to certain singular K3 surfaces which are double covers of the projective plane. Namely, the genus-2 curves under consideration move in the K3 surface and are just the inverse images of lines in the plane. A general explanation of this phenomenon is provided, and under some non-degeneracy conditions, any Lagrangian fibration of Jacobians of genus 2 curves is of this form. A local result of this type was obtained by Hurtubise–Markman. A question of description of one integrable system by different families of genus-2 curves is discussed, and the example of the restricted Kowalevski top (that is, the orbit (l,g)=0, where l is the angular momentum of the top, g the constant gravity vector) is worked out in detail. It is shown that an infinity of different families of genus-2 curves can be obtained from Kowalevski's by applying Richelot's transformations. Among these families, there is one coming from the Reyman–Semenov-Tian-Shansky Lax representation of the top. Some generalizations to families of Prym surfaces and examples of more abstract algebro-geometric systems are described, namely, those corresponding to symmetric powers of K3 surfaces, moduli of sheaves and families of intermediate Jacobians of threefolds.

Markush Ivan Ivanovich

Development of Asymptotic Methods in the Theory of Ordinary Linear Singularly Perturbed Differential Equations

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Newtonian Flow Through a Network of Thin Pipes

We study the fluid flow through a network of intersected thin pipes with prescribed pressure at their ends. Pipes are either thin or long and the ratio between the length and the cross-section, denoted as usual by ε, is considered as the smal l parameter. It is well known that the stationary Navier-Stokes system describing the viscous flow in straight pipes with impermeable walls governed by the prescribed pressure drop has a solution in the form of the Poiseuille flow with

perfectly parabolic velocity pr ofile. In real-life situations, often two (or several) pipes are interconnected (watering systems, water-works, system of blood vessels). Also, pipes can be curved and constricted (particularly in case of blood vessels). In such situation the flow is not so simple any more and the velocity profile is not necessarily parabolic. We study the above situations using the asymptotic analysis with respect to ε. Some numerical illustrations are given.

AMS subject classification. 35B25, 76D30, 76D05

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Matrosov Vladimir Mefodievich

Methods of Nonlinear dynamical analysis and its applications.

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On the Strongly Hyperbolic Systems

Introduction. The systematic study of the Cauchy problem was started by I. G. Petrovsky [19], [20]. When a scalar and higher order operator has constant coefficients, its characteristic roots must be simple for the strong hyperbolicity. On the other hand, O. A. Oleinik's results in [18] include the following example for which the Cauchy problem is well-posed in C^∞ class:

Example 2

$$(\frac{\partial}{\partial t})^2 - t^{2k}(\frac{\partial}{\partial x})^2 + b(t,x)t^{k-1}\frac{\partial}{\partial x} + c(t,x)\,, \qquad (x \in \mathbf{R},\ k \in \mathbf{Z}_+). \qquad (0.1)$$

In case of $k = 1$, the lower order part has no restriction, that is, $(\frac{\partial}{\partial t})^2 - t^2(\frac{\partial}{\partial x})^2$ is strongly hyperbolic even if it has the double characteristic at $t = 0$. After this paper, the study on the strong hyperbolicity of scalar equations made hardly and obtained the necessary and sufficient condition. (See, V. Ja. Ivrii and Petkov [5], V. Ja Ivrii[4], L. Hörmander [3], N. Iwasaki [6], [7], [8], etc.)

On the other hand, for first order systems, K. O. Friedrichs [2] showed that if the principal part is symmetric, it is strongly hyperbolic. Therefore, we hope that if the principal part of a system is diagonalizable, it is strongly hyperbolic. However, I. G. Petrovsky [20] gave a counter example: there is a system *with constant coefficients* and its principal part is *pointwisely diagonalizable* but it is not strongly hyperbolic. The uniform estimate of a diagonalizer is required. K. Kasahara and M. Yamaguti [9] characterized the strong hyperbolicity of first order systems with constant coefficients.

In case of first order systems with variable coefficients, the study of the strong hyperbolicity has been made many mathematicians, for example, K. Kajitani [10], T. Nishitani (many papers), etc. Unfortunately, we have not yet arrive at the perfect characterization. Here, assuming every coefficient depends only on the time variable t, we give a necessary and sufficient condition. The results have been announced partially in [15] by H.Yamahara and me and in [12].

Definition and Results

We consider the following Cauchy problem:

$$\begin{cases} (P_1(t,D_t,D_x)-B(t,x))u \equiv D_t u - \sum_{i=1}^{\ell} A_i(t)D_{x_i}u - B(t,x)u = f(t,x) \quad , \\ \\ u|_{t=t_0} = u_0(x) \quad , \end{cases} \tag{0.2}$$

where, $A_i(t)$ ($1 \leqslant i \leqslant \ell$) and $B(t,x)$ are $\mathtt{N}\times\mathtt{N}$ matrices of C^∞-class, u, u_0 and f are vectors of dimension $\mathtt{N}$, $D_t = \frac{1}{\sqrt{-1}}\frac{\partial}{\partial t}$ and $D_x = \frac{1}{\sqrt{-1}}\frac{\partial}{\partial x}$.

Definition 1 *(C^∞ well-posedness)*
(1) We say that (0.2) *is C^∞ well-posed at (t_0,x_0) in $\Omega \subset \mathbf{R}^{1+\ell}$ when there exists a neighborhood ω of (t_0,x_0) where a solution u in $C^\infty(\omega)$ of* (0.2) *exists for every $f(t,x) \in C^\infty(\omega)$ and every $u_0 \in C^\infty(\omega_{t_0})$ and it is unique in $C^\infty(\omega_c)$ for arbitrary positive c. Here, $\omega_c = \omega \cap \{|t-t_0| < c)\}$ and $\omega_{t_0} = \omega \cap \{t = t_0\}$.*
(2) When (0.2) *is C^∞ well-posed at every (t_0,x_0) in Ω, we say that* (0.2) *is C^∞ well-posed in Ω.*

We can easily obtain an a priori estimate.

Proposition 1 (A priori estimate)

If the Cauchy problem (0.2) *is C^∞ well-posed at (t_0,x_0), for each M and each positive c, there exist M' and positive C such that*

$$|u|_{M,\omega_c} \leqslant C(|u_0|_{M',\omega_c} + |f|_{M',\omega_{t_0}}), \tag{0.3}$$

where $|u|_{M,\omega} = \sum_{|\alpha|\leqslant M,\, \alpha\in \mathbf{Z}_+{}^{1+\ell}} \max_{(t,x)\in\omega} |(\frac{\partial}{\partial x})^\alpha u(t,x)|$.

Definition 2 *(Strong hyperbolicity)*
We say that P_1 is a strongly hyperbolic system when the Cauchy problem for $P_1 - B$ is C^∞ well-posed in Ω for arbitrary B in $M_N(C^\infty(\Omega))$.

We fix ω in $S^{\ell-1}$ and put $\xi = \rho\omega$ for positive ρ. We set $P_1(t, D_t, \rho) = P_1(t, D_t, \rho\omega)$. We count the order of $P_1(t, D_t, \rho)$ by the order of D_t plus the degree of ρ.

Lemma 1 *There exist $T_o > t_o$, $d \in \mathbf{N}$, $d_o \in \mathbf{Z}_+$, $\{m_j\}_{j=1}^d \in \mathbf{N}^d$ ($\sum_{j=1}^d m_j = N$), $\{p_j\}_{j=d_o+1}^d \in \mathbf{N}^{d-d_o}$, $\{d_j\}_{j=d_o+1}^d \in \mathbf{N}^{d-d_o}$, $\{n_{jk}\}_{k=1}^{d_j} \in \mathbf{N}^{d_j}$, λ_{ji} ($d_o+1 \leqslant j \leqslant d$, $i \geqslant 0$) and $N_o(t) = N_{o0} + tN_{o1} + t^2 N_{o2} + \cdots \in GL(tN; \mathbf{C}[[t]])$ such that, in $\Omega \cap \{t_o < t < T_o\}$,*

$$N_o{}^{-1} P_1(t, D_t, \rho) N_o \equiv \oplus_{1\leqslant j\leqslant d} P^j(t, D_t, \rho) \qquad mod\ O(t^\infty)\rho$$

$$P^j(t, D_t, \rho) = \begin{cases} I_{m_j}(D_t - \lambda_j(t)\rho) & (1 \leqslant j \leqslant d_o \ \ Case\, 1), \\ I_{m_j}(D_t - \lambda_j(t)\rho) + (t^{p_j-1}A_{j0} + t^{p_j}A_{j1} + O(t^{p_j+1}))\rho \\ \qquad\qquad\qquad (d_o + 1 \leqslant j \leqslant d \ \ Case\, 2), \end{cases} \tag{0.4}$$

where $\lambda_j(t) = \sum_{i=0}^\infty \lambda_{ji} t^i$ (in Case 2, $\sum_{i=0}^\infty$ is replaced to $\sum_{i=0}^{p_j-1}$), $\lambda_j \neq \lambda_k$ ($j \neq k$),

$$A_{j0} = \oplus_{1\leqslant k\leqslant d_j} J(n_{jk}),\ J(n) = \begin{pmatrix} 0 & 1 & & \\ & \ddots & \ddots & \\ & & & 1 \\ & & & 0 \end{pmatrix} : \ n \times n \text{ and } A_{j1} \in M_{m_j}(\mathbf{C}).$$

When the coefficients of P_1 are real analytic, $\lambda_j(t)$ converges and $O(t^\infty) = 0$.
Standing on Lemma 1 and Proposition 1, we obtain the following.

Proposition 2 *If P_1 is strongly hyperbolic, all λ_{ji} are real and A_{j0} and A_{j1} in Case 2 satisfy*

$$A_{j0}(A_{j1})^k A_{j0} = 0\,, \ \ (0 \leqslant k \leqslant m_j\,). \tag{0.5}$$

Allowing the singularity $\frac{1}{t}$ on the lower order term, we can reduce Case 2 to the form with new p_j and d greater than the original ones under the conditions (0.5). Further, we can establish a similar proposition as Proposition 2 corresponding to the operator with the singularity $\frac{1}{t}$ on the lower order term.

Definition 3 *(Diagonal Fuchsian) A diagonal Fuchsian system means an operator $P(t, x, D_t, \rho\omega)$ with a diagonal first order part of real symbol modulo $O(t^\infty)\rho$ and lower order term with the singularity $\frac{1}{t}$.*

Theorem 1 *If $P_1(t, D_t, D_x)$ is strongly hyperbolic, for each ω in $S^{\ell-1}$, $P(t, x, D_t, \rho\omega) = P_1(t, D_t, \rho\omega) - B(t, x)$ is reduced to a diagonal Fuchsian by a similar transformation by a matrix $N(t)$ in $M_N(\mathbf{C}[[t]]) \cap GL(N; \mathbf{C}[[t]][t^{-1}])$ for arbitrary $B(t, x)$ in $M_N(C^\infty(\Omega))$. When all coefficients of $P_1(t, D_t, D_x)$ are real analytic, $N(t)$ and $N(t)^{-1}$ converge and $O(t^\infty)\rho = 0$.*

Theorem 2 *If the dimension of x-space is one and $P_1(t, D_t, D_x)$ has real analytic coefficients, the converse of Theorem 1 holds good.*

We also discuss the case of general ℓ.

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Jump decay problem for Hamilton-Jacobi equation in non-selfsimilar case

An initial condition jump decay problem is considered for the general nonlinear first-order PDE: $F(x, y, u, p, q) = 0$ in the half-plane $y \geqslant 0$, $p = u_x$, $q = u_y$, the Hamilton-Jacobi equation $q = H(x, t, u, p)$ being a particular case (with substitution $y \to t$). This is a fully nonlinear analog of the Riemann problem [1] in terms of the weak waves. The derivative of the initial value function $w(x) = u(x, 0)$ is supposed to have a jump of the first kind at the origin O and the finite left and right limits denoted by p^+ and p^-. Consider the function $q = g(p)$ of the variable p running in the segment $[p^-, p^+]$, generated by the PDE at the origin. For the Hamilton-Jacobi case one has $g(p) \equiv H(0, 0, w(0), p)$. Consider the line L which convexifies the region $q \geqslant g(p)$ of the (p, q)-plane (the $epi\{g(p)\}$). Suppose that L consists of the finite number of straight line segments and the convex arcs of the graph of the function $g(p)$. Suppose that these segments are *simple*, i.e. do not have common points with the graph of $g(p)$ except for their ends. The segments tangent to the graph at the ends will be called *tangent*, those tangent to the graph at only one end are *semitangent*. Consider all the convex subarcs of the line L and put in correspondence to each arc a family of solutions of the characteristic system [2] emanating from the origin with the initial conditions in p ranging between the p-coordinates of an arc's ends. Such a family is called an *integral funnel*. In the (x, y)-plane an *integral funnel* forms a curvilinear finite angle at the origin O. The *right (left) family of the regular characteristics* is called the (x, y)-projection of the set of the solutions of the characteristic system starting at the semiaxis $x > 0$ $(x < 0)$. Left and right families and integral funnels may intersect each other or leave blank curvilinear angles. The following proposition related to the character of the discontinuity lines of the first derivatives of the generalized viscosity solution to the PDE under consideration (i.e. weak *shock waves*) represents the main result of the present paper.

Proposition. The jump decay problem has the unique solution in some neighborhood of the origin. The solution involves exactly M smooth lines of discontinuity of the derivatives (weak shock waves), where M is the number of simple segments arising in the above mentioned convexification. The shock waves can be constructed as the solutions of the certain ODE systems (*singular characteristics* [3]) whose right hand sides are defined by: a) the left and right families of regular characteristics or integral funnels (so-called *equivocal* lines); b) the Hamiltonian only (so-called *focal* lines), or as the solutions of certain algebraic equations defined by the left and right families of regular characteristics or integral funnels (so-called *dispersal* lines). An effective algorithm is developed to determine one of the aforementioned types of the shock waves to which the given shock wave belongs. The algorithm uses the signs of the certain functions of two variables

$A(p', p'')$, $A^e(p', p'')$, for which the explicit expressions are obtained in terms of the Hamiltonian and its derivatives (here p', p'' correspond to the ends of segments). All statements given above for the Hamilton-Jacobi case (for the sake of simplicity) are true for the general equation $F = 0$ with appropriate modification. For the definition of equivocal, focal and dispersal lines and singular (generalized) characteristics see [3].

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Low and high frequency convergence of the spectrum of some perturbed operator

An abstract scheme of investigation of the asymptotic behaviour of eigenvalues and eigenvectors of some family of self-adjoint positive compact operators $\{A_\varepsilon : \varepsilon > 0\}$ acting in a Hilbert space $\mathcal{H}_\varepsilon$ will be presented. This scheme generalizes the procedure of justification of the asymptotic behaviour of eigenvalues and eigenvectors of boundary value problems in thick junctions of different types [1-6]. Such junctions are the union of some domain (junction's body) and a large number of ε-periodically situated thin domains along some manifold on the boundary of junction's body. These problems have specific difficulties in the asymptotic investigation: the absence of extension operators that would be bounded uniformly in ε in the Sobolev space W_2^1, the loss of compactness in the limit passage as $\varepsilon \to 0$, as a result the limiting operator $A_0 : \mathcal{H}_0 \mapsto \mathcal{H}_0$ is noncompact and its spectrum has both the points of the discrete spectrum and the essential one. Our scheme with respect to his ideology is close to the scheme in [8, Sec. III.1], but there exist principal distinctions between these schemes. Some of them are the following ones. Firstly, the limiting operator A_0 in [8] is compact. Secondly, the family of operator $\{A_\varepsilon\}$ is uniformly compact. The facts of this condition for spectral problems in domains depending on a small parameter ε mean that there exists an extension operator in a domain, which is independent of ε, and this operator is bounded uniformly with respect to ε in the corresponding Sobolev norm. The uniformly boundedness of extension operators is the necessary condition in the statement of many problems. Such extension operators exist, for example, for domains that are ε-periodically perforated by holes with diameter of order ε. But for thick junctions, as was mentioned above, there exist no extension operators that are bounded uniformly in ε. We study the low and high frequency convergence of the spectrum of the operator A_ε. The asymptotic estimates of the differences between eigenvalues of A_ε and points of the spectrum $\sigma(A_0)$ (both of the discrete spectrum and essential one) are obtained. The asymptotic estimates for eigenvectors of A_ε are also proved. This scheme can apply to other perturbed spectral problems that satisfy conditions of the scheme. Some results have already published in [7].

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Asymptotics of eigenfunctions for the Schrödinger equation with a shallow potential well

It is well-known that the Schrödinger equation $(-\Delta+U)\psi = E\psi$ in the case when U describes a shallow potential well (i.e., $U = \varepsilon V(x)$, $V(x) \in C_0^\infty(\mathbf{R}^n)$, $\varepsilon \to 0$) has exactly one eigenvalue $E_0 = -\beta^2$, $\beta \in \mathbf{R}$, below the essential spectrum $[0,\infty)$ in the case when $\int_{\mathbf{R}^n} V(x)dx \leqslant 0$ and the dimension n of the configuration space is 1 or 2. This was established for $n = 1$ and in the radially symmetric case for $n = 2$ already in the famous textbook of Landau&Lifshitz [1] and later was demonstrated in the general case in dimension 2 by Simon [2]. The methods used by those authors are quite different and consist, in brief, in the following. Landau&Lifshitz construct the asymptotics of the eigenfunction in the domains where $V \equiv 0$ and $V \not\equiv 0$ separately and then glue then together; thus, the asymptotics of the eigenfunction is nonuniform and the method *per se* is applicable only in the radially symmetric case for $n = 2$. The asymptotics of the eigenvalues is obtained from the gluing conditions. On the other hand, Simon reduces the problem to an equation for the eigenvalues (secular equation) which he solves by means of a Taylor expansion using the implicit function theorem; thus in his approach the asymptotics of the eigenfunction does not appear at all. Moreover, Simon's method is by no means trivial because it uses, for example, the theory of nuclear operators. Our goal here is to construct a uniform asymptotics of the eigenfunction in this situation assuming that $\|\psi\| = O(1)$ as $\varepsilon \to 0$ (the norm is that of $L_2(\mathbf{R})$). It turns out that this construction is completely elementary when one passes to the momentum representation. More exactly, we prove the following theorem. Denote $\tilde{V}(p) = (2\pi)^{-n/2} \int_{\mathbf{R}^n} e^{-ipx} V(x)dx$.

Theorem. *(i) $n = 1$. 1) Let $\int_{\mathbf{R}} V(x)dx < 0$. Then $\psi_0 = \beta_0^{3/2} \int_{\mathbf{R}} e^{ipx} \frac{A_0(p)}{p^2+\beta_0^2} dp$, where $\beta_0 = -\varepsilon\sqrt{\frac{\pi}{2}}\tilde{V}(0)$, $A_0(p) = \tilde{V}(p)$, is the asymptotics of the eigenfunction belonging to the eigenvalue $E = -\beta_0^2(1+O(\varepsilon))$, i.e. $\|\psi-\psi_0\| = o(1)$, $\|\psi_0\| = O(1)$ as $\varepsilon \to 0$; 2) Let $\int_{\mathbf{R}} V(x)dx = 0$. Then $\psi_0 = \beta_0 \int_{\mathbf{R}} e^{ipx} \frac{A_0(p)+\varepsilon A_1(p)p^2+\beta_0^2}{d} p$, where $\beta_0 = \varepsilon^2 \sqrt{\frac{\pi}{2}} \int_{\mathbf{R}} \frac{|V(t)|^2}{t^2} dt$, $A_0(p) = \tilde{V}(p)$, $A_1(p)$ is a function from the Schwartz space whose explicit form we do not give here for lack of space, is the asymptotics of the eigenfunction belonging to the eigenvalue $E = -\beta_0^2(1+O(\varepsilon))$ in the above sense. (ii) $n = 2$. Let $\int_{\mathbf{R}^2} V(x)dx < 0$. Then $\psi_0 = \beta_0 \int_{\mathbf{R}^2} e^{ipx} \frac{A_0(p)}{p^2+\beta_0^2} dp$, where $\beta_0 =$*

$\exp(1/(\varepsilon\tilde{V}(0))$, $A_0(p)=\tilde{V}(p)$, is the asymptotics of the eigenfunction belonging to the eigenvalue $E=-\beta_0^2(c+O(\varepsilon))$, where c is a nonvanishing constant calculated in terms of V, in the above sense.

Remarks 1. A result analogous to (i) 2) is valid also in the case $n=2$, $\int_{\mathbb{R}^2}V(x)dx=0$; we do not give it here for the lack of space. 2. It is possible to construct corrections of any order to the asymptotic eigenfunction ψ_0. In fact, the proof of the theorem consists exactly in an explicit construction of these corrections using the same representation for ψ_0 with A_0 and β_0 changed to the corresponding expansions. The theorem on closeness of formal asymptotics to the exact solution [3] provides the final step of the proof. 3. The (strange at the first sight) presence of the correction εA_1 in the formula for ψ_0 in the case (i) 2) is due to the fact that $A_0(0)=0$, $A_1(0)=1$; and thus the L_2-norm of the "correction" turns out to be even greater than that of the leading term.

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On the spectrum of a positive elliptic operator

Let Ω be a bounded domain in $\mathbb{R}^n$, $n>1$, with a smooth boundary, and let $\mathcal{A}(x,D)$ be a strongly elliptic on $\overline{\Omega}$ formal differential operator of order $2m$ with smooth coefficients. We assume that $\mathcal{A}(x,D)=\mathcal{A}^+(x,D)$ and the minimal operator $A_{\min}>0$ in the Hilbert space $L_2(\Omega)$. Then $D(A_{\min})=\overset{\circ}{H}{}^{2m}(\Omega)$ and the maximal operator $A_{\max}=A^*_{\min}$ acts as the mapping $u\mapsto\mathcal{A}(x,D)u$ on the domain

$$D(A_{\max})=\{\,u\in H^{2m}_{\mathrm{loc}}(\Omega)\cap L_2(\Omega)\ :\ \mathcal{A}(x,D)u\in L_2(\Omega)\,\}.$$

This domain depends on coefficients of $\mathcal{A}(x,D)$ and contains functions with singularities near the boundary. The singularities are such that

$$D(A_{\max})\not\subseteq H^{\epsilon}(\Omega),\qquad \forall\epsilon>0.$$

Let A be a positive self-adjoint extension of the symmetric operator $A_{\min}$. In the general case, the spectrum of A is not discrete. If the operator A is generated by regular boundary conditions, then the spectrum of A is discrete and the corresponding counting function of the eigenvalues satisfies the asymptotic formula

$$N(\lambda, A) = w\lambda^{n/2m} + O(\lambda^{(n-1)/2m}), \qquad \lambda \to \infty, \tag{0.1}$$

with the standard Weyl coefficient $w = w(\mathcal{A}', \Omega) > 0$.
Theorem. *Under the assumption*

$$D(A) \subset H^s(\Omega), \qquad s \in (0, 2m],$$

the following asymptotic formulas hold:

$$N(\lambda, A) = \begin{cases} w\lambda^{n/2m} + O(\lambda^{(n-1)/s}), & s \in (s_0, 2m], \\ \asymp \lambda^{n/2m}, & s = s_0, \\ O(\lambda^{(n-1)/s}), & s \in (0, s_0), \end{cases}$$

where the critical exponent $s_0 = 2m(n-1)/n$.

These formulas are precise in the following sense: for any fixed Ω, $\mathcal{A}(x, D)$, and $s \in (0, s_0) \cup (s_0, 2m)$, we can not replace "$O$" with "$o$". The case $s = 2m$ has been studied by many authors. In this case, the asymptotic formula is precise in a different sense: under some additional assumptions on Ω and $\mathcal{A}(x, D)$, the operator A_D with homogeneous Dirichlet boundary conditions has the second term of the form $v\lambda^{(n-1)/2m}$, $v \neq 0$, in the asymptotic formula for $N(\lambda, A)$.

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Maximal real algebraic hypersurfaces

Consider non-singular real projective hypersurfaces of the same degree and dimension. If we were to consider the complex case they would all be diffeomorphic as smooth manifolds. However, over the real numbers, we have a finite number of distinct diffeomorphism types. Among these types we may consider those which have the highest possible total $\mathbb{Z}_2$-Betti number. It is known that there is a huge number of diffeomorphism types which are maximal in this sense. Nevertheless it turns out that most of them cannot be maximal with respect to the toric structure of the ambient projective $(n+1)$-space or, stating this in another language, with respect to $(n+2)$ distinct hyperplanes. (Recall that the projective n-space minus $(n+1)$ coordinate hyperplanes is the algebraic torus $(\mathbb{R}^*)^{n+1}$.) In the talk I present the uniqueness theorem for diffeomorphism types of torically maximal hypersurfaces of dimension 2 and a partial uniqueness result in higher dimensions. These results are higher-dimensional counterpart of the main theorem of [2]. The main tool of the proof are amoebas of algebraic varieties introduced in [1].

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Analytical construction of homoclinic orbits of two-and three-dimensional dynamical systems

New approach for a construction of homoclinic trajectories (HT) of nonlinear dynamical systems is proposed here. Let a solution under consideration is a function of some parameter (an amplitude or an energy of the system etc.). At small values of the parameter the solution can be presented as power series of the parameter while at large values, as power series of inverse parameter. In order to investigate the solution at arbitrary parameter values, PadeT approximants (PA) are used. Comparing it with reduced local expansions, one has a succession of linear algebraic systems for a determination of the PA coefficients. A neccessary condition for a convergence of the PA succession is the following: normalized determinants of the algebraic systems tend to zero. It is possible to adapt the condition for obtaining some unknown parameter or an initial value. On the closed HT a dynamical system behaves like a conservative one. It is proposed here a potentiality condition along the HT; a corresponding line integral, wich coincides with the Melnikov function, must be equal to zero. The condition was used earlier by author for a construction of closed trajectories in nonlinear n-DOF systems close to conservative ones. HT of the nonlinear Schrodinger equation is considered. The sought solution can be expressed in Taylor series. Substituting the reduced series to the line integral along the HT and integrating, we construct then PA which is an analytical continuation of the local expansion ad infinitum. A condition at infinity gives us an algebraic equation for computing the HT amplitude value. Quasi-Pade approximants (QPA) which contain exponential functions and powers of unknowns, joins together local expansions and describes the closed HT. The potentiality condition, convergence conditions and conditions at infinity give us algebraic equations which permit to obtain initial values of HT of the nonautonomous Duffing equation, just as an amplitude of the external action. There is a good correspondence of the analytical solution and a checking numerical computation. The approach proposed here is more exact than the generally accepted one, because it is not necessary to use here a separatrix trajectory of the autonomous Duffing equation. HT of the Lorenz system is analyzed too. Using the potentiality condition *along the HT with* the help of reorganization to PA, one obtains an algebraic equation for computing an initial value of HT. Condition of the real value existence give us points of two first bifurcations. Local expansions at zero and infinity can be used for a construction of HT in the form of QPA.

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Generalizations of the cahn-hilliard equation

The Cahn-Hilliard equation is very important in materials science. It is a conservation law which describes important qualitative behaviors of two-phase systems, namely the transport of atoms between unit cells. Although this equation is physically sound, it should not, according to several authors, be regarded as basic (for instance, it is not clear how the classical theory should be generalized in the presence of processes such as deformations, see [G]). Our aim is to present a more general theory proposed by M. Gurtin and to discuss the mathematical problems related to the models derived (boundary conditions, existence and uniqueness of solutions, existence of finite dimensional attractors). In particular, we shall consider models that take into account the working of internal microforces and the deformations of the material.

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Priznaki neopredelennosti Jacobievych matric s matrichnymi elementami (jointly with A.G.Kostyuchenko)

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Geometry of Lagrangian manifolds in thermodynamics

It is considered, that the classical thermodynamic properties of substance are defined by relations connecting volume, pressure, temperature, entropy and energy of the given substance. Generally substance is characterized by some number of magnitudes, wh ich half is intensive, and half — by extensive magnitudes. From this point of view pressure and temperature are considered as intensive magnitudes, and volume and entropy - extensive magnitudes. The modern point of view consists that in a condition of a thermodynamic equilibrium the substance should be characterized by a point in the space $R^{2n+1}(p,q,\Phi)$, where coordinates $q = (q_1, q_2, \dots, q_n)$ are intensive magnitudes, first two of which are pressure and temperature ($q_1 = P, \quad q_2 = -T$), and coordinate $p_1, p_2, \dots, p_n$) are extensive coordinates first two from which are volume and entropy ($p_1 = V, \quad p_2 = S$). First four coordinates (P, V, T, S) describe, so to tell, variables of a mechanical nature for homogeneous substances. generally follows to consider heterogeneous (i.e. multicomponent) systems, and also variables not mechanical nature (for example, electromagnetic properties). In any case, the space $R^{2n+1}(p,q,\Phi)$ is supplied by a contact structure, i.e. differential 1-form $\omega = d\Phi - pdq$, and the set of thermodynamic equilibrium states of substance is represented by a submanifold $L \subset R^{2n+1}(p,q,\Phi)$, such that $\omega = 0$. Hence tha projection $L_0 \subset R^{2n}(p,q)$ is a Lagrangian submanifold in symplectic space $R^{2n}(p,q)$ with the symplectic form $\Omega = dp \wedge dq$. Function Φ is function of action on Lagrangian manifold L_0, $d\Phi = pdq$. For classical thermodynamics it coincides with a thermodynamic potential ($\Phi = E + PV - TS$). By Gibbs ([1]) the energy E is a function of variables (V, S), as, however, and all remaining thermodynamic magnitudes. It hence, that Lagrangian manifold L_0 bijectively is projected on a domain in the space $R^2(P, S)$, i.e. the manifold L is defined by the graph of function $E = E(V, S)$. Implicitly Gibbs actually assumed, that the surface $E = E(V, S)$, being noncompact, its any plane of support has by property, that touches a surface in each common point. This condition ensures realization of the following statement: from minimization thermodynamic potential at fixed P and T the positiveness of Hessian of function $E = E(V,S)$, $\text{Hess}_{(V,S)}E(V,S) > 0$ follows. By Maslov ([3]) such condition are called essential. Then in essential condition are fulfilled local thermodynamic inequalities ([2]). Let's consider function

$$\tilde{\Phi}^L(q) = \min_{q(x)=q; x\in L} \Phi(x),$$

under condition of existence of the minimum in question. Consider a symplectic transformation φ of symplectic spaces

$$\varphi : R^{2n}(p,q) \longrightarrow R^{2n}(P,Q)$$

and Lagrangian manifold $\Gamma_\varphi \subset R_{4n}(P,p,Q,q)$, which is the graph of transformations φ. Let S - be function of action on Lagrangian manifold Γ_φ, $dS = PdQ - pdq$. Let's assume, that manifold Γ_φ is uniquely projected on the space $R^{2n}(Q,q)$. Then function S can be understood as function of variables (Q,q), $S = S(Q,q)$. In this case the function S is called generating function of transformation φ. Symplectic transformation naturally extends to contact transformation of contact spaces

$$\tilde{\varphi} : R^{2n+1}(p,q,\Phi) \longrightarrow R^{2n+1}(P,Q,\Phi_1).$$

L e t $L_1 = \tilde{\varphi}(L)$ and $\tilde{\Phi}^{L_1}(Q)$ be defined similar to manifold L_1.

Theorem 1. *At an approaching choice of boundary conditions on manifold L and transformation φ the following formula takes place*

$$\tilde{\Phi}^{L_1}(Q) = \min_q \left(S(Q,q) + \tilde{\Phi}^L(q) \right).$$

Similar, if $\varphi_1, \varphi_2, \ldots, \varphi_n$ is a sequence of symplectic transformations which admit generating functions

$$S_1(Q,q_1), S_2(q_1,q_2), \ldots, S_n(q_{n-1},q_n),$$

and $L_1 = \tilde{\varphi}_1\tilde{\varphi}_2 \cdots \tilde{\varphi}_n(L)$, then

$$\tilde{\Phi}^{L_1}(Q) = \min_{q_1,q_2,\ldots,q_n} \left(S_1(Q,q_1) + S_2(q_1,q_2) + \cdots + S_n(q_{n-1},q_n) + \tilde{\Phi}^L(q_n) \right).$$

The choice of boundary conditions should supply existence of a minimum at an evaluation of $\tilde{\Phi}^{L_1}$. The theorem 1 supplies the map of insignificant points of manifold L into insignificant points of manifold L_1 (compare [3]). the Lagrangian L.

Theorem 2. *Assume that the Lagrangian $L(q,\dot{q})$ satisfies the conditions that the index of inertia of $Hess_{q,\dot{q}}L$ equals to (n,n). Let $S(Q,q,t)$ be the generating* *function of transformation induced by Lagrangian L. Assume that there is a minimum*

$$\tilde{\Phi}^{L_1}(Q) = \min_q \left(S(Q,q) + \tilde{\Phi}^L(q) \right).$$

Then

$$Hess_Q \tilde{\Phi}^{L_1}(Q) < 0.$$

The theorem 2 ensures realization local thermodynamic inequalities in essential points of Lagrange manifold L_1.

Theorem 3. *Let L be a Lagrange manifold which uniquely projected onto p–coordinates. Then at an approaching choice of boundary conditions in essential points the local thermodynamic inequalities are fulfilled, i.e.*

$$Hess_q \Phi^L(q) < 0.$$

As approaching boundary conditions for the theorem 3 the following condition can serve:

Condition 1. *The function $E(p)$, $dE = -qdp$, $E = \Phi^L(p) - pq$ is locally convex upwards in all domain of definition $G(p) \subset R^n(p)$ behind elimination of some compact set $K \subset G(p)$.* The condition 1 is fulfilled for the majority of modelling examples of gases (ideal gas, Van der Waals gas , degenerated Fermi gas). The theorems 1 and 2 allow to construct such Lagrangian manifolds, which are not projected uniquely on p - coordinate, but in all essential points satisfie to local thermodynamic inequalities.

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Spectral gaps of periodic Schroedinger operator and smoothness of its potential

The rate of decay of instability zones (spectral gaps) of Schroedinger operator with periodic potential depends and is well determined by the smoothness of its potential. This relationship was well known for potentials of Sobolev classes (V.A.Marchenko, 1970's) or analytic potentials [E.Trubowitz, 1977]. We went beyond of these classes of potentials. Series of results (joint with T.Kappeler [1, 2] and P.Djakov [3, 4]) to this direction will be presented. As a typical example let us formulate the following statement. Let $V(x) = \sum v_k \exp(2\pi ikx)$ be a periodic potential of Schroedinger operator

$$L = -d^2/dx^2 + V(x)$$

such that

$$\sum |v_k|^2 \exp(2a|k|^\alpha) < \infty, \quad a > 0,\ 0 < \alpha < 1. \tag{0.1}$$

Then the gaps $\delta_n = \lambda_n^+ - \lambda_n^-$, where $\lambda_n^\pm$ are eigenvalues of periodic (or antiperiodic) boundary problem on $[0, 1]$, satisfy a condition

$$\sum |\delta_n|^2 \exp(2a(2n)^\alpha) < \infty. \tag{0.2}$$

Moreover, in the case of real-valued potential (0.2) implies (0.1).

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Inverse scattering for a nonselfadjoint small perturbation of the wave equation

We consider the wave equation of the form

$$w_{tt} + b(x)w_t - \Delta w = 0, \quad (x,t) \in \mathbf{R}^n \times \mathbf{R},$$

where $n \geqslant 3$ and $b(x)$ is a complex valued function decaying sufficiently fast at infinity. In this talk we discuss the following three scattering problems for this equation:

1) To show the existence of the scattering operator,

2) To obtain the expression of the scattering amplitude,

3) To develop the reconstruction procedure of $b(x)$ from the scattering amplitude.

1) is studied in [7] in case $b(x) \geqslant 0$. To obtain 2) we follow the argument of [6]. 3) is studied by Faddeev [2],[3] (cf. also [5]) for the Scrödinger operator. This approach is not directly applicable in our case, and we have to employ another approach based on the works of Faddeev [3], Eskin-Ralston [1] and Isozaki [4].

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О решении некоторых краевых задач для уравнений смешанного типа спектральным методом

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Flat pencils of metrics and integrable reductions of the Lamé equations

We solve the problem of description for nonsingular flat pencils of metrics in the general N-component case. Flat pencils of metrics or, in other words, compatible nondegenerate local Poisson structures of hydrodynamic type (compatible Dubrovin-Novikov structures) play an important role in the theory of integrable systems of hydrodynamic type and in the Dubrovin theory of Frobenius manifolds. In the general form, the problem of description for flat pencils of metrics was initially considered by Dubrovin in [1]. In [2] Dubrovin proved that the theory of Frobenius manifolds (they correspond to two-dimensional topological field theories) is equivalent to the theory of quasihomogeneous flat pencils of metrics. In our work, the nonlinear partial differential equations describing all nonsingular flat pencils of metrics are found and integrated by the inverse scattering method. First, the problem is reduced to integrating a special nonlinear differential reduction of the classical Lamé equations, and then we use the Zakharov method of differential reductions in the dressing method (a version of the inverse scattering method) (see [3], [4]).

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Sandwiched simplices and the topology of the space of explanations

This is joint work with Jim Smith (Statistics, University of Warwick, England) and Duco van Straten (Mathematics, University of Mainz, Germany). We use polyhedral Morse theory to describe the homotopy type of the space of n-dimensional simplices contained in a given convex solid W in n-space and containing a given convex solid V. The question is motivated by statistics: our space of simplices is in fact the space of stochastic factorisations of a given stochastic matrix. Determination of the number of connected components of this space, and of their geometry, is important for the understanding of the convergence of algorithms seeking stochastic factorisations. We use the term "Space of Explanations" since a stochastic factorisation of the (stochastic) matrix of conditional probabilities of random variable X with respect to random variable Y, is given by the existence of a third variable Z with respect to which the variables X and Y are conditionally independent. It is customary to regard Z as an explanation for the correlation between X and Y.

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On the Cauchy Problem for Higher Order Parabolic Equations

In the present work we consider the higher order linear parabolic equation

$$Lu \equiv u_t - \sum_{|k|\leqslant 2m} a_k(t,x) D_x^k u = f(t,x) \qquad (1)$$

in the layer $\Pi_T = [0,T] \times E_n$ with the initial condition

$$u\,|_{t=0} = \varphi(x). \qquad (2)$$

Here $x = (x_{1,\dots,}x_n)$ is a point of the n- dimensional Euclidean space E_n . $t \in [0,T]$, $k = (k_1, \dots, k_n)$, $|k| = k_1 + \dots + k_n$, $k_i \geqslant 0$, $i = 1, \dots n$,

$$u_t = \frac{\partial u}{\partial t}, D_x^k\, u = \frac{\partial^{|k|} u}{\partial x_1^{k_1} \dots \partial x_n^{k_n}}.$$

We establish new a priori estimates for solutions $u\,(t,x)$ to the problem (1), (2) in general anisotropic norms, under the assumption that the coefficients $a_k(t,x)$

and the independent term $f\ (t,x)$are continuous functions in the layer $\Pi_T = [0,T] \times E_n$ and they satisfy the general Hölder condition in $\Pi_T = [0,T] \times E_n$ of exponent $\alpha(l)$, $l > 0$ with respect to the space variables $x = (x_{1,\ldots,}x_n)$ only . In this connection, however, we also obtain an estimate for the modulus of continuity with respect to the time t of the leading derivatives $D_x^k\ u$, $|k| = 2m$. Note that in others works , the a priori estimates of this type have been obtained under the fulfillment of a (general) Hölder condition with respect to the totality of variables (t,x) on the coefficients and the independent term of equation (1). On the basis of our new a priori estimates for the solutions to the problem (1), (2), we establish the corresponding solvability theorems for this problem in general Hölder anisotropic spaces . We assume that the coefficients of Equation (1) satisfy the uniform parabolicity condition: for any non-zero vector

$$\xi = (\xi_{1,\ldots,}\xi_n) \in E_n$$

and

$$(t,x)\ \in\ \Pi_T,$$

$$(-1)^{m+1} \sum_{|k|=2m} a_k(t,x)\xi^k > \lambda\,|\xi|^{2m} \qquad (3).$$

$\lambda = const. > 0\xi^k = \left(\xi_1^{k_1},\ldots,\xi_n^{k_n}\right).$

We apply our results in the linear theory to establish the local solvability with respect to the time t, in general Hölder anisotropic spaces, of the Cauchy problem for the nonlinear parabolic equation,

$$u_t = A(t,x,u,D_xu,\ldots,D_x^{2m}u) \qquad (4)$$

in Π_T with the initial condition (2), where $D_xu = \left(\frac{\partial u}{\partial x_1},\ldots,\frac{\partial u}{\partial x_n}\right)$ and $D_x^r u$ is the set of all derivatives$D_x^k u$ of order $r = |k|, 1 \leqslant r \leqslant 2m$. In the present work, the equation (4) is linearized directly. No conditions are imposed here on the nature of the growth of the nonlinearity of the function $A\left(t,x,p^0,p^1\ldots,p^{2m}\right)$, (See [9]), where p^0-scalar, $p^r = (\ldots,p_k^r,\ldots)$, which is defined for $(t,x) \in \Pi_T$ and any $p^0,p^r,.1 \leqslant r \leqslant 2m$. The main assumption concerning to the function $A\left(t,x,p^0,p^1\ldots,p^{2m}\right)$ is the parabolicity condition: for any non zero vector $\xi = (\xi_{1,\ldots,}\xi_n) \in E_n$ and any $(t,x) \in \Pi_T$ and $p^0,p^1\ldots,p^{2m}$

$$(-1)^{m+1}\left\{\sum_{|k|=2m} A_{p_k^{2m}}\left(t,x,p^0,p^1\ldots,p^{2m}\right)\right\} > 0, \qquad (5)$$

(ξ,η) here and below denotes the usual scalar product in E_n . In all the work we suppose that in the equation (1), the function $f = f_1 + f_2$; the functions $a_k(t,x)$, $|k| = 2m$ and f_1 satisfy the general Hölder condition in Π_T of exponent $\beta(l)$; $l > 0$ with respect to the space variables $x = (x_1,...,x_n)$ only and f_2 satisfies the general Hölder condition in Π_T of exponent $\alpha(l)$, $l > 0$ with respect to the space variables $x = (x_1,...,x_n)$ only. All the coefficients and the independent terms of equation 1 are continuous in the layer Π_T. We require less smoothness conditions from the functions $A(t, x, v^0, v^1, ..., v^{2m})$ and $\omega(x)$ than in others works.

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Virasoro actions on quantum cohomology, following Dubrovin-Zhang and Madsen-Tillmann

Work of Kontsevich and Witten has exhibited a rather mysterious action of the Virasoro algebra on the stable rational cohomology of the moduli space of Riemann surfaces. Recently, I. Madsen and U. Tillmann have shown that the infinite loopspace $Q(CP^{\infty}_{+}$ provides a good model for the integral cohomology of the moduli space. We use their work to construct a Heisenberg group structure on the cohomology of the Mahowald prospectrum $CP^{\infty}_{-\infty}$; the Virasoro representation arises naturally from this, by the Segal-Sugawara construction. More generally, the Virasoro action on the quantum cohomology of algebraic varieties constructed recently by B. Dubrovin and Y. Zhang seems also to arise from this construction.

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Shtrihi k portretu Ivana Georgievicha Petrovskogo

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To a question on existence of the solution of a Tricomi problem for one class of systems of the mixed type equations [21]

We consider a system

$$L_iU \equiv K(y)u_{ixx} + u_{iyy} + A_i(x,y)u_{ix} + B_i(x,y)u_{iy} + \sum_{k=1}^{n} C_{ik}(x,y)u_k = 0, \quad (0.1)$$

where $yK(y) > 0$ for $y \neq 0$, $i = \overline{1,n}$, $n \geqslant 2$, $U = (u_1, u_2, ..., u_n)$ in bounded domain $D \subset R^2$, with boundary consisting for $y > 0$ of Lyapunov curve Γ with endpoints $A(0,0)$ and $B(l,0)$, $l = const > 0$ and for $y < 0$ of characterictics AC and CB of system (0.1). Let $x = x(s)$, $y = y(s)$ – parametric equations of a curve Γ, s - length of an arc curve Γ, counted from a point B, L – length of a curve Γ. We pose analogue of the Tricomi problem for system (0.1) in D. Problem T. Find a function $U(x,y)$ satisfying the conditions:

$$U(x,y) \in C(\overline{D}) \wedge C^1(D) \wedge C^2(D_+ \cup D_-);$$

$$L_iU(x,y) \equiv 0, \ (x,y) \in D_+ \cup D_-, \quad i = \overline{1,n};$$

$$U(x,y)\Big|_{\overline{\Gamma}} = \Phi(s), \ 0 \leqslant s \leqslant L;$$

$$U(x,y)\Big|_{\overline{AC}} = \Psi(x), \ 0 \leqslant x \leqslant l/2,$$

where $\Phi = (\varphi_1, \varphi_2, ..., \varphi_n)$ and $\Psi = (\psi_1, \psi_2, ..., \psi_n)$ – are given, sufficiently smooth vector - functions, $\varphi_i(L) = \psi_i(0)$, $D_+ = D \cap \{y > 0\}$, and $D_- = D \cap \{y < 0\}$ The existence theorem regular or generalized solution of the problem T for system of mixed type equations in many papers are established. This results were obtained under the additional condition that the curve Γ in points A and B ended in small arcs of a "normal" curve. Our theory of a maximum principle for the system allows to remove this limitation concerning a curve Γ. By a regular solution of the problem T for a system (0.1) in the domain D we mean a function $U(x,y)$ satisfying conditions of the problem T. A limit of a sequence of regular solutions of the problem T uniform in $\overline{D}$ is generalized solution of the problem T for a system

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(0.1). In domain D_- we introduce characteristic coordinates (ξ,η). System (0.1) in these coordinates transforms into system

$$\tilde{L}_i U(\xi,\eta) \equiv u_{i\xi\eta} + a_i(\xi,\eta)u_{i\xi} + b_i(\xi,\eta)u_{i\eta} + \sum_{k=1}^{n} c_{ik}(\xi,\eta)u_k = 0, \qquad (0.2)$$

and the domain D_- is mapped in $\Delta = \{(\xi,\eta) : 0 < \xi < \eta < l\}$. Vertex of a triangle Δ designate through $A_0 = (0,0)$, $B_0 = (l,l)$, $C_0 = (0,l)$; $\alpha_i = a_i\beta_i$, $\beta_i = \exp \int b_i d\xi$, $h_i = a_{i\xi} + a_i b_i - c_{ii}$. The coefficients a_i, $a_{i\xi}$, b_i, c_{ik} in $\overline{\Delta}$ except for, maybe, segment $\overline{A_0B_0}$ are assumed continuous and satysfying the condition:

$$\alpha_i(\xi,\eta) - \int_0^{\xi} \beta_i(t,\eta)\left(|h_i(t,\eta)| + \sum_{k\neq i}^{n} |c_{ik}(t,\eta)|\right) dt > 0,\ 0 < \xi < \eta \leqslant l. \qquad (0.3)$$

Theorem 1. *Let: 1) the coefficients of a system (0.1) in domain D_+ are limited and*

$$C_{ii}(x,y) + \sum_{k\neq i}^{n} |C_{ik}(x,y)| \leqslant 0;$$

2) the coefficients of a system (0.2) are smooth and satysfying the condition (0.3); 3) $U(x,y)$ - a generalized solution of system (0.1) in D, equal to zero on characteristic AC. Then, if $\max\limits_{1\leqslant i\leqslant n} \max\limits_{\overline{D}} |u_i(x,y)| > 0$, this maximum is reached only on a curve $\overline{\Gamma}$. We assume, that the coefficients of a system (0.1)

$$K(y) = sgny\,|y|^m,\ m = const > 0,\ C_{ik}(x,y) \in C(\overline{D}_+ \cup \overline{D}_-),$$

$$A_i(x,y),\ B_i(x,y) \in C^1(\overline{D}_+) \wedge C^2(\overline{D}_-)$$

and satisfy the conditions of theorem 1.

Theorem 2. *Let the regular solution of the problem T for a system (0.1) in domain D under the condition that the curve Γ in points A and B ended in small arcs of a "normal" curve are existed. Then if the function $\Phi(s)$ is continuous on Γ and $\Psi(x)$ is sufficiently smooth on AC, $\Psi(0) = \Phi(0) = \Phi(L) = 0$, there is a unique generalized solution $U(x,y)$ of problem T with the boundary data $U = \Phi$ on Γ and $U = \Psi$ on AC at the arbitrary approach curve Γ to an axis $y = 0$, except for a case, when in enough small neighbourhoods of the points A and B dx/ds changes a sign and $dy/ds = 0$.*

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Bounded solutions of dynamic systems

Consider the autonomous system

$$x' = F(x) \tag{0.1}$$

where $F : R^n \mapsto R^n$ - is a smooth vector field. Continuously differentiable on R^n, the function $V(x)$ is called the quiding function for system (1), if $(V'(x), F(x)) > 0$ when $|x| \geqslant \rho_0$, where $V'(x)$ - is a gradient of the function $V(x)$, and ρ_0 - some positive nember. We say, that guiding functions $V_1(x), \ldots, V_m(x)$ derivate a complete set, if $|V_1(x)| + \ldots |V_m(x)| \to \infty$ when $|x| \to \infty$. The guiding function $V(x)$ for system (1) we shall call own, if for any solution $x(t)$ of system (1) from boundedness of a sequence of values $V(x(t_k))$ of the function V the boundedness of a sequence of values $x(t_k)$ of solution $x(t)$ follows.

Theorem 1. *Let $V_1(x)$, ..., $V_m(x)$ is a complete set guiding functions for the systme (1). Then $V(x) = V_1(x)+ \ldots +V_m(x)$ is the own guiding function for the system (1).*

Theorem 2. *Following properties are equivalent: 1) system (1) has no saddle in infinity and its set of bounded on axis of solutions admits a prior estimation; 2) for the system (1) there is a complete set of guiding functions; 3) for the system (1) there is an own guiding function.* Let $f(x)$ - is a smooth function on R^n, which doesn't have critical points outside of some sphere $|x| \leqslant \rho_0$. Let's consider smooth prolongation $g(x)$ of funtcion $f(x)$ into the sphere $|x| \leqslant \rho$, $\rho \geqslant \rho_0$ with nondegenerate critical points (Morse's function). Through $M(g, \rho)$ we shall designate an amount of critical points of the function g in the sphere $|x| \leqslant \rho$, and through $M_*(f, \rho)$ we shall designate a minimum of numbers $M(g, \rho)$, when g transverses the every possible Morse's functions, which are smooth prolongations of the function f into the sphere $|x| \leqslant \rho$. The function $M_*(f, \rho)$ does not increase ρ. The limit $M_*(f)$ of the function $M_*(f, \rho)$ at $\rho \to \infty$ we shall term as a Morse's number of the function f at the infinity.

Theorem 3. *Morse's numbers of any two own quiding functions of the system (1) in infinity coincide and they are equal to zero, if the system (1) has not bounded solutions on an axis.* We shall formulate the important application of the theorem 3. **Theorem 4.** ***Let system (1) has no stationary solutions and Morse's number of the own guiding function of the system (1)in infinity is not equal to zero. Then there is at least one bounded at all axis solution of system (1).*** By Birkgof's theorem in conditions of the theorem 4, system (1) has a recutsion driving.

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Fundamental solutions and Cauchy problem solvability for parabolic differential-difference equations[22]

The following problem is investigated:

$$\frac{\partial u}{\partial t} = \Delta u + \sum_{k=1}^{m} a_k u(x - b_k h, t); \; x \in \mathbf{R}^n, t > 0 \tag{1}$$

$$u\Big|_{t=0} = u_0(x); \; x \in \mathbf{R}^n \tag{2}$$

Here u_0 is continuous and bounded, h is a parameter from $\mathbf{R}^n$, a,b are parameters from $\mathbf{R}^m$.

Theorem. *Let*

$$u(x,t) = \frac{1}{(2\pi)^n} \int_{\mathbf{R}^n} \mathcal{E}_{(n)}(x-\xi,t) u_0(\xi) d\xi, \tag{3}$$

where

$$\mathcal{E}_{(n)}(x,t) = \int_{\mathbf{R}^n} e^{-t\left(|\xi|^2 - \sum\limits_{k=1}^{m} a_k \cos b_k h \cdot \xi\right)} \cos\left(x \cdot \xi - t \sum_{k=1}^{m} a_k \sin b_k h \cdot \xi\right) d\xi. \tag{4}$$

Then (3) satisfies (1) in the classical sense in $\mathbf{R}^n \times \mathbf{R}^1_+$ and $u(x,t) \xrightarrow{t \to +0} u_0(x)$ for any $x \in \mathbf{R}^n$.

The principal steps of the proof are as follows. The fact, that the fundamental solution (4) satisfies (1), is checked by means of the direct substitution. Then we have to prove the convergence of (3). In the one-dimensional case we do that by means Wiener Tauberian Theorem. Then, in case of several spatial variables, we use the invariantness of Laplacian with respect to rotation and expand the integrand of (4) to plane waves. Finally, it is left to prove, that (3) satisfies also (2). For that purpose we fix an arbitrary x_0 from $\mathbf{R}^n$ and consider the integral over $\mathbf{R}^n$, presenting the difference $u(x_0,t) - u_0(x_0,t)$, and estimate it as a singular integral with a non-vanishing kernel, breaking $\mathbf{R}^n$ into two subsets and estimating the corresponding terms by means of two different ways.

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On resonance for systems of wave equations

We prove new resonance results for linearly coupled systems. The main concept is the matrix spectrum which is a natural extension of the standard definition. As an example we consider the system of wave and beam equations with linear coupling and perturbation of monotone type.

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Schrödinger operators with singular Gordon potentials

It is well known that the spectrum of a Schrödinger operator

$$S = -\frac{d^2}{dt^2} + q$$

with periodic locally integrable potential is absolutely continuous. If the potential q is quasiperiodic, then the spectral properties of S are far from being simple. However, Gordon [1] introduced a class of bounded below $L_{2,loc}(\mathbb{R})$-potentials that are well enough approximated by periodic ones and for which the corresponding Schrödinger operators do not possess eigenvalues. Damanik and Stolz [2] enlarged recently this class to $L_{1,loc}(\mathbb{R})$-valued functions and showed that the point spectrum of the Schrödinger operators with generalized Gordon potentials is empty. Here we extend these results to singular potentials $q \in W^{-1}_{2,loc}(\mathbb{R})$ to inlcude point-like interactions of one-dimensional quasicrystal theory. The exact definition of the operators S for such potentials can be found, e.g., in [3]. We observe that the spectrum of Schrödinger operators with periodic singular potentials $q \in W^{-1}_{2,loc}(\mathbb{R})$ is absolutely continuous as in the regular situation [4]. We introduce a class of *singular Gordon potentials* that are well approximated by singular periodic ones and prove the following result.

Theorem. *Suppose that q is a singular Gordon potential. Then the operator S does not have any eigenvalues.* Typical examples of singular Gordon potentials are potentials of one-dimensional quasicrystal theory of the form

$$q(x) = q_1(x) + q_2(\alpha x + \theta),$$

where the $q_1 \in W^{-1}_{2,loc}(\mathbb{R})$ and $q_2 \in L_{1,loc}(\mathbb{R})$ are 1-periodic and α is a Liouville number.

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Педагогическая деятельность И.Г.Петровского

Рассматривая многогранную личность И.Г.Петровского, следует специально остановиться на его преподавательской деятельности, его вкладе в совершенствование математического образования, его педагогических воззрениях. Это вполне естественно, поскольку вся его жизнь была неразрывно связана с самыми разнообразными и разноуровневыми проблемами реализации учебного процесса и подготовки научно-педагогических кадров - в масштабе кафедры, факультета, университета, всей страны. Пройдя в МГУ все ступени - от студента до ректора, он реально понимал нужды студентов и аспирантов, преподавателей и профессоров, был не понаслышке знаком с проблемами руководства педагогическими коллективами. И.Г.Петровский всегда вдумчиво относился к вопросам содержания учебных курсов и их преподавания, постоянно искал нетривиальные пути преподнесения материала на лекциях и семинарских занятиях, особое внимание уделял научному и профессиональному качеству учебников. Всемирную известность заслуженно получили учебники самого И.Г.Петровского. Он обладал особым даром подбирать наиболее перспективные кадры, неустанно добивался неразрывного единства научной и педагогической компонент в деятельности университетских преподавателей. Важнейшее значение он придавал основанию и развитию кафедр и специальностей. связанных с новыми актуальными научными направлениями: здесь ярко проявился широчайший круг его интересов в различных областях знаний. В работе кафедры, руководимой И.Г.Петровским, в разные голы активно участвовали такие блестящие ученые и педагоги, как И.Н.Векуа, С.К.Годунов, И.М.Лифшиц, Е.Ф.Мищенко, О.А.Олейник. Л.С.Понтрягин, С.Л.Соболев, Д.Н.Тихонов, другие хорошо известные специалисты. Усилиями кафедры были созданы современные концепции курсов дифференциальных уравнений - обыкновенных и с частными производными. Особую роль в подготовке научной молодежи сыграли научные семинары И.Г.Петровского, а также его обзорные статьи по актуальным проблемам математики. Большое внимание И.Г.Петровский уделял проолемам обучения юношества, считая ключевым звеном в подготовке кадров отыскание и всемерную поддержку талантливой молодежи, особенно в "глубинке". При его активном конкретном участии были реализованы такие образовательные проекты, как Физико-математическая школа-интернат при МГУ, Заочная математическая школа при МГУ, планомерная работа МГУ по повышению квалификации учителей и неооходимой дополнительной подготовке абитуриентов.

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Глобальная непрерывная разрешимость смешанной задачи для гиперболических систем квазилинейных уравнений

Интерес к вопросу о возможности существования глобальных непрерывных решений гиперболических систем квазилинейных уравнений усилился после работы Лакса [1]. В этой работе Лакс предложил новый подход к попыткам осмысления результатов знаменитого вычислительного эксперимента Ферми - Паста - Улама [2]. После работы Лакса [1] сформировались два направления. Одно из них связано с установлением условий образования разрывов решений (см., например, [3]). Другое направление связано с поиском достаточных условий глобальной непрерывной разрешимости (см., например, [4]). В докладе будет рассказано о полученных нами достаточных условиях глобальной непрерывной разрешимости смешанной задачи следующего общего вида:

$$\partial_t(u_i) + \lambda_i(x,t,u)\partial_x(u_i) = 0, \qquad i = 1,\ldots,m; \quad u = (u_1,\ldots,u_m)$$

$$u(x,0) = \alpha(x), \quad 0 \leqslant x \leqslant \ell, \quad \alpha = (\alpha_1,\ldots,\alpha_m),$$

$$u_i(0,t) = \gamma_i^0(t,u(0,t)), \quad i \in I_+^0 := \{i \mid \mathrm{sgn}\lambda_i(0,0,0) = 1\},$$

$$u_i(\ell,t) = \gamma_i^\ell(t,u(\ell,t)), \quad i \in I_-^\ell := \{i \mid \mathrm{sgn}\lambda_i(\ell,0,0) = -1\},$$

Полученные нами условия содержат некоторые естественные требования монотонности заданных функций, а также требование достаточно быстрого ослабления зависимости правых частей краевых условий от u при $t \to \infty$. Существенность последнего условия нам не ясна.

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On the spectral analysis of infinite selfadjoint Jacobi matrices

A class of unbounded selfadjoint Jacobi matrices is considered. For its spectral analysis we use various asymptotic mthods and Gilbert-Pearson subordinacy theory.Applications to spectral shift transition of 1-st and 2-nd order,quantum optics and Markov processes to be presented.

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Global solutions to the Kolmogorov-Petrovskii-Piskunov equation

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On Some Fundamental Properties of Riemann-Liouville Operaror and Their Application to Partial Differential Equations of Fractional Order

At the present time, when fractal structures arise in many branches of physics, chemistry and biology, it became evident that fractional calculus will be of importance in creating of mathematical models of different processes which proceed in fractal dimension media, on development analysis on fractals, on building correct analogues of ∇-operator $\nabla_x = (\partial/\partial x_1, \partial/\partial x_2, \dots, \partial/\partial x_n)$ – of laplacian Δ_x [1], of Cauchy-Riemann operator and of wave operator. In the report the monograph of author will be presented ([2]), which is devoted to basic elements of fractional calcalus, to essentially new properties of fractional integration differentiation operators and thier application to local and nonlocal partial differential equations of basic types; on the basis of fractional calcalus analogues of indissolublity equation, Fourier's, Fick's and Nernst laws for fractals will be offered, and corresnonding diffusion and transfer equation will be recieved. Important samples of these equations, which may become a basis of nonlocal processes mathematical physics, are: *generalized Laplace equation*

$$\Delta_x^\alpha u \equiv \sum_{k=1}^{n} D_{a_i x_i}^{\alpha_i} u = 0, \quad u = u(x),$$

where $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$ – fixed point in $\mathbb{R}^n$, $x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$, $D_{a_i x_i}^{\alpha_i}$ – fractional differentiation operator $\alpha_i \in]1, 2[$ with beginning at point x_i and with end at point a_i [2]; *generalized (or nonlocal) wave equation*

$$D_{0t}^{\beta} u(x, \eta) = c_\beta^2 \Delta_x^\alpha u(x, t), \; c_\beta = const > 0, \quad t > 0, \quad 1 < \beta < 2;$$

fractal diffusion equation

$$D_{0t}^{\nu} u(x, \eta) = c_\nu^2 \Delta_x^\alpha u(x, t), \; \int_0^1 D_{0t}^{\mu} u(x, \eta)\, d\mu = c^2 \sum_{k=1}^{n} x_k^{1-\alpha_k} \frac{\partial}{\partial x_k} \left(x_k^{\alpha_k - 1} \frac{\partial u(x,t)}{\partial x_k} \right),$$

$$D_{0t}^{\nu} u(x, \eta) = c^2 \sum_{k=1}^{n} x_k^{1-\alpha_k} \frac{\partial}{\partial x_k} \left(x_k^{\alpha_k - 1} \frac{\partial u(x,t)}{\partial x_k} \right),$$

$$\int_0^1 D_{0t}^{\mu} u(x, \eta)\, d\mu = c^2 \Delta_x^\alpha u(x, t),$$

where c, c_ν - constants, $t > 0$, $0 < \nu < 1$; *generalized Tricomi equation*

$$sign\, y \cdot |y|^m \frac{\partial^2 u(x,y)}{\partial x^2} + D_{0y}^{\nu} u(x, \eta) = 0.$$

These equations are loaded differential equations [3].

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Formulas for Witten's solutions of Gelfand-Dikii hierarchy

Among solutions of Gelfand-Dikii (n-KdV) hierarchy there exists a remarkable solution, which is also a solution of the string equation, generates a vacuum vector of W-algebra and has a representation on a form of matrix integral. We call it Witten's solution because according to the Witten conjecture it is the generating function for numbers of intersection of Mumford-Morita- Muller stable cohomological classes of moduli space of n-spin bundles on Riemann surfaces with punctures. In this talk we produce formulas, permiting to find the coefficients of formal power series expanded, which are the Mumford-Morita-Muller numbers if the Witten conjecture is true. Using these formulas we prove also some new special case of the Witten conjecture. a remarkable solution, which is also a solution of the string equation, generates a vacuum vector of W-algebra and has a representation on a form of matrix integral. We call it Witten's solution because according to the Witten conjecture it is the generating function for numbers of intersection of Mumford-Morita- Muller stable cohomological classes of moduli space of n-spin bundles on Riemann surfaces with punctures. In this talk we produce formulas, permiting to find the coefficients of formal power series expanded, which are the Mumford-Morita-Muller numbers if the Witten conjecture is true. Using these formulas we prove also some new special case of the Witten conjecture.

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Quasilinear elliptic Dirichlet problem in domains with smooth edges

Let $n \geqslant m \geqslant 2$. We consider the boundary value problem

$$-a^{ij}(x,u,Du)D_iD_ju + a(x,u,Du) = 0 \quad \text{in} \quad \Omega, \qquad u|_{\partial\Omega} = 0, \qquad (1)$$

where Ω is a domain in $\mathbb{R}^n$ with compact closure $\overline{\Omega}$. It is assumed that the boundary $\partial\Omega$ is smooth (belongs to $W^2_{q,loc}$, $q > n$) with the exception of $(n-m)$-dimensional closed submanifold $\mathcal{M}$ (an "edge" for $m < n$ or a conical point for

$m = n$). In a neighborhood of every point of $\mathcal{M}$ the boundary Ω is locally diffeomorphic to a wedge with edge of codimension m (a cone if $m = n$). Moreover, we suppose that all such wedges are "acute", i.e. are subsets of the half-space. The principal requirements on the coefficients of (1) are: 1. Uniform ellipticity of matrix (a^{ij}):

$$\nu|\xi|^2 \leqslant a^{ij}(x,z,p)\xi_i\xi_j \leqslant \nu^{-1}|\xi|^2, \quad \forall\xi \in \mathbb{R}^n, \quad \nu = \text{const} > 0,$$

2. Quadratic growth of a with respect to the gradient:

$$|a(x,z,p)| \leqslant \mu|p|^2 + b(x)|p| + \Phi_1(x), \qquad \mu = \text{const} > 0,$$

$$b, \Phi_1 \in \mathbb{L}_{r,(\alpha)}(\Omega), \quad \alpha < 1 - n/r, \quad n < r < \infty;$$

(here $\mathbb{L}_{r,(\alpha)}$ is the space L_r with the weight $(dist(x,\mathcal{M})^{\alpha/r})$. Under some natural structure conditions we prove the existence theorem for the problem (1) in Kondrat'ev spaces. A-priori estimates required for this theorem are based on a new variant of Aleksandrov-type maximum principle established in [1]. For details and proof we refer the reader to [2]. This work was partially supported by Russian Fund for Fundamental Research, grant no. 99-01-00684.

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Effect of localization for eigenfunctions in a thin domain near its edge

For the thin cylindrical domain $\Omega_0(\varepsilon) = \omega \times (-\varepsilon/2, \varepsilon/2) \subset \mathbb{R}^3$ with the lateral side $\Gamma_0(\varepsilon) = \partial\omega \times [-\varepsilon/2, \varepsilon/2]$, by separating variables, one can easily calculate the eigenvalues and the eigenfunctions

$$\lambda_j^{(m)}(\varepsilon) = \varepsilon^{-2} j^2 \pi^2 + \Lambda^{(m)},$$

$$u_j^{(m)}(\varepsilon, x) = w^{(m)}(y) \sin\{j\pi(\varepsilon^{-1} z + 1/2)\}, \; j, m = 1, 2, \ldots, \qquad (1)$$

of the mixed boundary value problem

$$-\Delta_x u(\varepsilon,x) = \lambda(\varepsilon)u(\varepsilon,x),\ x=(y,z)\in\Omega(\varepsilon),$$
$$u(\varepsilon,x)=0,\ x\in\Sigma^{\pm}(\varepsilon)=\omega\times\{\pm\varepsilon/2\}, \tag{2}$$
$$\partial_\nu u(\varepsilon,x)=0,\ x\in\Gamma(\varepsilon)=\partial\Omega(\varepsilon)\setminus(\Sigma^+(\varepsilon)\cup\Sigma^-(\varepsilon)).$$

Here $\varepsilon\in(0,1]$ is a small parameter, ∂_ν stands for differentiation along the inward normal and the couple $\{\Lambda^{(m)},w^{(m)}\}$ in (1) is a solution to the two-dimensional spectral problem

$$-\Delta_y w(y)=\Lambda w(y),\ y=(y_1,y_2)\in\omega;\ \partial_n w(y)=0,\ y\in\partial\omega. \tag{3}$$

The simple asymptotic structures (1) of eigenfunctions and eigenfunctions **are not preserved under any perturbation of the lateral side of the thin domain.** Moreover, the procedure reducing dimension can replace the two-dimensional resultant problem (3) by a one-dimensional ordinary differential equation on the contour $\partial\omega$ while eigenfunctions take a form of exponential boundary layer near $\Gamma(\varepsilon)$. Here we demonstrate those localization effects for the Laplacian only but, of course, they are observed for another differential operators as well. In particular, the isotropic plate $\Omega_0(\varepsilon)$ possesses localized elastic eigenmodes when its bases $\Sigma^{\pm}(\varepsilon)$ are clamped and the cylindrical (**unperturbed**) lateral side is free of tractions. Let s denote the arc length on $\partial\omega$ and n the distance from $\partial\omega$; $n>0$ in ω. The perturbed lateral side is given by

$$\Gamma(\varepsilon)=\{x: s\in\partial\omega, |z|<\varepsilon/2, n=-\varepsilon\Upsilon(\varepsilon^{-1}z)\}$$

where $\Upsilon\in C^\infty[-1/2,1/2]$, $\Upsilon(\zeta)>0$ as $|\zeta|<1/2$ and

$$\int_{-1/2}^{1/2}\Upsilon(\zeta)\cos(2\pi\zeta)d\zeta>0,\ \Upsilon\left(\pm\frac{1}{2}\right)=0.$$

The following assertions describe the localization phenomenon for eigenfunctions of the spectral problem (2) in the domain $\Omega(\varepsilon)$ bounded by the surfaces $\Gamma_0(\varepsilon)$ and $\Upsilon^{\pm}(\varepsilon)$: 1. *There exists $\mu\in(0,\pi^2)$ such that the problem*

$$-\Delta_\eta\Phi(\eta)=\mu\Phi(\eta),\ \eta=(\eta_1,\eta_2)\in\Pi,$$

$$\Phi(\eta_1,\pm1/2)=0, \eta_1>0;\ \partial_\nu\Phi(-\Upsilon(\eta_2),\eta_2)=0, |\eta_2|<1/2,$$

possesses an exponentially decaying solution Φ having $\|\Phi;L_2(\Pi)\|$ equal to one; here $\Pi=\{\eta:|\eta_2|<1/2,\eta_1>-\Upsilon(\eta_2)\}$. 2. *If ω is a circle, then solutions to* (1) *have the asymptotic form*

$$\lambda^{(m)}(\varepsilon)=\varepsilon^{-2}\mu-\varepsilon^{-1}b_0k_0-(b_1-m^2)k_0^{-2}+o(1),$$

$$u^{(m)}(\varepsilon, x) = \Phi(\varepsilon^{-1}n, \varepsilon^{-1}z)\exp(\mathrm{im}\, sk_0) + \ldots, m = 0, \pm 1, \ldots,$$

where k_0 is the curvature of the circle, b_1 depends on Υ and Φ,

$$b_0 = -\int_{\Pi} \Phi(\eta)\frac{\partial \Phi}{\partial \eta_1}(\eta)d\eta > 0.$$

3. *Let $k(s) = k(s_0) - K(s - s_0)^2 + O(|s - s_0|^3)$, $K > 0$, i.e., s_0 delivers a local maximum for the curvature k of $\partial\omega$. Then eigenvalues of problem* (2) *take the form*

$$\lambda^{(m)}(\varepsilon) = \varepsilon^{-2}\mu - \varepsilon^{-1}b_0k(s_0) + \varepsilon^{-1/2}(K_0b_0)^{1/2}(1 + 2m) + o(\varepsilon^{-1/2}),$$

$$m = 0, 1, 2, \ldots$$

and the corresponding eigenfunctions have the behavior $O(\exp[-\delta(\varepsilon^{-1}n + \varepsilon^{-1/2}|s - s_0|^2)])$, i.e., they decay exponentially at a distance from s_0. Asymptotic formulae for eigenfunctions are derived along with estimates of their remainders.

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Asymptotic method of differential inequalities and new spectral and geometric variational problems for reaction-diffusion systems.

We consider a spatially inhomogeneous reaction-diffusion equation

$$\begin{cases} \epsilon^2 \dfrac{\partial u}{\partial t} = \epsilon^2 \Delta u - f(u, x, \epsilon) & (x \in \mathcal{D} \subset R^N,\ t > 0), \\ \dfrac{\partial u}{\partial n} = 0 & (x \in \partial\mathcal{D},\ t > 0). \end{cases} \tag{0.1}$$

and investigate the existence of equilibrium internal layer solutions under the following important for applications assumptions.

(A1) *The equation $f(u,x,0)=0$ has exactly three solutions $u=\phi^{(\pm)}(x)$, $\phi^{(0)}(x)$, and*

$$\phi^{(-)}(x)<\phi^{(0)}(x)<\phi^{(+)}(x), \overline{f_u^{\pm}}(x)\equiv f_u(\phi^{(\pm)}(x),x,0)>0 \qquad x\in\overline{\mathcal{D}}$$

and the function $I(x)$, $I(x):=\int_{\phi^{(-)}(x)}^{\phi^{(+)}(x)} f(u,x,0)du$, satisfies $I(x)\equiv 0$ m $\overline{\mathcal{D}}$.

Let us define a function $V_1(x,\Gamma)$ for closed surfaces Γ by $V_1(x,\Gamma)\equiv -\kappa(x,\Gamma)m(x)+J(x;\Gamma)$ where $\kappa(x,\Gamma)$ is the mean curvature of Γ,

$$m(x)=\int_{-\infty}^{\infty}\left(\frac{\partial\tilde{Q}_0(\tau;x)}{\partial\tau}\right)^2 d\tau \qquad x\in\overline{\mathcal{D}},$$

$$J(x;\Gamma)=\int_{-\infty}^{\infty}\left[\tau\left(\nabla_x f(u,x,0)\Big|_{u=\tilde{Q}_0(\tau;x)}\cdot\nu(x;\Gamma)\right)\right.$$
$$\left.+f_\epsilon(\tilde{Q}_0(\tau;x),x,0)\right]\frac{\partial\tilde{Q}_0(\tau;x)}{\partial\tau}d\tau \qquad x\in\Gamma,$$

and $\tilde{Q}_0(\tau;x)$ is the solution for the zero order internal layer equation.

(A2) *There exists a Γ such that $V_1(x,\Gamma)\equiv 0 \qquad x\in\Gamma$.*

We also define an elliptic operator $\mathcal{A}^\Gamma$ by $\mathcal{A}^\Gamma R(x) = \mathrm{div}_\Gamma\left(m(x)\nabla_\Gamma R(x)\right) + G(x)R(x)$. The principal part of this operator is the *Laplace-Beltrami* operator on the surface Γ and $G(x)$ is known function which is determined by the nonlinearity.

(A3) The spectrum $\sigma(\mathcal{A}^\Gamma)$ does not contain 0.

Theorem. *Assume that the conditions $(A1)-(A3)$ are satisfied. Then there exist $\epsilon_0>0$ and a family of equilibrium solutions $u(x,\epsilon)$ of (1.1) such that for each $d_0>0$ fixed*

$$\lim_{\epsilon\to 0}u(x,\epsilon)=\begin{cases}\phi^{(-)}(x) & x\in\mathcal{D}_\Gamma^{(-)}\backslash\Gamma^{(d_0)}\\ \phi^{(+)}(x) & x\in\mathcal{D}_\Gamma^{(+)}\backslash\Gamma^{(d_0)}\end{cases} \tag{0.2}$$

uniformly where $\Gamma^{(d_0)}$ stands for the d_0-neighborhood of Γ. Moreover, if the principal eigenvalue of $\mathcal{A}^\Gamma$ in (A), λ_0 is negative, $u(x,\epsilon)$ is asymptotically stable. If there are a few positive eigenvalues of $\mathcal{A}^\Gamma$, then $u(x,\epsilon)$ is unstable with instability index equal to the number of positive eigenvalues.

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Neiman-zade M.I.
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Shroedinger operators with singular potentials

We consider operator $T+Q$, where T is the Laplace operator in $\mathbb{R}^n$, and Q is multiplication on a distribution q.

It can be shown that this operator is uniquely defined if the potential q belongs to the space of multipliers from Sobolev space H_2^1 to negative Sobolev space H_2^{-1}. Also, we can demonstrate that if a sequence of smooth functions q_n converges to q in the norm of the space multiplier from H_2^1 to H_2^{-1}, then the sequence of operators $T+Q_n$ converges to $T+Q$ in sence of uniform resolvent convergence.

We present several tests and inclusion theorems to verify belonging and convergence in $M[H_2^1 \to H_2^{-1}]$. In addition we give several known and new examples of admissible potentials.

This talk is based on the joint work with professor A.A. Shkalikov.

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Sidorenko V.V
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Treschev D.V.
(Moscow State University)

Stability islands in the domain of passage through a separatrix in systems with fast and slow motions

A Hamilonian system with two degrees of freedom and Hamiltonian function of the form $E = E(p,q,y,x)$ is considered. Here (p,q) and $(y, \varepsilon^{-1}x)$ are pairs of conjugated canonical variables, ε is a small positive parameter. Variables (p,q) are fast ones $(\dot{p}, \dot{q} \sim 1)$, and variables (y,x) are slow ones $(\dot{y}, \dot{x} \sim \varepsilon)$. It is supposed that for all fixed values of slow variables the sub-system for fast variables (so called fast system) possesses a saddle stationary point and two separatix loops passing through this point similarly to the motion in a potential with two minima. Separatrices divide the phase plane of the fast systems into three regions: G_1, G_2 (surrounded by the separatrix loops), and G_3. Under some assumptions it is proved that on the level surface of the Hamiltonian there are many, $\sim 1/\varepsilon$, periodic trajectories with periods of order of $1/\varepsilon$, and for any such trajectory the projection of the phase point onto the plane of fast variables passes periodically from the region G_1 to the region G_3 and back spending in any of these regions time of order of $1/\varepsilon$ (and similarly for passages between regions G_2 and G_3). Every such trajectory is surrounded by a stability island of measure $\sim \varepsilon$. Therefore, the total measure of the stability islands is bounded from below by a value which does not depend on ε. The conditions used are that in the system averaged over fast motions on a given level surface of the Hamiltonian periodic passages between the choosen regions take place, and some general position conditions are satisfied. The case of a Hamiltonian of the form $E(p,q,\varepsilon t)$ was considered in [1], [2].

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Nekhoroshev N.N.
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Generalizations of Gordon's theorem.

Gordon's theorem claim that period of solution of Hamiltonian system, which have only periodical solutions, depend only of value of Hamiltonian function on trajectory of this solution. Generalizations are obtain for case of invariant isotropic tori of arbitrary dimensions k instead $k = 1$, which fiber submanifold $N \subseteq M$ of all phase space M. One suppose that system have some collection $Z = (Z_1, \dots, Z_k)$ of k integrals in involution on N such that the corresponding vectorfields are tangent to this tori $\Lambda \subset N$. Then frequencies $\omega_1, \dots, \omega_k$ of quasiperiodic motion on such torus are depended only of values of these first integrals on this torus. It is true also for the circular action functions, but sufficient conditions are essentially different in this two cases. The formulation of main generalization is following. Theorem. Let additionally the collection $\tilde{Z} = (\tilde{Z}_1, \dots, \tilde{Z}_k)$ of restrictions $Z_i = Z_i|_N$) on N of functions of collection Z define the regular map $\tilde{Z} : N \to \mathbf{R}^k$, that is without critical points. Then frequencies $\omega_1, \dots, \omega_k$ are depend only of $\tilde{Z}$, that is $\omega_i = \omega_i(\tilde{Z})$.

Nesenenko G.A.
(Bauman Moscow State Technical University)

Solving of singularly perturbed nonlinear heat conductivity problems in unsmooth domains by modified "geometrical-optical" asymptotic method

The approximate analitycal method for obtaining the Poincare [1] asymptotic expansions of the solutions to the singularly perturbed nonlinear boundary value problems of a non-stationary thermal conduction in multidimensional domains with angular points is proposed. As is known [2] when the boundary of a multidimensional domain contains angular points, the singular solutions to the corresponding boundary value problem arise. Due to a small parameter at the Laplace operator the modified "geometrical-optical" asymptotic method [3], [4] allows to obtain the approximate solution to the nonlinear boundary value problem in the form of a Poincare asymptotic expansion in powers of both small parameters and corresponding boundary variables including angular boundary variables. The proposed method differs from ones developed by a number of authors for the same purpose, see for instance, [5], in that it is based on a Poincare asymptotic expansion instead of an Erdelyi one [1]. As is known, the coefficients of an Erdelyi asymptotic expansion are some functions of small parameters. An Erdelyi asymptotic expansions has serious disadvantages compared to a Poincare one. As an example, it does not possess uiqueness and causes considerable difficulties in practical calculations [6].

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Novikov I.Ya.
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Asymptotics of Compactly Supported Wavelets

The talk is devoted to investigation of zeros of Bernstein polynomials approximating piecewise linear function. Such polynomials are used in the construction of modified compactly supported wavelets which, in contrast to classical Daubechies wavelets, preserve localization with the growth of smoothness [1]. It is proved that the limiting curve for zeros is the boundary of the convergence region of the Bernstein polynomials on the complex plain. This result is used for investigation of asymptotics of corresponding scaling functions and wavelets.

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Novikov S.Ya.
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Similarity and difference between factorized and weak type operators

Two theorems will be presented, they give the possibility to distinct the factorized through the spaces $L_{p,\infty}$ (weak L_p-spaces) operators from the weak type operators (i.e. bounded from a quasi-Banach space X to the space $L_{p,\infty}$).

Theorem 1. *Let $0 < p < \infty$. For the operator $T : X \mapsto L_0$ with the only condition $|T(\gamma x)| = |\gamma||Tx|$ a.e. the following assertions are equivalent: 1) there exists a function $g \in L_0$, $g > 0$ such that*

$$\left\|\frac{Tx}{g}\right\|_{p,\infty} \leqslant 1, \quad \|x\| \leqslant 1, \quad \text{i.e. the operator } T = M_g \cdot T_1,$$

where the operator $T_1 : X \mapsto L_{p,\infty}$, $M_g y = g y$. 2) For any $\varepsilon > 0$ there exists $C_\varepsilon > 0$ such that for each finite sequence (x_i) from X we have

$$\left(\sup_i |Tx_i|\right)^* (\varepsilon) \leqslant C_\varepsilon \left(\sum \|x_i\|^p\right)^{1/p},$$

where x^* denotes the decreasing rearrangement of x. Two theorems of E. Nikishin [1] are the corollaries of theorem 1.

Theorem 2. *In notations of theorem 1 the following assertions are equivalent: 1) The function $g \in L_\infty$, i.e. the operaror $T : X \mapsto L_{p,\infty}$. 2) There exists $q \in (0, p]$ such that for any sequence $(x_i) \subset X$ with $(\|x_i\|) \in l_{p,q}$ and for any sequence of independent (in probabilistic sense) functions Y_i, equimeasurable with Tx_i, we have* $\sup_i |Y_i| < \infty$ a.e. *3) There exists $C > 0$ such that for each finite sequence (x_i) from X we have*

$$\left(\sup_i |Tx_i|\right)^* (t) \leqslant \frac{C}{t^{1/p}} \left(\sum \|x_i\|^p\right)^{1/p}, \quad t \in (0,1].$$

If a quasi-Banach space X is shift invariant, and the operator T is superlinear [1] and shift invariant, then the difference, marked in theorems 1, 2 disappears and all the five assertions are equivalent.

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Novokshenov V.Yu.

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Weak Dispersion Management in NLS Model. Inverse Scattering Approach

A simple recursion formula is derived for the amplitude and phase of the optical pulse propagating over a DM fibre with zero mean dispersion. Under the assumption of zero dissipation and constant dispersion along the adjacent legs of the waveguide, the integrable NLS models can be applied within each leg: $iu_t \pm u_{xx} + 2|u|^2u = 0, \quad u = u^\pm(t,x)$. The pulse is continuous through the interfaces of adjacent legs $u^-(T,x) = u^+(-T,x)$, where $2T$ is the length of each leg. Choosing the legs to be long enough ($T \gg 1$) to ensure the formation of a self-similar profile one can use the well-known asymptotic formulas for the non-soliton initial pulses. Matching them through the subsequent legs we get the recursion formulas for the pulse amplitude and chirp (phase modulation). The analytical results are well justified by numerical simulations (see [1]).

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A theory of interpolation of anisotropic spaces

Let A_1 be a Banach space, A_2 a Banach lattice. Let us designate by $A = (A_1, A_2)$ the space of measurable functions f with values in A_1 and such that $\|f(x)\|_{A_1} \in A_2$. The norm in A is defined by $\|f\| = \|\|f(x)\|_{A_1}\|_{A_2}$. The definition of the space $\mathbf{A} = (A_1, \dots, A_n)$ is inductive. We shall call it anisotropic space of dimension n.

Set $E = \{\varepsilon = (\varepsilon_1, ..., \varepsilon_n) : \varepsilon_j = 0 \text{ or } \varepsilon_j = 1, \ \ j = 1, ..., n\}$. Given any two anisotropic spaces $\mathbf{A_0} = (A_1^0, ..., A_n^0)$, $\mathbf{A_1} = (A_1^1, ..., A_n^1)$, then for every $\varepsilon \in E$ we define the space $\mathbf{A}_\varepsilon = (A_1^{\varepsilon_1}, ..., A_n^{\varepsilon_n})$ with the norm

$$\|a\|_{\mathbf{A}_\varepsilon} = \|...\|a\|_{A_1^{\varepsilon_1}} ...\|_{A_n^{\varepsilon_n}}.$$

The pair of anisotropic spaces $\mathbf{A_0} = (A_1^0, ..., A_n^0)$, $\mathbf{A_1} = (A_1^1, ..., A_n^1)$ is said to be consistent, if there exists a linear Hausdorff space which contains the spaces $\mathbf{A}_\varepsilon, \ \ \varepsilon \in E$.

Let $* = (j_1, ..., j_n)$ be a fixed permutation of the sequence $(1, 2, ..., n)$. For any $\varepsilon = (\varepsilon_1, ..., \varepsilon_n) \in E$ we denote by $\varepsilon^* = (\varepsilon_{j_1}, ..., \varepsilon_{j_n}) \in E$. Define the K^* - functional :

$$K^*(t, a; \mathbf{A_0}, \mathbf{A_1}) = \inf\{\sum_{\varepsilon \in E} t^\varepsilon \|a_{\varepsilon^*}\|_{\mathbf{A}_{\varepsilon^*}} \ : \ a = \sum_{\varepsilon \in E} a_\varepsilon, \ a_\varepsilon \in \mathbf{A}_\varepsilon\},$$

here $t^\varepsilon = t_1^{\varepsilon_1} \dots t_n^{\varepsilon_n}$. Let $0 < \theta = (\theta_1, \dots, \theta_n) < 1, \ \ 0 < \mathbf{q} = (\mathrm{q}_1, \dots, \mathrm{q}_n) \leqslant \infty$. Denote by $\mathbf{A}_{\theta\mathbf{q}}^* = (\mathbf{A_0}, \mathbf{A_1})_{\theta\mathbf{q}}^*$ the linear subset of $\sum_{\varepsilon \in E} \mathbf{A}_\varepsilon$, for which:

$$\|a\|_{A_{\theta\mathbf{q}}^*} = \Phi_{\theta\mathbf{q}}(K^*(t, a)) =$$

$$= \left(\int_0^\infty \dots \left(\int_0^\infty (t_1^{-\theta_1} \dots t_n^{-\theta_n} (K^*(t, a))^{q_1} \frac{dt_1}{t_1}\right)^{\frac{q_2}{q_1}} \dots \frac{dt_n}{t_n}\right)^{\frac{1}{q_n}} < \infty,$$

, here $\left(\int_0^\infty (G(t))^q \frac{dt}{t}\right)^{1/q} = \sup_{t>0} G(t)$, if $q = \infty$. Let $\mathbf{m} = (m_1, ..., m_n)$, $* = (j_1, ..., j_n)$ be a fixed permutation of the sequence $\{1, 2, \dots, n\}$. For any measurable function $f(x_1, ..., x_n)$ on $\mathbb{R}^\mathbf{m} = \mathbb{R}^{m_1} \times ... \times \mathbb{R}^{m_n}$ we denote by $f^*(t) = f^{*j_1, \dots, *j_n}(t_1, ..., t_n)$ the function, obtained by applying decreasing rearrangement, consecutively with respect to the variables $x_{j_1} \in \mathbb{R}^{m_{j_1}}, ..., x_{j_n} \in \mathbb{R}^{m_{j_n}}$, considering the other variables fixed. The space $L_{\mathbf{pq}}^*(\mathbb{R}^\mathbf{m})$ is defined as the set of all functions for which

$$\|f\|_{L_{\mathbf{pq}}^*(\mathbb{R}^\mathbf{m})} = \Phi_{\theta\mathbf{q}}(t f^*(t)) < \infty,$$

here $\theta = 1 - 1/\mathbf{p}$.
Theorem. *Let* $1 \leqslant \mathbf{p}_i = (p_1^i, \dots, p_n^i), \sigma_i = (\sigma_1^i, \dots, \sigma_n^i) \leqslant \infty,\ i = 0, 1, \mathbf{p}_0 \neq \mathbf{p}_1$, $0 < \mathbf{q} = (q_1, \dots, q_n) \leqslant \infty$, $0 < \theta = (\theta_0, \dots, \theta_n) < 1$, $1/\mathbf{p} = (1-\theta)/\mathbf{p}_0 + \theta/\mathbf{p}_1$, $* = (j_1, \dots, j_n)$ *- be a certain permutation of the sequence* $\{1, 2, \dots, n\}$, $\mathbf{A}^*_{\mathbf{p}_0\sigma_0} = \left(L_{p^0_{j_1}\sigma^0_{j_1}}, \dots, L_{p^0_{j_n}\sigma^0_{j_n}}\right)$, $\mathbf{A}^*_{\mathbf{p}_1\sigma_1} = \left(L_{p^1_{j_1}\sigma^1_{j_1}}, \dots, L_{p^1_{j_n}\sigma^1_{j_n}}\right)$, *then*

$$L^*_{\mathbf{pq}} = (\mathbf{A}^*_{\mathbf{p}_0\sigma_0}, \mathbf{A}^*_{\mathbf{p}_1\sigma_1})^*_{\theta\mathbf{q}}, \quad L^*_{\mathbf{pq}} = (L^*_{\mathbf{p}_0\infty}, L^*_{\mathbf{p}_1\infty})^*_{\theta\mathbf{q}}$$

Nobuaki Obata
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Quadratic Quantum White Noises and the Lévy Laplacian

On the basis of *white noise theory* we shall discuss quantum noises which are singular from a usual aspect of quantum stochastic calculus and an unexpected relation with the Lévy Laplacian known for its peculiar properties. The white noise theory is based on a particular Gelfand triple:

$$\mathcal{W} \subset \Gamma(L^2(\mathbf{R})) \cong L^2(E^*, \mu) \subset \mathcal{W}^*,$$

where the middle is the space of L^2-functions on a Gaussian space (E^*, μ) which is isomorphic to the Boson Fock space over $L^2(\mathbf{R})$ due to the Wiener–Itô–Segal theorem. The annihilation and creation operators at a time point t are denoted by a_t and a_t^*, respectively. These are also called the *quantum white noise.* It is known that $a_t \in \mathcal{L}(\mathcal{W}, \mathcal{W})$ and $a_t^* \in \mathcal{L}(\mathcal{W}^*, \mathcal{W}^*)$. Thus, the Brownian motion and the white noise process (time-derivative of Brownian motion) are respectively represented as

$$B_t = \int_0^t (a_s + a_s^*)\, ds, \qquad W_t = a_t + a_t^*. \tag{0.1}$$

In general, a continuous operator $\Xi \in \mathcal{L}(\mathcal{W}, \mathcal{W}^*)$ is called a white noise operator. With the help of the general theory for white noise operators [3] many concepts in quantum Itô theory can be generalized. In particular, a quantum stochastic differential equation of Hudson–Parthasarathy type:

$$dX = (L_1 dA_t + L_2 dA_t^* + L_3 d\Lambda_t + L_4 dt)\, X, \qquad X(0) = I,$$

is brought into an ordinary differential equation for white noise operators:

$$\frac{dX}{dt} = (L_1 a_t + L_2 a_t^* + L_3 a_t^* a_t + L_4) \diamond X, \qquad X(0) = I,$$

where $\diamond$ stands for the Wick product (normal-ordered product). ¿From a mathematical aspect it is more natural to consider a linear differential equation for white noise operators of the form:

$$\frac{d}{dt}\Xi = L_t \diamond \Xi, \qquad \Xi(0) = I, \tag{0.2}$$

where $\{L_t\}$ is a quantum stochastic process, i.e., $t \mapsto L_t \in \mathcal{L}(\mathcal{W}, \mathcal{W}^*)$ is a continuous map defined on an interval. An equation of the above form is called a *normal-ordered white noise differential equation.* A unique existence of a solution to (0.2) is proved in terms of weighted Fock space [2,4]. The normal-ordered white noise differential equations cover a wide class of quantum stochastic differential equations with very singular driving noises that are not reached by the traditional Itô theory. For example, the simple case of $L_t = a_t^2 + a_t^{*2}$ is already non-trivial; moreover, the solution $\{\Xi_t\}$ is unexpectedly related to the Lévy Laplacian. In fact, the solution to a heat (or Schrödinger) type equation associated with the Lévy Laplacian:

$$\frac{\partial}{\partial t}\Psi_t = \frac{\alpha}{2}\Delta_L \Psi_t, \qquad t \in \mathbf{R},$$

where $\alpha \in \mathbf{C}$ is a constant, is obtained by the Laplace transform of the classical stochastic process $\{\Phi_t\}$ corresponding to $\{\Xi_t\}$ in such a way that $\Phi_t = \Xi\phi_0$, where ϕ_0 is the vacuum vector. Further properties are examined.

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Modulated periodic waves appearing in a perturbed KdV or a convection problem

A family of periodic travelling wave solutions parameterized by the wavenumber is shown to bifurcate from the trivial solution in a perturbed KdV equation. Studying linearized eigenvalue problem about each periodic travelling wave solution, all of them are shown to be unstable immediately after the bifurcation in contrast to the Eckhaus stability/instability. Analysis from a dynamical viewpoint suggests that "modulated periodic waves" are obtained by a secondary bifurcation from periodic travelling waves as a super critical Hopf bifurcation. The normal form approach will be introduded to understand the dynamics about the unstable periodic travelling wave solution.

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Ohya M.
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NP-complete problems with chaos dynamics

In theory of computational complexity, there is a famous unsolved problem whether NP-complete problem can be P problem. In [1], we discussed this in SAT (satisfiability) problem, and it is shown that the SAT problem can be solved in polynomial time by means of quantum algorithm if the superposition of two orthogonal vectors is physically detected. However such a physical detection is not easy to realize. In the case that one of the amplitudes for two vectors is very small so that its measured value can not be distinguished from 0, we have to amplify the small value so as to distinguish from zero. For this purpose, we proposed in [2] quantum chaos algorithm combining usual quantum algorithm with chaotic dynamics amplifier based on the logistic map [3], by which we showed that the SAT problem (so all NP-complete problems) is solved in polynomial time.

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Oinarov R. O.

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Postanovka kraevyh zadach dlya vyrojdennyh differencialnyh operatorov

Olshanski M.A.

(Moscow State University)

Stabilized Galerkin method for the steady incompressible Navier-Stokes equations and an iterative solver

The talk concerns with the accuracy of a finite element (FE) method for the incompressible Navier-Stokes equations and numerical performance of linear algebra solvers for discrete systems resulting from a FE approximations. Apart from the standard (SUPG type) stabilization another consistent stabilization method is studied numerically and understood from the theoretical point of view. The method is intend to reduce the loss of accuracy, which the FE solution may suffer for small viscosity values due to the presence of the first order pressure term. To solve non-linear problem we apply quasi-Newton iterations for the coupled system. Every non-linear iteration requires a linearized problem to be solved. Therefore we use a Krylov subspace iterative method to solve linearized Navier-Stokes problem (Oseen type problem). The latter has non-symmetric and non-definite matrix of coefficients. However the saddle-point structure of the matrix admits a specially designed preconditioning. Two elements of the preconditioning are of major importance. They are multigrid method for velocity problem and a preconditioner for the Schur complement (pressure operator) of the system. The design of both elements (but especially of the second one) is a non-trivial task and a topic of on-going research in several research groups. We will present our results in this direction. Additionally we will see that the stabilization mentioned above is important also for the advanced performance of the linear solver. One more point of interest is the rotation form of Navier-Stokes equations. We use it as a predictor with a more 'traditional' convection form and on its own right. It appears to be of much help both as a starting point for a FE approximation to the Navier-Stokes system and as a convenient form for a numerical analysis of iterative solvers. The theory is complimented with numerical experiments with two type of problems. One problem has analytical smooth solution, another one is the driven-cavity problem for moderate and high Reynolds numbers.

Olver P.
(School of Mathematics, University of Minnesota)

Moving Frames – in geometry, the calculus of variations, computer vision and numerical analysis

In this talk, I will describe a new approach to the powerful Cartan theory of moving frames. The method is completely algorithmic, and applies to very general Lie group actions. I hope to discuss a wide variety of new applications, including classical geometry, classification and syzygies of differential invariants, invariant variational problems and their Euler-Lagrange equations, object recognition in computer vision, and the design of symmetry-preserving numerical approximations.

Omel'yanov G.A.
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Dynamics and interactions of nonlinear waves: multidimensional case

This paper deals with the problem of describing propagation and interaction of singularities of solutions to nonlinear equations in the multidimensional case $(x, t) \in \mathbf{R}^{n+1}$, $n \geqslant 1$. The main result, the model equation describing the evolution of singularity support, can also be treated as the result of averaging in nonlinear nonintegrable models. Our basic approach is the weak asymptotics method; the main examples are the phase field system and the Kadomtsev–Petviashvili type equation. If the dimension increases, the problem becomes significantly more meaningful and new interesting effects appear. One of the main features that are new as compared with the one-dimensional case ($n = 1$) is the appearance of geometric effects, namely, the dynamics of the singularity support Γ_t is closely related to its geometry. As an example, consider the KP type equation

$$\{u_t + f(u)_x + \varepsilon^2 u_{xxx}\}_x + \sigma u_{yy} = 0, \qquad \varepsilon \ll 1, \tag{1}$$

for some $f(z) \in C^\infty$, $f(0) = 0$. The weak asymptotic $\bmod O_{\mathcal{D}'(R^2)}(\varepsilon^2)$ solution of Eq. (1) has the form

$$\begin{aligned} u &= A\omega_0\Big(\beta\frac{x-\psi}{\varepsilon}, A\Big) + \varepsilon u^- \omega_2\Big(\beta\frac{x-\psi}{\varepsilon}, A\Big) \\ &= \varepsilon a\frac{A}{\beta}\delta(x-\psi) + \varepsilon u^-(1 - H(x-\psi)) + O_{\mathcal{D}'}(\varepsilon^2), \end{aligned}$$

where $H(z)$ is the Heaviside function and δ is the Dirac δ-function. It was proved that the amplitude $A = A(y,t)$ and singularity support $\Gamma_t = \{(x,y), x = \psi(y,t)\}$ can be uniquely determined by the solution of equations of gas dynamics

$$\rho_t + (\rho w)_y = 0, \qquad \rho(w_t + w w_y) + p_y = 0,$$

$a = a(A)$, $\beta = \beta(A)$, and u^- can be found from the linear equation in $\{(x,y,t), x < \psi(y,t), t > 0\}$ with conditions on Γ_t, $t > 0$ and for $t = 0$. In the special case of $f(z) = z^m$, $m \geqslant 2$, $m \neq 5$ we have $\beta^2 = \lambda^2 A^{m-1}$, $a = 1$, $p = \kappa_0 \rho^\kappa$, $\kappa = (m+3)/(5-m)$, $\rho = \rho_0 A^{(5-m)/2}$, $w = -2\sigma\psi_y$, and $\lambda, \kappa_0, \rho_0$ are some constants. Another fact that is no less important is that for $n > 1$ there may exist singularities with $\operatorname{codim}\Gamma_t > 1$ along with the traditional case $\operatorname{codim}\Gamma_t = 1$, which is the only possible for $n = 1$.

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On the deficiency indices of ordinary higher–order symmetric differential operator degenerating inside interval

Let H be the minimal operator in $L^2(I)$ corresponding to real differential expression $s[f] = (-1)^n(c(x)f^{(n)})^{(n)}$, $n \geqslant 2$, defined on some interval $I \subseteq \mathbb{R}$. Suppose $c(x)$ vanishes in a finite or countable set of interior points of I. Then the meaning of the defociency indices $\operatorname{Def} H$ of operator H may be any nonnegative integer and even $+\infty$. We present results concerning the dependence of $\operatorname{Def} H$ on the orders of zero's of coefficient $c(x)$ of $s[f]$.

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Orudzhev E.G.

On a kernel of resolvent of a special class of bundles of differential operators.

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Let's consider in the space $L_2(-\infty,\infty)$ a bundle of the differential operators L_λ generated by the differential expression

$$L_\lambda(Y) \equiv y^{(2n)}(x) + \sum_{l=1}^{2n}\left(\sum_{k=0}^{l}(p_{lk}(x)+q_{lk}(x))\lambda^k\right)y^{(2n-1)} \tag{1}$$

where p_{ll}, q_{ll} are real numbers, and the functions $p_{lk}(x)$, $k<l$ contain only positive exponents of Fourier series

$$p_{lk}(x) = \sum_{m=1}^{\infty} p_{lkm}e^{imx}, \ \ l=\overline{1,2n},\ k=\overline{0,l-1}$$

and it is supposed that the series $\sum\limits_{l=1}^{2n-1}\sum\limits_{m=1}^{\infty} m^l\,|p_{lkm}|$ converge. Relative to introduced in $p_{lk}(x)$ perturbed functions $q_{lk}(x)$, $k<l$ we assume that $|q_{lk}(x)|e^{\varepsilon|x|}<c_{lk}$, $\varepsilon>0$, c_{lk} are constant numbers. We also assume that the roots θ_ν of the equation $F(\theta)=\sum\limits_{i=0}^{2n}(p_{ii}+q_{ii})\,\theta^{2n-i}$, $p_{00}+q_{00}=1$, are various and different from zero and such that the plane λ is divided into the sectors R_k, $k\leqslant 4n$, in each of which for suitable numaration of these roots, the Tamarkin type inequalities are satisfied. In work [1] meromorphic by λ solutions of the equation $L_\lambda(Y)=0$ is constructed, the structure of resolvent is investigated, it is shown that the spectrum of the bundle L^0_λ (corresponding to the case $q_{lk}(x)=0$, $l=\overline{1,2n}$ in expression (1)) consists of some rays for which infinitly many spectral singularities are located. Here the resolvent R_λ of the bundle L_λ is studied. By using the meromorphic Fredholm theorem, it is obtained that the kernel of the resolvent R_λ is the solution of the integro-differential equation

$$K(x,\xi,\lambda) + \int_{-\infty}^{\infty} K_0(x,\xi,\lambda)\sum_{l=1}^{2n}\left(\sum_{k=0}^{l-1}q_{lk}(t)\lambda^k\right)\frac{\partial^{2n-l}}{\partial t^{2n-l}}K(t,\xi,\lambda)dt = K_0(x,\xi,\lambda)$$

belonging to the special chosen Banach space with poles at points of discrete set. In addition, at every sector R_k in a small neighborhood of any sufficiently distant solutions λ^0_{mk} of the equation $\sum\limits_{\nu=0}^{2n-1} \frac{(im)^\nu}{(\nu+1)!} F^{(\nu+1)}(\theta_\nu)\lambda^{2n-1-\nu} = 0,\ m = 1,2,3,\dots,$ the kernel of the resolvent R_λ has poles different from λ^0_{mk} on $o(1)$. The poles of a kernel lying on the boundary R_k will be a spectral singularity of the bundle L_λ.

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On some properties of the infinite band matrices with operator elements

We consider the band matrix $A = (A_{i,k})_{i,k=0}^{\infty}$ the elements of which are bounded operators in a separable Hilbert space H:

$$A_{i,k} \in L(H), \quad A_{i,k} = O \text{ (zero operator)}, \quad k > i + r, \quad i > k + q,$$
$$A_{i,i+r},\ A_{i+q,i} \text{ invertible}, \quad i \geqslant 0. \tag{0.1}$$

If $sup_{i,k}\|A_{i,k}\| \leqslant C < \infty$ then matrix A generates the bounded operator in $\mathbf{l}^2 = L(H, l^2(H;[0,\infty)))$ with the pseudoscalar product $[U,V] = U^*V,\ U,V \in \mathbf{l}^2$ [1] .The following analogue of the Stone theorem gives the characterization of operators in $\mathbf{l}^2$ which admit a band matrix representation (band operators).

Theorem 1 *A bounded operator in $\mathbf{l}^2$ has in some pseudoorthonormal basis the matrix representation (1) iff there exist pseudovectors $V^0,\dots,V^{q-1};\ \ U^0,\dots,U^{r-1}$ such that the system $\{V^k\}_{k=0}^{\infty}$ where $V^k = A^sV^j$, for $k = qs + j,\ s \geqslant 0,\ j = mod(k,q)$ is dense in $\mathbf{l}^2$, admits a pseudoorthogonalization and for any $n \geqslant 0$*

$$span\{V^0,\dots,V^n\} = span\{U^0,\dots,U^n\};$$

where $U^k = (A^*)^sU^i,\ k = rs + i,\ i = mod(k,r)$ and operator A^* is adjoint to A with respect to the pseudoscalar product $[\ ,]$. For the band operators in $\mathbf{l}^2$ the following criterion of resolvent set is valid:

Theorem 2 *Let operator A in $\mathbf{l}^2$ admits a matrix representation (1) such that $sup_{i,j;i\neq j}\|A_{i,j}\| \leqslant C < \infty$. Then an arbitrary complex λ belongs to the resolvent set of A iff there are constants $C > 0$, $0 < q < 1$ and matrix*

$\mathcal{M} = \{\mathcal{M}_{i,j}(\lambda)\}_{i=1,\dots,r}^{j=1,\dots,q}$, $\mathcal{M}_{i,j}(\lambda) \in L(H)$ *such that*

$$\|\sum_{i=1}^{r} Q_m^i(\lambda)\Phi_n^{i,+}(\lambda)\| \leqslant Cq^{n-m}, \quad 0 \leqslant m \leqslant n;$$

$$\|\sum_{j=1}^{q} \Phi_m^j(\lambda)Q_n^{j,+}(\lambda)\| \leqslant Cq^{m-n}, \quad 0 \leqslant n \leqslant m, \quad m,n \geqslant 0,$$

where $\{\Phi_n^j(\lambda) = Q_n^i(\lambda)\mathcal{M} - P_n^j(\lambda)\}_{n=-q}^{\infty}$, $\{\Phi_n^{i,+}(\lambda) = \mathcal{M}Q_n^{j,+}(\lambda) - P_n^{i,+}(\lambda)\}_{n=-r}^{\infty}$, *and* $\{Q_n^i(\lambda)\}_{n=-q}^{\infty}$, $\{P_n^j(\lambda)\}_{n=-q}^{\infty}$; $\{Q_n^{j,+}(\lambda)\}_{n=-r}^{\infty}$, $\{P_n^{i,+}(\lambda)\}_{n=-r}^{\infty}$ *are the fundamental systems of polynomial solutions of the difference equations*

$$A_{n,n-q}Y_{n-q} + \dots + A_{n,n+r}Y_{n+r} = \lambda Y_n,$$
$$Y_{n-r}^{+}A_{n-r,n} + \dots + Y_{n+q}^{+}A_{n+q,n} = \lambda Y_n^{+}, \quad n \geqslant 0.$$

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Гладкость решения обобщенной системы Коши-Римана в неограниченной области

Работа посвящена исследованию неоднородной обобщенной системы Коши-Римана

$$Lw = \partial_{\bar z}w + A(z)w + B(z)\bar w = F(z), \quad z \in E, \tag{1}$$

где E– комплексная плоскость,$\partial_{\bar z} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)$. (1) представляет собой комплексную запись системы двух вещественных уравнений в частных производных

$$\begin{cases} u_x - v_y + a(x,y)u + b(x,y)v = f(x,y) \\ u_y + v_x + c(x,y)u + d(x,y)v = g(x,y), \end{cases}$$

которая играет важную роль в теории двумерных систем уравнений первого порядка. Отмечены применения системы (1) в гидродинамике, теории поверхностей и оболочек. Когда

$$A(z), B(z), |z|^{-2}A\left(\frac{1}{z}\right), |z|^{-2}B\left(\frac{1}{z}\right) \in L_p(E_1)\ (p > 2), E_1 = \{z \in E : |z| \leqslant 1\} -$$

единичный круг, она систематически изучалась в работах И.Н.Векуа и его последователей. Относительно мало исследован случай, когда область задания бесконечна, а коэффициенты $A(z), B(z)$ необязательно суммируемы. Тогда для однородной системы, соответствующей (1) может не иметь места известная теорема Лиувилля, более того ядро оператора L может оказаться бесконечномерным. Например, системе $\partial_{\bar{z}} w + 2z(1+|z|^2)w = 0$ удовлетворяют функций $w_n(z) = z^n \exp[-(1+|z|^2)^2]$ $(n = 1, 2, \dots)$. Для системы (1), когда $B(z)$– гладкая ограниченная функция, $A(z) \equiv 0$ В.С.Виноградовым найдено условие единственности решения, а для ограниченных и гельдеровых $A(z), B(z)$, удовлетворяющих условию $\lim\limits_{z\to\infty} \max\limits_{|z-\eta|\leqslant 1} (|A(z) - A(\eta)| + |B(z) - B(\eta)|) = 0$ Э.Мухамадиевым и С.Байзаевым в классе $C(E)$ установлены существование и структурные свойства решения, нетеровость оператора L. В работе система (1) исследуется в случае, когда коэффициенты могут расти на бесконечности. На основе детального изучения краевых задач и метода локальных оценок, при близких к минимальным условиях на коэффициенты, доказана однозначная ее разрешимость для всех правых частей из $L_2(E)$. При некоторых дополнительных ограничениях локального характера показано, что для решения имеют место коэрцитивные оценки в норме весового пространства Соболева, а также найдены условия компактности и слабая асимптотика функции распределения аппроксимативных чисел резольвенты оператора L.

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Vychislenie sobstvennyh chisel differencialnyh operatorov

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Techniques for existence theorems in problems with free boundary.

The problem of evolution of a fluid with free boundaries constists in the determination of the domain Ω_t, the velocity vector $\mathbf{v}$ and a scalar pressure p satisfying the equation of motion, and stress-free boundary conditions. This problem has been studied for compressible and incompressible fluids, when the domain is a layer with rigid bottom and free upper surface, or when the boundary is totally free (drop of fluid). Several existence theorems have been proven local or global in time for small initial data, and explicit steady solutions have been computed when there is a stabilizing surface tension. Open problems concern the existence, uniqueness and stability of the se steady solutions at least locally, and some hint has been proposed by Padula & Solonnikov.

Here, we reconsider different techniques used to solve some global existence probelms, and propose some coordinate change that suit better the problem for a rotating drop of an incompressible fluid, and for a piece of horizontal layer of compressible flu id having upper free surface. In both cases we provide first uniqueness (for small rotation in the first case) of the given steady solution, and then existence of global regular solutions, starting by a small perturbation of the steady solution, finally, the asymptotic decay of these solutions to the corresponding steady one. Interesting enough, the method changes deeply with the geometry of the problem, below I quote a result in collaboration with V.A. Solonnikov.

Theorem *Let $\Gamma_0 = \partial\Omega_0$ be given by $R = R_0(y)$, $y \in S_1$, $R_0 \in C^{3+\alpha}(S_1)$, $\alpha \in (0,1)$, and let $\mathbf{v}_0 \in C^{2+\alpha}(\Omega_0)$ satisfy the boundary compatibility conditions. Assume additionally that*

$$|\mathbf{v}_0|_{C^{2+\alpha}(\Omega_0)} + |R_0 - 1|_{C^{3+\alpha}(S_1)} \leqslant \epsilon, \tag{0.1}$$

with $\epsilon > 0$ sufficiently small. Then there exists a unique solution $(R, \mathbf{v}, p)$ defined in an infinite time interval $t > 0$ and possessing the following properties: $R(\cdot,t) \in C^{3+\alpha}(S_1)$, $\mathbf{v}(\cdot,t) \in C^{2+\alpha}(\Omega_t)$, $p(\cdot,t) \in C^{1+\alpha}(\Omega_t)$, $\forall t > 0$, $\Gamma_t = \partial\Omega_t$ is given by equation (1.3). The solution satisfies the inequality

$$\begin{aligned} &\sup_{t\in(0,T)} |\mathbf{v}_t(\cdot,t)|_{C^{\alpha}(\Omega_t)} + \sup_{t\in(0,T)} |\mathbf{v}(\cdot,t) - \mathbf{v}_\infty|_{C^{2+\alpha}(\Omega_t)} \\ &+ \sup_{t\in(0,T)} |p(\cdot,t) - p_\infty|_{C^{1+\alpha}(\Omega_t)} + \sup_{t\in(0,T)} |R(\cdot,t) - R_\infty|_{C^{3+\alpha}(S_1)} \\ &\leqslant c\left(|\mathbf{v}_0 - \mathbf{v}_\infty|_{C^{2+\alpha}(\Omega_0)} + |R_0 - R_\infty|_{C^{3+\alpha}(S_1)}\right) e^{-bt}, \quad \forall T \in (0,\infty], \end{aligned} \tag{0.2}$$

with some constants $b > 0$ and c independent of T.

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О численном методе с расщеплением граничных условий для стационароной системы Навье-Стокса в шаровом слое в случае осевой симметрии[23]

В [1], [2] были предложены и исследованы новые быстросходящиеся итерационные методы с неполным и полным расщеплением граничных условий (ГУ) решения 1-й краевой задачи для многомерной сингулярно возмущенной

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системы типа Стокса с большим параметром μ

$$-\Delta \mathbf{u} + \mu^2 \mathbf{u} + grad\, p = \mathbf{f}, \quad div\, \mathbf{u} = 0 \text{ в } \Omega,$$
$$\mathbf{u}|_{\partial\Omega} = \mathbf{g}, \quad \int\limits_{\partial\Omega} (\mathbf{g}, \mathbf{n}) ds = 0, \quad \int\limits_{\Omega} p dx = 0.$$

Методы приводят на итерациях к существенно более простым скалярным или эквивалентным таковым по трудности решения векторным эллиптическим краевым задачам, и наиболее быстрые их варианты сходятся тем быстрее, чем бо́льшие значения принимает сингулярный параметр μ. В [3]–[5] были построены в случае полосы при условии периодичности билинейные конечно-элементные (КЭ)-реализации этих методов, которые (после преодоления трудностей, см. [5], вызванных существенным падением по сравнению с дифференциальным случаем скорости сходимости на высоких гармониках у непосредственных КЭ-реализаций) наследуют в основном все лучшие качества методов и обеспечивают второй порядок точности и для скоростей и для давления. В [6] была разработана модификация методов с полным расщеплением ГУ в шаровом слое для системы Стокса ($\mu = 0$), т.е. в том случае, когда уже нет большого параметра μ как фактора, благоприятствующего повышению скорости сходимости. Тем не менее, высокой скорости сходимости такой модификации – уменьшения ошибки за 1 итерацию не менее, чем в 10 раз – удалось достичь за счет усложнения формул пересчета, введения в них трех релаксационных параметров и специальных операторов типа Пуанкаре-Стеклова пересадки с одной граничной сферы на другую. Итерационный процесс был двухшаговым для тонких и трехшаговым для толстых слоев. В [7] и [8] были построены и исследовались численные реализации на основе билинейных КЭ этих итерационных методов для системы Стокса, а также типа Стокса в шаровом слое в случае осевой симметрии. Методы также обнаруживают 2-й порядок точности. Содержание настоящего доклада. 1. Получены способы оптимизации значений трех релаксационных параметров вычислительно более эффективного одношагового варианта метода из [6] для системы Стокса во всех случаях: тонких, толстых и промежуточной толщины шаровых слоев. 2. Установлено (численными экспериментами), что разработанный в [6] метод обладает следующим замечательным качеством: у непосредственных его КЭ-реализаций, осуществленных в [7] и [8], в отличие от таковых для методов для системы типа Стокса, отсутствует такое весьма нежелательное явление как существенное падение скорости на высоких гармониках, причем даже для очень толстых шаровых слоев, когда коэффициент уменьшения ошибки достигает многих сотен раз за 1 итерацию. 3. На основе этих алгоритмов и метода последовательных приближений создан численный метод решения задачи о стационарных течениях вязкой несжимаемой жидкости при небольших числах Рейнольдса в шаровом слое в случае осевой симметрии.

Разработаны два варианта КЭ-аппроксимации конвективных членов – полная и более простая, основанная на консервативной их форме. Численные эксперименты, проведенные на серии тестов обнаруживают общий второй порядок точности по шагу сетки этого метода для системы Навье-Стокса. Метод позволяет производить расчеты на сетках с высоким разрешением – до нескольких миллионов элементов. Получен ряд высокоточных численных решений задачи о течении вязкой несжимаемой жидкости между двумя соосно вращающимися сферами при некоторых зазорах слоя и различных режимах вращения. Эти результаты уточняют и дополняют некоторые, имеющиеся к настоящему времени, ср., например, работы, цитированные в статьях в [9].

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Method of asymptotic partial decomposition of domain for singular problems.

Method of asymptotic partial decomposition of domain was proposed in [1,2] for thin domains and tube structures (finite connected unions of thin cylinders). The main idea of this method is to replace the original 3D or 2D problem by a hybrid 3D-1D, 3D-2D or 2D-1D model; i.e. we reduce the dimension of some parts of the domain where the solution has a regular asymptotic behavior of the solution while the dimension in subdomains of singular asymptotic behavior of the solution are kept unchanged. Usually subdomains of singular asymptotic behavior of the solution are the domains of boundary layers. This approach permits to reduce essentially the number of nodes in the grid when a numerical analysis is applied. The same approach can be applied when the domain is not singular but the equation is singular. Of cours the most important question is: what are the interface conditions between subdomains of singular behaviour and subdomains of regular behaviour (usually with reduced dimension of the problem)? The main principles of construction of such conditions are as follows:

a) the asymptotic expansion of the exact solution of the initial problem should satisfy these interface conditions with great accuracy; b) the reduced problem of hybrid dimension with the interface conditions (i.e. partially decomposed problem) should be well posed, i.e. it should have the unique solution and it should be stable with respect to small perturbations in the right hand side. Some similar hybrid models appeared earlier in mechanics and computations. They are based on some heuristic approaches (see for example, a numerical simulation of shallow water equation in the system "lake-river" [3]). Some other approachs of specially precised boundary conditions were considered in [4,5]. In the presented paper we give a variational version of the method of asymptotic partial decomposition of domain [6]. In this version we consider the variational formulation of the original problem stated in some Hilbert space. The main idea is to replace this Hilbert space by a simple Hilbert subspace which contains the asymptotic solution (or asymptotic expansion of the solution). The variational formulation of the initial problem restricted onto this subspace corresponds to some hybrid 3D-1D, 3D-2D or 2D-1D model. It is well adopted to an application of finite element method with some elements of various dimensions and some "super-elements" of combined dimension. We consider also partial domain decomposition for some singular problems for integral and differential equations.

1. General description of the variational version.

Let H_ε be a family of Hilbert spaces (depending on a small parameter ε). Consider a variational problem: find $u_\varepsilon \in H_\varepsilon$ such that

$$\forall w \in H_\varepsilon, B(u_\varepsilon, w) = (f, w). \tag{1}$$

Here $B(.,.)$ is a bilinear symmetric coercive form and $(f,.)$ is a linear bounded functional. We suppose that

$$\forall w \in H_\varepsilon, B(w, w) \geqslant c_1 \varepsilon^r \|w\|^2, \tag{2}$$

where $c_1 > 0$ and $r \geqslant 0$ do not depend on ε, and that $\forall u,\ w \in H_\varepsilon,\ B(u,w) = B(w,u)$, $\forall w \in H_\varepsilon$, and $|(f,w)| \leqslant \|f\|\|w\|$. Let $H_{\varepsilon,dec}$ be a subspace of H_ε. Let u_ε^a

be an asymptotic solution such that (i) $u_\varepsilon^a \in H_{\varepsilon,dec}$, and such that (ii) there exist $\psi_\varepsilon \in H_\varepsilon$ such that $\|\psi_\varepsilon\| \leqslant c_2$, where c_2 does not depend on ε, and such that

$$\forall w \in H_\varepsilon, B(u_\varepsilon^a, w) = (f, w) + \varepsilon^K(\psi_\varepsilon, w), \tag{3}$$

where $K > r$. Subtracting (1) from (3) we obtain $\forall w \in H_\varepsilon$, $B(u_\varepsilon^a - u_\varepsilon, w) = \varepsilon^K(\psi_\varepsilon, w)$, i.e. for $w = u_\varepsilon^a - u_\varepsilon$ we obtain $c_1\varepsilon^r\|u_\varepsilon^a - u_\varepsilon\| \leqslant \varepsilon^K\|\psi_\varepsilon\| \leqslant \varepsilon^K c_2$, i.e.

$$\|u_\varepsilon^a - u_\varepsilon\| = O(\varepsilon^{K-r}). \tag{4}$$

Let $H_{\varepsilon,dec}$ be a subspace of H_ε and let u_ε^d be the solution of the partially decomposed problem, i.e. of the identity (1) restricted onto the subspace $H_{\varepsilon,dec}$:

$$\forall w \in H_{\varepsilon,dec}, B(u_\varepsilon, w) = (f, w). \tag{5}$$

We assume that this subspace has a more simple structure than H_ε, for example the functions of $H_{\varepsilon,dec}$ are polynomials on the regular part of the domain. Therefore the problem (5) is easier than the problem (1). So the variational version is related to the special choice of a simple subspace $H_{\varepsilon,dec}$ (i.e. special restriction of the original problem), satisfying the conditions (i), (ii). To justify the variational version let us subtract the identity (5) from (3) for any $w \in H_{\varepsilon,dec}$. Then we obtain $\forall w \in H_{\varepsilon,dec}$, $B(u_\varepsilon^a - u_\varepsilon^d, w) = \varepsilon^K(\psi_\varepsilon, w)$, i.e. for $w = u_\varepsilon^a - u_\varepsilon^d$ we obtain $c_1\varepsilon^r\|u_\varepsilon^a - u_\varepsilon^d\| \leqslant \varepsilon^K\|\psi_\varepsilon\| \leqslant \varepsilon^K c_2$, i.e.

$$\|u_\varepsilon^a - u_\varepsilon^d\| = O(\varepsilon^{K-r}). \tag{6}$$

Comparing the estimates (4) and (6) we obtain

$$\|u_\varepsilon - u_\varepsilon^d\| = O(\varepsilon^{K-r}). \tag{7}$$

This estimate justifies the method.

2. Model example.

Consider the elasticity system of equations stated in

$$G_\varepsilon = (0,1) \times (-\frac{\varepsilon}{2}, \frac{\varepsilon}{2})$$

(below the convention of the summation from 1 to 2 in repeating indices is accepted):

$$\frac{\partial}{\partial x_r}(A_{rm}\frac{\partial u_\varepsilon}{\partial x_m}) = f(\frac{x}{\varepsilon}), x \in G_\varepsilon, \tag{8}$$

where A_{rm} are the constant 2×2 matrices with the components a_{rm}^{kl}:

$$a_{rm}^{kl} = \lambda\delta_{rk}\delta_{lm} + \mu(\delta_{rm}\delta_{kl} + \delta_{rl}\delta_{km}),$$

λ and μ are the positive constants ; here u_ε, f are two- dimensional vector-valued functions, $suppf(\xi)$ belongs to the square $(0,1)\times(-\frac{1}{2},\frac{1}{2})$, and $f \in L^2$. Let us state the following boundary conditions: free lateral boundary and fixed ends:

$$A_{2m}\frac{\partial u_\varepsilon}{\partial x_m} = 0, x_2 = \pm\varepsilon/2 \tag{9}$$

$$u_\varepsilon = 0, x_1 = 0, 1. \tag{10}$$

The variational formulation is the integral identity (1) where

$$B(u_\varepsilon, w) = \int_{G_\varepsilon} (\frac{\partial w}{\partial \xi_i})^T A_{ij}\frac{\partial u_\varepsilon}{\partial \xi_j}dx, (f,w) = -\int_{G_\varepsilon} fwdx, \tag{11}$$

H_ε is the subspace of vector valued functions of $[H^1(G_\varepsilon)]^2$ vanishing on the segments $\{x_1 = 0\}$ and $\{x_1 = 1\}$. As it follows from [5,6], the asymptotic solution is almost a polynomial at some distance δ of order $O(\varepsilon|ln(\varepsilon)|$ from the extremities, i.e. for any K there exist such $\hat{K} \in \mathbb{R}$ independent of ε that if $\delta = \hat{K}\varepsilon|ln(\varepsilon)|$ then x of the rectangle $(\delta, 1-\delta) \times (-\frac{\varepsilon}{2}, \frac{\varepsilon}{2})$ the discrepancy is of order $O(\varepsilon^K)$.

This polynomial solution has a form

$$\begin{array}{l} v_1 - x_2 v_2' + \{\frac{1}{6}(\frac{E}{\mu} - \frac{\lambda}{\lambda+2\mu})x_2^3 + \varepsilon^2\frac{1}{8}(\frac{\lambda}{3(\lambda+2\mu)} - \frac{E}{\mu})x_2\}v_2''' \\ v_2 - \frac{\lambda}{\lambda+2\mu}x_2 v_1' + \frac{\lambda}{2(\lambda+2\mu)}(x_2^2 - \frac{1}{12}\varepsilon^2)v_2'' \end{array} \tag{12}$$

where

$$v_1(x_1) = ax_1 + b, \tag{13}$$

$$v_2(x_1) = cx_1^3 + dx_1^2 + ex_1 + g \tag{14}$$

are polynomials with some undetermined coefficients a, b, c, d, e, g, $E = \frac{(\lambda+2\mu)^2-\lambda^2}{\lambda+2\mu}$. We introduce the subspace $H_{\varepsilon,dec}$ of partially decomposed problem as the subspace of H_ε, such that its elements have the form (12)-(14) for all x of the rectangle $(\delta, 1-\delta) \times (-\frac{\varepsilon}{2}, \frac{\varepsilon}{2})$. It can be proved that slightly corrected asymptotic solution [7,8] satisfies the conditions (i) and (ii) of the section 1 and therefore as in the section 1 we can obtain the estimate (7), i.e. for any K there exist such $\hat{K} \in \mathbb{R}$ independent of ε that if $\delta = \hat{K}\varepsilon|ln(\varepsilon)|$ then the estimate (7) holds true. (In order to prove the coercivity (2) we should use the Korn inequality for thin domains; it gives the estimate (2) with some $r \leqslant 3$, see [9]).

This result can be generalized for the rod structures described in [7] and tube structures from [2]. The case when the right hand side f is different from zero and depends on the longitudal variable in a regular part can be reduced to the previous case.

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Wave type solutions of hyperbolic systems of conservation laws with relaxation terms

In this paper we consider the system of PDEs

$$\partial_t \rho + \partial_x j = 0$$

$$\partial_t j + \frac{1}{\epsilon^2}\partial_x \rho = -\frac{1}{\epsilon^2} k(\rho) j$$

with relaxation terms. Here the relaxation time is given by ϵ, the nonnegative function $k(\rho)$ represents a collisional cernel, whereas the quantities ρ and j denote, respectively, the density mass and the flux of the particles. Such systems occur in the kinetic theory of gases, in gas flows with relaxation, multiphase and phase transition, turbulence, water waves viscoelasticity, and reactive flows (for references and more information see [1]). We study the problem for existence and uniqueness of wave type solutions of the above system (usually called simple states, or solutions, constructed by means of Riemann invariants; see [2]). It turns out that the problem stated above reduces to the problem of investigation of the existence of integral manifolds of a certain involutive system of Frobenius type, which is equivalent to a system of ordinary DEs and can be treated by standard methods.

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First Boundary Problem for High Order Hyperbolic Equations and Related Topics in Analysis

Boundary value problems for linear PDE in bounded domains with data on a whole boundary are studied well in the case of elliptic equations of an arbitrary order. Almost nothing is known about solvability of such problems in nonelliptic case. Some separate results which appear from time to time have a negative character. In particular the Dirichlet problem for second order hyperbolic differential equations in a bounded domain is usually regarded as an "unnatural" problem of mathematical physics. Its solution may neither exist, nor be uniquely determined, nor depend continuously on the data. When dealing with a boundary problem for hyperbolic differential equations of an arbitrary order in a bounded domain a part of the boundary remains usually free of a priori information about unknown solutions. The evolutionary character of hyperbolic equations seems to impose taboo on a priori information about a solution on the *whole* boundary of a domain. On this background the results to be represented in this talk turned out to be unexpected to a large extent. With any *third* order strictly hyperbolic differential operator P with constant coefficients in the plane we associate a curvilinear (two-dimensional) triangle $D = OS_1S_2$. Its sides OS_1 and OS_2 coincide with two characteristics of P and the side $\Gamma = S_1S_2$ is a generic smooth curve, connecting points S_1 and S_2. The angle S_1OS_2 is supposed to be less then π and it contains a segment of the third characteristic of P. The first boundary problem we deal with is as follows:

$$Pu = f \quad \text{in } D, \qquad u = \varphi \quad \text{on } \partial D. \tag{1}$$

To solve this problem we introduce some noncommutative finitely generated semigroup Θ_Γ of maps in Γ intimately connected with P and Γ. This semigroup generates naturally a set $\mathcal{O}_\Gamma$ of orbits in Γ. Let $\mathcal{T}$ be a set of characteristic (with respect to the operator P) points in Γ. In terms of the sets $\mathcal{O}_\Gamma$ and $\mathcal{T}$ we formulate, as the main result, the necessary and sufficient condition of the unique solvability of problem (1) in $C^{2+k}(\overline{D})$ for arbitrary functions $f \in C^k(\overline{D})$ and $\varphi \in C^{2+k}(\partial D)$, $k = 0, 1, \dots$, (φ is continuous on ∂D and $k+2$ times differentiable on each side of triangle OS_1S_2). In particular *if $\mathcal{T} = \Omega$ (noncharacteristic first boundary problem!) then problem (1) is uniquely solvable for all above functions f and φ. The inverse operator $(f, \varphi) \to u$ is continuous in the corresponding pair of spaces.* If the above mentioned condition is violated then for some Γ problem (1) becomes Fredholm problem of index zero. It is worth noting that the first boundary problem (1) turns out to be equivalent to some new problem in Integral geometry, reminding the well known Radon problem. In a domain D of the described type let us consider a parallelogram $D_q = Oq_1qq_2$ where q is an arbitrary point in Γ and $q_1 \in OS_1$, $q_2 \in OS_2$. Then the problem in question is: given a function $h \in C(\Gamma)$ to find a function $f \in C(\overline{D})$ such that

$$\int_{D_q} f d\sigma = h(q), \quad q \in \Gamma. \tag{2}$$

We give the exhaustive answer to the following question (which is known to be the main problem in Integral geometry): for which classes of functions f is the map $f \mapsto h$ is one-to-one, and which functions $h(q)$ can be represented by the integral (2). When solving this problem we reduce it to a functional equation on the curve Γ of the following form:

$$F(z) - \alpha_1(z)F \circ \delta_1(z) - \alpha_2(z)F \circ \delta_2(z) = G(z), \quad z \in \Gamma. \tag{3}$$

Here G and F are given and unknown (respectively) functions on Γ, δ_1 and δ_2 are two maps in Γ and α_1, α_2 are given functions. To the best of author's knowledge such equations (except for the case of linear functions δ_1 and δ_2) has never been investigated. It is in this part of the work that the above semigroup Θ_Γ (generated by δ_1 and δ_2) of maps in Γ and all accompanying dynamic conceptions are used to the large extent. The main result related to equation (3) consists of conditions (in terms of $\mathcal{O}_\Gamma$) which guarantee the unique solvability (or Fredholm property) in $C(\Gamma)$ of the equation for all functions $G \in C(\Gamma)$. Some preliminary results one can find in author's publications: B.Paneah, "On a Problem in Integral Geometry Connected with the Dirichlet Problem for Hyperbolic Equations", IMRN, Intern.Math.Res.Notes 5(1997), pp.213 - 222. B.Paneah, "A Problem in Integral Geometry with Application to the Dirichlet Problem for a Class of Hyperbolic Differential Equations", *Differential Equations, Asymptotic Analysis and Mathematical Physics*, Math.Res.100, Academic Verlag, Berlin(1997), pp.260 - 267.

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On Minkowski decompositions of polytopes

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Global semiclassical description of the spectrum for two-dimensional magnetic Schrödinger operator

We study an asymptotic of the spectrum for the operator

$$\hat{H} = \frac{1}{2}\left(-ih\frac{\partial}{\partial x_1} + x_2\right)^2 + \frac{1}{2}\left(-ih\frac{\partial}{\partial x_2}\right)^2 + \varepsilon v(x_1, x_2),$$

where v is a real-analytic two-periodic relative vectors $a_1 = (2\pi, 0)$ and $a_2 = (a_{21}, a_{22})$ function, as h, ε tend to 0. The corresponding Hamiltonian is

$$H(p, x) = \frac{1}{2}(p_1 + x_2)^2 + \frac{1}{2}p_2^2 + \varepsilon v(x_1, x_2),$$

and it corresponds to a non-integrable Hamiltonian system. Nevertheless, by using modifications of Krylov–Bogolyubov–van Alfven–Neishtadt averaging methods we show almost integrability of the Hamiltonian system for H with exponentially small disrepancy relative ε, and reduce it to a one-dimensional one on a torus. Using the topological methods for integrable Hamiltonian systems and some elementary facts from the Morse theory, we give a general classification of the classical motion defined by H. According this classification, the classical motion is separated into *regimes* with different topological characteristics (like rotation numbers and Maslov indices). Now using these regimes, the correspondence principle, the semiclassical approximation, and the Bohr-Sommerfeld quantization rule we give a global semiclassical description of the spectrum for operator $\hat{H}$. If the flux $\eta = a_{22}/h$ is rational, by some heuristic considerations we can give a more complete picture of the spectrum for $\hat{H}$. In particular, the estimation of the number of subbands in each Landau level is obtained. From the point of view of this description for the spectrum, the above mentionad regimes are classical preimages of spectral series for the operator $\hat{H}$. The work is partially supported by the project DFG–RAS 436 RUS113/572.

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On locally integrable entropy solutions of the Cauchy problem for a first order quasilinear equation [1]

Consider the Cauchy problem for a first order quasilinear equation

$$u_t + \operatorname{div}_x \varphi(u) = g(t,x) \in L^1_{loc}(\Pi_T), \quad u(0,x) = u_0(x) \in L^1_{loc}(\mathbb{R}^n), \qquad (1)$$

$\varphi = (\varphi_1, \dots, \varphi_n)$, $u = u(t,x)$, $(t,x) \in \Pi_T = [0,T) \times \mathbb{R}^n$, $T > 0$. The flux vector $\varphi(u)$ is assumed to be only continuous and to satisfy the growth restriction: $|\varphi(u)| \leqslant C_0(1+|u|)$, $C_0 = \text{const}$. We study generalized entropy solutions (g.e.s.) $u(t,x) \in L^1_{loc}(\Pi_T)$ of the Cauchy problem (1) in the sense of S.N.Kruzhkov (see [1]) and present the following results.

Theorem 1. *A g.e.s. of the problem* (1) *exists. Moreover there exist the maximal and the minimal g.e.s. of this problem.*

Theorem 2. *Let* $u_0 \in L^p(\mathbb{R}^n)$, $g(t,x) \in L^1((0,T), L^p(\mathbb{R}^n))$, $1 \leqslant p \leqslant \infty$, $u = u(t,x)$ *be a g.e.s. of the problem* (1). *Then for a.e.* $t \in (0,T)$ $u(t,\cdot) \in L^p(\mathbb{R}^n)$ *and* $\|u(t,\cdot)\|_p \leqslant \|u_0\|_p + \int_0^t \|g(\tau,\cdot)\|_p d\tau$.

Theorem 3 (uniqueness). *Suppose that* $|\varphi_i(u) - \varphi_i(v)| \leqslant \omega_i(|u-v|)$, $i = 1,\dots,n$, *where* $\omega_i(r)$ *are nondecreasing subadditive functions on* $\mathbb{R}_+$, $\omega_i(r) > 0$ *for* $r > 0$ *and* $\liminf_{r\to 0+} r^{1-n} \prod_{i=1}^{n} \omega_i(r) < \infty$. *Then the g.e.s. of the problem* (1) *is unique.* One could define a g.e.s. $u \in L^p_{loc}$ for any $p > 1$ changing the growth condition to the requirement $|\varphi(u)| \leqslant C_0(1+|u|^p)$ (with no restriction for $p = \infty$). But in this case the Cauchy problem seems to be ill-posed. Even for $n = 1$, $g \equiv 0$ all of the positive results above are no longer valid.

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Homogenization for some problems with inequality type boundary condition in elasticity theory

Let $\varepsilon \in (0,1)$, $Y = (0,1)^3$, and let V be a smooth bounded domain in $\mathbb{R}^3$, $\partial V = S$. Suppose that the unit cube Y contains a closed piece of a smooth surface M. If $T_\varepsilon = \{z \in \mathbb{Z}^3 : \varepsilon(\bar{Y} + z) \in V\}$, then we set $M_\varepsilon = \cup_{z \in T_\varepsilon} \varepsilon(M + z)$ and $V_\varepsilon = V \setminus M_\varepsilon$. The inner boundary of $Y_M = Y \setminus M$ consists of two copies of the surface $M : M^+$ и M^-. In accordance with this the inner boundary of V_ε consists of M_ε^+ and M_ε^-. We define a unit outward normal vector $n^+(n^-)$ on $M_\varepsilon^+(M_\varepsilon^-)$. For an elastic body with fissures occupying the domain V_ε we study its displacement field u^ε when the body is subjected to the external forces having volumetric density vector f while it is fixed on its outer boundary S and when sides of fissures do not overlap. The latter condition supposes some system of one-side constraints on the inner boundary of V_ε. Let $\sigma(u)$ be a commonplace stress tensor considered for a displacement field u within the framework of the linear elasticity theory. It defines on $M_\varepsilon^+(M_\varepsilon^-)$ a stress vector $\sigma_n^+(\sigma_n^-)$, which decomposes into mutually ortogonal components $\sigma_n^+ = \sigma_{nn}^+ n^+ + \sigma_\tau^+$ $(\sigma_n^- = \sigma_{nn}^- n^- + \sigma_\tau^-)$. If $u \cdot v$ denotes a scalar product of vectors $u, v \in \mathbb{R}^3$, then $[u \cdot n]|_{M_\varepsilon} = u|_{M_\varepsilon^+} \cdot n^+ + u|_{M_\varepsilon^-} \cdot n^-$ is mutual approach of two sides of fissures across the normal for given displacement field u in V_ε. We consider in V_ε the folowing mixed problem (1_ε) for the system of Lame equations: $-div\, \sigma(u^\varepsilon) + f = 0$ in V_ε, $u^\varepsilon = 0$ on S; $[u^\varepsilon \cdot n] \leqslant \varepsilon h$, $\sigma_\tau^+ = \sigma_\tau^- = 0$, $\sigma_{nn}^+ = \sigma_{nn}^- \leqslant 0$, $([u^\varepsilon \cdot n] - \varepsilon h)\sigma_{nn}^+ = 0$ on M_ε.
Here $h = const \geqslant 0$ and εh stands for width of fissures in an initial state. The solution u^ε is treated in the generalized sense as a solution of some variational inequality considered in a set of geometrically possible displacements. We prove that as $\varepsilon \to 0$ solution u^ε and energy functional of the problem (1_ε) converge to the displacement field and energy functional for the homogeneous body without fissures which occupies the domain V and is subjected to the same external effect and fixation on S but with nonlinear state equation (defined with a help of auxiliary variational problem on Y_M.) Thus, we verify the hypothesis stated in [1, ch.6] about the hyperelastic law of deformation of linear-elastic body with numerous small periodically distributed fissures.

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Regularized traces of differential operators: method of Lidskii–Sadovnichii

Eigenvalues for wide classes of differential operators are zeros of characteristic determinant $\Delta(\lambda)$ wich is an entire function (see [1]–[3]), having asymptotic representation as $\lambda \to \infty$:

$$\Delta(\lambda) \sim \sum_{k=1}^{H} e^{\sum_{\ell=p}^{h-1} \theta_{k\ell} \lambda^{\frac{h-\ell}{h}}} \lambda^{\frac{n_k}{h}} \sum_{\nu=0}^{\infty} \beta_{\nu}^{(k)} \lambda^{-\frac{\nu}{h}}, \tag{0.1}$$

where $h \in \mathbf{N}$, p, $n_k \in \mathbf{Z}$, $p < h$, $\theta_{k\ell}$, $\beta_{\nu}^{(k)} \in \mathbf{C}$, with $\beta_0^{(k)} \neq 0$. If $\theta_{k\ell} = 0$ for $\ell \neq 0$ and $\theta_{k0} \neq 0$, then $\Delta(\lambda)$ is an entire function of class K. V.B.Lidskii and V.A.Sadovnichii have suggested a method of calculating regularized sums of roots λ_ℓ, $\ell = 1, 2, 3, \ldots$ of entire function of class K., i.e. sums

$$\sum_{\ell=1}^{\infty} (\lambda_\ell^m - A_m(\ell)) = S_m\,, \quad m \in \mathbf{N} \tag{0.2}$$

where $A_m(\ell)$ are concrete numbers, guaranteeing the convergence of the series. The method of Lidskii–Sadovnichii [1] of introducing zeta-function, associated with class K is extended on entire functions $\Delta(\lambda)$, having asymtotic representation (1). Zeta-function associated with the function $\Delta(\lambda)$ is introduced via integral

$$Z(\sigma) = \frac{1}{2\pi i} \int_{\Gamma} \lambda^{-\sigma} \frac{\Delta'(\lambda)}{\Delta(\lambda)} d\lambda, \quad Re\sigma > 1 - \frac{p}{h},$$

where the contour Γ is chosen on the Riemann surface of $\sqrt[h]{\lambda}$ and the multifunction $\lambda^{-\sigma}$ is determined by fixing a regular branch of the logarithm in exterior of Γ. Regularized sums of roots (2) are expressed in terms of the values of the analytic extention of $Z(\sigma)$ to the left half-plane. Example. Consider boundary value problem second order

$$y'' - 2\lambda y + (\varphi(x) - \lambda + \lambda^2) y = 0, \quad y(0) = y(1)\,, \; y'(0) = y'(1), \; \varphi(x) \in C^\infty[0,1].$$

Characteristic determinant $\Delta(\lambda)$ of this problem has asymptotic representation as $\lambda \to \infty$

$$\Delta(\lambda) \sim e^{2\lambda} - e^{\lambda + \lambda^{1/2}} \left(1 + \sum_{\nu=1}^{\infty} \frac{\alpha_{2\nu}}{\lambda^{\nu/2}}\right) - e^{\lambda - \lambda^{1/2}} \left(1 + \sum_{\nu=1}^{\infty} \frac{\alpha_{1\nu}}{\lambda^{\nu/2}}\right) + 1,$$

where the coefficients $\alpha_{1\nu}$, $\alpha_{2\nu}$, $\nu = 1,2,\ldots$ are computed successively by recurrence relation. The formula for the first trace is

$$\sum_{k=1}^{\infty}\sum_{s=1}^{4}\left(\lambda_{ks} - a_s k - (-1)^s\sqrt{a_s}k^{1/2} - \frac{1}{2} - \frac{\eta_1^{[s]}}{k^{1/2}} - \frac{\eta_2^{[s]}}{k}\right) = 1\,,$$

where $a_1 = a_2 = 2\pi i$, $a_3 = a_4 = -2\pi i$,

$$\eta_1^{[r]} = \frac{(-1)^r}{2\sqrt{2\pi i}}\left(\int_0^1 \varphi(t)dt - \frac{1}{4}\right),\ r = 1,2\,,\ \eta_1^{[3]} = \bar{\eta}_1^{[2]}\,,\ \eta_1^{[4]} = \bar{\eta}_1^{[1]}\,,$$

$$\eta_2^{[s]} = \frac{(-1)^{s-1}}{8\pi i}\left(\int_0^1 \varphi(t)\int_0^t \varphi(\xi)d\xi dt - \frac{1}{2}\left(\int_0^1 \varphi(t)dt\right)^2\right),\ s = \overline{1,4}\,.$$

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Эллиптические неравенства на стратифицированных множествах.

Подробное описание затрагиваемых здесь понятий имеется в [1-3]. Стратифицированное множество Ω – связное подмножество $\mathbb{R}^n$, составленное из конечного числа гладких многообразий σ_{ki} (стратов), "правильно" примыкающих друг к другу. Векторное поле $\vec{F}$ называется касательным к Ω, если его сужения на страты касаются их. Лебеговы меры на стратах задают меру μ на Ω. В терминах этой меры можно определить понятие дивергенции $\nabla\vec{F}$ касательного векторного поля на Ω; в точке $x \in \sigma_{k-1j}$ она определяется по формуле

$$\nabla\vec{F}(x) = \nabla_{k-1}\vec{F}(x) + \sum_{\sigma_{ki}\succ\sigma_{k-1j}} (\vec{F}\cdot\vec{\nu})\Big|_{\overline{ki}}(x),$$

в правой части которой стоит классическая $k-1$-мерная дивергенция ∇_{k-1} (на страте σ_{k-1j} как на римановом многообразии) и сумма проекций $\vec{F}$ на нормали в точке x, направленные внутрь стратов σ_{ki}, примыкающих к σ_{k-1j};

примыкание записывается в виде $\sigma_{ki} \succ \sigma_{k-1j}$). Запись $u\Big|_{\overline{ki}}$ означает продолжение на $\overline{\sigma}_{ki}$ сужения u на σ_{ki}. Оператор $(\Delta_p u)(x) = \nabla(p\nabla u)$ – аналог оператора дивергентного типа. В каждом страте σ_{ki} предполагается либо $p > 0$, либо $p \equiv 0$. Содержательная эллиптическая теория с оператором Δ_p получается в предположении прочности стратифицированного множества. Прочность означает, что любые два страта можно соединить такой связной цепочкой стратов $\sigma_{k_1 i_1}, \ldots, \sigma_{k_p i_p}$, что $|k_{q+1} - k_q| = 1$ при $q = 1, \ldots p-1$. Для постановки краевых задач множество Ω разбивается на две части; Ω_0 – любое открытое (в топологии на Ω, индуцированной из $\mathbb{R}^n$) и связное подмножество Ω, составленное из его стратов, и $\Omega \setminus \Omega_0 = \partial\Omega_0$. Задача Дирихле имеет вид:

$$(\Delta_p u - qu)(x) = f(x), \quad (x \in \Omega_0), \quad u\Big|_{\partial\Omega_0} = \phi.$$

При естественных требованиях на коэффициенты она однозначно разрешима в пространствах Соболевского типа. Можно показать, что оператор Δ_p порождает аналитическую полугруппу в $L^2_\mu(\Omega)$; что позволяет получать результаты о разрешимости параболических краевых задач. Получен слабый принцип максимума для эллиптических неравенств в общем случае и сильный принцип максимума для двумерного стратифицированного множества. При $\partial\Omega_0 = \varnothing$ обнаруживаются аналогии с классическим случаем эллиптического оператора на замкнутом римановом многообразии. К примеру, неравенство $(\Delta_p u - qu)(x) \geqslant 0$ при $q \geqslant 0$ допускает лишь постоянные решения (лемма Бохнера). Работа частично поддержана РФФИ (Гранты 01-01-00417, 01-01-00418)

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Highly localized solutions of the Klein-Gordon equation

For the Klein-Gordon equation

$$h^2\left(u_{tt}-u_{xx}-u_{yy}\right)+u=0, \quad h=\text{const} \tag{0.1}$$

we present here two new precise explicit solutions u_b, u_p

$$u_b=\phi_b e^{iS_b/h}, \qquad u_p=\phi_p e^{iS_p/h}, \tag{0.2}$$

$$S_b=\varkappa\theta-\frac{\beta}{4\varkappa}, \qquad S_p=i\left[(\theta+i\varepsilon)(\beta-i4\varkappa^2\varepsilon)\right]^{1/2}, \tag{0.3}$$

$$\phi_b=(\beta-i\varepsilon)^{-1/2}, \qquad \phi_p=\left[(\beta-i\varepsilon)(\beta-i4\varkappa^2\varepsilon)\right]^{-1/2}, \tag{0.4}$$

where

$$\theta=x-t+\frac{y^2}{x+t-i\varepsilon}, \quad \beta=x+t. \tag{0.5}$$

Functions $u=u_b$ and $u=u_p$ satisfy (0.1) for any non-zero constants ε, $\varkappa$ and for any choice of square root branch in (0.3). Both complex phases $S=S_b$ and $S=S_p$ are the precise solutions of the Hamilton-Jacobi (eikonal) equation

$$\left(\frac{\partial S}{\partial t}\right)^2-\left(\frac{\partial S}{\partial x}\right)^2-\left(\frac{\partial S}{\partial y}\right)^2=1. \tag{0.6}$$

Therefore solutions $u=u_b$ and $u=u_p$ can be considered as complex ray expansions of the solutions of (0.1) at $h\to 0$ consisting only of the zeroth order term. If $\varepsilon>0$ is fixed and $\varkappa\to+\infty$, the asymptotics of solutions are as follows

$$u_b \sim C_b\exp\left(i\frac{(kx-\omega t)}{h}-\frac{y^2}{h\Delta_y^2}\right), \qquad |t+x|\ll\varepsilon, \tag{0.7}$$

$$u_p \sim C_p\exp\left(i\frac{(kx-\omega t)}{h}-\frac{(x-vt)^2}{h\Delta_x^2}-\frac{y^2}{h\Delta_y^2}\right), \qquad |t|\ll\varepsilon, \tag{0.8}$$

where $C_b=(-i\varepsilon)^{1/2}$, $C_p=i\exp\left(-2\varkappa\varepsilon/h\right)2\varkappa\varepsilon$ and

$$k = \varkappa - \frac{1}{4\varkappa}, \qquad \omega = \sqrt{k^2+1}, \qquad v = \frac{\partial \omega}{\partial k}, \tag{0.9}$$

$$\Delta_x = \frac{2\sqrt{\varepsilon\varkappa}}{\omega}, \qquad \Delta_y = \sqrt{\frac{\varepsilon}{\varkappa}}.$$

The formula (0.8) is valid under the additional assumption that we choose such a branch root in (0.3) that $\operatorname{Im} S_p > 0$. Thus u_b is localized near the axis $y = 0$ and behaves like a beam. The solution u_p is localized near the point moving along the axis $y = 0$ with the speed v and behaves like a particle. The parameter $\varkappa$ governs the degree of localization of both u_s and u_p as it can be seen from the evident asymptotics $\Delta_x \sim 2\Delta_y$ if $\varkappa \gg 1$. Because of this the greater is $\varkappa$ the less are Δ_x and Δ_y and the stronger becomes the localization. It should be mentioned that the beam-like solution u_b can be viewed as a reference one for the Maslov asymptotic construction [1]. Large-time asymptotics of u_b, u_p are also gained. The construction of these solutions is based on the beam-like [2], [3] and particle-like [4] solutions of the wave equation belonging to the class of the so called "relatively undistorted progressing waves" introduced by Courant and Gilbert [5]. We thank Prof. A.P. Kiselev for his interest in our work and for the stimulating discussion. The work is supported by the RFBR grant $\mathcal{N}$ 00-01-00485.

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Признаки устойчивости в критических случаях

Предлагаются различные подходы к установлению устойчивости в критических случаях для линейных и нелинейных систем разностных или дифференциальных уравнений. Предполагается, что квадратная матрица (оператор) A удовлетворяет условиям: $AX = XS$, $YA = TY$, $YX = 0$, $\mathrm{sp}S \cap \mathrm{sp}T = \varnothing$, где X и Y - прямоугольные матрицы максимального ранга, S и T — квадратные матрицы простой структуры с чисто критическими спектрами (sp служит для обозначения спектра).

Первый подход основан на изучении нормы $\|A + \Delta A\|$ (она должна быть меньше единицы) в дискретном случае или логарифмической нормы lgn $\|A + \Delta A\|$ (она должна быть отрицательной) в непрерывном случае, где $\Delta A \equiv XU + VY$ - произвольное допустимое возмущение. Отметим, что Im $X \subseteq$ kernel Y, что даёт возможность построить фактор-пространство kernel Y/Im X; здесь Im X - подпространство, натянутое на столбцы матрицы X, а kernel Y - ядро матрицы Y. Фактор-оператор $R \equiv (A|\mathrm{kernel}\, Y)/\mathrm{Im}\, X$, действующий в фактор-пространстве kernel Y/Im X, называется остаточным; здесь $A|$kernel Y - сужение оператора A на подпространство kernel Y.

Второй подход основан на непосредственном вычислении (или оценке) нормы или логарифмической нормы остаточного оператора и использовании соотношений: spr $R \leqslant \|R\| < 1$ или spa $R \leqslant$ lgn $\|R\| < 0$ (spr - спектральный радиус и spa - спекральная абсцисса).

Третий подход основан на оценках спектрального радиуса и спектральной абсциссы, вытекающих из локализационных теорем Гершгорина и Островского. Основная трудность - доказательство простоты элементарных делителей, отвечающих критическим собственным значениям, - преодолевается с помощью полученных недавно оценок элементов обратных матриц в условиях критериев регулярности Адамара, Брауэра и др. Работа выполнена при поддержке РФФИ (грант N0001-00664).

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Ground state asymptotics for singularly perturbed locally periodic operators

Given a smooth bounded domain $Q \subset R^n$, we consider a family of eigenproblems

$$A^\varepsilon p = \lambda p, \qquad p \in H_0^1(Q), \tag{1}$$

for singularly perturbed locally periodic elliptic operators of the form

$$A^\varepsilon = -\varepsilon^2 \frac{\partial}{\partial x_i} a_{ij}(x, \frac{x}{\varepsilon}) \frac{\partial}{\partial x_j} + c(x, \frac{x}{\varepsilon}),$$

and study the asymptotic behaviour of its ground state (the principal eigenfunction and eigenvalue) as $\varepsilon \downarrow 0$. Our assumptions are as follows: the coefficients $a_{ij}(x, z)$ and $c(x, z)$ are $[0,1]^n$-periodic in z sufficiently smooth functions, the matrix $\{a_{ij}(x, z)\}$ is uniformly positive definite. In order to formulate one more condition, we identify periodic in z functions with functions on the torus T^n, and consider an auxiliary eigenproblem

$$\mathcal{A}(x)q \equiv -\frac{\partial}{\partial z_i} a_{ij}(x, z) \frac{\partial}{\partial z_j} q + c(x, z)q = \mu q, \qquad z \in T^n; \tag{2}$$

here x appears as a parameter. Denote by $\mu(x)$ the principal eigenvalue in the latter problem. We suppose that $\mu(x)$ has only one global minimum point in $\bar{Q}$ which is attained at an interior point of Q, and that the quadratic form in the Taylor series for $\mu(x)$ about the minimum point, does not degenerate. We use the following notation: $p_0^\varepsilon(x)$ is the normalized principal eigenfunction in problem (1), $\|p_0^\varepsilon\|_{L^2(Q)} = 1$, x_0 is the global minimum point of $\mu(x)$, and $q_0(z)$ is the principal eigenfunction in (2) at $x = x_0$.

Theorem. *Under the above assumptions the eigenfunction $p_0^\varepsilon(x)$ admits the asymptotics*

$$p_0^\varepsilon(x) \sim C_\varepsilon q_0(\frac{x}{\varepsilon}) \exp\Big(-\frac{D(x - x_0) \cdot (x - x_0)}{\varepsilon}\Big),$$

where D is a positive definite constant matrix, and C_ε is a normalizing constant. The estimate holds

$$\Big\| p_0^\varepsilon - C_\varepsilon q_0(\frac{x}{\varepsilon}) \exp\Big(-\frac{D(x - x_0) \cdot (x - x_0)}{\varepsilon}\Big)\Big\|_{L^2(Q)} \leqslant c\sqrt{\varepsilon}.$$

This is a joint work with G.Allaire.

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On the solvability of some nonlinear systems of elliptic equations.

Setting by Ω a bounded connected open set of $\mathbb{R}^N$ with locally Lipschitz boundary, we study the existence of solutions to the following problems :

$$\begin{aligned} -\Delta_p u_1 + \lambda_1 |u_1|^{p-2} u_1 &= |a_1 u_1 + a_2 u_2|^{\gamma-2}(a_1 u_1 + a_2 u_2) a_1 + f_1 \\ -\Delta_p u_2 + \lambda_2 |u_2|^{p-2} u_2 &= |a_1 u_1 + a_2 u_2|^{\gamma-2}(a_1 u_1 + a_2 u_2) a_2 + f_2 \end{aligned} \quad \in \Omega, \qquad (1)$$

$u_1 = u_2 = 0$ on $\partial\Omega$,

$$\begin{aligned} -\Delta_p u_1 - \lambda_1 |u_1|^{p-2} u_1 &= |a_1 u_1^2 + a_2 u_2^2|^{\frac{\gamma}{2}-1} a_1 u_1 - b_1 |u_1|^{\gamma-2} \\ -\Delta_p u_2 - \lambda_2 |u_2|^{p-2} u_2 &= |a_1 u_1^2 + a_2 u_2^2|^{\frac{\gamma}{2}-1} a_2 u_2 - b_2 |u_2|^{\gamma-2} \end{aligned} \quad \in \Omega, \qquad (2)$$

$u_1 = u_2 = 0$ on $\partial\Omega$.

The functions a_i, b_i are essentially bounded in Ω and b_i is nonnegative, $f_j \in W^{-1,p'}$, λ_j are real parameters.

The range of exponents is $1 < p < \gamma < p^*$ ($p^* = Np/(N-p)$ if) $N > p$, $+\infty$ otherwise in the case (1), $1 < p < \frac{\gamma}{2} < p^*$ in the case (2).

The analysis of the problems starts from the comparison between the parameters λ_j and the first eigenvalue of the p-Laplacian operator. Under suitable assumptions it was shown that there exists at least one solution (u_1, u_2) with $u_j \geqslant 0$ and in some particular cases it was obtained the existence of multiple solutions.Our approach is based on the Lagrange multipliers and, more generally, on the "fibering method" introduced by Pohozaev a few years ago.

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Stokes and Navier–Stokes problems in domains that are layer–like at infinity

The Stokes problem is studied in a domain Ω which, outside a ball, coincides with three-dimensional layer $\{x \in \mathbb{R}^3 : x = (y, z) \in \mathbb{R}^2 \times (0, 1)\}$. Asymptotic formulae for solutions are derived. In order to justify the asymptotic expansions the procedure of dimension reduction is employed together with estimates of solutions in a certain weighted function space $\mathcal{D}^l_\beta(\Omega)$ with the norm determined by a stepwise anisotropic distribution of weight factors (the direction z is distinguished). The smoothness exponent l is allowed to be a positive integer, and the weight exponent β is an arbitrary real number. It is shown that the Stokes problem is of Fredholm type for all β except for the integer set $\mathbb{Z}$ where the Fredholm property is lost. Dimensions of kernel and co-kernel of the Stokes operator are calculated in dependence of β. It turns out, that at any admissible β, the operator index does not vanish. Based on generalized Green formula, asymptotic conditions at infinity are imposed to provide the problem with index zero. Weak solutions to stationary Navier-Stokes problem with either finite or infinite Dirichlet integral are proved to exist. Solutions to Navier-Stokes problem that drives a nonzero flux over cylindrical sections of the layer are found. It is shown that for arbitrary large data such solutions have the same asymptotics at infinity as that of solutions to the linear Stokes problem. Results concerning the linear Stokes problem and the existence of weak solutions to the nonlinear Navier-Stokes problem are obtained jointly with S.A. Nazarov.

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Usual, inverse, and weak shadowing properties

Consider a dynamical system generated by a diffeomorphism f of a closed smooth n-dimensional manifold M. Let dist be a Riemannian metric on M. Denote by $N(a, A)$ the a-neighborhood of a set A and by $O(p)$ the trajectory of a point p in the dynamical system f. We say that a sequence $\xi = \{x_k \in M\}$ is a d-pseudotrajectory of f if the inequalities $\mathrm{dist}(f(x_k), x_{k+1}) < d$ hold. We say that a point $p \in M$ ϵ-shadows the pseudotrajectory ξ if the inequalities

$$\mathrm{dist}(f^k(p), x_k) < \epsilon \tag{0.1}$$

hold. The usual shadowing property of the system f is formulated as follows: given $\epsilon > 0$ there exists $d > 0$ such that any d-pseudotrajectory of f is ϵ-shadowed by a point p. We say that a family T_d of d-pseudotrajectories is a d-method for f. A family $\Theta = \{T\}$ of methods is called a class. We say that a system f has the inverse shadowing property with respect to a class Θ if, given $\epsilon > 0$, there exists $d > 0$ such that, for any point $p \in M$ and for any method $T_d \in \Theta$, there exists a d-pseudotrajectory $\xi = \{x_k\} \in T_d$ satisfying inequalities (0.1). We say that a system f has the first (second) weak shadowing property if, given $\epsilon > 0$, there exists $d > 0$ such that, for any d-pseudotrajectory $\xi = \{x_k\}$, there exists a point p with the property $\xi \subset N(\epsilon, O(p))$ (with the property $O(p) \subset N(\epsilon, \xi)$, respectively). In the talk, we discuss relations between the introduced shadowing properties and the properties of structural stability and Ω-stability. We show that the usual shadowing property, the first weak shadowing property, and the second weak shadowing property are related to qualitatively different characteristics of the dynamical system.

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Inverse Sturm-Liouville Problem on a Simple Graph

The spectrum of small vibrations of a graph consisting of three joint smooth stringswith the free ends fixed can be reduced to the Sturm-Liouville boundary problem on the graph.The spectrum of such a problem is investigated in comparison with the union of spectra ofDirichlet problems on the rays of the graph. It is shown that the spectra interlace in some sense, thus an analogue of Sturm theorem is established. The inverse problem of recoveringthe potentials from the given spectra is solved.

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On the Neumann problem for hyperbolic systems in a wedge

A class of hyperbolic systems including the dynamical equations of elasticity is considered in a wedge. The original problem is reduced to a problem with parameters in a cone. Solutions are estimated in "combined" weighted norms. Near the vertex of the cone (the edge of the wedge) the estimates are coercive elliptic; far from the singularities of the boundary they become hyperbolic. This allows to derive the asymptotics of solutions near the vertex (the edge). Formulas are given for the coefficients in the asymptotics (that are functions on the edge depending on time). Sharp estimates on the coefficients are obtained in Sobolev's norms. The result obtained allow to establish similar facts for problems with variable coefficients in an arbitrary domain with smooth edges on the boundary.

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Existence of an Abel basis consisting of root functions for some nonlocal problem

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Lebesgue-petrovskii derivatives in sturm-liouville problem

Oscillation spectral properties (the number of eigenfunction zeros and interchanging of zeros) of the Sturm-Liouville problem for the equation

$$-(pu')' + Q'u = \lambda M'u \quad (a \leqslant x \leqslant b) \tag{1}$$

with generalized coeffitients appear to be unaccessible in the frames of the distribution theory. But if p, Q и M are the functions of bounded variation then for a certain strictly positive $\mu(x)$ the equation (1) is equivalent to the following

$$-\frac{d}{d\mu}(pu') + \left(\frac{dQ}{d\mu}\right)u = \lambda\left(\frac{dM}{d\mu}\right)u \tag{2}$$

where $\frac{d}{d\mu}$ denotes the μ-derivation in the Radon-Nikodim sence. The basis of the analysis of respective derivatives $\frac{df}{d\mu}$ was made by A.Lebesgue and I.G.Petrovskii for the case of continuous $\mu(x)$. The technique of the derivation of respectively discontinuous function was introduced by Feller who defines the operation $Lu = -\frac{d}{dM}u'_+$ (where u'_+ denotes the right derivative). The same technique was used by M.G.Krein, B.S.Kats and others for the analysis of the equation connected to the Stiltjes string: the equation $-\frac{d}{dM}u'_+ = \lambda u$ means the symbolic denotion of the integrodifferential one

$$u'_+(x) = u'_-(a) - \lambda \int_a^{x+0} u(s)dM(s).$$

The form (2) of the equation (1) allows to consider the usual Sturm-Liouville conditions as the relization of the equation (1) at the endpoints of the interval $[a, b]$ and to establish for the corresponding spectral problem (using the natural modifications of traditional methods of ODE) the complete list of classical oscillation (Tchebyshev) spectral properties which are similar to harmonic properties of the usual string.

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An approach to Fučik spectrum

medskip The elliptic boundary value problem with jumping nonlinearity

$$\begin{cases} -\Delta u - \lambda_+ u^+ + \lambda_- u^- = 0, \quad x \in \Omega, \\ u\Big|_{\partial\Omega} = 0, \end{cases} \tag{1}$$

where $u^{\pm} \equiv \max\{\pm u, 0\}$, $\Omega \subset R^N$ is bounded domain, posed by S. Fučik [1], is considered. The suggested approach to the study of the spectrum, that is the set of pairs (λ_+, λ_-), when the problem (1) has nontrivial solutions, is based on the construction of the special functional $\lambda(u)$, which can be referred to a spectral parameter. Furthermore for any point $\Lambda(\lambda_+, \lambda_-)$ let's define the set

$$M(\Lambda) \equiv \left\{ u \in W_2^{1,0}(\Omega) : \ \|\nabla u\| = 1, \ \lambda_+ \|u^+\|^2 - \lambda_- \|u^-\|^2 \geqslant 1 \right\}.$$

The application of the Morse - Palais - Smale theory allows us to recieve the following results.

T h e o r e m 1. On any connected component ρ of the resolvent set Re of the problem (1) the homotopic type of the set $M(\Lambda)$, $\Lambda \in \rho$, is constant [3].

C o r o l l a r y 1. Let the points A and B belong to the set R. Let the sets $M(A)$ and $M(B)$ are homotopically unequivalent, $M(A) \not\approx M(B)$. Then on any continuous curve, connecting the points A and B, there are some points of spec trum of the problem (1).

Let us consider the linear problem along with problem (1)

$$\begin{cases} -\Delta u - \lambda u = 0, \quad x \in \Omega, \\ u\Big|_{\partial\Omega} = 0, \end{cases} \tag{2}$$

obtained from (1) when $\lambda_+ = \lambda_-$. The following result is also proved.

T h e o r e m 2. Let $\lambda_n < c < \lambda_{n+1}$, where λ_k is the eigenvalue of the problem (2). Then the set $M(C)$ for the point $C(c, c)$ has the homotopic type of the $(n-1)$ - dimensional sphere.

Formulated assertions allow us to establish for the problem (1) the existence of points of spectrum in the domains more wide in comparison with the well known ones [2].

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On location of spectrum of a mixed boundary value problem in a square

In [1-3] there were studied spectral problems in two-dimensional domains for equation $\Delta u + \lambda u = 0$ under boundary conditions with a slope derivative. In [1-2] one considered mixed boundary value problems in domains with a special boundary, and one proved that under some limitations on the domain boundary the spectrum lies in the Carleman parabola; in [3] there was investigated a spectral problem in a circle with the slope derivative given on the entire boundary, and it was proved that the spectrum does not lie in the Carleman parabola. Here we pose a mixed boundary value problem in a square. Let D be square with vertices $O(0;0)$, $A(\pi;0)$, $B(\pi;\pi)$, $C(0;\pi)$. We consider there the following spectral problem:

$$u_{xx} + u_{yy} + \lambda^2 u = 0,$$

$$u\Big|_{CO} = u\Big|_{OA} = u\Big|_{AB} = 0,$$

$$u_x - ku_y\Big|_{BC} = 0, \quad k \neq 0, \quad k \in R,$$

where $u = u(x,y) \in \mathrm{C}\left(\bar{D}\right) \bigcap \mathrm{C}^2\left(D\right)$. Main theorem: the spectrum of the problem lies in the Carleman parabola $|\mathrm{Im}\lambda| \leqslant const$. The proof is based on application of the Fourier transform and reduction to a conjugate problem for a pair of piece-analytic functions and then to a singular integral equation on a finite segment, which has no nontrivial solutions if $|\mathrm{Im}\lambda|$ is large enough.

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Topology of decomposable real plane algebraic curves

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О нормальной форме элемента свободной алгебры Сташефа

В алгебраической топологии часто встречаются результаты, описывающие строение тех или иных колец, групп и т.п., связанных с топологическими объектами. Имеется принципиальное отличие результатов, так называемого, классического периода алгебраической топологии от результатов последнего времени. Это отличие состоит в характере используемых алгебраических структур. Например, известно (см. [?]), что кольцо комплексных бордизмов Ω_*^U является кольцом многочленов с целыми коэффициентами от образующих, расположенных по одной в каждой четной размерности. Такое описание кольца Ω_*^U позволяет получить окончательный ответ на любой вопрос, касающийся алгебраического устройства кольца Ω_*^U. Результаты статей [?, ?, ?] относятся к результатам последнего времени. В этих статьях было получено описание когомологий алгебры Стинрода в терминах алгебр Сташефа, которые представляют из себя в достаточной степени изощренный алгебраический объект. Это описание не позволяет, как в предыдущем примере, сразу ответить на вопрос какова размерность, скажем, 17-й группы когомологий. Именно необходимость дополнительной работы отличает эти результаты от классических. Однако эти результаты позволяют вычислять когомологии алгебры Стинрода гораздо быстрее и с много меньшими затратами ресурсов по сравнению с вычислением, которое использует стандартную резольвенту алгебры Стинрода. Предварительно следует построить теорию алгоритмических вычислений в алгебрах Сташефа, аналогичную теории вычислений в ассоциативных алгебрах. В докладе мы ответим на некоторые вопросы, связанные с использованием упомянутого описания для точного вычисления когомологий алгебры Стинрода, и обозначим несколько проблем. В частности, предлагается определение записи элемента свободной алгебры Сташефа и нормальной формы записи. Предлагается алгоритм, который по произвольной записи данного элемента строит запись в нормальной форме. Доказывается теорема о том, что среди записей данного элемента существует и единственна запись в нормальной форме.

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Нули функций Миттаг-Леффлера и обратные задачи

Функции Миттаг-Леффлера $E_\rho(z,\mu)$ – целые функции комплексного переменного z с параметрами $\rho > 0$ и $\mu \in \mathbb{C}$ – определяются как суммы степенных рядов $E_\rho(z,\mu) = \sum\limits_{k=0}^{\infty} z^k/\Gamma(\mu+k/\rho)$. Они находят различные приложения в дифференциальных уравнениях (см. [1,2]). В статье [3] возник вопрос об описании множества W, состоящего по определению из всех пар положительных чисел (ρ,μ) таких, что функция $E_\rho(z,\mu)$ имеет в $\mathbb{C}$ только вещественные и отрицательные простые нули. Этот вопрос приобрел дополнительную актуальность в связи с найденным недавно И.В.Тихоновым и Ю.С.Эйдельманом критерием единственности решения одного класса обратных задач для линейных автономных дифференциальных уравнений

$$\frac{d^N u(t)}{dt^N} = Au(t) + p, \quad 0 \leqslant t \leqslant 1, \quad n \in \mathbb{N}, \tag{0.1}$$

в произвольном банаховом пространстве $\mathcal{E}$ (здесь A – заданный линейный замкнутый оператор с областью определения $\mathfrak{D}(A) \subset \mathcal{E}$). Теорема И.В.Тихонова – Ю.С.Эйдельмана состоит в том, что уравнение (1) относительно неизвестных $p \in \mathcal{E}$ и $u : [0,1] \to \mathfrak{D}(A)$ имеет при условиях $u(0) = u'(0) = \cdots = u^{N-1}(0) = u(1) = 0$ лишь тривиальное решение $u(t) \equiv 0$ и $p = 0$ тогда и только тогда, когда ни один нуль функции $\chi_N(z) \equiv E_{1/N}(z, N+1)$ не является собственным числом оператора A (цитированный результат опубликован с полным доказательством в [4] пока лишь для $N = 1$). Нули функции $\chi_1(z) = (e^z - 1)/z$, $\chi_2(z) = (ch\sqrt{z} - 1)/z$ тривиально находятся, но при $N \geqslant 3$ множество нулей $\chi_N(z)$, по-видимому, не допускает явного описания. Асимптотики нулей функций Миттаг-Леффлера при всех значениях параметров были найдены в [5,6]. В [3] была высказана гипотеза, состоящая в том, что $\mathcal{W} = \{(\rho, \mu) | 0 < \rho \leqslant 1/2,\ 0 < \mu < 1 + 1/\rho\}$; интересующий нас случай $\mu = 1 + 1/\rho$ исследован не был. Автором доказано, что эта гипотеза неверна и множество $\mathcal{W}$ шире, чем предполагалось в [3].

Теорема. *Справедливо включение* $\{(\rho, \mu) | 0 < \rho < 1/2,\ 0 < \mu < 2/\rho - 1\} \subset \mathcal{W}$. *В частности, все нули функции* $\chi_N(z)$ *при* $N \geqslant 3$ *лежат на луче* $(-\infty, -(2N)!/N!)$ *и просты.*

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Asymptotics of bound states and bands for waveguides and layers coupled through small windows

Spectral problem for the Dirichlet Laplacian in two-dimensional strips (three-dimensional layers) of widths d_+, d_-, $d_+ > d_-$ coupled through small openings $a\omega_i, i = 1, ...n$, (a is small parameter) is considered. Method of matching of asymptotic expansions of solutions of boundary value problem is used. The asymptotics (in a) of a bound state λ_a close to the threshold π^2/d_+^2 is obtained:

$$\lambda_a = \begin{cases} \frac{\pi^2}{d_+^2} - (\frac{\pi^3}{d_+^3}\sum_{i=1}^n c_{\omega_i})^2 a^4 + o(a^4), & \mathbf{R}^2, \\ \frac{\pi^2}{d_+^2} - (\frac{1}{d_+^2}\exp(-\frac{2d_+^3}{3\pi^2}(\sum_{i=1}^n b_{\omega_i})^{-1}a^{-3}(1+o(1))), & \mathbf{R}^3. \end{cases}$$

Here c_{ω_i} is a capacity of ω_i in $\mathbf{R}^2$, b_{ω_i} is an average virtual mass of ω_i in $\mathbf{R}^3$. The case of two identical waveguides (layers) is considered too. Asymptotics of bands for the case of periodic system of coupling windows (period L) for two waveguides is constructed:

$$[\frac{\pi^2}{d_+^2} - \frac{3\pi^3}{2Ld_+^3}a^2 + o(a^2), \frac{\pi^2}{d_+^2} - \frac{\pi^3}{2Ld_+^3}a^2 + o(a^2)], \quad \mathbf{R}^2.$$

$$[\lambda_1^+ - \frac{18\pi(\psi_1^{+0})^2 b_\omega}{L}a^3 + o(a^3), \lambda_1^+ - \frac{6\pi(\psi_1^{+0})^2 b_\omega}{L}a^3 + o(a^3)], \quad , \mathbf{R}^3.$$

Here λ_1^+ is the first transversal eigenvalue, ψ_1^{+0} is the value of the normal derivative of the corresponding transversal eigenfunction at the centre of the opening. There is a gap for suffisiently small a. For the case of layers coupled through singly (Λ_1) and doubly (Λ_2) periodic system of windows there is no gap, and the asymptotics of the lower bound for the continuous spectrum is:

$$\lambda_{min} = \frac{\pi^2}{d_+^2} - \frac{\pi^6 b_\omega^2}{Ld_+^6}a^6 + o(a^6), \quad \text{for} \quad \Lambda_1,$$

$$\lambda_{min} = \frac{\pi^2}{d_+^2} - \frac{\pi b_\omega|\hat{\Lambda}|}{d_+^3}a^3 + o(a^3), \quad \text{for} \quad \Lambda_2,$$

where $|\hat{\Lambda}|$ is a square of the Brillouin zone for Λ_2.

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On nondegenerated differential-geometric Poisson brackets of the third order

The homogeneous differential-geometric Poisson brackets of the order n define a certaine geometric structure on the manifold M with local coordinates u. The classification of such brackets is the nontrivial differential-geometric problem even on condition that the leading coefficient is nondegenerate. It is known the complete classification of such Poisson brackets for n=1 and n=2 and the conditions for Poisson brackets of the third order . The main new result is the construction of a special flat conection(with nonzero torsion) defined in the flat coordinates of the so-called "last" connection. Investigating the geometry of this connection, author prove that Doyle's anzatz actually provides a general form of the Poisson brackets in question. Some examples of algebras associated with brackets of the third order are considered. It is demonstrated how known examples fit into the sceme proposed.

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Nonlinear Models in Elasticity and Hydrodynamics

We want to study the motion of viscous, incompressible fluid in a tube with elastic wall, in particular the flow of blood in arteries. A simplified two-dimensional model is introduced and it is shown that, under appropriate assumptions on the data, a local existence theorem holds. The model considered may be taken as a first approximation of more sophisticated models, in particular in 3 dimensions. The proof is based on the Tychonov fixed point theorem.

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A nonlocal problem for hyperbolic equations

Consider the equation

$$Lu \equiv u_{xy} + (Au)_x + (Bu)_y + Cu = f(x,y,u) \qquad (0.1)$$

in the rectangular domain $D = \{(x,y) : 0 < x < a,\ 0 < y < b\}$ and pose the problem for (0.1) with the following non-local conditions:

$$\int_0^a u(x,y)dx = 0, \quad \int_0^b u(x,y)dy = 0. \qquad (0.2)$$

We define the function space $\tilde{H}^1(D)$ as the completion of the set $U = \{u|u \in C^1(D),\ u_{xy} \in C(D),\ \int_0^a udx = 0,\ \int_0^b udy = 0\}$ with respect to the norm $||u||_1^2 = \int\int_D (u^2 + u_x^2 + u_y^2)dxdy$. Let operator $l : \tilde{H}^1 \to L_2$ be defined by

$$lv = \int_0^y \int_0^x v(t,\tau)dtd\tau - \int_0^y v_x(x,\tau)d\tau - \int_0^x v_y(t,y)dt.$$

Applying integration by parts we perform $(Lu, lv)_0$, $u, v \in U$, and resulting expression denote by $B(u,v)$.

Definition. *A function $u(x,y) \in \tilde{H}^1(D)$ is called a generalized solution of the problem (0.1)-(0.2), if for every $v(x,y) \in \tilde{H}^1(D)$ $B(u,v) = (f, lv)_0$.* In order to prove the solvability of the problem (0.1)-(0.2), we at first establish a priori estimates for the corresponding linear problem.

Lemma. *If the coefficients of $Lu = f(x,y)$ and their first order derivatives are bounded, moreover $|A| \leqslant M < 1$, $|B| \leqslant M < 1$, C_{xy} is bounded in D and $C_{xy} \geqslant 0$, $A_yB_x - C^2 \geqslant 0$, $MA_y - 2(A_x + C)^2 \geqslant 0$, $MB_x - (B_y + C)^2 \geqslant 0$, then there exist $c_1 > 0$, $c_2 > 0$, such that $c_1||u||_1^2 \leqslant B(u,u)$, $||u||_1 \leqslant c_2||f||_0$.* Using a priori estimates established in Lemma and Schauder's fixed point theorem we have

Theorem. *Let $f(x,y,u) \in L_2(D)$ for every $u \in \tilde{H}^1(D)$, $|f(x,y,u_1) - f(x,y,u_2)| \leqslant L_0|u_1 - u_2|$, $|f(x,y,u)| \leqslant \frac{1}{\sqrt{2}}(|p(x,y)| + \sqrt{c_1^2 - \eta^2}|u|)$, where $p(x,y) \in L_2(D)$, $0 < \eta \leqslant c_1^2$, and the conditions of Lemma hold. Then* *there exists at least one generalized solution to the problem (0.1)-(0.2) such that* $\|u\|_1^2 \leqslant \frac{\|p\|_0^2 ab}{\eta^2}$. *If* $L_0 < c_1$, *then this solution is unique.*

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The proof of quantum chaos conjecture and the distribution of distances between neighboring fractional parts of the polynomial values

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Boundary Value Problems for degenerate operator-differential equations

We study boundary value problems for the operator-differential equations

$$Mu = B(t)u_t - L(t)u = f, \quad t \in (0,T), \quad T \leqslant \infty, \tag{1}$$

where $B(t)$ and $L(t)$ $(t \in (0,T))$ are linear operators in a Hilbert space E endowed with the inner product $(\cdot,\cdot)$ and the norm $\|\cdot\|$. We do not assume that B is invertible; in particular, B may have a nontrivial kernel. Equations of this type arise in physics, geometry, population dynamics, and in some other fields. The main assumptions for the operators $L(t)$ and $B(t)$ are as follows. I. *There exists a Hilbert space* H_1 *densely embedded in* E *such that* $L(t) \in L_\infty(0,T;L(H_1;H_1'))$ *and* $B(t) \in W_\infty^1(0,T;L(H_1,H_1'))$ *(i.e., without loss generality, we may assume that* $B(t) \in C([0,T];L(H_1,H_1'))$*). The operators* $B(0)$ *and* $B(T)$ *(if* $T < \infty$*) are selfadjoint in* E. *The operators* $B(t)$ *are symmetric on* $(0,T)$, *i.e.* $(B(t)u,v) = (u,B(t)v)$ *for* $u,v \in H_1$; H_1 *is densely embedded into* $D(|B(0)|^{1/2})$ *and into* $D(|B(T)|^{1/2})$. II. *There exists a constant* $\delta > 0$ *such that*

$$\mathrm{Re}\Big(\Big(-L(t) - \frac{1}{2}B_t(t)\Big)u,u\Big) \geqslant \delta\|u\|_{H_1}^2$$

for $u \in H_1$ *almost everywhere on* $(0,T)$. Let $E^\pm(0)$ and $E^\pm(T)$ be the spectral projections of $B(0)$ and $B(T)$ corresponding to the positive and the negative parts of the spectrum. Equation (1) is furnished with the boundary conditions

$$E^+(0)u(0) = u_0^+, \quad \lim_{t\to\infty} u(t) = 0 \quad (T = \infty), \tag{2}$$

$$E^+(0)u(0) = h_{11}E^-(0)u(0) + h_{12}E^+(T)u(T) + u_0^+,$$

$$E^{-}(T)u(T) = h_{21}E^{-}(0)u(0) + h_{22}E^{+}(T)u(T) + u_T^{-} \quad (T < \infty), \qquad (3)$$

where h_{ij} are bounded operators in the corresponding spaces.

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Some odd thin sets of integers in Harmonic Analysis (joint work with D.Li and L.Rodriguez-Piazza)

Relying on results of F. Piquard and J.Bourgain, we randomly construct subsets Λ of the integers which have both smallness and largeness properties. On the one hand, they are small because they are very close in some sense to Sidon sets : the continuous functions with spectrum in Λ have a uniformly convergent Fourier series, and the sequence of their Fourier coefficients belongs to ℓ_p f

or every $p > 1$; moreover, all the Lebesgue spaces L^q_Λ are the same for $0 < q < \infty$. On the other hand, they are big because they are dense in the Bohr group of the integers, and because the space of bounded functions with spectrum in Λ is non-separable. So, those sets are very different from the sets of integers previously known in this area.

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A Spectrum of Vladimirov Operator on the Group of Finite Adels.

Let $P = \{2, 3, ..\}$ be a set of all prime numbers, $\mathbb{Q}_p$ be a field of p-adic numbers with p-adic morm $|\cdot|_p$, $\mathbb{Z}_p$ be a ring p-adic integers, $S(\mathbb{Q}_p)$ be a set of locally constant functions with compact supports. The set of sequences $x = (x_2, x_3, ..., x_p, ...)$ ($x_p \in \mathbb{Q}_p$, every $x_p \in \mathbb{Z}_p$ except for a finite number) is called an adels group $\mathbb{A}$. A base of neighbourhoods 0 in $\mathbb{A}$ is given by subgroups: $\prod_{p\in\pi} V_p \times \prod_{p\notin\pi} \mathbb{Z}_p$, where V_p is a neighbourhood 0 in $\mathbb{Q}_p$ for all π, where π is a finite subset of P. The set $\mathbb{A}$ with such topology is totally disconnected and locally compact Abel topological group. Any character of $\mathbb{A}$ has a form $\chi_0(\xi x) = \exp 2\pi i(\sum_p \{\xi_p x_p\}_p (mod 1))$. There is a Haar measure dx on $\mathbb{A}$, that is connected with measures on $\mathbb{Q}_p$ by $dx = dx_2 \cdot dx_3 \cdot ... \cdot dx_p \cdot ....$ Spaces $L_2(\mathbb{A})$ and $L_1^{loc}(\mathbb{A})$ are defined by standard way. We consider on $\mathbb{A}$ functions $\phi(x) = \prod_{p\in P} \phi_p(x_p)$, such that 1) $\phi_p \in S(\mathbb{Q}_p)$; 2)

$\phi_p(x_p) = \Delta_{\mathbb{Z}_p}(x_p)$ are characteristic functions of $\mathbb{Z}_p$ for all p with the exception of a finite number. Vector space of all linear combinations of such functions is a space of Schwartz–Bruhat functions $S(\mathbb{A})$ that is dense in $L_2(\mathbb{A})$. We define a spectral topology on $S(\mathbb{A})$, in that the space is a separable, complete, nuclear and locally convex. Dual space $S'(\mathbb{A})$ is called a space of distributions on $\mathbb{A}$. Fourier transform of functions from $S(\mathbb{A})$ is given by

$$(F\phi)(\xi) \equiv \hat{\phi}(\xi) = \int_{\mathbb{A}} \phi(x)\chi_0(-\xi x)dx$$

and this is an isomorphism $F : S(\mathbb{A}) \to S(\mathbb{A})$. **Theorem 1.** *Let* $\xi = (\xi_2, \xi_3, ...) \in \mathbb{A}$, $\alpha = (\alpha_2, \alpha_3, ...) \in \mathbb{R}^\infty$ *be a multiindex. Formula* $|\xi|^\alpha = \prod_{p\in P} |\xi_p|_p^{\alpha_p}$ *defines a function from* $L_1^{loc}(\mathbb{A}) \setminus L_2(\mathbb{A})$ *if* $\lim_{p\to\infty} \alpha_p \ln p = 0$. (*) Denote by V_α (if assumption (*) is true) Vladimirov operator that is defined by

$$(V_\alpha\phi)(x) = \int_{\mathbb{A}} |\xi|^\alpha \hat{\phi}(\xi)\chi_0(-\xi x)d\xi, \forall\phi \in S(\mathbb{A}).$$

Theorem 2. *Operator* V_α *is an essentially self-adjoint in* $L_2(\mathbb{A})$. *Its closure* $\overline{V_\alpha}$ *has a range of definition*
$D(\overline{V_\alpha}) = \{\phi \in L_2(\mathbb{A}) : |\xi|^\alpha\hat{\phi}(\xi) \in L_2(\mathbb{A})\}$ *and* $\sigma(\overline{V_\alpha}) = \mathbb{R}_+$.

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О радиусе голоморфности функции $F(zA)f$ в терминах расстояния вектора f от векторов конечной степени оператора A

Везде далее A – линейный оператор, действующий в банаховом пространстве X, $R(\lambda)$ – его резольвента, а $\mathfrak{D}_\infty$ – пересечение областей определений операторов A^j при $j \in \mathbb{N}$. Введем $\mathfrak{G}_\xi := \{g \in \mathfrak{D}_\infty : \|A^j g\| \leqslant c(g)\xi^j, j \in \mathbb{N}\}$ – множество векторов степени не выше $\xi > 0$ относительно оператора A и $E_\xi(f) := \inf\{\|f - g\| : g \in \mathfrak{G}_\xi\}$ – расстояние элемента f из X от $\mathfrak{G}_\xi$. Для целой функции F зададим $M(\xi) := \max\{|F(z)| : |z| = \xi\}$, $\xi \geqslant 0$, и G – обратную к M функцию. Если $f \in \mathfrak{D}_\infty$, то определен формальный степенной ряд $F(zA)f$, радиус голоморфности которого обозначим через $r(f)$. В следующей теореме установлены соотношения между $r(f)$ и числом

$$s(f) := \liminf_{\xi\to\infty} \frac{1}{\xi} G\left(\frac{1}{E_\xi(f)}\right).$$

Теорема. *Пусть*

$$\limsup_{\xi\to\infty} \frac{(\ln\xi)^2}{\ln M(\xi)} < \infty \qquad (1)$$

и существует такая возрастающая последовательность положительных чисел $\{\zeta_m\}_{m\in\mathbb{N}}$, *что*

$$\lim_{n\to\infty} \frac{\zeta_{m+1}}{\zeta_m} < \infty, \qquad (2)$$

почти все точки окружностей $\{\lambda : |\lambda| = \zeta_m\}$, $m \in \mathbb{N}$, *принадлежат резольвентному множеству оператора* A *и*

$$\limsup_{m\to\infty} \int_0^{2\pi} \ln^+ \left(\frac{M(\zeta_m/\eta)}{M(c_0\zeta_m/\eta)} \|R(\zeta_m e^{i\theta})\| \right) d\theta < \infty \qquad (3)$$

при некотором $c_0 > 1$ *и всех* $\eta > 0$.

Тогда найдется такая постоянная $c > 1$, *что* $c^{-1}s(f) \leqslant r(f) \leqslant cs(f)$ *для всех* $f \in \mathfrak{D}_\infty$. *Если же левые части в неравенствах (1) и (3) равны нулю, причем в (3) при произвольных* $c_0 > 1$ *и* $\eta > 0$, *а левая часть в (2) равна 1, то* $r(f) = s(f)$ *для всех* $f \in \mathfrak{D}_\infty$.

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Conserved Quantities And Entropy In General Relativity

The notion of entropy of exact solutions of General Relativity and, more generally, of gauge covariant field theories, is reviewed. A definition resembling the Clausius formulation of classical gas thermodynamics is considered and analyzed by an extensive use of the geometrical framework for field theories as well as Nöther theorem. This new definition of entropy applies in particular to all covariant theories of gravitation, in any dimension and signature. A complete correspondence with Brown-York original formulation of the first principle of black hole thermodynamics is finally established.

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On character of bifurcation of exponential stability

It is known that the Lyapunov exponents of linear differential system depending even smoothly on parameter may be discontinues functions of this parameter.

This lack of continuity can occur for every value of the parameter in a certain interval (see [1]). Such behavior of Lyapunov exponents can reduce to a violation of stability on the dense set of values of parameters on a some interval. The purpose of given studies is to determine the possibility of similar behavior for exponential stability of linear differential system depending linearly on a parameter and establish the most typical events. We consider the linear system depending on the parameter ω

$$x' = \omega A(t)x, \quad x \in R^n, \quad \omega \in [0,1], \tag{0.1}$$

where $A(.) : R \to Hom(R^n, R^n)$ is a bounded continuous mapping. We say that the value of parameter $\omega \in [0,1]$ is the bifurcation point of exponential stability of the system (1), if in any its neighborhood the values exist for which there is the property of exponential stability, and values for which it is absent.

Theorem 1. *If $n > 1$ then a system (1) exists such that each value of parameter $\omega \in [0,1]$ is the bifurcation point of exponential stability .*

Theorem 2. *Let any value of parameter $\omega \in [0,1]$ of system (1) is the bifurcation point of exponential stability. Then the set of such ω , for which the system is exponentially stable, is denumerable.*

Theorem 3. *Let the system (1) is exponential stabile almost everywhere on $[0,1]$ (in the sense of Lebesgue measure) . Then the set of parameters values where exponential stability can be broken is not more, than denumerable nowhere dense set on [0,1].* Proofs of theorems 2,3 are based on assertion of theorem 3* in [2].

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Analog of Bitsatze-Samarskii problem for mixed-type equation with degeneracy of the second kind

Consider the equation

$$u_{xx} + sgny|y|^m u_{yy} = 0 \qquad (0 < m < 1) \tag{1}$$

in the domain D, bounded by lines AA_∞ $(x = 0)$, BB_∞ $(x = 1)$ $y \geqslant 0$ and characteristics

$$AC: \ \xi = x - \frac{2}{2-m}(-y)^{\frac{2-m}{2}} = 0, \qquad BC: \ \eta = x + \frac{2}{2-m}(-y)^{\frac{2-m}{2}} = 1$$

of the equation (1), issuing from points $A(0,0)$ и $B(1,0)$. Let us introduce the following notations:

$$J \equiv AB,$$

$$D_1 = D \cap \{(x,y) : y > 0\},$$

$$D_2 = D \cap \{(x,y) : y < 0\},$$

$$\Theta_0(x) = \frac{x}{2} - (\frac{1}{1-2\beta}\frac{x}{2})^{1-2\beta}, \qquad \beta = \frac{m}{2(m-2)}, \qquad -\frac{1}{2} < \beta < 0.$$

$$(I_{0+}^{\alpha,\beta,\eta}\varphi)(x) = \begin{cases} \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)} \int\limits_0^x (x-t)^{\alpha-1} F(\alpha+\beta, -\eta; \alpha; 1-\frac{t}{x})\varphi(t)dt, \\ \qquad (0 < x < 1,\ \alpha > 0,\ \beta,\eta \in C) \\ (\frac{d}{dx})^n (I_{0+}^{\alpha+n,\beta-n,\eta-n}\varphi)(x), \\ \qquad (0 < x < 1,\ \alpha < 0,\ \beta,\eta \in C,\ n = [-\alpha]+1) \end{cases} \text{—}$$

— generalized operator of fractional integro-defferentiation in meaning of M.Saigo [1, pp.135-136], (see also [2, pp.326-327], [3, pp.14-15]).

Analog of Bitsatze-Samarskii problem. Find a function

$$u(x,y) \in C(\overline{D}) \cap C^1(D_1 \cup J) \cap C^1(D_2 \cup J) \cap C^2(D_1 \cup D_2),$$

satisfying the equation (1) in all of the domains D_i, $i = 1,2$ ans the boundary conditions

$$u(0,y) = u(1,y) = 0, \qquad 0 \leqslant y < \infty,$$

$\lim\limits_{y\to+\infty} u(x,y) = 0$ unformly with respect to $x \in \overline{J}$,

$$u(x,+0) = u(x,-0), \qquad u_y(x,+0) = -u_y(x,-0),$$

$$A(I_{0+}^{a,-a-\beta,\beta-a}(\frac{d}{dt}u[\Theta_0(t)]))(x) = B(I_{0+}^{a-\beta}u_y(t,-0))(x) + b(x), \qquad x \in J,$$

where $(I_{0+}^{\alpha}\varphi)(x)$ — is a Riemann–Liouville operator, $b(x)$ — given function, continious on $[0,1]$, a — a real number, $0 < a - \beta < 1$, A и B — certain real constants, such that A, $B > 0$ or A, $B < 0$. The unique solvability of the posed problem is proved.

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A posteriori estimates of the accuracy of variational methods for elliptic type variational inequalities

Mathematical models related to variational inequalities often arise in applied sciences. Existence and differentiability properties of their solutions were analyzed by many authors. Also, much efforts has been directed toward creating numerical methods for variational inequalities. We are focused on the problem, which is important for verifying the accuracy of numerical approximations of variational inequalities. Let u be the exact solution of a problem and v be an admissible function from the "energy" functional space V of the considered problem. Our goal is to derive a functional defined by v and by the given data of our problem that measures the deviation of $v - u$ in the norm of V. This functional must be nonnegative and vanish if and only if v coincides with exact solution u. Besides, it must be explicitly computable and possess proper continuity properties. We present a new method of deriving such type majorants of the deviation. It can be regarded as an extension of the duality technique earlier used for getting a posteriori error majorants in variational problems with uniformly convex functionals (see, e.g., [1,2]). The performance of the above method is demonstrated for three classical problems related to variational inequalities:
(a) problem with an obstacle,
(b) elasto–plastic torsion problem,
(c) a problem with friction type boundary conditions. Properties of the majorants and practical implementation of the proposed techniques are discussed.

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Differential equations for some recurrent Polynomials

We consider the factorization method, introduced by the physicists L. Infeld and T.E. Hull, in the fifties, and recalled in the classical book of W. Miller Jr.: "Lie Theory and Special Functions", in order to find the differential equation satisfied by some special functions. This method, under the name of "monomiality principle", has been recently used by Dr. G. Dattoli (E.N.E.A. - Frascati, Italy) and his school, in connection with the solution of problems in the field of quantum optics. In our approach, the factorization method can be applied to the general case of hypergeometric and confluent hypergeometric functions, including consequently the most part of special functions occurring in applications. As an application, we construct the differential equations satisfied by some recurrent polynomials: the Appel polynomials (including the Bernoulli and Euler polynomials), and lastly the 2-orthogonal polynomials, $B_n^{\alpha,\beta}(x) = {}_1F_2(-n; 1+\alpha, 1+\beta; x)$, which are related to Bateman's functions.

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Newton-Kantorovich method for conformal representation of plane domains

Recently (see [1]) a new approach to the study of conformal representation of a simply connected domain was proposed. It was based on the system of integral-functional equations which was analysed in [1] in the framework of Schauder spaces. In our report we show that reparametrization $g_{\zeta,w}^{-1} \circ \zeta$ of a curve ζ can be computed by applying the Newton-Kantorovich method to the above system. Here $g_{\zeta,w}$ is the normalized Riemann mapping of the unit disc onto the domain encircled by the curve ζ.

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Romanov A.V.

On Alternative Conceptions of Reducing of Nonlinear Parabolic Equations to Ordinary Differential Equations

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On a class of globally smooth solutions to Euler equations in several dimensions

Consider a system of gas dynamics with a forcing term $\mathbf{f}$ in $\mathcal{R} \times \mathcal{R}^n$, $n \geqslant 1$ for unknown functions $M(t,\mathbf{x})$, $\mathbf{V}(t,\mathbf{x})$, $S(t,\mathbf{x})$ (the density, velocity vector and entropy, correspondingly) of the form

$$M(\partial_t \mathbf{V} + (\mathbf{V}, \nabla)\mathbf{V}) + \nabla P = M\mathbf{f}(\mathbf{x}, t, \mathbf{V}, M, P),$$

$$\partial_t M + div\,(M\mathbf{V}) = 0, \qquad \partial_t S + (\mathbf{V}, \nabla S), \tag{E}$$

where the pressure $P = e^S M^\gamma$, $\gamma = const > 1$, $\mathbf{f}$ is a smooth in all its arguments function. It is well known that solutions to the Cauchy problem for the system may lose the initial smoothness, often there is a possibility to estimate the time on the singularity formation from above (see, f.e. [1] and references therein). Nevertheless it is interesting to find some nontrivial classes of globally in time smooth solutions. The class is not empty. In [3] (a generalization of [2]) for $\mathbf{f} = 0$ it was proved the following. Let $\bar{\mathbf{V}}(t,\mathbf{x})$ be a globally smooth solution to $\partial_t \mathbf{V} + (\mathbf{V}, \nabla)\mathbf{V} = 0$ such that initially the specrum of the solution Jacobian is uniformly bounded away from the real negative numbers. If for the initial data $(M_0, \mathbf{V}_0, S_0)$ the Sobolev norm

$$\|M_0^{(\gamma-1)/2}, \mathbf{V}_0 - \bar{\mathbf{V}}(0,\mathbf{x}), S_0\|_{H^m(\mathcal{R}^n)}, \; m > 1 + n/2$$

is sufficiently small then the solution to the corresponding Cauchy problem will be globally smooth as well. We announce a supplement of the result.

Theorem. *Suppose $(\bar{M}(t,\mathbf{x}), \bar{\mathbf{V}}(t,\mathbf{x}), \bar{S}(t,\mathbf{x}))$, is a globally in time smooth solution to system (E) and $\bar{\mathbf{V}}(t,\mathbf{x}) = A(t)\mathbf{r}$ with a matrix $A(t)$, $det A(t) \neq 0$ ($\mathbf{r}$ is the raduis-vector of point). If the norm*

$$\|(M_0^{(\gamma-1)/2} - \bar{M}^{(\gamma-1)/2}(0,\mathbf{x}), \mathbf{V}_0 - \bar{\mathbf{V}}(0,\mathbf{x}), S_0 - \bar{S}(0,\mathbf{x})\|_{H^m(\mathcal{R}^n)}$$

is sufficiently small then the Cauchy problem with initial data $(M_0, \mathbf{V}_0, S_0)$ for system (E) has a unique solution $(M, \mathbf{V}, S)$ such that $(M^{(\gamma-1)/2} - \bar{M}^{(\gamma-1)/2}$, $S - \bar{S}$, $v_i - \bar{v}_i$, $i = 1,..n) \in \cap_{j=0}^m C^j([0,\infty); H^{m-j}(\mathcal{R}^n))$, $m > 1 + n/2$. In some physically important cases one can construct explicitly the exact smooth solutions $(\bar{M}, \bar{\mathbf{V}}, \bar{S})$ with the convenient properties. Partially supported by the grant of RFBR no. 00-02-16337.

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The solvability of a nonlinear degenerate differential equation

The Cauchy problem $A(t)u'(t) + B(t)u(t) = f(t,u)$, $u(0) = u_0$ is considered. Here $A(t)$, $B(t)$ are closed linear operators from a Banach space X into another Banach space Y and $f : [0,\tau] \times S \to Y$ is a continuously differentiable mapping (S is an open ball in X). Generally, $KerA(t) \neq \{0\}$ for all $t \in [0,\tau]$. The domain D of the sheaf $L(\lambda, t) = \lambda A(t) + B(t)$ is independent of t. For all $d \in D$, the operator functions $A(t)d, B(t)d$ are continuously differentiable on $[0,\tau]$. The main assumption reads: the point $\mu = 0$ is a simple pole of the resolvent $(A(t)+\mu B(t))^{-1}$ for all t. Under compatibility conditions on the initial vector $u_0 \in D \cap S$, the vector $f(0, u_0)$ and the Fréchet derivative $\frac{\partial f}{\partial u}(0, u_0)$ there exists a solution of the Cauchy problem on a non-trivial segment. To prove the result, the original equation is transformed into a non-degenerate differential equation and an operator one. The transform is realized with the help of two families of spectral projectors $P(t) = \frac{1}{2\pi i} \oint\limits_{\gamma(t)} L^{-1}(\lambda, t) d\lambda A(t)$ and $Q(t) = \frac{1}{2\pi i} \oint\limits_{\gamma(t)} A(t) L^{-1}(\lambda, t) d\lambda$ in the spaces X and Y respectively. The projectors and the corresponding spectral decompositions of the sheaf $L(\lambda)$ were introduced in [1] for time-independent operators A, B. The time-dependent projectors $P(t), Q(t)$ were considered in [2]. The degenerate equation with time-dependent operators A, B was investigated in [3]. In the linear case (f is independent of u) conditions for the existence and uniqueness of the solution on the whole segment $[0,\tau]$ are indicated. The results are applied to the problem of the liquid filtration in fractured porous rocks. The equation for the liquid pressure $u(t,x)$ is the following: $(\zeta - \Delta)\left(a(t,x)\frac{\partial u(t,x)}{\partial t}\right) - (c(t)\Delta - b(t))u(t,x) = f(t,x,u)$. We shall restrict ourselves to the case of the filtration in layered rocks. Then we have a one-dimensional equation ($0 \leqslant x \leqslant \pi$) with Dirichlet boundary conditions. If $\zeta = -1$, the equation is degenerate. For an initial function $u_0(x) \in C^2[0,\pi]$ such that $u_0(0) = u_0(\pi) = 0$ and the compatibility conditions hold $\int\limits_0^\pi \left[\frac{\partial f}{\partial u}(0, x, u_0(x)) - \gamma\right] a^{-1}(0,x) \sin^2 x dx \neq 0$,

$\int_0^\pi [f(0,x,u_0(x)) - \gamma u_0(x)] \sin x dx = 0$ there exists a solution of the filtration equation. Here $\gamma = b(0) + c(0)$.

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Method of difference potentials

The basic construction of the method — the difference potential — comprises some features of the classical Cauchy–type integral with the universality and effectivity of finite difference schemes. This allows to use the methods for the numerical modelling and solution of a number of problems of mathematical physics. In the report on the model example one is acquainted with method main concepts. Current state of the method is reflected in [1,2].

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Homogeneous and divergent equations of plane potential flows and related topics

Consider the homogeneous equations of potential compressibale plane flows

$$q_N - q\theta_L = 0, \qquad q_L(M^2-1) - q\theta_N = 0, \tag{1}$$

where q is the absolute value of velocity vector, θ is the angle of velocity vector, M is Mach number, q_L, θ_L, q_N and θ_N are derivatives taken along the streamlines and them normals. We esteblish that for the angle φ_q between the line $q = const$ and the velocity vector and for the angle φ_θ between the line $\theta = const$ and the velocity vector the following equation holds

$$\tan\varphi_q \tan\varphi_\theta = (M^2 - 1)^{-1} \tag{2}$$

If $M > 1$, then Mach angle $W = \arcsin M^{-1}$ exists and the following formula holds

$$\tan\varphi_q \tan\varphi_\theta = \tan^2 W$$

Hence *the geometric mean of* $\tan\varphi_q$ *and* $\tan\varphi_\theta$ *equals tangent of Mach angle.* Relation (2) is useful for study topological properties of mixed (subsonic and supersonic) flows. The line $\varphi_q = \varphi_\theta = W$ is called a *branch line.*

Definition. *We say that equations of the plane or axially symmetric flows with the dependent variables f and g are similar to equations (1) if*

$$\tan\varphi_f \tan\varphi_g = (M^2 - 1)^{-1}.$$

The examples of the similar equations of the plane vortex and potential and axially symmetric potential flows are given in this work. In the second part, we demonstrate an algorithm for transformation of system (1) to infinitely number of independent divergent forms

$$(A_i(q,\theta))_x + (B_i(q,\theta))_y = 0, \quad i = 1, 2, \ldots.$$

This algorithm uses the exact solutions of Chaplygin equation

$$k\psi_{\theta\theta} + \psi_{zz} = 0,$$

where ψ is stream function, ρ is density, $z = \int(\rho/q)dq$, $k = k(z) = (1 - M^2)\rho^{-2}$.
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The Tricomi type problem for the mixed type equation with nonsmoth line of degeneration

We consider the equation

$$Lu \equiv \operatorname{sgn} y|y|^m u_{xx} + x^m u_{yy} = 0, \quad m > 0, \tag{1}$$

in bounded domain D with boundary consisting for $x, y > 0$ of simple curve Γ with endpoints $A(1,0)$, $B(0,1)$, of segment OB of axis Oy and for $y < 0$ of characterictics OC and CA of equation (1). We consider the next problem for equation (1) in D which was posed in [1].

Problem TN. Find a function $u(x,y)$ satisfying the conditions:

$$u(x,y) \in C(\overline{D}) \cap C^1(D \cup OB) \cap C^2(D_+ \cup D_-), \tag{2}$$

$$Lu(x,y) \equiv 0,\ (x,y) \in D_+ \cup D_-, \quad u(x,y) = \hat{f}(x,y),\ (x,y) \in \Gamma, \tag{3}$$

$$u_x(0+0,y) = 0,\ y \in (0,b), \quad u(x,y) = 0,\ (x,y) \in OC, \tag{4}$$

where $D_+ = D \cap \{y > 0\}$, $D_- = D \cap \{y < 0\}$, $\hat{f}$ is given sufficiently smooth function. The uniqueness of solution of problem (2) – (4) was proved in [2]. The existense of solution of problem (2) – (4) was reduced to nonlocal elliptic problem. In case $n = m > 0$ and Γ is $\Gamma_0 : x^{2\alpha} + y^{2\alpha} = 1$, the solution of elliptic problem was built as the sum of series on eigenfunctions of corresponding spectral problem as in [3, 4]. Then the solution of problem TN was built in hyperbolic domain.

Theorem. *Let* $f(\varphi) \in C^1[0,\pi/2]$, $f(\varphi)$ *is a twice continuous-differentiable function in neighbourhoods of the points* $\varphi = 0$ *and* $\varphi = \pi/2$ *and* $f(0) = f'(0) = f(\pi/2) = f'(\pi/2) = 0$, $\Gamma \equiv \Gamma_0$. *Then there is unique solution of problem* TN *in* D *and it is defined by formulas*

$$u(x,y) = \begin{cases} \sum_{n=0}^{\infty} f_n r^{2\rho_n - 2q} \sin^{1/2-q} 2\varphi \, \mathrm{P}^{1/2-q}_{\rho_n - 1/2}(-\cos 2\varphi),\ (r,\varphi) \in D_+, \\ -2^{q+1/2}\sqrt{\pi}\Big(x^\alpha + (-y)^\alpha\Big)^{-2q} \sum_{n=0}^{\infty} \frac{f_n \Gamma(1-q)}{\Gamma(1+\rho_n)\Gamma(q-\rho_n)} \times \\ \times \Big(x^\alpha - (-y)^\alpha\Big)^{2\rho_n} F\left(\rho_n + q, q, 1+\rho_n; \left(\frac{x^\alpha - (-y)^\alpha}{x^\alpha + (-y)^\alpha}\right)^2\right),\ (x,y) \in D_-. \end{cases}$$

where $q = m/(2(m+2))$, $\rho_n = n + q/2 + 1/4$, $\alpha = (m+2)/2$,

$$f_n = \frac{\Gamma(q)\sin(\pi q)}{\sqrt{2\pi}} \int_0^\pi h_n(\theta)\omega(\theta)d\theta,\ h_n(\theta) = \frac{2}{\pi}\frac{(2\cos(\theta/2))^{q-3/2}}{\operatorname{tg}(\theta/2)} \sum_{i=1}^{n} \sin(i\theta) B_{n-i},$$

$$\omega(\theta) = \sin\theta \int\limits_0^\theta \left(f\left(\frac{\pi - t}{2}\right) \sin^{2q-1} t \right)' (\cos t - \cos\theta)^{-q} dt,$$

$$B_l = \sum_{m=0}^{l} (-1)^{l-m} C_1^{l-m} C_{1/2-q}^m, \quad C_l^n = \frac{l(l-1)\cdots(l-n+1)}{n!},$$

$f(\varphi) = \hat{f}(\cos^{1/\alpha}\varphi, \sin^{1/\alpha}\varphi)$, $F(\cdot)$ – *hypergeometric function,* P_ν^μ – *modified Legendre function.*

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On the Caushy problem Schrodinger equation with degeneration on subdomain

We consider the following Caushy problem for degenerating operator:

$$i\frac{\partial u}{dt} = L_\epsilon u \equiv -\frac{\partial}{\partial x}(\epsilon(x)\frac{\partial u}{\partial x}) + \frac{i}{2}(\frac{\partial}{\partial x}(a(x)u) + a(x)\frac{\partial u}{dx}), \quad t > 0, \ x \in R, \qquad (1)$$

$$u(x, +0) = u_0(x), \quad x \in R, \qquad (2)$$

where $\epsilon(x) = \epsilon\theta(x) + 1 - theta(x)$, $\epsilon \in [0, 1]$, и $a(x) = a\theta(x)$, $a \in R$, $\theta(x)$ is the characteristic function of semiaxis $R_+ = (0, +\infty)$. The difficulties of the boundary problems for the equations of different type similar to (1), (2) have been described in the article [1]. We find the maximal domain $D(L_\epsilon)$ of operator L_ϵ and obtain that if $\epsilon = 0$ then the operator $L = L_0$ is degenerate on R_+, is symmetric in the space $L_2(R)$, its spectrum is axe R and it has different defect indexes. But if $\epsilon > 0$ then the regularizing operator L_ϵ is self-adjoint in $L_2(R)$. Definition: The solution of problem (1), (2) is the function $u(t, x) \in C(R_+, L_2(R))$ such that for any $t > 0$ the following equation is correct for any $\phi \in D(L^*)$:

$$(u(t, x), \phi(x)) = (u_0(x), \phi(x)) - i\int\limits_0^t (u(s, x), L^*\phi(x))ds.$$

The following results on the well-posedness of Caushy problem are obtained.
Theorem 1 *Let $a \leqslant 0$ and $u_0(x) \in L_2(R)$. Then there is a unique solution of problem (1), (2).* In the case $a > 0$ there is a condition on the function $u_0(x)$ which is necessary and sufficient to existence and uniqueness of solution of problem (1), (2) (see [1]). We obtain the following statement on the convergence of the sequence of solutions of regularizing problems to the solutions of degenerate problem.
Theorem 2 *Let the conditions of theorem 1 are satisfied. Then for any $\sigma > 0$ there is a function $u_1(x) \in C_0^\infty(R)$ such that for any $T > 0$ there is $\epsilon_0 > 0$ such that for any $\epsilon \in (0, \epsilon_0)$*

$$\sup_{t\in[0,T]} \|u(t,x) - u_{1,\epsilon}(t,x)\|_{L_2(R)} \leqslant \sigma,$$

where $u(t,x)$ is the solution of problem (1), (2) with degenerate operator L and $u_{1,\epsilon}$ is the solution of equation (1) with operator L_ϵ and initial data $u_1(x)$.

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Delta-Subharmonic Functions of Completely Regular Growth in the Half-Plane

A positive increasing function $\gamma(r)$, $r \in [0, \infty)$, is called a function of growth. Kondratyuk A. [1] studied the behavior at infinity of Fourier coefficients

$$c_k(r,f) = \frac{1}{2\pi}\int_0^{2\pi} \ln|f(re^{i\phi})| e^{-ik\phi}\, d\phi, \quad k \in \mathbf{Z},$$

of meromorphic function f of completely regular growth with respect $\gamma(r)$. We prove analogous proposition for δ-subharmonic functions v of finite γ-type on the upper half-plane ($v \in J\delta$). We now define the Fourier coefficients of function $v \in J\delta$ as

$$c_k(r,v) = \frac{2}{\pi}\int_0^{\pi} v(re^{i\phi}) \sin k\phi\, d\phi, \quad k \in \mathbf{N}.$$

The function $v \in J\delta$ is called the function of finite γ-type if there exist constants A and B such that

$$T(r,v) \leqslant \frac{A}{r}\gamma(Br)$$

for all $r > r_0 > 0$, where $T(r,v)$ is the characteristic of Nevanlinna of the function v. The function $v \in J\delta$ is called the function of completely regular growth with respect $\gamma(r)$ if there exists

$$\lim_{r\to\infty} \frac{1}{\gamma(r)} \int_{\eta}^{\varphi} v(re^{i\phi}) \sin\phi \, d\phi \tag{1}$$

for all η and φ from $[0,\pi]$. We denote by $J\delta(\gamma(r))^o$ the class of functions which satisfy (1). By $J\delta(\gamma(r))$, we denote the class of δ-s.h. functions of finite γ-type.
Theorem. *Let v be δ-subharmonic function. The following are equivalent: a) $v \in J\delta(\gamma(r))^o$; b) $v \in J\delta(\gamma(r))$, and there exists*

$$\lim_{r\to\infty} \frac{c_k(r,f)}{\gamma(r)}, \quad for \quad all \quad k \in \mathbf{N};$$

c) the measure $\lambda_-(v)$ has the finite γ-density, and there exists

$$\lim_{r\to\infty} \frac{1}{\gamma(r)} \int_{\eta}^{\varphi} \psi(\phi) v(re^{i\phi}) \sin\phi \, d\phi$$

for all functions ψ from $C[0,\pi]$. The details will be given at the talk.

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Новая оценка приближения решений уравнения Штурма–Лиувилля с аналитическим потенциалом частичными суммами асимптотических рядов

На отрезке $-a \leqslant x \leqslant a$, $a > 0$ рассмотрим дифференциальное уравнение

$$-y'' - q(x)y = \lambda^2 y, \qquad \lambda > 0, \tag{0.1}$$

с потенциалом q, аналитическим в некоторой окрестности отрезка $[-a,a]$. Хорошо известно (см. [1,2]), что решения уравнения (1) $y_0(x,\lambda)$ и $y_1(x,\lambda)$, удовлетворяющие начальным условиям

$$y_0(0,\lambda)=1, \quad y_0'(0,\lambda)=0, \quad y_1(0,\lambda)=0, \quad y_1'(0,\lambda)=\lambda i, \qquad (0.2)$$

разлагаются в формальные ряды, которые являются асимптотическими при $\lambda \to +\infty$:

$$y_j(x,\lambda)\sim\frac{1}{2}\left(e^{i\lambda x}\sum_{k=0}^{\infty}\frac{B_{k,j}(x)}{(-2i\lambda)^k}+(-1)^j e^{-i\lambda x}\sum_{k=0}^{\infty}\frac{B_{k,j}(x)}{(2i\lambda)^k}\right), \quad j=0,1. \qquad (0.3)$$

Коэффициенты рядов (3) вычисляются по рекуррентным формулам. Ряды (3) являются асимптотическими для функций $y_j(x,\lambda)$ в том смысле, что при любых $n\in\mathbb{N}$ справедлива равномерная по $x\in[-a,a]$ асимптотика

$$y_j(q,x,\lambda)=S_{n,j}(q,x,\lambda)+O_{q,n}(\lambda^{-n-1}), \quad (\lambda\to+\infty) \qquad (0.4)$$

с постоянной в символе O, зависящей только от потенциала q и номера n. Через $S_{n,j}(q,x,\lambda)$ здесь обозначена n-я частичная сумма асимптотического ряда (3). Возникает вопрос о возможности приближенного вычисления значений $y_j(q,x,\lambda)$ при $\lambda\geqslant 1$, $x\in[-a,a]$ с помощью асимптотических рядов (3). В работе [3] В. А. Садовничим и А. Ю. Поповым для потенциалов, аналитических в круге $|z|<R$, $R>a$ и удовлетворяющих условию $q(0)=0$, а также для потенциалов, аналитических в некоторой ρ-окрестности отрезка $[-a,a]$, получены оценки для погрешности наилучшего приближения фундаментальной системы решений уравнения (1), удовлетворяющей начальным условиям (2), суммами (6), экспоненциально убывающие с ростом λ.

Цель настоящей работы — улучшить оценку, приведенную в статье [3] для потенциалов, аналитических в $O(\rho,[-a,a])$ — ρ-окрестности отрезка $[-a,a]$, замыкание которой представляет собой объединение двух полукругов $\{|z-a|\leqslant\rho,\ \operatorname{Re}z\geqslant a\}$, $\{|z+a|\leqslant\rho,\ \operatorname{Re}z\leqslant -a\}$ и прямоугольника $\{|\operatorname{Re}z|\leqslant a,\ |\operatorname{Im}z|\leqslant\rho\}$.

Пусть $M_0=\max\left\{\int\limits_0^a|q(t)|\,dt,\ \int\limits_{-a}^0|q(t)|\,dt\right\}$; через $\phi_{n,j}(q,x,\lambda)$ обозначим невязку при приближении решения y_j суммой $S_{n,j}$: $\phi_{n,j}(q,x,\lambda)=y_j(q,x,\lambda)-S_{n,j}(q,x,\lambda)$. Основной результат статьи заключается в следующем.

Теорема 1. *Пусть функция $q(z)$ аналитична в $O(\rho,[-a,a])$ и следующая норма конечна*

$$\max_{\alpha\in[-a,a]}\sum_{n=0}^{\infty}\frac{|q_{n,\alpha}|\rho^{n+1}}{n+1}=M_1<+\infty, \qquad (0.5)$$

где $q_{n,\alpha} = \dfrac{q^{(n)}(\alpha)}{n!}$. Для $\lambda > 0$ положим $N = N(\lambda) = [2\rho\lambda] - 1$. Тогда при $N > 1$ имеем

$$\sup_{\eta\geqslant\lambda}\max_{j=0,1}\max_{x\in[-a,a]}|\phi_{N,j}(q,x,\eta)| \leqslant$$
$$\leqslant \frac{2e}{3}\sqrt{2\pi}aM(2\lambda\rho+2)^{3.5}\exp(M_0/\lambda + M\rho e - 2\rho\lambda), \quad (0.6)$$

где $M = M_1 + M_0$.

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Generalized Bäcklund-Darboux Transformations

The Bäcklund-Darboux transformations (BDTs) are named after the pioneering discoveries by A.V. Bäcklund and G. Darboux. The generalized Bäcklund-Darboux transformation (GBDT) develops this approach. We consider the first order systems $w'(x,\lambda) = G(x,\lambda)w(x,\lambda)$ $(w' = \frac{d}{dx}w)$. The solutions $\tilde{w}$ of the transformed systems $\tilde{w}'(x,\lambda) = \tilde{G}(x,\lambda)\tilde{w}(x,\lambda)$ are connected with the solutions w by the relation $\tilde{w} = w_A w$. The matrix function w_A (so called gauge transformation) is written down for each x in the form of the transfer matrix function $w_A(x,\lambda) = I + C(x)(\lambda I - A)^{-1}B(x)$ that goes back to Kallman. In various examples w_A proves to be a characteristic matrix function by Livšic. The particular form of the transfer matrix function and at the same time a generalization of the characteristic matrix function that is used in GBDT was introduced by L. Sakhnovich. Parameter matrices and the corresponding "generalized" eigenfunctions are used in constructing w_A instead of the eigenvalues and eigenfuctions in the standard BDTs. The transfer matrix function type representation of w_A allows, in particular, to separate the dependence on the variable x and spectral parameter λ. In this way the solutions of the spectral problems and matrix nonlinear equations are expressed in a general and analytically and computationally optimal form. Such a representation proves to be important in the study of the explicit solutions of the spectral and scattering problems and bispectrality also. The GBDT approach was developed, in particular, in the papers [1] and [2] (see also references in [3]).

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On Closed Extensions of Nonlinear Partial Differential Operators

In this report we present a complete metric space consisting of genuine functions (more precisely of classes of equivalent functions) such that very wide class of nonlinear p.d. expressions allow closed extension there. For one of such extension solutions of the corresponding p.d.e. coincide with so-called viscosity (discontinuous) solutions. Let X be a closed domain in $\mathbb{R}^n$, Y be a locally compact metric space with a metric d, $f : X \to Y$ be a map. We say that f belongs to the class S if $\forall x \in X$ $\forall \varepsilon > 0$ $\exists U$ – an open subset of X such that $x \in clU$ (closure of U) and $y \in U \Rightarrow d(f(x), f(y)) < \varepsilon$. On the set of the maps of the class S from X to Y there is the following equivalence relation: $f \sim g$ iff $\exists X'$ – a dense subset of X, such that $f = g$ on X' . Let note by $S(X, Y)$ the set of equivalence classes . If $Y = Y_1 \times \ldots \times Y_n$ is a product of metric spaces than the natural map $S(X,Y) \ni f \mapsto (f_1, \ldots f_n) \in S(X, Y_1) \times \ldots \times S(X, Y_n)$ is bijective. Therefore we will deal with $S(X, \mathbb{R})$, it is a vector space.

Proposition *In every equivalence class $f \in S(X, \mathbb{R})$ there is a unique lower semicontinouos (s.c.) representative f_* and a unique upper s.c. representative f^* and in addition $(f_*)^* = f^*$, $(f^*)_* = f_*$ (* notes lower or upper s.c. envelope). $S(X, \mathbb{R})$ is completely characterized by this property.*

Convergence. $f_i \to f$ iff

$$f_*(x) = \lim \inf_{i \to \infty, y \to x} f_{i*}(y), f^*(x) = \lim \sup_{i \to \infty, y \to x} f_i^*(y)$$

Theorem *There exists a metric in $S(X,\mathbb{R})$ which generates the introduced convergence and makes $S'(X,\mathbb{R})$ a complete metric space*

We study a differential expression $\mathcal{D}$ in $S(X,\mathbb{R})$. If f is differentiable on some dense subset X' of X then the map $x \to \mathcal{D}f(x)$ is well defined on X'. If also there exists F from the class S' such that on X' $F(x) = \mathcal{D}f(x)$ than F determines the unique element from $S(X,\mathbb{R})$ depending only on f. Let us note this element by $\mathcal{D}f$. We construct this way an extension of differential expressions defined by Hamiltonians. This extension dermines a pre-closed operator in $S(X,\mathbb{R})$ that generates a semigroup of continuous transformations. We obtain a compactness criterion in S-spaces and hence, some results on the closure of image and on compactness of the set of solutions of some boundary-values problems.

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"Exact and approached finite-dimensional reductions in analysis of functionals of variational calculation"

Investigation of extremals of smooth functional in smooth Banach manifold often can be reduced to similar problem of analysis of extremals of key function (in a finite-dimensional manifold of key parameters) [1]. Through this function it is conveniently to introduce all topological and analytic notions that characterize in some way the type of extremals (the multiplicity, the local ring of the singularity, the versal deformation, the bifurcation diagram etc.) and also it is possible to realize application of elements of a Morse theory. For example, it is conveniently to reprezent bif—decompositions (allowable sets of Morse extremals, bifurcated from complicated critical points) by characteristic CW—complexes, in which the dimension of cells are equal to Morse indexes of appropriate critical points of key function and adjoining cells correspond to adjoining critical points (as stationary points of gradient dynamic system). For spreaded in theory of crystals singularity of n—dimensional pleat type (defined by a quartic part of Taylor decomposition of key function) the rather complete lists of bif—decompositions had been received at $n \leqslant 3$ [2] - [3]. The characteristic complexes can be received on the basis of diverse exact and approached reduction schems. Exact reductions result to uniform class of equivalent complexes, and approached reductions "catch" a type of a characteristic complex only at fulfilment of some auxiliary conditions [4].

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Sturm–Liouville operators with potentials–distributions. Generalized Gelfand–Levitan trace formula.

It was shown in [1] that the Sturm-Liouville operator $Ay = y'' + q(x)y$ with certain boundary conditions on a finite or infinite interval can be well defined if $q(x)$ belongs to the Sobolev space H_2^{-1}, i.e. $q(x)$ coincides with the derivative in the distributional sense of a function $u(x) \in L_2$. In the case when $u(x)$ is a function of bounded variation the corresponding operator was defined by M.Krein and M.Kaz and independently by F.Atkinson. In this particular case we are able to establish a modification of Gelfand-Levitan trace formula [2] which is well known for smooth potentials.

Denote by λ_n the eigenvalues of the operator $Ay = y'' + q(x)y$ with Dirichlet boundary conditions on the interval $[0, \pi]$. We prove the following interesting identities (see [3]).

Theorem 1 *Let $u(x)$ be a function of bounded variation, continuous at the end points 0 and π, and $u(\pi) - u(0) = 0$. Let the potential $q(x)$ be defined by the identity $q(x) = u'(x)$ which is understood in the distributional sense. Denote*

$$b_k = \frac{1}{\pi}\int_0^\pi \cos kx\, du(x), \quad k = 1, 2, \ldots$$

Then

$$\sum_{n=1}^{\infty}(\lambda_n - n^2 + b_{2n}) = -\frac{1}{8}\sum_j h_j^2,$$

where h_j are the jumps of $u(x)$.

Theorem 2 *Let in the addition to the conditions of the previous theorem there exist the left and the right derivatives of $u(x)$ at the points 0 and π correspondingly, i.e.*

$$u(x) = xu'(0) + o(x), \quad u(\pi - x) = -xu'(\pi) + o(x) \quad npu\ x \to +0.$$

Then the series $\sum \lambda_n - n^2$ is summable by the Cesaro method of order 1 and

$$(C,1)\sum_{n=1}^{\infty} \lambda_n - n^2 = -\frac{u'(0) + u'(\pi)}{4} - \frac{1}{8}\sum h_i^2.$$

In particular, if $u(x)$ is absolutely continious in neighbourhoods of the points 0 and π, and the function $q(x) = u'(x)$ is continuous in these points, then $\sum \lambda_n - n^2$ is summable by the Cesaro method of order 1 and its sum is equal to $-(q(0) + q(\pi))/4$.

This result goes along with the Gelfand-Levitan formula.

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Tartar Equation In Homogenization Of Problem On Motion Of Small Asymptotic Mixture

Fulfilling a homogenization procedure for a differential equation, say $\mathcal{P}_\varepsilon u_\varepsilon = 0$, where $\mathcal{P}_\varepsilon$ is a differential operator which contains some small parameter ε, researchers often face the problem on how to find effective coefficients in homogenized equation which appears as $\varepsilon \to 0$. This problem in many cases is very complicated due to the circumstance that one should deal with a product of two weakly convergent sequences of functions. To this end, the powerful methods of formal asymptotic representations and two-scale convergence were created. However, they work only in cases when coefficients in $\mathcal{P}_\varepsilon$ have some ordered structure, such as periodic or quasi-periodic. So, what can be done if these properties are absent? In the present work, the new approach for treating homogenization problems is introduced. We consider

Problem A (on nonstationary Stokes-type flow of small asymptotic mixture provided with oscillatory initial data). The mixture fills a bounded container $\Omega \subset \mathbb{R}^2$. In $Q_T = \Omega \times (0,T)$ ($T = \text{const} > 0$) it is necessary to find $\vec{v} \in L_2(0,T; J_0^1(\Omega))$, $\vec{v}^{(0)} \in L_2(0,T; J_0^{2+\alpha}(\Omega))$, and $\nu \in L_\infty(Q_T)$ satisfying in distribution sense the following system.

$$\partial_t \vec{v} - div_x(2\nu\mathbb{D}(\vec{v})) = \vec{f}, \quad \partial_t \nu + \vec{v}^{(0)} \cdot \nabla_x \nu = 0, \quad \partial_t \vec{v}^{(0)} - a_0 \Delta_x \vec{v}^{(0)} = \vec{f},$$
$$\vec{v}|_{t=0} = \vec{v}_0^{(0)} + \lambda \vec{v}_{0\varepsilon}^{(1)}, \quad \nu|_{t=0} = a_0 + \lambda b_{0\varepsilon}, \quad \vec{v}^{(0)}|_{t=0} = \vec{v}_0^{(0)}.$$

Here, $\vec{f} \in L_2(0,T; J_0^{2\alpha}(\Omega))$, $\alpha \in (0,1/2)$, $\vec{v}_0^{(0)} \in J_0^{1+\alpha}(\Omega)$, $\vec{v}_{0\varepsilon}^{(1)} \in J(\Omega)$, $-c_- \leqslant b_{0\varepsilon} \leqslant c_+$ in Ω, $\vec{v}_{0\varepsilon}^{(1)} \underset{\varepsilon\to 0}{\longrightarrow} \vec{v}_0^{(1)}$ in $J(\Omega)$. Constants a_0, c_-, c_+ admit the condition $a_0 - c_- > 0$. Small parameter $\varepsilon > 0$ characterize oscillation effects in initial data, and small parameter $\lambda > 0$ indicates that these oscillations have small amplitudes in neighbourhood of some smooth state of a fluid which is described by the velocity field $\vec{v}_0^{(0)}$ and viscosity distributions $\nu_0^{(0)} \equiv a_0$. Due to standard bounds on solutions $\vec{v}_\varepsilon(\lambda)$ and $\nu_\varepsilon(\lambda)$ of Problem A, after extraction of proper subsequences one has $\vec{v}_\varepsilon \underset{\varepsilon\to 0}{\longrightarrow} \vec{v}_*$ weakly in $L_2(0,T; J_0^1(\Omega))$, $\nu_\varepsilon \underset{\varepsilon\to 0}{\longrightarrow} \nu_*$ weak-star in $L_\infty(Q_T)$, and, in view of H-convergence theory, $\nu_\varepsilon \mathbb{D}(\vec{v}_\varepsilon) \underset{\varepsilon\to 0}{\longrightarrow} \mathbb{M}_* : \nabla_x \vec{v}_*$ weakly in $L_2(Q_T)$. Here, $(\mathbb{M}_* : \nabla_x \vec{v}_*)_{ik} = \sum_{j,l=1}^{2} M_*^{ijkl} \partial_j v_l$. $\mathbb{M}_* = \mathbb{M}_*(\vec{x}, t, \lambda)$ cannot be derived explicitly directly from the limiting transition because neither $\vec{v}_\varepsilon$ nor ν_ε do not possess any ordered structure in the sense mentioned above. Thus, the system of homogenized equations (let us call it SYSTEM H) does not compose the closed model because it contains too many unknown functions. In order to complete the system, we utilize the notions of H-measure [1] and Tartar equation. Namely, we introduce H-measure μ_t associated with the sequence $\{\nu_\varepsilon\}$, and derive the explicit representation for $\mathbb{M}_*$ in the form $\mathbb{M}_* : \nabla_x \vec{\varphi} = \Lambda \mathbb{D}(\vec{\varphi}) + \mathbb{D}(\vec{\varphi})\Lambda$, $\Lambda = \mathbb{I}\nu_* + \lambda^2 a_0^{-1} W(\mu_t) + O(\lambda^3)$, where $\vec{\varphi}$ is a smooth test function, $W = W(\vec{x}, t)$ is 2x2-matrix which is defined by μ_t solely. Next, we establish the evolutionary *Tartar equation*

$$\partial_t \mu_t + \vec{v}^{(0)} \cdot \nabla_x \mu_t + \partial_y (\mu_t Y : \nabla_x \vec{v}^{(0)}) = 0.$$

Here, $Y = Y(y)$ is 2x2-matrix with smooth given components. H-measure μ_t is the unique solution of Cauchy problem for Tartar equation provided with Cauchy data $\mu_t|_{t=0} = \mu_0$, where H-measure μ_0 is associated with $\{\nu_{0\varepsilon}\}$. Thus, System H together with representation for $\mathbb{M}_*$ and Tartar equation compose the desired model. *The work was supported by Russian Fund for Basic Researches. Grant code 0001-00911.*

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Schłomiuk D.

Some algebro-geometric aspects of planar polynomial vector fields.

In this lecture we focus our attention on some global problems about low degree polynomial systems and show how the use of some algebro-geometric concepts provide us with a good framework for some chart independent studies of families of these differential systems. We also discuss some integrability criteria. Connections of these concepts with the algebraic invariant theory of differential systems will be made.

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A stable time discretization of the Stefan problem with surface tension

We present a time discretization for the single phase Stefan problem with Gibbs-Thomson law. The aim is to obtain a discretization in physical variables that is stable, that is, it satisfies a priori bounds independent of the time-step size. The method resembles an operator splitting scheme with an evolution step for the temperature distribution and a transport step for the dynamics of the free boundary. The evolution step only involves the solution of a linear equation that is posed on the old domain. The linearity of this step makes the scheme useful for a numerical implementation. We prove that the proposed scheme is stable in function spaces of high regularity. In the limit $\Delta t \to 0$ we find strong solutions of the continuous problem. This proves consistency of the scheme and it additionally yields a new short-time existence result for the continuous problem.

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Hausdorff Geometry of Polynomials

Problems from the Geometry of polynomials concerning

Conjecture 1 *If all the zeros of the polynomial* $p(z) := \prod_{k=1}^{n}(z - z_k)$, $(n \geqslant 2)$ *lie in the unit disk* $D(0,1) = \{z : |z| \leqslant 1\}$, *then for every* z_k *the disk* $D(z_k, 1)$ *contains at least one zero of* $p'(z)$;

are discussed. Conjecture 1 is formulated as an estimate of the Hausdorff deviation of the set of the zeros from the set of the critical points of a polynomial. Some new conjectures in Hausdorff geometry of polynomials are formulated.

Serdyukova S.I.

On numerical solution of $u_t = u_x - \mu\delta(x)sin(u)$.

An asymptotic expression for numerical solution is constructed. This implies that for the numerical solution to approximate the solution of the original problem we have to perform the computation with a fictitious μ^*.

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Условия квазирегулярности и неквазирегулярности операторов, порожденных системами сингулярных квазидифференциальных выражений.

На полуоси $I = [0, \infty)$ рассмотрим систему квазидифференциальных выражений $\ell_1[y] = -(q_0(x)y_2')' + q_1(x)y_1 + q(x)y_2$, $\ell_2[y] = -(q_0(x)y_1')' + q(x)y_1 + q_2(x)y_2$, в которой $y = (y_1(x), y_2(x))$ — двухкомпонентная вектор-функция, а функции q_0^{-1}, q_1, q_2 и q локально суммируемы на I минимальный замкнутый симметрический оператор, порожденный этой системой в пространстве $L^2(I)$ двухкомпонентных вектор-функций. Положим $q_-(x) = -\min(q(x), 0)$, $\sigma(x_1, x_2) = \int_{x_1}^{x_2} \{q_0(x)\}^{-1}\, dx$. Следствием теорем для более общих операторов, которые будут изложены в докладе, является, в частности, следующая

Теорема. *Пусть существует последовательность попарно непересекающихся конечных интервалов $I_n = (a_n, b_n) \subset I$ $(n = 1, 2, \ldots)$, такая, что q_0 положительна п.в. на $\bigcup_n I_n$, q_0^{-2} суммируема на каждом I_n, и выполняется одно из двух условий: 1) п.в. на $\bigcup_n I_n$ $q(x)$ неотрицательна, $q_1(x)$ и $q_2(x)$ либо обе неотрицательны, либо обе неположительны, и расходится хотя бы один из четырех рядов*

(A) $\sum_{n=1}^{\infty}\{\int_{I_n}\{q_0\}^{-2}dx\}^{-1}\{\sigma(a_n, b_n)\}^3$,

$\sum_{n=1}^{\infty}\int_{I_n}\{q_0\}^{-2}dx\}^{-1}\int_{I_n}\{\sigma(a_n, x)\sigma(x, b_n)\}^2\eta(x)\, dx$, где $\eta(x) = q(x), |q_1(x)|$ *или* $|q_2(x)|$; *2) на $\bigcup_n I_n$ $q_1(x), q_2(x)$ существенно ограничены либо обе снизу, либо обе сверху, ряд (A) расходится и имеет место неравенство*

$$\int_{I_n} \sigma(a_n, x)\sigma(x, b_n)q_-(x)\, dx \leqslant \sigma(a_n, b_n)$$

$(n = 1, 2, \ldots)$. Тогда **L** *неквазирегулярен.*

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Recent Regularity Results for Weak Solutions to the Three-Dimensional Navier-Stokes Equations

We are going to discuss recent results on regularity of the so-called suitable weak solutions to the non-stationary Navier-Stokes equations in dimension three. In particular, we show that the Caffarelli-Kohn-Nirenberg condition is valid near the boundary.

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Seiberg–Witten Equations and Complex Abrikosov Strings

We discuss a relation between the Seiberg-Witten equations (SW-equations, for short) and their 2-dimensional analogue — the vortex equations. There exists a correspondence, proposed by Taubes, between solutions of SW-equations on a 4-dimensional compact symplectic manifold and pseudoholomorphic curves, lying on it. This correspondence is established in the following way. We plug into the equations a scale parameter λ (note that SW-equations are not scale invariant). Then for $\lambda \to \infty$ one of the components of the SW-solution tends to become pseudoholomorphic (w.r. to a fixed almost complex structure, compatible with the symplectic form), while the other component tends to zero. The zeros of the first component converge to a pseudoholomorphic curve. Moreover, the SW-equations themselves for $\lambda \to \infty$ reduce to a family of vortex equations, defined in the normal planes to the pseudoholomorphic curve. Conversely, given a pseudoholomorphic curve on a 4-dimensional compact symplectic manifold and a family of solutions of vortex equations in the normal planes to this curve, one can construct data for the SW-equations on the manifold. These data provide an approximate solution of SW-equations for $\lambda \to \infty$ iff the family of vortex solutions satisfies a non-linear equation, which is a complex analogue of the equation for Abrikosov strings from the superconductivity theory.

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Old problems and recent progress in the KAM theory

The KAM theory named after its founders A.N.Kolmogorov, V.I.Arnol'd, and J.K.Moser studies quasi-periodic motions in non-integrable dynamical systems. During the last decade and a half, a considerable and versatile progress in this theory has been achieved. The talk will be devoted to some particular developments closely connected with the context of the original 1954 Kolmogorov theorem. The following topics will be discussed. A) Weak nondegeneracy conditions. A completely integrable Hamiltonian system with n degrees of freedom is said to be *KAM-stable* if any small Hamiltonian perturbation of this system admits many invariant n-tori carrying quasi-periodic motions and close to the unperturbed n-tori. H.Rüssmann discovered in the eighties that in the analytic category, the following geometric condition is sufficient for the KAM-stability: the image of the frequency map $I \mapsto \omega(I)$ does not lie in any hyperplane of $\mathbf{R}^n$ passing through the origin (provided that the action variables I range in a connected domain). This condition is much weaker than the original Kolmogorov nondegeneracy condition. Later on, the Rüssmann theorem was proven by various authors (M.R.Herman, Ch.-Q.Cheng and Y.-S.Sun, M.B.Sevryuk, and others) via different methods. Rüssmann's nondegeneracy condition is also necessary for the KAM-stability (Sevryuk, 1995). B) Exponential "condensation" of invariant tori. In the ρ-neighborhood of a perturbed invariant n-torus in the Kolmogorov theorem, the measure of the complement to the union of the perturbed invariant n-tori is *exponentially small* in $\rho > 0$ (for a fixed and sufficiently small analytic perturbation). This was first proven by A.Morbidelli and A.Giorgilli in 1995. C) Destruction of resonant tori. An unperturbed invariant n-torus $\{I = I^* = \text{const}\}$ in the Kolmogorov theorem is said to be *resonant* if among its frequencies $\omega_1(I^*), \dots, \omega_n(I^*)$, there are only $d < n$ rationally independent numbers. Resonant tori are destroyed by an arbitrarily small generic Hamiltonian perturbation. Nevertheless, such tori give rise (under some further conditions) to finite collections of invariant d-tori carrying quasi-periodic motions. This mechanism of the break-up of resonant unperturbed tori was first studied by D.V.Treshchëv in 1989. However, Treshchëv described the so-called hyperbolic d-tori only. The elliptic d-tori and d-tori of mixed type were constructed no earlier than in 1999 by Ch.-Q.Cheng and Sh.Wang and in 2000 by F.Cong, T.Küpper, Y.Li, and J.You. D) Excitation of elliptic normal modes of lower-dimensional tori. This phenomenon whose precise content will be explained in the talk pertains to Cantor families of invariant m-tori around families of invariant tori of dimensions $l < m$ in Hamiltonian systems with $n \geqslant m$ degrees of freedom. The first results in this direction were obtained by V.I.Arnol'd in 1962–63 (for $m = n$) and by A.D.Bryuno in 1974 (for arbitrary l, m, and n). General theorems were proven by M.B.Sevryuk in 1996 and by À.Jorba and J.Villanueva in 1997.

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Theory of parametric resonance for periodic systems with small damping

A linear multi-degree-of-freedom oscillatory system with periodic coefficients is considered. It is assumed that the system depends on three independent parameters: a frequency and amplitude of the periodic excitation and a damping parameter. The last two parameters are assumed to be small. The instability (resonance) of a trivial equilibrium of the system is studied. For an arbitrary matrix of the periodic excitation and positive definite matrix of dissipative forces stability conditions are derived for the cases of parametric and combination resonances. Then, two cases of the parametric excitation matrix that typically occur in applications are considered: symmetric matrix and a stationary matrix multiplied by a scalar periodic function. It is shown that in both cases the resonance domains are cones in the three-parameter space (in the first approximation). The derived formulae allow analyzing dependence of the form of the instability domains on the eigenfrequencies, corresponding to the unexcited system, and on the number of the resonance zone. The method of analysis of the parametric resonance domains, used in the paper, is based on the analysis of perturbations of multipliers and uses formulae for derivatives of a monodromy matrix with respect to parameters. As an example, the problem of dynamic stability of a plane form of a beam, loaded by periodic momenta, is considered. This work was done together with Alexei A. Mailybaev and was supported by Russian Foundation for Basic Research, grant RFFI 99-01-39129.

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Asymptotical solutions for equations of hydrodynamics and topological invariants of divergence - free vector fields and liouville foliations

Asymptotic theory of equations of hydrodynamics and magnetohydrodynamics is old and highly developed branch of mathematical physics and theory of nonlinear PDE's. Among first and most famous results in this area is the classical

Prandtl description of boundary layers; one of the most recent ones is general asymptotical theory of coherent microstructures, developed by V.P. Maslov and G.A. Omel'yanov. Topological hydrodynamics is a modern and rapidly developing area of mathematics; the most famous results of this theory describe geometrical properties of Euler equations of ideal fluids (which are Euler equations on Lie algebra of divergence-free vector fields) and topological properties of steady flows (these properties are analogous to those of completely integrable Hamiltonian systems with one or two degrees of freedom). The main aim of the talk is to describe certain connections between these two parts of mathematical hydrodynamics as well as between each of them and topological theory of integrable Hamiltonian systems. As an example consider solitary vortex in external flow, localized in a small vicinity of a moving point $x = R(t) \in \mathbb{R}^3$. Such vortex can be described by asymptotic as $\varepsilon \to 0$ solution $v(x, t, \varepsilon)$ of 3D Navier — Stokes equations with small viscosity $\varepsilon^2 \nu$, where

$$v = V(x,t) + u(\frac{x - R(t)}{\varepsilon}, t) + \varepsilon u_1 + \dots \qquad (1)$$

Here $x \in \mathbb{R}^3$, $v \in \mathbb{R}^3$, $V(x,t)$ is a smooth vector field (external flow); the vortex itself is described by the smooth vector field $u(\tau, t)$, where $\tau = (x - R)/\varepsilon$ are "rapid" coordinates and $u(\tau, t) \to 0$ as $|\tau| \to \infty$. Below we obtain and study equations governing the behavior of u; we look at this function as a 3-D vector field in the τ-space, depending smoothly on the parameter t.

Assertion 1 *Let vector field (1) satisfies Navier — Stokes equations* mod $O(1)$ *as* $\varepsilon \to 0$. *Then three-dimensional vector-field* u *satisfies steady Euler equations.*

Evolution of the vortex (i.e. dependence of u on the "slow" coordinate t) can be described in terms of topological invariants of the three-dimensional divergence-free vector field u. Namely, remind, that "in general position " solution of steady Euler equations define Liouville-like foliation of $\mathbb{R}^3$ by 2D tori; we will restrict ourselves to the fields of this type. Consider the quotient space of the τ - space by the tori. This topological space is a graph Γ — the Fomenko invariant of the Liouville foliation. Consider parameterization of this graph by the "action variable"; namely, to each point of the arbitrary edge of Γ (i.e. 2D torus, invariant with respect to the vector field u) we associate a number I, equal to the volume inside the torus. Note that this parameter (as well as the graph itself) is the invariant of the field u with respect to the volume-preserving diffeomorphisms of the τ- space. The motion along the trajectories of u on the tori is conditionally periodic; denote by $\omega = (\omega_1, \omega_2)$ the frequency vector of the motion associated with some smooth basis of cycles on the tori (ω can be treated as a smooth vector-function on the edges of Γ, depending on a parameter t).

Theorem 1 *Let vector field (1) satisfy Navier-Stokes equations* mod $o(1)$ *as* $\varepsilon \to 0$. *Then the function* ω *satisfy the following system of equations*

$$\frac{\partial\omega}{\partial t} + Q\frac{\partial\omega}{\partial I} + R\omega = \nu\Big(\mathcal{D}^2\frac{\partial^2\omega}{\partial I^2} + M\frac{\partial\omega}{\partial I} + Z\omega\Big). \tag{2}$$

Here the scalar function $\mathcal{D}^2$ *and the entries of the* 2×2 *matrices* Q, R, M *and* Z *can be expressed via the coefficients of the Euclidean metric in the three-dimensional space of the fast variables* τ *and their derivatives at points of Liouville tori.*

Remark 1 *The role of equations (2) in the description of solitary vortices is analogous to the role of Prandtl equations in the boundary-layer theory and of Maslov equations describing periodic coherent structures.*

Remark 2 *Equations (2) with respect to the variable* I *are defined on the edges of the graph* Γ. *At the vertices of this graph* ω *satisfies certain additional conditions, which are analogous to Kirghoff conditions for electric chains. These conditions can be expressed in terms of topological characteristics of singular fibers of the Liouville foliation, defined by* u.

Remark 3 *Connections between asymptotical and topòlogical parts of mathematical hydrodynamics appear also in the theory of magnetic fields in conducting fluid.*

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The spectrum structure of Shturm-Liouville boundary problem on the bounded segment

Next result was proved in the work. **Theorem.** *The spectrum of Shturm-Liouville operator can be either an empty multitude, either all complex flatness, or an accoun multitude; this spectrum can not be a finite multitude.* The main idea of the truth of this theorem is the using of integer functions theory and the properties of integer functions zeros.

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Spaces of Initial Data for Parabolic Functional Differential Equations

Let V and H be separable Hilbert spaces. Let V be dense in H, and let the embedding $V \dot{\subset} H$ is continuous. Denote by V' the adjoint space. Clearly, $V \subset H \subset V'$. We consider bounded linear operator $\mathrm{A} : V \to V'$.

Definition. An operator A is called V*-coercive* if for each $v \in V$

$$\mathrm{Re} < \mathrm{A}v, v > \geqslant c_1 \|v\|_V^2, \tag{1}$$

where $c_1 > 0$ does not depend on v. By Theorem 9.1 [1, Chapter 2] the operator A has a bounded inverse. We difine unbounded operator $\mathcal{A} : H \to H$ with domain $\mathcal{D}(\mathcal{A}) = \{u \in V : \mathrm{A}u \in H\}$ by the formula $\mathcal{A}u = \mathrm{A}u$. We consider the Hilbert space $\mathcal{D}(\mathcal{A})$ with the norm $(u, v)_{\mathcal{D}(\mathcal{A})} = (\mathcal{A}u, \mathcal{A}v)_H + (u, v)_H$. We denote by $L_2((0,1), H)$ the Hilbert space of all measurable H-valued functions $u(t)$ on $(0,1)$, for which the norm $\|u\|_{L_2((0,1),H)} = \left(\int\limits_0^1 \|u(t)\|_H^2 dt \right)^{1/2}$ is finite. The inner product in $L_2((0,1), H)$ is definied as following: $(u, v)_{L_2((0,1),H)} = \int\limits_0^1 (u(t), v(t))_H dt$. Consider the Cauchy problem in the space H

$$u'(t) + \mathcal{A}u(t) = 0, \quad (t \in (0,1)), \quad u(0) = \varphi. \tag{2}$$

A function $u(t)$ is said to be a *strong solution of problem (2)* if it is absolutely continuous, $u'(t) \in L_2((0,1), H)$, the equation in (2) is satisfied for a.e. $t \in (0,1)$, and $u(0) = \varphi$. Let $\mathcal{F}_{\frac{1}{2}}(\mathcal{A})$ be a *space of initial data for the problem (2)* as . It consists of functions $\varphi \in H$ such that problem (2) has a strong solution.

Theorem. *Let the operator* A *be* V*-coercive. Suppose* $V \subset \mathcal{F}_{\frac{1}{2}}(\mathcal{A})$ *and this embedding is continuous. Then* $\mathcal{F}_{\frac{1}{2}}(\mathcal{A}) = V$. Now we consider some examples of parabolic functional differential equations. Let $Q \subset \mathbb{R}^n$ be a bounded domain with Lipschitz boundary. Let $H = \mathcal{L}_2(Q)$, $V = \mathring{W}_2^1(Q)$ and $V' = W_2^{-1}(Q)$. We consider a bounded operator $A_B : \mathring{W}_2^1(Q) \to W_2^{-1}(Q)$ define by the formula $A_B u = -\operatorname{div}(B \operatorname{grad} u)$, where an operator $B : \{\mathcal{L}_2(Q)\}_{k=1}^n \to \{\Longleftrightarrow\}_{k=1}^n$ is bounded. Assume that the following conditions hold: 1. The operator $B : \left\{\mathring{W}_2^1(Q)\right\}_{k=1}^n \to \{W_2^1(Q)\}_{k=1}^n$ is a bounded operator. 2. The operator A_B is $\mathring{W}_2^1(Q)$-coercive. Denote by $\mathcal{A}_B$ an unbounded restriction of the operator A_B on $\mathcal{D}(\mathcal{A}_B) = \{u \in \mathring{W}_2^1(Q) : A_B u \in \mathcal{L}_2(Q)\}$.

Corollary. *Suppose that conditions 1, 2 hold. Then* $\mathcal{F}_{\frac{1}{2}}(\mathcal{A}_B) = \mathring{W}_2^1(Q)$.

Example 1. We define difference operators $R_{ij} : L_2(\mathbb{R}^n) \to L_2(\mathbb{R}^n)$ by the formula $R_{ij}u(x) = \sum_{h\in M} a_{ijh}(x)u(x+h)$, where $a_{ijh} \in C^\infty(\mathbb{R}^n)$ are complex-valued functions; $M \subset \mathbb{R}^n$ is a finite set of vectors with integer coordinates. We introduce operators $R_{ijQ} = P_Q R_{ij} I_Q : \mathcal{L}_2(Q) \to \mathcal{L}_2(Q)$, where $I_Q : \mathcal{L}(Q) \to L_2(\mathbb{R}^n)$ is an operator of extension of functions from $\mathcal{L}_2(Q)$ by zero in $\mathbb{R}^n \setminus Q$; $P_Q : L_2(\mathbb{R}^n) \to \mathcal{L}(Q)$ is the operator of restriction of functions from $L_2(\mathbb{R}^n)$ to Q. Let $B = R$, where $R : \{\mathcal{L}_2(Q)\}_{k=1}^n \to \{\Longleftrightarrow\}_{k=1}^n$ is defined by the formula $(Ru)_i = \sum_{j=1}^n R_{ijQ}u_j$; $i = 1, 2, \ldots, n$, where $u = \{u_j\}$ id a vector-valued function. We consider differential-difference operator A_R. Necessary and sufficient conditions of strong ellipticity of operator A_R in algebraic form are obtained in [2]. The conditions of strong ellipticity correspond to $\mathring{W}_2^1(Q)$-coerciveness. The first mixed problem for parabolic differential-difference equations was studied in [3]. If operator A_R is strongly elliptic, then by virtue of Corollary problem (2) has a strong solution iff $\varphi \in \mathring{W}_2^1(Q)$.

Example 2. We define operators with contracted and expanded arguments $T_{ij} : L_2(\mathbb{R}^n) \to L_2(\mathbb{R}^n)$ by the formula $T_{ij}u(x) = \sum_{l\in N} a_{ijl}u(q^{-l}x)$, where $a_{ijl} \in \mathbb{C}$, $N \subset \mathbb{R}^n$ is a finite set of vectors with integer coordinates, $q > 1$. We introduce the operators $T_{ijQ} = P_Q T_{ij} I_Q : \mathcal{L}_2(Q) \to \mathcal{L}_2(Q)$, where operators I_Q, P_Q are defined in Example 1. Let $B = T$, where $T : \{\mathcal{L}_2(Q)\}_{k=1}^n \to \{\mathcal{L}_2(Q)\}_{k=1}^n$ defined by the formula $(Tu)_i = \sum_{j=1}^n T_{ijQ}u_j$, $i = 1, 2, \ldots, n$, where $u = \{u_j\}$ id a vector-valued function. We consider a functional differential operator A_T with contracted and expanded argument. Necessary and sufficient conditions of $\mathring{W}_2^1(Q)$-coerciveness for operator A_T are obtained [4]. If operator A_T is $\mathring{W}_2^1(Q)$-coercive then by virtue of Corollary problem (2) has a strong solution iff $\varphi \in \mathring{W}_2^1(Q)$.

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Shapiro M.

Hurwitz Numbers and Hodge Integrals.

We shall discuss a relation between classical problem of counting ramified covers of a 2D sphere and intersection theory on moduli space of complex curves with marked points.

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Об усреднении решений задачи Неймана для стационарной системы линейной теории упругости в областях с перфорированной внутренней границей

В работе рассматривается задача усреднения решений u_ε стационарной системы линейной теории упругости в областях, состоящих из нескольких частей, разделенных $\varepsilon-$ перфорированными границами ($\varepsilon > 0-$ малый параметр) при различном размере полостей. На внешней границе задано нулевое условие Дирихле, на внутренней перфорированной границе - условие Неймана. Методами, разработанными в [1], в работе исследовано поведение u_ε при $\varepsilon \to 0$, выписана предельная задача и доказана сильная в H^1 сходимость u_ε к решению усредненной задачи. Пусть Ω - ограниченная область в $R^n_x, n \geqslant 3$, с гладкой границей $\partial\Omega = \Gamma$, $\bar{x} = (x_2, ..., x_n), x = (x_1, \bar{x})$. Положим $\Omega^+ = \Omega \cap \{x_1 > 0\}$, $\Omega^- = \Omega \cap \{x_1 < 0\}$, $\gamma = \Omega \cap \{x_1 = 0\}$. Пусть точки $P^j \in \gamma, j = 1, ..., N(\varepsilon)$, $N(\varepsilon) \leqslant K_1\varepsilon^{1-n}$, $K_1 = const > 0$. Обозначим через G^j_ε область на γ, $P^j \in G^j_\varepsilon$, $G^j_\varepsilon \subset \{\bar{x} : |\bar{x} - P^j| \leqslant a^j_\varepsilon\}$, $diam G^j_\varepsilon = a^j_\varepsilon$, $a^j_\varepsilon \leqslant K_0\varepsilon$, $K_0 = const > 0$. Пусть $a_\varepsilon = \max_j a^j_\varepsilon$. Положим $G_\varepsilon = \bigcup\limits_{j=1}^{N(\varepsilon)} G^j_\varepsilon$, $\gamma_\varepsilon = \gamma \setminus G_\varepsilon$, $\Omega_\varepsilon = \Omega^+ \cup \Omega^- \cup G_\varepsilon$, $\Gamma^+ = \Gamma \cap \{x_1 > 0\}, \Gamma^- = \Gamma \cap \{x_1 < 0\}$. Пусть $f(x) = (f_1(x), ..., f_n(x))$- гладкая вектор-функция, а $A^{ij}(x)$ — действительные симметрические матрицы, элементы которых $a^{ij}_{kl}(x)$ — гладкие в Ω функции, удовлетворяющие условию $a^{ij}_{kl}(x) = a^{ji}_{kl}(x) = a^{kj}_{il}(x)$ и условию эллиптичности $C_1\xi_{ki}\xi_{ki} \leqslant a^{ij}_{kl}(x)\xi_{ki}\xi_{lj} \leqslant C_2\xi_{ki}\xi_{ki}$, где $C_1, C_2 = const > 0$. В области Ω_ε изучается краевая задача

$$\begin{cases} \Lambda(u_\varepsilon) \equiv -\dfrac{\partial}{\partial x_m}\left(A^{mk}(x)\dfrac{\partial u_\varepsilon}{\partial x_k}\right) = f(x) \quad \text{в} \quad \Omega_\varepsilon, \\ u_\varepsilon = 0 \quad \text{на} \quad \Gamma_\varepsilon, \quad \sigma(u_\varepsilon) \equiv \nu_m A^{mk}(x)\dfrac{\partial u_\varepsilon}{\partial x_k} = 0 \quad \text{на} \quad \gamma_\varepsilon, \end{cases} \tag{0.1}$$

где $\nu = (\nu_1, ..., \nu_n)-$ внешняя нормаль к γ_ε. Под решением задачи (1) понимается вектор-функция $u_\varepsilon \in (H_1(\Omega_\varepsilon, \Gamma_\varepsilon))^n$. Пусть функции $v^\pm$ и $v-$ гладкие решения задач

$$\Lambda(v^{\pm}) = f(x) \quad \text{в} \quad \Omega^{\pm}, \qquad v^{\pm} = 0 \quad \text{на} \quad \Gamma^{\pm}, \qquad \sigma(v^{\pm}) = 0 \quad \text{на} \quad \gamma. \qquad (0.2)$$

$$\Lambda(v) = f(x) \quad \text{в} \quad \Omega, \qquad v = 0 \quad \text{на} \qquad \Gamma. \qquad (0.3)$$

Theorem 1 *Пусть u_ε– решение задачи (1), а v и $v^{\pm}$– гладкие решения задач (2) и (3). Тогда, если $a_\varepsilon^{n-2}\varepsilon^{1-n} \to 0$, то* $\|u_\varepsilon - v^+\|^2_{(H_1(\Omega^+,\Gamma^+))^n} + \|u_\varepsilon - v^-\|^2_{(H_1(\Omega^-,\Gamma^-))^n} \leqslant K_2 a_\varepsilon^{n-2}\varepsilon^{1-n} \to 0$. *Если $a_\varepsilon^{n-2}\varepsilon^{1-n} \to \infty$, то* $\|u_\varepsilon - v\|^2_{(H_1(\Omega,\Gamma))^n} \leqslant K_3\sqrt{a_\varepsilon^{2-n}\varepsilon^{n-1} + \varepsilon} \to 0$.

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Growth of the solutions of a non-linear degenerating elliptic inequality posed in the half-space.

An inequality

$$\sum_{i,j=1}^{n} a_{ij}(x)\frac{\partial^2 u}{\partial x_i \partial x_j} \geqslant f(u), \quad u\big|_{x_n=0} \leqslant 0 \qquad (0.1)$$

is considered in a half-space $\Pi = \{x \in \mathbb{R}^n : x_n > 0\}$. Here $a_{ij}(x) = a_{ji}(x)$ are measurable functions. Suppose that a lattice $A = \left(a_{ij}(x)\right)_{i,j=1}^{n}$ is positive definite:

$$\sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j > 0 \quad \forall\xi \in \mathbb{R}^n, \ |\xi| > 0, \ x \in \Pi.$$

Let the lattice A not degenerate in the direction x_n such that $\sup_{x\in\Pi} a_{nn}(x)$ is bounded. Let $|x|_{n-1} = \sqrt{x_1^2 + \ldots + x_{n-1}^2}$. Then define $a(r)$ as

$$a(r) = \sup_{x_n\in\mathbb{R}_+} \sup_{|x|_{n-1}\leqslant r} \sup_{|\xi|=1} \sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j.$$

Let

$$h(r) = \begin{cases} \int_1^r \frac{\zeta\, d\zeta}{a(\zeta)}, & \text{if } \frac{\zeta^2}{a(\zeta)} = \bar{o}(1) \text{ as } \zeta \to \infty \\ \int_1^r \frac{d\zeta}{\sqrt{a(\zeta)}}, & \text{otherwise.} \end{cases}$$

Assume that the coefficients $a_{ij}(x)$ are such that $h(r) \to \infty$ as $r \to \infty$. Suppose that the function $f(v) \in \mathbb{C}(\mathbb{R})$ introduced in (0.1) is such a monotone nondecreasing function that $f(v) > 0$ if $v > 0$ and

$$\Phi(r) \stackrel{\text{def}}{=} \int_0^r \frac{d\eta}{\sqrt{\int_0^r f(\zeta)d\zeta}} \to \infty \quad \text{as} \quad r \to \infty. \tag{0.2}$$

A function u is a solution of (0.1) if u has the continious partial derivatives up to the second order and satisfies to the inequalites (0.1). Denote $\sup_{\substack{|x|=r\\ x\in\Pi}} u(x)$ by $M(r)$.

Theorem. *Let the cited above conditions be fulfilled. Then either $u \leqslant 0$ in Π or*

$$\overline{\lim} \frac{M(r)}{[\Phi(r)]^{-1}[h(r)]^{-1}} > 0,$$

where by $[H]^{-1}$ it is denoted the map inversed to H.

Example. Let $f(v) = v$, $a(\zeta) = \underline{O}(\zeta^\alpha)$, $0 < \alpha < 2$. Then

$$\overline{\lim} \frac{M(r)}{e^r r^{\frac{2}{2-\alpha}}} > 0.$$

Remark. The condition (0.2) is sharp. If the function $\Phi(r)$ is bounded then $u \leqslant 0$ in Π.

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Newtonian Dynamical Systems Admitting Normal Blow-Up of Points

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Spartial-temporal chaos in boundary value problems

When studying evolutionary problems, the occurrence of fractal sets is usually linked to the very complicated structure of attractors. At the same time in our investigation into boundary value problems (BVP), fractal sets appear in describing just the interior structure of attractor elements but not attractors (see, for example, [1-4]). It is *a cascade process of birth of coherent structures of decreasing scales* and forming in the limit (at $t = \infty$) *fractal structures* or even *random structures* (when limiting for a solution are random functions) that can really be observed in (deterministic) dynamical systems generated by "simple" BVP involving linear partial differential equations (PDE) and nonlinear boundary conditions. Among such BVP, there is a wide class of problems reducible to difference, differential-difference and other relevant equations, which allows one to employ profitably the achievements in the field of low-dimensional dynamical systems (DS), especially, one-dimensional DS. Modern theory of DS enables one in many cases to preform a deep analysis of properties of BVP. A.A.Witt was the first to apply this approach for the study of BVP (1936, Zhurn. Tekn. Fiz., 6). At that time, however, the theory of one-dimensional DS (the iteration theory of real functions) was yet nonexistent, and on I.G.Petrovsky's offer, his student S.P.Pulkin launched investigations into iterations of functions. Several S.Pulkin's works on this subject was published over the forties. These works were likely pioneering in iteration theory of real functions. The wave equation

$$w_{tt} - w_{xx} = 0, \qquad 0 \leqslant x \leqslant 1,$$

with the local boundary conditions

$$H_0(w, w_t, w_x)\,|_{x=0} \;=\; 0, \qquad H_1(w, w_t, w_x)\,|_{x=1} \;=\; 0,$$

or with similar nonlocal boundary conditions, is a classic example of BVP reducible to difference or differential-difference equations (depending on the particular form of boundary conditions). Varied possibilities for initiation of deterministic chaos in "reducible" BVP of this kind and different characteristics of such chaos are discussed in the talk.

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On modification of Rayleigh Criterion of Stability in One Hydrodynamic Problem

A circular flow of an inviscid, incomressible fluid between two coaxial cylinders is considered. Investigation of perturbations depending on time exponentially reduces the stability question to localization of spectrum of corresponding problem ([1],[2]):

$$(mA + k^2B)u = \lambda Cu. \tag{0.1}$$

where A, B and C are the matrix-operators with commom domain
$\mathfrak{D} = \{u = (u_1, u_2), u_1 \in W_2^2(a;b), u_1(a) = u_2(b) = 0; u_2 \in W_2^1(a;b)\}$
in Hilbert space $H = L_2((a;b);\sqrt{r}) \times L_2((a;b);\sqrt{r})$. The Hilbert space $L_2((a;b);\sqrt{r})$ is the space with inner product $(f,g) = \int_a^b f(r)\overline{g(r)}r\,dr$.

The spectral properties of operator pencil (1) were considered in [3], [4].

The equation (1) is equivalent to the eigenvalue non-linear problem (see [4])

$$\frac{d}{dr}\left(\frac{r}{m^2+k^2r^2}v'(r)\right) - \frac{v(r)}{r} + \frac{2k^2\omega(r)q(r)}{p^2(r,\lambda)}\frac{v(r)}{r} - \frac{mq'(r)}{p(r,\lambda)}\frac{v(r)}{r} = 0 \tag{0.2}$$

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with boundary conditions for $v(r)$: $v(a) = v(b) = 0$, here $\omega(r)$ is angle velocity.

The stability of nonaxisymmetric perturbation of circular flow of an inviscid, incomressible fluid between two coaxial cylinders is defined as absence of eigenvalues of problem (1) or (2) in upper half-plane.

We define the notion of neutral stability in case that all eigenfunctions are real.

In case $m = 0$ the Rayleigh criterion of stability is $\frac{d}{dr}(r^2\omega(r))^2 > 0$.

For equation (1) or (2) the next statement is the generalization of Rayleigh criterion in case $m \neq 0$.

Theorem 1. *The nonaxisymmetric perturbation of circular flow of an inviscid, incomressible fluid between two coaxial cylinders is neutral stable for sufficient large values of parameter m if condition*

$$(2\omega(r) + r\omega'(r))' > 0$$

holds.

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Shilkin T.N.

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Full Internal Regularity for Solutions of the Two-Dimensional Modified Navier-Stokes System

We study regularity for solutions of the Modified Navier-Stokes system (MNS) describing the flow of a generalized Newtonian liquid in the two-dimensional case. We obtain a local criterion of Holder continuity of the spatial gradient of a solution at the neighbourhood of a given point. This result is applied to investigation of solutions to the first initial-boundary value problem for MNS.

Holder continuity of the spatial gradient of a solution on internal subdomains of a parabolic cylinder is proved under natural assumptions on the data of the problem.

Shilnikov L.P.
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On analytic symplectic maps with infinitely many elliptic periodic points

The existence of homoclinic and heteroclinic tangencies at symplectic maps is a sources of many complicated dynamical phenomena. This fact is well-understood up to the present for the case of dissipative diffeomorphisms. Here, we will discuss the problem on the existence of infinitely many elliptic periodic points in the case of two- and four-dimensional symplectic maps having non-transversal (a quadratic tangency) homoclinic orbits to fixed points of saddle and saddle-focus type , respectively. We will discuss also the case of the simplest two-dimensional structurally unstable heteroclinic cycle.

Two-dimensional case. Let a map T has a fixed saddle point O with multipliers λ, λ^{-1} where $0 < \lambda < 1$. In a small neighbourhood U of the point O the map T can be written in the following normal form $\bar{x} = \lambda x(1 + f(xy))$, $\bar{y} = \lambda^{-1} y(1 + f(xy))^{-1}$ where $f(0) = 0$. Let $M^-(0, y^-) \in U$ and $M^+(x^+, 0) \in U$ be points of a non-transversal homoclinic orbit Γ where $x^+ > 0, y^- > 0$. Let $T^q(M^-) = M^+$ for an integer q. The map T^q near the point M^- can be written in the form

$$\bar{x} = x^+ + ax - c^{-1}(y - y^-) + \dots \ , \ \bar{y} = cx + ex^2 + hx(y - y^-) + d(y - y^-)^2 + \dots$$

where $d \neq 0, c \neq 0$. Consider the following invariants of T

$$\nu = \frac{1}{\ln \lambda} \ln \frac{cx^+}{y^-} \ , \ s = dx^+(ac + ex^+) + hx^+(1 - \frac{1}{4}hx^+) \ .$$

Theorem 1 (Gonchenko,Shilnikov). *In the case $c > 0$ at $\nu = 0$, $-3 < s < 1$ and $s \neq 0, -5/4$ the map T has infinitely many single-round generic elliptic periodic orbits (of all periods beginning with some $\bar{k}$).* Maps T from theorem 1 form a codimension two bifurcation set. In the case of codimension one we have proved that,in general, either the map T have no periodic orbits in a small neighbourhood $V(O \cup \Gamma)$ or a countable set of periodic orbit exists and all of them are saddle. Another situation take place in the case of symplectic maps with non-transversal heteroclinic cycles of the third class. Let g be such a map. In the simplest case it has two saddle fixed points O_1 and O_2 and two heteroclinic orbits

$\Gamma_{12} \subset W^u(O_1) \cap W^s(O_2)$ and $\Gamma_{21} \subset W^u(O_2) \cap W^s(O_1)$ where one of them, Γ_{21}, is non-transversal. Let $\theta = \ln \lambda_2 / \ln \lambda_1$ where λ_i is the stable multiplier of O_i.

Theorem 2 (Gonchenko,Shilnikov). *Let f_ρ be a one parameter family of maps with the non-transversal heteroclinic cycle of the third class where $\partial\rho/\partial\theta \neq 0$. Then in any interval of varying ρ values of ρ are dense such that the map g_ρ has infinitely many generic elliptic periodic orbits.* In the homoclinic case we consider a two-parameter family $T_{\mu\nu}$ of maps where μ is the splitting parameter. Then on the parameter plane (μ, ν) a countable set of curves $l_n : \mu = \varphi_n(\nu)$ exists such that 1) $\|\varphi_n\|_{C^1} \to 0$ as $n \to \infty$; 2) the map $T_{\mu\nu}$ at $(\mu, \nu) \in l_n$ has a non-transversal heteroclinic cycle of the third class; 3) the corresponding value θ_n changes monotonically under moving along the curve l_n.

Theorem 3 (Gonchenko,Shilnikov) *In any neighbourhood of a point $(0, \nu_0)$ of the parameter plane (μ, ν) there exist values (μ^*, ν^*) such that $T_{\mu^*\nu^*}$ has infinitely many generic elliptic periodic orbits. Four-dimensional case.* Among codimension one four-dimensional symplectic maps with homoclinic tangencies, only bifurcations of a homoclinic tangency to a saddle-focus fixed (periodic) point can lead to elliptic points. It is connected with the fact that in other cases a global center invariant manifolds of saddle type exists. Let F be a four-dimensional symplectic map with a saddle-focus fixed point O with multipliers $\lambda e^{\pm i\varphi_0}$ and $\lambda^{-1} e^{\pm i\varphi_0}$. Let F have a homoclinic orbit Γ_0 in whose points the manifolds $W^s(O)$ and $W^u(O)$ have a quadratic tangency. Denote by H a codimension one bifurcation surface of such maps.

Theorem 4. (Gonchenko,Shilnikov,Turaev) *Maps with infinitely many KAM-generic elliptic periodic points are dense in H.*

These results are true also in a smooth case.

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Motion of interfaces in thin film theory

There is considered the following Cauchy problem for fourth order degenerate parabolic equation:

$$Lu \equiv u_t + \operatorname{div}\left(|u|^n \nabla \Delta u - |u|^m \nabla u\right) = 0 \text{ in } \mathbb{R}^N \times \mathbb{R}^+_1. \tag{1}$$

$$u(x, u) = u \cdot (x) \geqslant 0 \text{ in } \mathbb{R}^N,\ N \leqslant 3,\ n > 0,\ m > 0. \tag{2}$$

Equation (1) was introduced to describe the evolution of the height u of a liquid film spreading on a solid surface. The fourth order term accounts for the effect of surface tension. The second-order term describes effect of gravity or may be considered as "porous-media cut-off" of Van der Waals forces. The care $n = m = 1$ describes the extent u of the region occupied by a liquid in the half-space Hele-Shaw cell in the lubrication regime. We study solvability of problem (1), (2). There is proved existence of nonnegative generalized strong solution $u(x,t)$ in the case $n \in (1/8, 2)$ and u_0 - arbitrary nonnegative Radon measure with finite mass and such that $\operatorname{supp} u_0$ is compact. It is established that this solution has finite speed propagation property. There are obtained precise estimates of the speed of solution's support propagation for small and large t. There is investigated the dependence of initial evolution of interface on local properties of initial function. We study effect of waiting time in propagation of support of solution too. Some of these results obtained jointy with R.Dal Passo and L.Giacomelli.

Shishmarev I.
(Moscow State University)

Large time asymptotics of solutions for periodic problem to wide class of nonlinear evolution equations

Large time asymptotics of solutions for periodic problem to wide class of nonlinear evolution equations I. A. Shishmarev (Moscow State University, Russia) We study the periodic problem for wide class of nonlinear (local and nonlocal) evolutions equations, including among others such well-known equations, as the generalized Kolmogorov-Petrovsky-Piskunov equation, the Korteweg-de Vries-Burgers equation, the Kuramoto-Sivashinsky equation, the Ott-Saden-Ostrovsky equation, the Landau-Ginzburg equation, the derivative nonlinear Schredinger equation. The main goal is investigation of large time asymptotic behavior of solution the periodic problem for this class of equations. We find out the principal term in explicit form and give the estimate of remind term for different kind of asymptotic behavior, that is decaying, growing or oscillating in the course of time. We consider both interesting cases of small and arbitrary initial data. These results were obtained in collaboration with E. Kaikina and P. Naumkin.

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Perseverance of continuous symmetry in 2D: the case of singular interactions.

It was known for a long time that the discrete symmetry of the interaction can be broken starting from dimension 2, while the continuous symmetry can be broken only starting from dimension 3. The statement about the absence of breakdown of continuous symmetry in 2D for the first correlation function is the content of the Mermin-Wagner Theorem. Physically this fact is expressed by saying that there are no Goldstone bosons in 2D. The following generalization of the Mermin-Wagner Theorem was proven in [1]. Let G be a compact connected Lie group, which acts on a single spin state space S of a 2D system. Suppose that the nearest neighbour translation-invariant interaction U between the spins is G-invariant: for every $g \in G$

$$U(s_x, s_y) = U(gs_x, gs_y).$$

Then every Gibbs state corresponding to U is G-invariant as well, provided the interaction U is C^{ϵ}. For a long time the common belief was that the smoothness assumption is necessary in the above statement. In a recent paper coauthred by D. Ioffe (Haifa) and Y. Velenik (Marseille) we were able to show that in fact this is not the case, and the same G-invariance of the Gibbs states holds even for the singular G-invariant interactions.

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Shnirelman A.
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Braids, Flows, and Magnetic Equilibria

The talk is devoted to a general method of construction of flows of an ideal incompressible fluid. Consider the motion of an ideal incompressible fluid in a bounded domain M. Configuration space of the fluid is the group D of volume-preserving diffeomorphisms of M, and the fluid moves along geodesics on D. If we want to construct interesting geodesics, the obvious idea is to look for the shortest path connecting two given fluid configurations. But this method generally fails if the dimension of M is greater than 2. In the 2-dimensional case we have an additional structure: every flow may be regarded as a braid with continuum of threads. Many features of these continual braids have no analogies with the well-known finite braids. After a careful analysis of the structure of braids we prove that the above variational problem has a generalized solution, so that the velocity field of the fluid is a weak solution of the Euler equations. This means that velocity field may have singularities. Examples show that singularities can really occur. Formally, the problem described above may be regarded as a particular case of equilibrium problem of a perfectly conducting plasma with frozen-in magnetic field, which is important for astrophysics. Singularities, arising in weak solutions of this problem, are materialized as solar flares and magnetic storms.

Dijksma Aad
(University of Groningen)
Shondin Yu.G.
(Nizhny Novgorod Pedagogical University)

Singular (and regular) point-like perturbations of the Laguerre operator in a Pontryagin space

The function $Q_\alpha(z) = -\frac{\pi}{\sin \pi\alpha}\frac{\Gamma(-z)}{\Gamma(-z-\alpha)}$, $\alpha \in \mathbf{R} \setminus \mathbf{Z}$, $z \in \mathbf{C} \setminus \mathbf{Z}^+$ belongs to the class N_m of generalized Nevanlinna functions with $m = [\frac{|\alpha|+1}{2}]$ negative squares. We describe an operator representation of Q_α as a Q-function for some symmetric operator S_α in a Pontryagin space Π_m and a canonical selfadjoint extension A of S_α and discuss related topics. The case $0 < |\alpha| < 1$ is known: Then $Q_\alpha \in N_0$ and we can take for Π_0 the space $\mathcal{H}_0 = L^2(\mathbf{R}^+, w_\alpha)$, $w_\alpha(x) = x^\alpha e^{-x}$, for S_α the minimal realization in $\mathcal{H}_0$ of the Laguerre expression $\ell_\alpha = -x\frac{d^2}{dx^2} - (1+\alpha-x)\frac{d}{dx}$, for A the selfadjoint operator L_α in $\mathcal{H}_0$ associated with orthogonal system of the generalized Laguerre polynomials $L_n^\alpha(x)$. For $\alpha < -1$, $\alpha \notin \mathbf{Z}^-$, R.D. Morton and A.M. Krall (1978) observed that the $L_n^\alpha(x)$'s are orthogonal relative to some indefinite inner product. V.A. Derkach (1998) obtained an operator representation in a Pontryagin space Π_m^α for a related function. For the case $|\alpha| > 1$, $\alpha \notin \mathbf{Z}$ we follow the line of [1] and start with the formal perturbation (FP): $A_\alpha^t = A_0 + t^{-1}\langle\cdot, \chi_\alpha\rangle\chi_\alpha$, $t \in \mathbf{R} \cup \{\infty\}$, where $A_0 \leftrightarrow L_\alpha$ and $\chi_\alpha \leftrightarrow \sum_{n=0}^{\infty} L_n^\alpha(x)$. If $\alpha > 1$ the series converges in the scale space $\mathcal{H}_{-m-1}(L_\alpha)$ and determines $\chi_\alpha \in \mathcal{H}_{-m-1}(L_\alpha) \setminus \mathcal{H}_{-m}(L_\alpha)$. In this case (FP) is an example of the singular perturbations in [1] and can be realized accordingly by a canonical selfadjoint extension A_α^t of some symmetric operator S_α in a Pontryagin space Π_m^α. If $\alpha < -1$ the series for χ_α converges in the space Π_m^α of Morton-Krall and (FP) is a regular rank one perturbation, if we identify $A_0 \equiv A_\alpha^\infty$ with the selfadjoint operator in Π_m^α associated with orthogonal polynomials $L_n^\alpha(x)$. With A_α^∞ and A_α^0 we associate the decomposition (D): $\Pi_m^\alpha = \mathcal{H}_0^\alpha \oplus (\mathcal{L}^\alpha \dot{+} \mathcal{M}^\alpha)$.

Theorem 1. *Assume $\alpha > 1$, $\alpha \neq 2, 3, \ldots$, and $m = [\frac{\alpha+1}{2}]$.*

(i) *$Q_\alpha(z)$ is the Q-function for S_α and A_α^∞ in Π_m^α, where $\Pi_m^\alpha = L^2(\mathbb{R}^+, w_\alpha) \oplus \mathbb{C}^m \oplus \oplus \mathbb{C}^m$ is a Pontryagin space with negative index m and a G-space with Gram operator $\mathbf{G} = I_0 \oplus \begin{pmatrix} 0 & I_m \\ I_m & (g_{jk}) \end{pmatrix}$, $g_{jk} = \frac{\Gamma(\alpha-j+1)\Gamma(\alpha-k+1)}{(j-1)!(k-1)!}\Gamma(j+k-\alpha-1)$, A_α^∞ is a "lifting" of L_α to Π_m^α, in particular, $\sigma(A_\alpha^\infty) = \sigma(L_\alpha) \cup \{\infty\}$, and $S_\alpha = \{\{f, f'\} \in A^\infty \mid \langle (f' + \alpha f), (0, 0, e_1)^T \rangle = 0\}$.*

(ii) *In the decomposition (D) $\mathcal{H}_0^\alpha = L^2(\mathbb{R}^+, w_\alpha)$, $\mathcal{L}^\alpha = \{0\} \oplus \mathbb{C}^m \oplus \{0\}$ is the root subspace of A_α^∞ at ∞ and $\mathcal{M}^\alpha = \{0\} \oplus \mathbb{C}^m \oplus \{0\}$ coincides with the span of the first m eigenvectors of A_α^0.*

Theorem 2. *Assume $\alpha < -1$, $\alpha \neq -2, -3, \ldots$, and $m = [\frac{|\alpha|+1}{2}]$.*

(i) *In* (FP) $\chi \in \mathrm{dom}\,(A_0^{m-1})$ and $\chi \notin \mathrm{dom}\,(A_0^m)$. $Q_\alpha(z) = \langle (A_\alpha^\infty - z)^{-1}\chi, \chi \rangle$ and *is the Q-function for A_α^∞ and $S_\alpha = A_\alpha^\infty |_{\{f \in \mathrm{dom}\, A_\alpha^\infty | \langle f, \chi_\alpha \rangle = 0\}}$;* and *$Q_{-\alpha}(z + \alpha)$ is the Q-function for S_α and A_α^0.*

(ii) *The space Π_m^α admits the decomposition (D), where $\mathcal{H}_0^\alpha$ is isomorphic to $L^2(\mathbb{R}^+, w_\alpha)$, $\mathcal{L}^\alpha = \ker\,(A_\alpha^0 - z)^{-m}$, and $\mathcal{M}^\alpha$ is spanned by the first m eigenvectors of A_α^∞.* The research of Yu.Shondin was supported by NWO (047-008-008) and RFBR (0001-00544).

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Shubin M.

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Spectra of magnetic Schrödinger operators

I will review some recent results connecting classical and quantum dynamics of a particle in an electromagnetic field. The first type of a result says that classical completeness implies quantum completeness for magnetic Schrödinger operators. Here the classical completeness means that the solution of the corresponding Hamiltonian system does not go to infinity in a finite time, whereas the quantum completeness means that the Schrödinger operator is essentially self-adjoint. The most advanced recent results in this direction are due to I. Oleinik and the speaker.

The second type of a result gives conditions for the spectrum of the magnetic Schrödinger operator to be discrete. It is well known that the condition $V(x) \to \infty$

as $x \to \infty$ implies that the Schrödinger operator $H = -\Delta + V(x)$ in $\mathbf{R}^n$ has a discrete spectrum (K. Friedrichs, 1934). In physical language this means that if a classical particle can not escape to infinity (being forced to remain in a potential well), then the corresponding quantum particle is also localized. Similar results about magnetic Schrödinger operators (in $\mathbf{R}^n$ or on manifolds) were obtained in recent papers by V. Kondratiev and the speaker and will be explained in the talk. They are formulated in terms of effective potentials which are constructed from both electric and magnetic fields. The most advanced of these results use the Wiener capacity and in case of vanishing magnetic field coincide with the necessary and sufficient conditions given by A.M.Molchanov in 1953.

Skubachevskii A.L.
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On smoothness of generalized solutions for nonlocal elliptic problems

A classical result concerning smoothness of solutions for elliptic problems can be formulated as follows. If $u \in W_2^1(Q)$ is a generalized solution of a second order elliptic equation with smooth coefficients and the homogeneous Dirichlet boundary condition on smooth boundary ∂Q, then $u \in W_2^2(Q)$. In the case of nonlocal elliptic boundary value problem, the situation is quite different. For the simplicity, in this lecture we consider a model problem. However, the results were obtained for general nonlocal elliptic boundary value problems in arbitrary bounded plane domains. We consider the nonlocal elliptic boundary value problem

$$-\Delta u(x) = f_0(x) \qquad (x \in Q), \tag{1}$$

$$\left.\begin{array}{ll} u(x)|_{\Gamma_i} - \gamma_i (x + h_i)|_{\Gamma_i} = 0 & (x \in \Gamma_i;\ i = 1, 2), \\ u(x)|_{\Gamma_3} = 0 & (x \in \Gamma_3). \end{array}\right\} \tag{2}$$

Here $Q \subset \mathbb{R}^2$ is a domain with boundary $\partial Q \in C^\infty$, which outside the disks $B_{1/8}((i4/3, j4/3))$ $(i, j = 0, 1)$ coincides with the boundary of the square $(0, 4/3) \times (0, 4/3)$; $\gamma_1, \gamma_2 \in \mathbb{R}$; $x = (x_1, x_2)$; $\Gamma_1 = \{x \in \partial Q : x_1 < 1/3, x_2 < 1/3\}$, $\Gamma_2 = \{x \in \partial Q : 1 < x_1, 1 < x_2\}$, $\Gamma_3 = \partial Q \setminus (\overline{\Gamma}_1 \cup \overline{\Gamma}_2)$; $h_1 = (1, 1)$, $h_2 = (-1, -1)$; $f_0 \in \mathcal{L}_2(Q)$. Let $W_2^k(Q)$ be the complex Sobolev space of order k. Denote by $W_{2,\gamma}^1(Q)$ a subspace of $W_2^1(Q)$ consisting of functions with nonlocal boundary conditions (2). We introduce an unbounded linear operator $A_\gamma : \mathcal{D}(A_\gamma) \subset \mathcal{L}_2(Q) \to \mathcal{L}_2(Q)$ acting in the space of distributions $\mathcal{D}'(Q)$ by the formula

$$A_\gamma u = -\Delta u \qquad (u \in \mathcal{D}(A_\gamma) = \{u \in W_{2,\gamma}^1(Q) :\ -\Delta u \in \mathcal{L}_2(Q)\}).$$

A function u is called a generalized solution of problem (1) and (2) if $u \in \mathcal{D}(A_\gamma)$ and

$$A_\gamma u = f_0. \tag{3}$$

If $\gamma_1 = \gamma_2 = 0$, we obtain the Dirichlet problem, which is a "local" one. It is well know, that if u is a generalized solution of problem (1) and (2), then $u \in W_2^2(Q)$. However, arbitrary small coefficients γ_i in nonlocal terms can lead to disturbance for smoothness of generalized solutions. On the other hand, for sufficiently large coefficients γ_i smoothness of solutions preserves.

Theorem 1. *Let $\gamma_1\gamma_2 < 4$ and $\gamma_1^2 + \gamma_2^2 \neq 0$. Then there exist $f_0 \in\Longleftrightarrow$ and a generalized solution u of problem (1) and (2) such that $u \notin W_2^2(Q)$.*

Theorem 2. *Let $\gamma_1\gamma_2 \geqslant 4$, and let u be a generalized solution of problem (1) and (2). Then $u \in W_2^2(Q)$.*

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Feynman and Wiener type integrals over trajectories in Riemannian manifolds.

One presents an approach, based on an idea of surface pseudomeasures [1], generated by Feynman pseudomeasures, both to obtaining some representations of solutions for Schroedinger equations on (compact) Riemannian manifolds by integrals with respect to Feynman pseudomeasures (= Feynman integrals) over (sets of) trajectories in the manifolds and also to obtaining some Cameron-Martin-Girsanov-Maruyama-Ramer (CMGMR) type formulas for Feynman integrals generated by these pseudomeasures. Actually there exist two types of surface (pseudo) measures respectively adapted to these two problems. The discussed constructions use and develop some ideas of papers [1]-[3], [6] (see [7]). Some results of the present paper related to Feynman-Kac formulas for Schroedinger equations on Riemannian manifolds can be considered as answers to questions posed in [5]. On the other hand some results related to CMGMR formulas generalize, to the case of trajectories on compact Riemannian manifolds, corresponding results of papers [4],[6]. The developed approach is completely different from a traditional one (which was used however only for usual measures generated by diffusion processes) where an essential role is played by the notion of stochastic parallel transport described by corresponding stochastic differential equations. Some of the presented results are obtained in collaboration with A.Truman and H.v.Weizsaecker.

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Spatially non-local homogenization for double porosity type media and applications

In a number of important applications "non-local" effects are observed in the overall behaviour of heterogeneous solid media. Those cannot be modelled within the classical homogenization theory which predicts that for PDE with uniformly elliptic rapidly oscillating coefficients the homogenized constitutive relations preserve the (local) structure of the original constitutive relations. On the other hand, if the assumption of uniform ellipticity is relaxed (for example, in the case of "high contrast" media of double porosity type), the homogenized behaviour may display a non-classical "two-scale" structure described by the theory of two scale convergence (e.g. [1-3]). The two scale limit $(u_0(x), w(x,y))$, $y := x/\varepsilon$ is described by a coupled system of (still local) equations. In this talk we give first a simple example of a variant of double porosity models where the two scale limit is described by a spatially *non-local* equations [4]. It is as follows. Consider an elliptic equation with vanishing ellipticity constant for a periodic array of "highly anisotropic fibres" as follows

$$-\frac{\partial}{\partial x_i}\left(A^{\varepsilon}_{ij}(x/\varepsilon)\frac{\partial}{\partial x_j}u^{\varepsilon}\right)=f(x),\quad x\in\mathbf{R}^3, \tag{0.1}$$

where $f(x)\in C^{\infty}$ is $\mathbf{T}$-periodic, $\mathbf{T}=[-T,T]^3$, $T>0$, and has zero mean over $\mathbf{T}$; the matrix $(A^{\varepsilon}_{ij}(y))$ is defined by $A^{\varepsilon}(y)=\operatorname{diag}\{\varepsilon^2,\varepsilon^2,1\}$ if $y\in F_0$ (in the "fibres"), and $A^{\varepsilon}(y)=I$ if $y\in F_1$ (in the "matrix"). Here $F_0=\tilde{F}_0\times[0,1]$ and $F_1=\tilde{F}_1\times[0,1]$; $\tilde{F}_0$ and $\tilde{F}_1$ are $[0,1]^2$-periodic sets; $\tilde{F}_0\cap\tilde{F}_1=\varnothing$ and $\tilde{F}_0\cup\tilde{F}_1=\mathbf{R}^2$. The set F_1 is connected and has Lipschitz boundary; T is fixed and T/ε is a large integer number. The solution $u^{\varepsilon}(x)$ is assumed $\mathbf{T}$-periodic with zero mean (such solution exists and is unique). We establish first (developing certain ideas of [5]) a Poincaré type inequality for high contrast media, as follows:

$$\|u\|_{L^2(T)}\leqslant C\left(\|\nabla u\|_{L^2(T\cap F_1^{\varepsilon})}+\varepsilon\|\nabla u\|_{L^2(T\cap F_0^{\varepsilon})}\right),\ u\in H^1(\mathbf{T}),\ \int_{\mathbf{T}}u(x)\mathrm{d}x=0, \tag{0.2}$$

where the constant C does not depend on ε. Then the use of two-scale compactness arguments allows us to conclude that u^{ε} has a two scale limit: $u^{\varepsilon}(x)\overset{2}{\rightharpoonup}u_0(x)+$ $+w(x,y_1,y_2)$, where $\overset{2}{\rightharpoonup}$ denotes two-scale convergence. The functions u_1 and w satisfy a coupled system of differential equations. In particular, eliminating from it w results in the following nonlocal homogenised equation for the averaged field u_0 in the matrix:

$$-\operatorname{div}(A^{hom}\nabla u_0)-|F_0\cap Q|^2\langle G\rangle_{\tilde{y}',\tilde{y}}\overset{x_3}{*}\frac{\partial^4u_0}{\partial x_3^4}=f+|F_0\cap Q|^2\langle G\rangle_{\tilde{y}',\tilde{y}}\overset{x_3}{*}\frac{\partial^2f}{\partial x_3^2}, \tag{0.3}$$

where $G=G(\tilde{y},\tilde{y}',x_3-x_3')$, $\tilde{y},\tilde{y}'\in\tilde{F}_0, x_3,x_3'\in\mathbb{R}$, is the Green's function for the "rescaled" fibres; A^{hom} is the standard homogenised matrix for the "void" fibres. It is interesting that this rigorous result can also be re-derived formally, from the "higher-gradient" asymptotics of the solution to the problem

$$-\frac{\partial}{\partial x_i}\left(A^{\delta}_{ij}(x/\varepsilon)\frac{\partial}{\partial x_j}u^{\varepsilon,\delta}\right)=f(x),$$

where the matrix of coefficients is initially determined by $A^{\delta}(y)=\operatorname{diag}\{\delta,\delta,1\}$ if $y\in F_0$, $A^{\delta}(y)=I$ if $y\in F_1$. Fixing first δ and treating ε as a parameter we arrive at a full asymptotic expansion for $u^{\varepsilon,\delta}(x)$ (see [6], section 4.2 and [7]) whose terms depend on δ. Further, letting δ be small we observe that when δ is of order ε^2 the asymptotic series "breaks up": all the terms become of equal "strength". It turns out that the main order terms constitute a certain expansion to the two-scale limit $u_0(x) + w(x, x_1/\varepsilon, x_2/\varepsilon)$ and is closely related to the "gradient" approximation of the above non-local operator. From the latter point of view, even the "local" classical two-dimensional double porosity model [1-3] (when treated in the above T-periodic setting) leads to non-local equations for $U(x)$ which is the weak L^2-limit of $u^\varepsilon(x)$ ($U = u_0 + \langle w\rangle_y$), and the full asymptotic expansion of [6] corresponds to the gradient expansion for this non-local operator. This will be further illustrated and certain implications will be discussed in the talk. We will also discuss further implications, applications and prospects.

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Bifurcations of slow integral manifolds

In this paper we report on the construction on slow integral manifolds of singularly perturbed differential systems in the events that the usual condition of structurally hyperbolic fast subsystem is violated. The situation along these lines have appeared in many problems of chemical kinetics and combustion, mechanics of gyroscopes, satellites and manipulators, automatic control and economic dynamics. Certain of these problems are discussed in our paper.

The statement of problems. Consider the differential system $\dot{x} = f(x,y,\varepsilon)$, $\varepsilon\dot{y} = g(x,y,\varepsilon)$ with vector variables x and y and small positive parameter ε. Let $y = \phi(x)$ be the solution of equation $g(x,y,0) = 0$. The fast subsystem $\varepsilon\dot{y} = g(x,y,\varepsilon)$ is structurally hyperbolic if the eigenvalues of the matrix $A(x) = \frac{\partial g}{\partial y}(x,\phi(x),0)$ are separated from the imaginary axis. In this case the system under consideration possesses an integral manifold of slow motions (slow integral manifold) in the ε-neighborhood of slow manifold $y = \phi(x)$. It is common knowledge that the slow integral manifolds are used as a building block to study of singularly perturbed systems. Consider now the situations when the condition of structural hyperbolicity is violated but differential systems under consideration possess slow integral manifolds.

Nonsingular matrix $A(x) = \frac{\partial g}{\partial y}(x,\phi(x),0)$ with eigenvalues situated on the imaginary axis and problems of mechanics. Let the part of eigenvalues are pure imaginary but under taking into account the perturbations of order $O(\varepsilon)$ they move to the left complex half-plane. In this case the considered system has the stable slow integral manifolds. Some important problems of mechanics of gyroscopes, satellites and manipulators with high-frequency and weakly damped transient regimes are studied.

Singular matrix $A(x) = \frac{\partial g}{\partial y}(x,\phi(x),0)$ and problems of chemical kinetics and automatic control. In the case of identity degeneration of the matrix $A(x)$ (singular singularly perturbed systems) it is possible to show that the original differential system has the slow integral manifold of higher dimension. The problem of chemical reactor dynamics and the high-gain control problem are investigated.

The branching of slow integral manifolds and control problems. If $y = \phi(x)$ is the multiple root of $g(x,y,0) = 0$ then the branching of slow integral manifolds is possible. In this case we use the asymptotic expansions with respect to fractional powers of ε to construct the slow integral manifolds. The "cheap control" problem is considered as application.

Degenerating of $A(x)$ on submanifold of $y = \phi(x)$, canards and black swans and problems of chemical kinetics, combustion and economic dynamics. The trajectory of differential equations with singular perturbations is called the canard if it at first moves over stable integral manifolds and then over the unstable one. The black swan (slow integral manifold of changing stability) is the natural many-dimensional generalization of canard. Black swans and canards occur in models of catalytic reactors, laser and economic dynamics, in models of combustion in an inert porous medium. We found the control function corresponding to the black swan to obtain the critical conditions of thermal explosion.

Sokołowski Jan
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Tangent sets in Banach spaces and applications to variational inequalities

In the talk the general results on the polyhedricity of convex sets in Banach spaces are applied to the shape sensitivity analysis of the following model problem.

Let $D \subset R^2$ be a bounded domain with smooth boundary Γ, and $\Sigma_{l+\delta}$ be the set $\{(x_1, x_2) | \, 0 < x_1 < l + \delta, \ x_2 = 0\}$. For any value of the parametr $\delta \in [-\delta_0, \delta_0]$, $\delta_0 > 0$, the function u^δ is the solution of the variational inequality

$$u^\delta \in K_\delta : \quad \int_{\Omega_\delta} \langle \nabla u^\delta, \nabla v - \nabla u^\delta \rangle \geqslant \int_{\Omega_\delta} f(v - u^\delta) \quad \forall v \in K_\delta \, , \tag{26}$$

where

$$K_\delta = \{w \in H^1(\Omega_\delta) | \ [w] \geqslant 0 \quad \text{on} \quad \Sigma_{l+\delta}; \ w = 0 \quad \text{on} \quad \Gamma\}.$$

The energy functional for the problem is equal to

$$J(\Omega_\delta) = \frac{1}{2} \int_{\Omega_\delta} |\nabla u^\delta|^2 - \int_{\Omega_\delta} f u^\delta . \tag{28}$$

The form of the derivative of the energy functional $J(\Omega_\delta)$ with respect to the variations of the crack's length

$$\frac{dJ(\Omega_\delta)}{d\delta}\Big|_{\delta=0} = \lim_{\delta \to 0} \frac{J(\Omega_\delta) - J(\Omega)}{\delta} \tag{29}$$

is obtained in [1].

Theorem 1 *The derivative of $J(\Omega_l)$ with respect to l is given by*

$$\frac{dJ(\Omega_l)}{dl} = -\frac{1}{2} \int_\Omega (\theta_{y_1} (u_{y_1}^2 - u_{y_2}^2) + 2\theta_{y_2} u_{y_1} u_{y_2}) - \int_\Omega (\theta f)_{y_1} u. \tag{39}$$

The first derivative is independent of the choice of θ with the required properties [1]. We can obtain the second order shape derivative of the energy functional in the directions θ and ψ using the following result which seems to be new and follows by the arguments given in [4].

Theorem 2 *The set K_0 is polyhedric.*

The directional derivative Q of u_δ in direction θ is given by the unique solution to the following variational inequality

$$Q \in S : \quad \int_\Omega \langle \nabla Q, \nabla (v - Q) \rangle_{\mathbb{R}^2} \geqslant$$

$$-\int_\Omega \langle A'(\psi)\nabla u, \nabla(v-Q)\rangle_{\mathbb{R}^2} + \int_\Omega f'(\psi)(v-Q) \quad \forall v \in S\,,$$

where

$$S = \{v \in H^1_\Gamma(\Omega) | [v] \geqslant 0 \quad \text{on } \{x \in \Sigma_l | [u(x)] = 0\}\,, \quad \int_\Omega \langle \nabla u, \nabla v\rangle = \int_\Omega fv\}\,.$$

Therefore, taking ψ with the support included in the set $\theta = 1$ we obtain the second order shape derivative of the energy functional $J(\Omega_l)$ in the directions θ, ψ.

Theorem 3 *The second order directional derivative of the energy functional $J(\Omega_l)$ with respect to the crack length is given by*

$$\frac{d^2 J(\Omega_l)}{dl^2} = \frac{1}{2}\int_\Omega \theta_{y_1}\psi_{y_1}[u^2_{y_1} + u^2_{y_2}] - \int_\Omega [\theta_{y_1}\psi_{y_2}u_{y_1}u_{y_2} + \theta_{y_2}\psi_{y_2}u^2_{y_1}]$$

$$-\int_\Omega (\theta_{y_1}[u_{y_1}Q_{y_1} - u_{y_2}Q_{y_2}] - \int_\Omega \theta_{y_2}[u_{y_1}Q_{y_2} + u_{y_2}Q_{y_1}]$$

$$-\int_\Omega u(\theta(\psi f)_{y_1})_{y_1} - \int_\Omega Q(\psi)(\theta f)_{y_1}\,,$$

where $Q = Q(\psi)$ solves the variational inequality for $A'(y) = A'(\psi)(y)$ and $f'(y) = (\psi f)_{y_1}(y)$.

The same result can be obtained in the case of elasticity system [2] with the frictionless contact conditions on the crack faces, i.e. the convex set is polyhedric and the second order directional differentiabilty of the energy functional follows by the same argument as above for the scalar equation.

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To well posed problems for equations of mixed type

Let us consider Lavrent'ev-Bitsadze system

$$\operatorname{sgn} u_x = v_y, \quad u_y = -v_x \tag{1}$$

in the mixed domain D such that elliptic part $D^+ = D \cap \{y > 0\}$ is bounded by the smooth arc σ and the segment $J = [0, 1]$ of real axis but hyperbolic part $D^- = D \cap \{y < 0\}$ lies within characteristic triangle Δ with the base J. A smooth curve is called non characteristic, if it has no characteristic directions in each of its points. Let domain D^- be bounded by J, smooth non characteristic γ with endpoints on the lateral sides of the triangle Δ, and two segments $l^\pm$ of these sides where l^+ (l^-) has as its ends the points τ^+ and $x^+ = 0$ (τ^- and $x^- = 1$). It is convinent to assume that $l^\pm = \varnothing$ if $x^\pm = \tau^\pm$. So the contour ∂D is composed of σ, γ and l, $l = l^+ \cup l^-$. Let as consider two types of Riemann-Hilbert problems $R^\pm$ for system (1). Namely the problem $R^\pm$ is definined by the following boundary value conditions:

$$(au + bv)\big|_{\sigma\cup\gamma} = f, \quad (v \pm u)\big|_{l^\pm} = g. \tag{2}$$

Here the coificients a, b are continuous on the arcs σ, γ and satisfy the conditions

$$(a + ib)(t) \neq 0, \quad t \in \sigma; \quad (a \pm ib)(t) \neq 0, \quad t \in \gamma. \tag{3}$$

If the boundary ∂D has no characteristic directions i.e. $l^+ = l^- = \varnothing$, then $R^\pm$ is the usual Riemann - Hilbert problem for system (1). In the case $l^\pm = \varnothing$, $l^{(p)} \neq \varnothing$ this problem is analogous to the generalized Tricomi problem. In the case $l^\pm \neq \varnothing$, $l^{(p)} = \varnothing$ we have the new problem. At last the case $l^+ \neq \varnothing$, $l^- \neq \varnothing$ corresponds to Ovsaynnicov problem. Note that in the first and third cases the conditions (2) ocupy all boundary ∂D. In the report the Fredholm solvability of the problem $R^\pm$ in appropriate weighted Hardy or Holder spaces is received and the index formula is founded. The solvability of the problem $R^\pm$ is described with the help of the homogeneuous conjugate problem $\tilde{R}^{(p)}$ (with that combination of sign). It follows from (3) that boundary conditions (2) are uniquly definded by the functions $G = a - ib$ on σ and $\rho = (b - a)/(b + a)$ on γ. In these notations the problem $\tilde{R}^{(p)}$ corresponds to the functions $\tilde{G}$ and $\tilde{\rho}$ from the equalitis $\tilde{G}G = n_2 - in_1$ on σ and $\tilde{\rho}\rho = (n_2 - n_1)/(n_2 + n_1)$ on γ, where n_1, n_2 are components of the exterior unite normal to the boundary ∂D.

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Artificial boundary conditions for Petrovsky systems of second order problems

Let $\Omega \subset \mathbb{R}^n$, $n \geqslant 2$ be a smoothly surrounded unbounded domain, which coincides with a cone $\mathbb{K}$ outside a fixed ball of radius R_0, $\Omega \backslash B(0, R_0) = \mathbb{K} \backslash B(0, R_0)$. On Ω we consider a boundary value problem for the following Petrovsky system of second order for $u = (u_1, \ldots, u_J)$:

$$\mathcal{L}u(x) = \mathcal{D}(\nabla_x)^* A(x) \mathcal{D}(\nabla_x) u(x) = f(x), \qquad x \in \Omega, \tag{0.1}$$

$$B(x, \nabla_x) u(x) = 0, \qquad x \in \partial\Omega \tag{0.2}$$

Suppose we have found a frame where a unique solution to this problem exists, then to apply numerical schemes this problem has to be reduced to a bounded problem, or at least to a sequence of bounded problems. A fairly used method to do this is to consider a family of boundary value problems on bounded domains Ω_R, where Ω_R is a sequence of domains exhausting Ω as R tends to infinity. If the approximating solution (v^R, p^R) solves the original problem restricted to Ω_R, an additional boundary condition has to be prescribed on the artificial boundary $\partial\Omega_R \backslash \partial\Omega$. This boundary condition has to be designed in such a way, that on one hand the approximation problem is uniquely solvable and on the other hand the difference $(v, p) - (v^R, p^R)$ decays as quick as possible as R tends to infinity

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On a motion of a solid body in a viscous non-homogeneous fluid

This work is devoted to investigation of the problem on a motion of an absolutely solid body in a viscous incompressible fluid filling a bounded container. There are few papers by N.A.Yudakov (1974), B.Desjardins and M.J.Esteban (1999), C.Conca, J. San Martin and M.Tucsnak (1999), M.D.Gunzburger, H.-C.Lee and G.A.Seregin (2000), where existence of a local–in–time solution (up to collisions of the body with the boundary) is proved. As it was recently shown by K.–H.Hoffmann and V.N.Starovoitov (1996), this problem has a global weak solution under the assumption that the boundaries of the body and of the container are curves of the class C^2 in 2-D case and spheres in 3-D case. The fluid was assumed to be homogeneous. In that work a method, which can be called "method of solidification", was applied. Namely, the body is considered as a part of the fluid, where the viscosity tends to infinity. If the boundary of the body and of the container satisfy the smoothness conditions mentioned above, then the body hits the wall with zero speed. This property enables to prove the global solvability of the problem. In the present work the same result for a non-homogeneous fluid is obtained. Let $\Omega \subset R^2$ be a bounded domain and $S(t)$ be its subdomain occupied by the body. We denote by φ the characteristic function of S. Let us introduce the following function space

$$K(S) = \{\vec{\psi} \in H_0^1(\Omega) \mid \mathcal{D}(\vec{\psi})(\vec{x}) = 0 \quad \text{for} \quad \vec{x} \in S, \quad div\,\vec{\psi} = 0\},$$

where $\mathcal{D}(\vec{\psi})$ is the tensor with the components $\mathcal{D}_{ij}(\vec{\psi}) = (\partial\psi_i/\partial x_j + \partial\psi_j/\partial x_i)/2$. Denote by $L_p(0,T;K(S(t)))$, $p \geqslant 1$, the set of functions from $L_p(0,T;H_0^1(\Omega))$ belonging to $K(S(t))$ for almost all $t \in [0,T]$.

Definition. *The triple of functions* $\vec{v} \in L_\infty(0,T;L_2(\Omega)) \cap L_2(0,T;K(S(t)))$, $\rho \in L_\infty(\Omega_T)$ *and* $\varphi \in C^{1/p}(0,T;L_p(\Omega))$, $1 \leqslant p < \infty$, *where* $\Omega_T = [0,T] \times \Omega$, $T < \infty$, *is said to be a generalized solution of the problem if the following integral identities*

$$\int_{\Omega_T} \Big(\rho\vec{v}\,(\vec{\psi}_t + (\vec{v}\cdot\nabla)\vec{\psi}) - \mathcal{D}(\vec{v}) : \mathcal{D}(\vec{\psi})\Big)\, d\vec{x}dt = -\int_{\Omega} \rho_0\,\vec{v}_0 \cdot \vec{\psi}_0\, d\vec{x},$$

$$\int_{\Omega_T} \rho\,(\eta_t + \vec{v}\cdot\nabla\eta)\, d\vec{x}dt = -\int_{\Omega} \rho_0\,\eta_0\, d\vec{x}, \quad \int_{\Omega_T} \varphi\,(\eta_t + \vec{v}\cdot\nabla\eta)\, d\vec{x}dt = -\int_{\Omega} \varphi_0\,\eta_0\, d\vec{x}$$

hold for any functions $\eta \in C^1(\Omega_T)$, $\eta(T) = 0$, $\vec{\psi} \in H^1(\Omega_T) \cap L_2(0,T;K(S(t)))$, $\vec{\psi}(T) = 0$. In this definition the solid body is specified by the condition that the deformation rate tensor $\mathcal{D}(\vec{v})$ is equal to zero in $S(t)$.

Theorem. *Let* $\vec{v}_0 \in L_2(\Omega)$, $0 < m \leqslant \rho_0 \leqslant M < \infty$ *and the boundaries* $\partial S(0))$ *and* $\partial\Omega$ *are curves of the class* C^2. *Then there exists a generalized solution of the problem. Moreover,*

1. there exists a family of isometries $\mathcal{A}_{s,t} : R^2 \to R^2$, $s,t \in [0,T]$, *such that* $S(t) = \mathcal{A}_{s,t}(S(s))$ *(in particular* $S(t) = \mathcal{A}_{0,t}(S(0))$*) and* $\mathcal{A}_{s,t}$ *is Lipschitz-continuous with respect to* s *and* t;

2. if $h(t) = dist(\partial\Omega, S(t))$ *and* $h(t_0) = 0$ *for some* $t_0 \in [0,T]$ *then* $\lim_{t\to t_0} h(t)\,|t-t_0|^{-2} = 0$;

3. if $E = \{t \in [0,T] \mid h(t) = 0\}$ *then* $\vec{v}(\vec{x},t) = 0$ *as* $\vec{x} \in S(t)$ *for almost all* $t \in E$. The same results can be obtain for the case of a few bodies in the fluid.

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Infinite-dimensional elliptic coordinates

We study the generalization of the elliptic coordinates (e.c.) to the infinite-dimensional case, assuming that the corresponding self-adjoint operator A has a purely discrete spectrum.

The e.c. of a vector x are given by the set $\{\lambda_i\}$ of roots of the equation:

$$1/2((A - \lambda E)^{-1}x, x) = 1.$$

Let $\{a_i\}$ be the ordered sequence of the eigenvalues of A.

Theorem 1. *Suppose that A is a self-adjoint lower bounded operator with compact inverse. A sequence $\{\lambda_n\}_{n=1}^{n=\infty}$ such that $a_n < \lambda_{n+1} < a_{n+1}$ and $\lambda_1 < a_1$ is the sequence of e.c. of some vector x in and only if*

$$\sum_{k=1}^{\infty}(a_k - \lambda_k) < \infty;$$

moreover, $||x||^2 = 2\sum_{k=1}^{\infty}(a_k - \lambda_k)$.

Theorem 2. *Let $A > 0$ be a compact operator.*

1. *A sequence $\{\lambda_n\}_{n=1}^{n=\infty}$ such that $a_{k+1} < \lambda_k < a_k$ is the sequence of e.c. of some vector if and only if $\prod_{n=1}^{\infty} \lambda_n/a_n > 0$.*

2. *A sequence $\{\lambda_n\}_{n=0}^{n=\infty}$ such that $\lambda_0 \leqslant 0$ and $a_{k+1} < \lambda_k < a_k$ is the sequence of e.c. of some vector if and only if $\prod_{n=1}^{\infty} \lambda_n/a_n = 0$.*

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On point spectrum of a class of transport operators

In the Hilbert space $L^2(\Omega \times \mathbb{R}^3)$ we consider operator

$$L\psi(\mathbf{r}, \mathbf{v}) = -\mathbf{v}\nabla_{\mathbf{r}}\psi(\mathbf{r}, \mathbf{v}) - h(|\mathbf{v}|)\psi(\mathbf{r}, \mathbf{v}) + \int_{\mathbb{R}^3} S(|\mathbf{v}|, |\mathbf{v}'|)\psi(\mathbf{r}, \mathbf{v}')\frac{d\mathbf{v}'}{|\mathbf{v}'|^2}, \qquad (0.1)$$

where $\Omega \subset \mathbb{R}^3$ is a bounded convex domain and on the boundary of Ω the following condition is filfilled: $\psi(\mathbf{r}, \mathbf{v}) = 0$ if $\mathbf{r} \in \partial\Omega$ and $(\mathbf{v}, \mathbf{n}) < 0$, $\mathbf{n}$ being exterior normal to the boundary at the point $\mathbf{r}$. It is assumed that $h(x) \geqslant 0 = \lim_{x\to 0} h(x)$, kernel $S(x, y) \geqslant 0$ is symmetric and the following conditions hold:

$$\sup_{x\in\mathbb{R}_+} \int_0^{\infty} S(x, y)\, dy < \infty, \quad K(x, y) := \int_0^{\infty} \frac{S(x, z)\, S(z, y)}{z^2}\, dz \in L^2(\mathbb{R}_+^2).$$

Operator of the type (0.1) appears in the particle transport theory [1] and is related to linearized Boltzmann equation.

Theorem 1. *Essential spectrum of operator L occupies the closed left half-plane, while discrete spectrum of L belongs to $\mathbb{R}_+$.*

By the use of a nonselfadjoint version of Birman-Schwinger principle (cf.[2]) an upper bound on the number of eigenvalues of L is obtained.

Theorem 2. *The total multiplicity $N(L)$ of eigenvalues of L admits the estimate*

$$N(L) \leqslant \iint\limits_{\Omega\times\Omega} |J(\mathbf{r},\mathbf{r}')|^2 d\mathbf{r}d\mathbf{r}' \iint\limits_{\mathbb{R}_+\times\mathbb{R}_+} |K(x,y)|\, dxdy, \qquad (0.2)$$

where $J(\mathbf{r},\mathbf{r}') \in L^2(\Omega\times\Omega)$ is the first iterate of the kernel $|\mathbf{r}-\mathbf{r}'|^{-2}, \mathbf{r},\mathbf{r}' \in \Omega$.

Estimate (0.2) confirms and qulitatively supplements Nelkin's hypothesis from the theory of neutron transport [3], which asserts that the discrete spectrum of L is empty if the size of Ω is small enough.

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A non-symplectic KAM phenomena for singular holomorphic vector fields

Let X be a germ of holomorphic vector field at $0 \in \mathbb{C}^n$ vanishing at this point. We assume that X is a "perturbation" of singular completely integrable system and whose linear part admits non-trivial polynomial first integrals. We show that it admits a lot of invariant analytic subsets in a neighbourhood of 0. They are biholomorphic to the intersection of a polydisc and an analytic set of the form "resonnant monomials= constants". Moreover, despite the fact that X is not holomorphically normalizable, we show that it has a holomorphic normal form in a neighbourhood of 0. This normal form is tangent to the set "resonnant monomials= constants", its restriction is a linear diagonal vector field and is conjugated to the restriction of X on its invariant set. If X is non-degenerated, we show that the set of "frequencies" defining the invariant sets is of positive measure

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Primenenie vspleskov pri reshenii kraevykh zadach

V doklade rech' pojdet o primenenii garmonicheskikh vspleskov (Ju.N.Subbotin, N.I.CHernykh, Izvestija RAN, ser.matem., t.64(1), 2000, s.145-174) pri reshenii zadachi Dirikhle i Puassona dlja koncetricheskogo i ne koncetricheskogo kol'ca. S ispol'zovaniem garmonicheskikh vspleskov vypisyvajutsja novye reshenija taikh zadach i privodjatsja asimptoticheskie razlozhenija v skhodjachshiesja rjady takikh reshenij v predpolozhenii, chto diametr vnutrennego kruga javljaetsja malym parametrom. V etom chastnom sluchae rezul'taty usilivajut izvestnye rezul'taty A.M.Il'ina ob asimptoticheskikh razlozhenijakh reshenij ellipticheskikh kraevykh zadach dlja oblastej s malym otverstiem.

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On a p-basicity of eigenvalues of differentiable operator-functions

This talk is based on a joint work with T.Ya. Azizov and A. Dijksma. Let $\mathcal{H}$ be a separable Hilbert space and let $L(\cdot)$ be a continuous operator function on $[a,b]$ which values are bounded selfadjoint operators. We give sufficient conditions under that there is in the closed linear span of eigenvectors of $L(\cdot)$ a p-basis consisting of eigenvectors of $L(\cdot)$.

Definition. *The system $\{e_j\}_{j=1}^{\infty} \subset \mathcal{H}$ is called a p-basis if there are both an orthonormal basis $\{f_j\}_{j=1}^{\infty}$ and an operator $T \in \mathfrak{S}_p$ such that $e_j = (I+T)f_j$,* $j = \overline{1,\infty}$. In an article of V.A. Grinstein (1991) there is considered a holomorphic operator function $L(\cdot)$ in a neighborhood of $[a,b]$:

$$L(\lambda) = \lambda + \sum_{k=0}^{\infty} \lambda^k A_k, \tag{0.1}$$

$A_k = A_k^*$, $k = \overline{0,\infty}$, and $A_k \in \mathfrak{S}_{p_k}$, $0 < p_k \leqslant \infty$ for $k \leqslant m$, $m > 0$ and A_k are bounded for $k > m$, and the following result is proved.

Theorem 1. *Let $a < 0 < b$, let the function* (0.1) *be holomorphic in a neighborhood $U \supset [a,b]$ and let $L'(\lambda) \gg 0$ for all $\lambda \in [a,b]$. Then the eigenvectors system* *of (0.1) related to eigenvalues lied on $[a,b]$ forms in a finite codimentional subspace of $\mathcal{H}$ a p-basis with*

$$p \geqslant \left[\min\left(\frac{1}{p_1}, \frac{1}{p_0}+\frac{1}{p_2}, \frac{2}{p_0}+\frac{1}{p_3}, \dots, \frac{m-1}{p_0}+\frac{1}{p_m}, \frac{m}{p_0}\right)\right]^{-1}. \tag{0.2}$$

If in addition

$$L(a) \ll 0, \quad L(b) \gg 0,$$

then the eigenvectors system of (0.1) related to eigenvalues lied on $[a,b]$ forms a p-basis in $\mathcal{H}$ with the same p as in (0.2). Let $L(\cdot)$ be a holomorphic on U operator function which satisfies the assumptions of the Virozub-Matsaev factorization theorem (1974). Then

$$L(\lambda) = M(\lambda)(\lambda - Z), \tag{0.3}$$

and the following properties hold:(a) $M(\lambda)$ is a holomorphic operator function, $M(\lambda)$, $M(\lambda)^{-1} \in L(\mathcal{H})$ for all $\lambda \in U$;
(b) there is a bounded and boundedly invertible positive operator G such that ZG is selfadjoint;
(c) G satisfies the identities

$$M(x) = (I - L(x)F(x))G^{-1}, \quad M^{-1}(x) = G + (x - Z)F(x), \; x \in [a,b], \tag{0.4}$$

where $F(\cdot)$ is a regular part of the representation of $L(\cdot)$:

$$L^{-1}(x) = (x - Z)^{-1}G + F(x). \tag{0.5}$$

Below in the case when in general $L(\cdot)$ is nonholomorphic but $m+1 \geqslant 1$ times continuously differentiable we assume instead (a) that $L(\cdot)$ has a representation (0.3) with a compact operator Z, and also assume (b) and (0.4) with $F(\cdot)$ which is the extension by continuity on $[a,b]$ of $F(x) = L^{-1}(\lambda) - (\lambda - Z)^{-1}G$ (see (0.5)). The main result is the following:

Theorem 2. *Let m times continuously differentiable on $[a,b]$ selfadjoint operator function $L(\cdot)$ besides the factorization conditions mentioned above satisfy the folowing assumptions:*

$$\begin{cases} A_0 := L(0) \in \mathfrak{S}_{p_0}, \quad A_1 := L'(0) - I \in \mathfrak{S}_{p_1}, \\ A_k := L^{(k)}(0) \in \mathfrak{S}_{p_k}, \; k = \overline{2,m}. \end{cases}$$

Then there is in the closed linear span of eigenvectors of $L(\cdot)$ a p-basis consisting of eigenvectors of $L(\cdot)$ with

$$p \geqslant \min_{s=\overline{1,m}} \left[\min\left(\frac{1}{p_1}, \frac{1}{p_0}+\frac{1}{p_2}, \frac{2}{p_0}+\frac{1}{p_3}, \dots, \frac{s-1}{p_0}+\frac{1}{p_s}, \frac{s}{p_0}, 1\right)\right]^{-1}.$$

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Eigenvalues distribution of one pencil of differential operators in nonkeldysh situation

Let A is the self-adjoint differential operator, generated in $H = L_2(-\infty;+\infty)$ by the differential expression $y^{(4)} + q(x)y$ with a positive function $q(x)$. B is a self-adjoint operator generated in the space H by the differential expression $\frac{i}{2}[(p(x)y'')' + (p(x)y')'']$, where $p(x)$ is twice continously differentiated positive function. Cosider an operator pencil $L(\lambda) = A + \lambda B - \lambda^2 I$. In the present work we have received asymptotic formulae for distribution function$N(\lambda)$ of eigenvalues of the operator pensil $L(\lambda)$. We consider the case when $L(\lambda)$ is not Keldysh pencil, that is both function $p(x)$ and function $q(x)$ influence on behaviour of the function $N(\lambda)$.

Theorem. *Let functions $p(x)$ and $q(x)$ satisfy the following demands*

$$q(x) \to +\infty \quad \text{on condition that} \quad |x| \to \infty, \tag{0.1}$$

$$|p(x)-p(y)| \leqslant cp(x)|x-y|,$$
$$|q(x) - q(y)| \leqslant cq^{5/4-\varepsilon_1}(x)|x-y|, \tag{0.2}$$

$$\text{when} \quad |x-y|r(y) \leqslant 1, \quad \text{where} \quad r(y) = q^{\chi}(y), \quad 0 < \chi < 1/4, \quad \varepsilon_1 > 0$$

$$p(x) \leqslant q^{1/4-\varepsilon}(x), \quad \varepsilon > 0, \tag{0.3}$$

and is fulfilled:

$$\psi(\lambda) = \frac{3(m-1)!}{2^{m+4}m\pi} \int\limits_0^\lambda s^{2m} ds \int\limits_\Omega \frac{p^{2m+1}(x)}{|\alpha(x,s)|^m} dx, \quad \psi(-\lambda) = -\psi(\lambda), \quad \lambda > 0,$$

satisfies the conditions of Tauberian theorem [1]. Ω is the set in the space of variables x, s, defined by the inequality $27s^4p^4(x) > 256(q(x) - s^2)$.

$$\alpha(x,s) = \left(\frac{-s^2p^2(x)(q(x)-s^2)}{16} - \sqrt{\frac{s^4p^4(x)(q(x)-s^2)^2}{16^2} - \frac{(q(x)-s^2)^3}{27}} \right)^{1/3} +$$

$$+\left(\frac{-s^2p^2(x)(q(x)-s^2)}{16}+\sqrt{\frac{s^4p^4(x)(q(x)-s^2)^2}{16^2}-\frac{(q(x)-s^2)^3}{27}}\right)^{1/3}-\frac{s^2p^2(x)}{8}.$$

Then $N(\lambda)\sim\psi(\lambda)$, *as* $\lambda\to\pm\infty$.

The functions $p(x)$ and $q(x)$ satisfy the usual conditions of growth's regularity of the type Levitan-Titchmarsh. It is important to note, that in the case of Keldysh only the function $q(x)$ influences on behaviour of $N(\lambda)$.

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Suslina L.A.
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Absolute continuity of the spectrum of a periodic Schrödinger operator with singular potential

In $L_2(\mathbf{R}^2)$ we study a periodic magnetic Schrödinger operator
$H=(\mathbf{D}-\mathbf{A}(\mathbf{x}))^*g(\mathbf{x})(\mathbf{D}-\mathbf{A}(\mathbf{x}))+V(\mathbf{x})+\sigma(\mathbf{x})\delta_\Sigma(\mathbf{x})$
with variable metric $g(\mathbf{x})$, magnetic potential $\mathbf{A}(\mathbf{x})$ and electric potential $V(\mathbf{x})+\sigma(\mathbf{x})\delta_\Sigma(\mathbf{x})$. The coefficients g, $\mathbf{A}$, V are periodic with respect to a lattice of periods $\Gamma\subset\mathbf{R}^2$. The electric potential is a sum of regular term V and singular term $\sigma\delta_\Sigma$. Here Σ is a Γ-periodic system of piecewise smooth curves and $\sigma(\mathbf{x})$ is a periodic real-valued function on Σ. Under rather wide assumptions on g, $\mathbf{A}$, V, Σ and σ we prove that the spectrum of H is absolutely continuous. This is joint result with M. Birman and R. Shterenberg.

Taimanov I.A.
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On lower bounds for the Willmore functional.

We shall discuss lower bounds for the value of the Willmore functional (the integral of the squared mean curvature) for compact surfaces in the three-space. Some time ago we had discovered such a bound which is quadratic in the dimension of the kernel of the Dirac operator associated with the surface. We proved this inequality for a wide class of spheres including spheres of revolution by using the inverse scattering theory of the one-dimensional Dirac operator and conjectured this inequality for all spheres. Recently Pinkall and his collaborators proved our conjecture together with its generalization for all topological types of surfaces. This implies interesting new inequalities in the theory of harmonic tori.

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On a criterion of presence of lacuna around singular loci of wave propagation.

We give an algebraic descriptions of (wave) fronts that appear in strictly hyperbolic Cauchy problem. Concrete form of defining function of wave front issued from initial algebraic variety is obtained by the aid of Gauss-Manin systems satisfied by Leray's residues associated to certain isolated complete intersection singularities. Further, we shall discuss asymptotic behaviour of solutions to the Cauchy problem around the wave front and thus establish a criterion of presence of lacuna by studying certain cohomology group in contrast with the method using homology groups established by I.G.Petrovsky in 1945.

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Интегрируемый случай Ковалевской-Горячева

Классическая задача о движении твёрдого тела вокруг неподвижной точки даёт много примеров интегрируемых гамильтоновых систем, изучению топологии которых посвящен большой цикл работ [1]-[7]. Одним из наиболее знаменитых интегрируемых случаев является так называемый волчок Ковалевской [1], в котором дополнительный интеграл имеет четвёртую степень. Его топология в настоящий момент полностью изучена (см. [4], [5]). В работе [2] Д.Н.Горячев частично обобщил этот случай, найдя гамильтониан с более общим потенциалом, который при нулевой константе площадей интегрируется также при помощи интеграла четвёртой степени. В безразмерных переменных S_1, S_2, S_3, R_1, R_2, R_3, связанных с обычными переменными Эйлера-Пуассона при помощи линейного преобразования ([3], с.28) полученные Д.Н.Горячевым гамильтониан H и дополнительный интеграл K записываются следующим образом:

$$H = \frac{1}{2}(S_1^2 + S_2^2 + 2S_3^2) + \frac{b}{2}(R_1^2 - R_2^2) + cR_1 + dR_2,$$

$$K = \left(\frac{S_1^2 - S_2^2}{2} + \frac{b}{2}R_3^2 - cR_1 + dR_2\right)^2 + (S_1S_2 - cR_1 - dR_2)^2.$$

. В настоящей работе проведён топологический анализ для случая, когда $d = 0$. Для него исследованы изоэнергетические поверхности, построена бифуркационная диаграмма отображения момента, а также в соответствии с качественной теорией, разработанной А.Т.Фоменко, А.В.Болсиновым и другими авторами в 80-е годы, выписаны молекулы дополнительного интеграла

K на изоэнергетических поверхностях. Теоретический аппарат работы описан в [6], [7].

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Greedy Algorithms in Banach Spaces

We study efficiency of approximation and convergence of two greedy type algorithms in uniformly smooth Banach spaces. The Weak Chebyshev Greedy Algorithm (WCGA) is defined for an arbitrary dictionary $\mathcal{D}$ and provides nonlinear m-term approximation with regard to $\mathcal{D}$. This algorithm is defined inductively with the m-th step consisting of two basic substeps: 1) selection of an m-th element φ_m^c from $\mathcal{D}$ and 2) constructing an m-term approximant G_m^c. We include the name of Chebyshev in the name of this algorithm because at the substep 2) the approximant

G^c_m is chosen as the best approximant from $\mathrm{sp}(\varphi^c_1, \ldots, \varphi^c_m)$. The term Weak Greedy Algorithm indicates that at each substep 1) we choose φ^c_m as an element of $\mathcal{D}$ that satisfies some condition which is "t_m-times weaker" than the condition for φ^c_m to be optimal ($t_m = 1$). We got error estimates for Banach spaces with modulus of smoothness $\rho(u) \leqslant \gamma u^q$, $1 < q \leqslant 2$. We proved that for any f from the closure of the convex hull of $\mathcal{D}$ the error of m-term approximation by WCGA is of order $(1+t_1^p+\cdots+t_m^p)^{-1/p}$, $1/p+1/q = 1$. Similar results are obtained for Weak Relaxed Greedy Algorithm (WRGA) and its modification. In this case an approximant G^r_m is a convex linear combination of $0, \varphi^r_1, \ldots, \varphi^r_m$. We also proved some convergence results for WCGA and WRGA.

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The Cauchy problem for integrodifferential systems arising in Fluid Mechanics

Integrodifferential systems of equations of special form appear in many problems related to fluid mechanics and physics. For example, the propagation of the long waves on a rotational flow of a perfect incompressible or compressible liquid is governed by the integrodifferential system of equations, which generalizes classical shallow water model. Analogous sytems arise in kinetic modeling of bubbly flows and plasma flows when the approximation of "quasineutrality" is applied. All above mentioned systems have common mathematical structure and can be considered as generalizations of conventional hyperbolic conservation laws. In the paper generalizing the theory of characteristics and hyperbolicity concept, we formulate the conditions for the Cauchy data that guarantee the hyperbolicity of integrodifferential systems. Then we prove local correctness in Sobolev spaces of the Cauchy problem with initial data satisfying the hyperbolicity conditions. It is shown that a smooth solution of quasilinear integrodifferential system exists only on bounded interval of time and examples of the solutions demonstrating "breaking of the waves" are presented. The theory of simple waves and discontinuous solutions is developed for some special models of fluid mechanics. The statement of the Riemann problem is proposed and the solution in specific classes of initial data is given. We discuss possible applications of the methods developed for dissipative models which describe viscous effects in a "boundary layer" approximation.

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Maximum norm estimates for parabolic finite element equations

We consider the initial-boundary value problem

$$u_t - \Delta u = 0 \quad \text{in } \Omega, \quad \text{for } t > 0, \qquad (0.1)$$
$$u = 0 \quad \text{on } \partial\Omega, \quad \text{for } t > 0, \quad \text{with } u(\cdot, 0) = v \quad \text{in } \Omega.$$

where Ω is a bounded domain in R^2 with sufficiently smooth boundary We first consider the spatially semidiscrete finite element approximation based on continuous, piecewise linear approximating functions on a quasiuniform family $\mathcal{T}_h$ of triangulations of Ω. Denoting the finite element space corresponding to $\mathcal{T}_h$ by S_h this semidiscrete problem is to find $u_h(t) \in S_h$ for $t > 0$ such that, with $(\cdot, \cdot)$ the standard inner product in $L_2(\Omega)$,

$$(u_{h,t}, \chi) + (\nabla u_h, \nabla \chi) = 0, \quad \forall \chi \in S_h, \ t > 0, \quad \text{with } u_h(0) = v_h \in S_h. \qquad (0.2)$$

In semigroup notation $u(t) = E(t)v = e^{\Delta t}v$ and $u_h(t) = E_h(t)v = e^{\Delta_h t}v_h$, where $\Delta_h : S_h \to S_h$ is a discrete analogue of the Laplacian Δ. For (0.1) the maximum-principle shows $\|E(t)v\|_\infty \leqslant \|v\|_\infty = \|v\|_{L_\infty(\Omega)}$. However, the analogous inequality does not hold for (2). (With respect to the norm in $L_2(\Omega)$ the corresponding results hold trivially for both (0.1) and (0.2).) Schatz, Thomée, and Wahlbin (1980) showed the weaker result

$$\|E_h(t)v_h\|_\infty \leqslant C\log(1/h)\|v_h\|_\infty. \qquad (0.3)$$

The derivation uses estimates in weighted norms of a discrete Green's function. The solution operator $E(t)$ of (0.1) also has the smoothing property

$$\|E'(t)v\|_\infty \leqslant Ct^{-1}\|v\|_\infty,$$

and a corresponding result for (0.2) was also shown in Schatz et al. (1980),

$$\|E'_h(t)v_h\|_\infty \leqslant Ct^{-1}\log(1/h)\|v_h\|_\infty. \qquad (0.4)$$

Estimates of the type (0.3) and (0.4) without a logarithmic factor, have been shown recently, first by Schatz, Thomée, and Wahlbin (1998) for a problem of type (0.1) with Neumann boundary conditions and then in Thomée and Wahlbin (1999) for Dirichlet boundary conditions. Together they are equivalent to a resolvent estimate

$$\|(zI+\Delta_h)^{-1}v_h\|_\infty \leqslant M|z|^{-1}\|v_h\|_\infty, \quad \text{for } \varphi \leqslant |\arg z| \leqslant \pi, \ \varphi \in (0,\pi/2). \quad (0.5)$$

In recent work by Bakaev and Thomée (2001) (0.5) has been shown for any $\varphi \in (0,\pi/2)$. Such estimates may be used in the analysis of time stepping schemes of the form $U^n = r(k\Delta_h)^n v_h$ where k is the time step and $U^n \approx u(nk)$, and $r(z)$ is a rational function with certain stability and approximation properties, using an integral representation of $r(k\Delta_h)^n$ in terms of $(zI+\Delta_h)^{-1}$ in the complex plane. Some of the reference quoted also contain results for higher order finite elements than linear and in higher space dimension than two.

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Spectral theory for operators with spectrum on a curve

A trace class perturbation S of normal operator with spectrum on a closed smooth curve C are considered. Let $e_S^{\pm}(f,g,w) = \lim\limits_{z\to w}((S-z)^{-1}f,g)$ be interior and exterior angular boundary values, where $f,g \in H$. We define weak spectral components

$$N_\pm(S) = \operatorname{clos}\widetilde{N}_\pm(S); \ \widetilde{N}_\pm(S) = \{f \in H : e_S^\pm(f,g,w) \in E_\pm^2 \ \forall g \in H\},$$
$$M(S) = \{f \in H : e_S^+(f,g,w) = e_S^-(f,g,w) \ \forall g \in H\},$$
$$D_\pm(S) = \{f \in H : e_S^\pm(f,g,w) \in D_\pm \ \forall g \in H\},$$

where $E_\pm^2$ are Smirnov spaces, $D_\pm$ are Nevanlinna N^+ (Smirnov) classes for interior and exterior domains. These components and its combinations play important role for scattering theory [1], [2] and for extremal factorizations of J-contractive-valued functions (J-inner-outer and A-singular-regular) [3], [4]. We establish the following orthogonal decompositions

$$H = N(S) \oplus M(S^*) = D_\pm(S) \oplus \operatorname{clos}(\widetilde{N}_{(p)}(S^*) \cap M(S^*)) =$$
$$= N_\pm(S) \oplus (D_{(p)}(S^*) \cap M(S^*)).$$

Correspomding theory is developed for two cases: 1) curve C is analytical; 2) operator S is the special trace class perturbation of $\varphi(T)$, where T is a contraction and φ is a conformal map of the unit disk onto a domain with $C^{2+\epsilon}$ smooth boundary. We extend function model technique [1] and obtain explicit formulas for spectral components in the last case.

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Continuous selectors of fixed point sets of non-convex multifunctions with decomposable values and their applications

Existence and relaxation (density) theorems of continuous selectors whose values are fixed points of non-convex multivalued contractions are proved. Using these results the topological properties of fixed point sets are obtained. Applications to the parabolic and hyperbolic inclusions are presented

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Indicial equations of two coupled second-order differential equations

Let us consider the two homogeneous coupled differential equations

$$x^2 y_1'' + x p_1(x)\, y_1' + q_1(x)\, y_1 + x M_2(x)\, y_2' + N_2(x)\, y_2 = 0\,, \qquad (0.1)$$

$$x^2 y_2'' + x p_2(x)\, y_2' + q_2(x)\, y_2 + x M_1(x)\, y_1' + N_1(x)\, y_1 = 0\,. \qquad (0.2)$$

$x = 0$ is a regular singular point of (1) and (2). ($x = 0$ is called a *regular singular point* of (1), if all the functions $p_1(x)$, $q_1(x)$, $M_2(x)$ and $N_2(x)$ are analytic at

$x = 0$.) Using the method of Frobenius we assume a power series solution of the form

$$y_1(x) = x^{\lambda_1} \sum_{j\geqslant 0} a_j\, x^j, \quad a_0 \neq 0 \quad \text{and} \quad y_2(x) = x^{\lambda_2} \sum_{j\geqslant 0} b_j\, x^j, \quad b_0 \neq 0\,. \qquad (0.3)$$

We immediately see, that the Frobenius' theory is unsufficient, because we cannot find the two indicial equations. The Frobenius' Method is only evaluated for *one* nth order differential equation of *one* unknown function, but not for coupled equations. These indicial equations we want to derive now. In the classical case of Frobenius (here $M_n \equiv N_n \equiv 0$, $n = 1, 2$) the indicial equation of (1) is the factor of the smallest power $a_0 x^{\lambda_1}$. This we get after inserting all series into equation (1). But in our case, the smallest power x^{λ_1} of y_1 resp. x^{λ_2} of y_2 can appear in both equations (1) and (2). To get all terms with the smallest power we add (1) and (2) and separate the y_1 and y_2 derivatives. This implies

$$x^2\, y_1'' + x\,[p_1(x) + M_1(x)]\, y_1' + [q_1(x) + N_1(x)]\, y_1 = \qquad (0.4)$$

$$= -\,x^2\, y_2'' - x\,[p_2(x) + M_2(x)]\, y_2' - [q_2(x) + N_2(x)]\, y_1\,.$$

A very important point is that the factors of y_1'' and y_2'' are the same. We differentiate (3) term by term and insert all these series and the power series

$$p_n = \sum_{j\geqslant 0} p_{n;j} x^j, \quad q_n = \sum_{j\geqslant 0} q_{n;j} x^j, \quad M_n = \sum_{j\geqslant 0} M_{n;j} x^j, \quad N_n = \sum_{j\geqslant 0} N_{n;j} x^j. \quad (0.5)$$

for the coefficients in (4). For the multiplication of power series we use the Cauchy product. Furthermore, we split off the first term of all series in equation (4), because the first term is this with the smallest x power. Then (4) becomes

$$\lambda_1(\lambda_1 - 1) a_0\, x^{\lambda_1} + \sum_{j\geqslant 1} (j + \lambda_1)(j + \lambda_1 - 1) a_j x^{j+\lambda_1} + \lambda_1 (p_{1;0} + M_{1;0}) a_0\, x^{\lambda_1} +$$

$$+ \sum_{j\geqslant 1} [\sum_{0\leqslant m\leqslant j} (m + \lambda_1)(p_{1;j-m} + M_{1;j-m}) a_m] x^{j+\lambda_1} + (q_{1;0} + N_{1;0}) a_0\, x^{\lambda_1} +$$

$$+ \sum_{j\geqslant 1} [\sum_{0\leqslant m\leqslant j} (q_{1;j-m} + N_{1;j-m}) a_m] x^{j+\lambda_1} =$$

$$-\lambda_2(\lambda_2 - 1) b_0\, x^{\lambda_2} - \sum_{j\geqslant 1} (j + \lambda_2)(j + \lambda_2 - 1) b_j x^{j+\lambda_2} -$$

$$-\lambda_2 (p_{2;0} + M_{2;0}) b_0\, x^{\lambda_2} - \sum_{j\geqslant 1} [\sum_{0\leqslant m\leqslant j} (m + \lambda_2)(p_{2;j-m} + M_{2;j-m}) b_m] x^{j+\lambda_2} -$$

$$-(q_{2;0}+N_{2;0})b_0\, x^{\lambda_2} - \sum_{j\geqslant 1}\Big[\sum_{0\leqslant m\leqslant j}(q_{2;j-m}+N_{2;j-m})b_m\Big]x^{j+\lambda_2}.$$

We now equate the coefficient of the smallest power x^{λ_n} to zero. The corresponding equation is

$$[\lambda_n(\lambda_n-1)+(p_{n;0}+M_{n;0})\,\lambda_n+q_{n;0}+N_{n;0}]^{a_0,\ n=1}_{b_0,\ n=2}=0\,.$$

Since by assumption $a_0\neq 0$ and $b_0\neq 0$, we obtain the indicial equation

$$\lambda_n{}^2+(p_{n;0}+M_{n;0}-1)\,\lambda_n+q_{n;0}+N_{n;0}=0\,. \qquad (0.6)$$

In answer to our question we state the following theorem.

Theorem 1 *We assume, that $x=0$ is a regular singular point of the differential equations* (1) *and* (2). *$p_n(x)$, $q_n(x)$, $M_n(x)$ and $N_n(x)$ are analytic functions at $x=0$. Let us assume series solutions in the form* (3)

$$y_1(x)=x^{\lambda_1}\left[a_0+a_1x+a_2x^2+\cdots\right]\ \text{and}\ y_2(x)=x^{\lambda_2}\left[b_0+b_1x+b_2x^2+\cdots\right].$$

The a_0 and b_0 are chosen so that $a_0\neq 0$ and $b_0\neq 0$. Then holds: I) The two values for λ_n are the roots of the indicial equation given by (6). *II) The coefficients of the indicial equations are obtained by*

$$p_{n;0}=p_n(0)\,,\quad q_{n;0}=q_n(0)\,,\quad M_{n;0}=M_n(0)\,,\quad N_{n;0}=N_n(0)\,. \qquad (0.7)$$

Relation (7) easily can be seen because the series (5) are Maclaurin series and the $p_{n;j}$, $q_{n;j}$, etc. are the corresponding Maclaurin coefficients. From this point onward vorwaerts, fortschreitend) the classical Method of Frobenius can be used again:

Corollary 1 *Any differential equations of the form* (1) *and* (2) *with the same assumptions as in* **Theorem 1**, *have at least one solution which can be represented in the form* (3). *The exponents λ_1, λ_2 may be any (real or complex) numbers. Let the roots of the indicial equation differ by an integer. Taking in both cases the larger exponents (or exponents with the larger real parts) $\lambda_{1,1}$ ($\geqslant\lambda_{1,2}$) and $\lambda_{2,1}$ ($\geqslant\lambda_{2,2}$), such a solution not involving the logarithmic term always exists.*

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К обратной задаче спектрального анализа для степени оператора лапласа с потенциалом на прямоугольнике

Настоящая работа по своей тематике и методам примыкает к [1]. Пусть $a > 0$, $b > 0$, $\Pi = [0,a] \times [0,b]$ — прямоугольник в $\mathbb{R}^2$ со сторонами $l_1 = \{(a,t) \mid 0 \leqslant t \leqslant b\}$, $l_2 = \{(t,b) \mid 0 \leqslant t \leqslant a\}$, $l_3 = \{(0,t) \mid 0 \leqslant t \leqslant b\}$, $l_4 = \{(t,0) \mid 0 \leqslant t \leqslant a\}$. Для оператора Лапласа $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$ на Π рассмотрим всевозможные смешанные граничные задачи следующего вида: на некоторых сторонах прямоугольника Π задается условие Дирихле, а на оставшихся — условие Неймана. Для описания этих задач мы будем использовать удобные обозначения из [2]. Любое целое число $s = \overline{0,15}$ однозначно представляется в виде $s = 8i_1 + 4i_2 + 2i_3 + i_4$, где i_1, i_2, i_3, i_4 равны 0 или 1. Полагаем $r_1 = |i_1 - i_3|$, $r_2 = |i_2 - i_4|$, $r_3 = i_3$, $r_4 = i_4$. Пусть при $s = \overline{0,15}$ T_s есть самосопряженный неотрицательный оператор в $L^2(\Pi)$, порожденный спектральной граничной задачей

$$\Delta u + \lambda u = 0 \text{ на } \Pi, \quad r_j u + (1 - r_j)\partial u/\partial\nu = 0 \text{ на } l_j \ (j = \overline{1,4}), \tag{1}$$

где ν — внутренняя нормаль к границе $\partial\Pi$ прямоугольника Π. Оператор T_3 соответствует задаче Дирихле, T_0 — задаче Неймана. Пусть $s = \overline{0,15}$,

$$u_{mn}(s) = u_{mn}(x,y;s) =$$
$$2\sqrt{\gamma_{m+i_1}\gamma_{n+i_2}/(ab)}\cos[\pi(m + i_1/2)x/a - \pi i_3/2]\cos[\pi(n + i_2/2)y/b - \pi i_4/2],$$

где $\gamma_0 = 1/2$ и $\gamma_n = 1$ при $n \neq 0$. Для любого $\alpha > 0$ $\{u_{mn}(s) \mid (m,n) \in J_s\}$ есть ортонормированная полная система в $L^2(\Pi)$, состоящая из собственных функций $u_{mn}(s)$ оператора T_s^α, соответствующих собственным числам $\lambda_{mn}^\alpha(s)$, где $\lambda_{mn}(s) = \pi^2(M^2/a^2 + N^2/b^2)/4$. Здесь и далее $J_s = \{(m,n) \in \mathbb{Z}^2 \mid m \geqslant i_3(1-i_1)$ и $n \geqslant i_4(1-i_2)\}$,

$$M = 2m + i_1, \quad N = 2n + i_2. \tag{2}$$

Пусть P — оператор умножения на функцию $p \in L^2(\Pi)$ такую, что

$$\iint\limits_{\Pi} p(x,y)\cos\frac{\pi n x}{a}\,dx\,dy = \iint\limits_{\Pi} p(x,y)\cos\frac{\pi n y}{b}\,dx\,dy = 0 \quad (n = 0,1,2,\ldots). \tag{3}$$

Обозначим через $\mu_{mn}(s,\alpha,p)$ собственные числа оператора $T_s^\alpha + P$, занумерованные так, что $|\mu_{mn}(s,\alpha,p) - \lambda_{mn}^\alpha(s)| \leqslant \text{const}$ при любых $(m,n) \in J_s$; это всегда можно сделать при $\alpha > 2$.

Теорема. *Пусть для любых натуральных чисел M и N имеем*

$$s_{MN} \in Q_k \equiv \{4k + j \mid j = 0, 1, 2, 3\}$$

при $k = 2i_1 + i_2$, где целые числа $i_1 \in \{0, 1\}$, $i_2 \in \{0, 1\}$, m, n однозначно определяются по M, N согласно (2). Тогда, если a^2/b^2 — иррациональное алгебраическое число, $\alpha > 5/2$ и $\sum_{M,N=1}^{\infty} |\xi_{MN} - \lambda_{mn}^{\alpha}(s_{MN})|^2 < \varepsilon$, где ε — достаточно малое положительное число, то в некотором шаре $\|u\| \leqslant \delta = \delta(\varepsilon)$ пространства $L^2(\Pi)$ существует один и только один потенциал p со свойством (3) такой, что $\mu_{mn}(s_{MN}, \alpha, p) = \xi_{MN}$ при всех натуральных M и N.

Доказательство проводится аналогично доказательству теоремы из [1] и состоит в построении оператора сжатия $A(p)$ в $L^2(\Pi)$, а именно, $p = q + A(p)$, где $q = \sum_{M,N=1}^{\infty} \theta(s_{MN})[\xi_{MN} - \lambda_{mn}^{\alpha}(s_{MN})] \cos(\pi Mx/a) \cos(\pi Ny/b)$,

$A(p) = \sum_{M,N=1}^{\infty} \theta(s_{MN}) A_{MN}(p) \cos(\pi Mx/a) \cos(\pi Ny/b)$,

$$A_{MN}(p) = \frac{1}{2\pi i} \int\limits_{\Gamma_{MN}} \lambda \ \mathrm{Sp}\ [R_\lambda(T^{\alpha}_{s_{MN}} + P)(PR_\lambda(T^{\alpha}_{s_{MN}}))^2]\, d\lambda,$$

$\theta(s_{MN}) = (-1)^{i_3+i_4} 2\sqrt{ab}$, $R_\lambda(V) \equiv (V - \lambda E)^{-1}$ — резольвента оператора V, $\Gamma_{MN} = \{\lambda \in \mathbb{C} \mid |\lambda - \lambda_{mn}^{\alpha}(s_{MN})| = r_{MN}\}$, $r_{MN} = \min\{|\lambda_{mn}^{\alpha}(s_{MN}) - \lambda_{m'n'}^{\alpha}(s_{MN})| \mid J_{s_{MN}} \ni (m', n') \neq (m, n)\}/2$.

Замечание 1. Из теоремы следует, что можно восстановить потенциал p из $L^2(\Pi)$ при условии (3) по частям спектров любых четырех операторов $T^{\alpha}_{s_k}$ $(k = \overline{0, 3})$, где $s_k \in Q_k$ при всех $k = \overline{0, 3}$.

Замечание 2. Условие (3) может быть снято, однако формулировка теоремы при этом усложняется.

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On the solvability of some nonlinear Neumann problems

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Blow-Up Results For Some Classes Of Nonlinear Systems

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Algorithmic solution of linear and nonlinear differential equations: 100 years of research

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Model spectrum problem for Poizeile profile. Critical spectrum curve.

We study the behavior of the spectrum of one model problem

$$i\varepsilon y'' + x^2 y = \lambda y, \quad y(-1) = y(1) = 0, \tag{0.1}$$

where ε is a small positive parameter.

It's easy to show, that the spectrum consists of infinite number of eigenvalues. Also it's known, that real parts of eigenvalues tend to 1/3 for every fixed ε, when the absolut values of the eigenvalues tend to infinity [1].

Let us assume

$$\mathcal{T}_{(-\frac{\pi}{2};-\frac{\pi}{4}]} = \{\lambda \in \Pi \,|\, \mathrm{Re}\, e^{\frac{\pi}{4}i} \int_{-1}^{1} \sqrt{\zeta^2 - \lambda}\, d\zeta = 0,\ \arg\lambda \in (-\frac{\pi}{2}; -\frac{\pi}{4}]\};$$

$$\mathcal{T}_{-\frac{\pi}{4}} = [0, \lambda_0],$$

where $\lambda_0 \in \mathcal{T}_{(-\frac{\pi}{2};-\frac{\pi}{4}]}$ and $\arg\lambda_0 = -\pi/4$;

$$\mathcal{T}_{[-\frac{\pi}{4};0]} = \left\{\lambda \in \Pi \,\middle|\, \operatorname{Re} e^{\frac{\pi}{4}i} \int_{\sqrt{\lambda}}^{1} \sqrt{\zeta^2 - \lambda}\, d\zeta = 0,\ \arg\lambda \in [-\frac{\pi}{4}; 0]\right\},$$

$$\mathcal{T} = \mathcal{T}_{(-\frac{\pi}{2};-\frac{\pi}{4}]} \cup \mathcal{T}_{-\frac{\pi}{4}} \cup \mathcal{T}_{[-\frac{\pi}{4};0]};$$

everywhere

$$\Pi = \left\{\lambda \in \mathbb{C} \,\middle|\, \operatorname{Im}\lambda < 0,\ 0 < \operatorname{Re}\lambda < 1\right\}.$$

Theorem 1 *All eigenvalues of (0.1) are kept in* Π. *For every neighbourhood* $O \subset \Pi \setminus \mathcal{T}$ *exists* $\varepsilon_0 > 0$ *and for each* $\varepsilon < \varepsilon_0$ *the neighbourhood* O *doesn't contain any eigenvalue. For every open set* Υ *for wich* $\Upsilon \cap \mathcal{T} \neq \varnothing$ *exists* $\varepsilon_0 > 0$ *and for each* $\varepsilon < \varepsilon_0$ *the neighbourhood* Υ *contains eigenvalues.*

We call curves with properties of $\mathcal{T}$ as a critical spectrum curves, because they determine the ultimate behavior of spectrum when a parameter is close to a critical one.

The behaviour of the eigenvalues depends on topology of Stocks' graphs (look [2]) of the equation (0.1).

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Steiner ratio for Riemannian manifolds

New estimates for Steiner ratio for Riemannian manifolds are obtained. In particular, the Steiner ratio of flat tori, flat Klein bottles, and projective plane are calculated (modulo the Du and Hwang theorem on the Steiner ratio of the Euclidean plane).

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Regularity through intrinsic scaling for degenerate parabolic PDEs

Several physical phenomena, like phase transitions or the flow of immiscible fluids in a porous matrix, are described by partial differential equations having either a singular or a degenerate character. We address the question of the regularity of local weak solutions for equations of this type, interpreting them in a geometry dictated by their own structure.

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Spectral L-2 conjecture and first KdV integrals for 1-D Schrodinger operator

We consider 1-D Schrodinger operators on the semiaxis with slow decaying potentials. A sequence of conditions on the potential is found such that under each of them the a.c. spectrum of the operator coincides with the positive semiaxis and the singular spectrum is unstable. The conditions are related to the first KdV integrals, and the first condition gives a justification of the spectral L-2 conjecture. Examples show that for a special class of sparse potentials these results can not be improved.

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Singular differential operators with regularly oscillating coefficients.

We consider the differential expression

$$ly = \sum_{k=0}^{n} q_k(x)p_k(\varphi(x))y^{(k)}(x), 0 < x < \infty, \tag{0.1}$$

where $q_k(x)$ satisfy to so-called conditions of regular increasing at infinity, $p_k(t + 2\pi) = p_k(t)$, $\varphi''(x) \neq 0$, $x \gg 1$. Let L_0 -minimal operator,generated in

$L^2[0;+\infty)$ by differential expression (0.1). It is known, that the spectral properties of the operator L_0 and its self-adjoint extensions are closely related to the asymptotic behavior of the fundamental system of solution of the equation $ly = \lambda y$, $\lambda \in C$ for $x \to \infty$ The properties of the operator L_0 are sufficiently well studied in the case, when $p_k(t) \equiv 1, k = 1...n$. As for operators with oscilating coefficients the most part of their properties is uknown. We propose the new method for asymptotic formulas constructing of the equation $ly = \lambda y$. Let H -the separable Hilbert space of n-dimensional vector-functions defined on $[0, 2\pi]$ with scalar product $(\overrightarrow{u}, \overrightarrow{v})_H = \int_0^{2\pi}(\sum_{k=0}^{n} u_k(t)\overline{v_k(t)})dt$. Consider the operator $\Lambda : H \to H$ defined by the next formula

$$\Lambda\overrightarrow{u} = (\frac{d}{dt} + A(t, x, \lambda))\overrightarrow{u} \tag{0.2}$$

with domain $D_\Lambda = \{\overrightarrow{u}, \overrightarrow{u_t} \in H | \overrightarrow{u}(0) = \overrightarrow{u}(2\pi)\}$ and the operator $F(x, \lambda)$ redusing Λ to canonical form. The elements of matrix $A(t, x, \lambda)$ depend on the coefficient of differential expression (0.1). We established that it is possible to find the asymptotic formulas for solutions of the equation (0.1), if the asymtotic behavior of the operator $F(x, \lambda)$ and eigenvalues of the of the operator Λ are known. Then we use our asymptotic formulas for the spectral properties of the singular differential operators investigating.

Vasiliev V. B.
(Novgorod State University) **Classes of elliptic symbols admitting a wave factorization**

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Periodic Solutions of Syngularly Perturbed Parabolic Problems

We consider the equation:

$$\varepsilon^2 u_{xx} - \varepsilon^p u_t = F(u, x, t), \quad 0 < x < 1, \tag{1}$$

where $\varepsilon > 0$ is a small parameter, $p = 1, 2$, the function $F(u, x, t)$ is $T-$periodic with respect to t. The boundary conditions are:

$$u(0, t, \varepsilon) = u^0, \quad u(1, t, \varepsilon) = u^1. \tag{2}$$

The existence of $T-$periodic solution of the problem (1) – (2) is proved. Its asymptotic expansion with respect to ε by means of boundary function method is constructed. The solution may have the pure boundary layer form or it may have the form of contrast structure (the form with interior layer). The solution can change its form by increasing of t from pure boundary layer type to the contrast structure and backwards. In this case the solution is called the alternating contrast structure. The $T-$periodic solution of the equation:

$$\varepsilon(u_{xx} - u_t) = A(u, x, t)u_x + B(u, x, t) \tag{3}$$

with boundary conditions (2) is also investigated. The details will be given at the talk.

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Hierarchical Bases generated by Extremal Functions of Cubature Formulas

Let Ω be a bounded domain in R^n and let $\mathbf{\Delta} = \{\Delta_k\}_{k=0}^{\infty}$ be a sequence of finite subsets of $\overline{\Omega}$;

$$\Delta_0 = \{\tilde{x}_j^{(0)} \mid j = 1, 2, \ldots, \sigma(0)\}, \Delta_k = \Delta_{k-1} \cup \{\tilde{x}_j^{(k)} \mid j = 1, 2, \ldots, \sigma(k)\},$$
$$\tilde{x}_j^{(k)} \notin \Delta_{k-1}, \quad k = 1, 2, \ldots.$$

We assume that the union of all Δ_k is dense in $\overline{\Omega}$. Let $X(\Omega)$ be a separable Hilbert space; the members of $X(\Omega)$ are real functions with domain Ω; the space $X(\Omega)$ is embedded in $C(\Omega)$; and the embedding of $X(\Omega)$ to $C(\Omega)$ is compact. Hence, the error functional of the form

$$(l_j^{(k)},\varphi)=\int_\Omega \varphi(x)\,dx-\sum_{m=1}^{k-1}\sum_{i=1}^{\sigma(m)} c_i^{(m)}\varphi(\tilde{x}_i^{(m)})-\sum_{i=1}^{j-1} c_i^{(k)}\varphi(\tilde{x}_i^{(k)}) \qquad (1)$$

is a member of the dual space $X^*(\Omega)$. For given j and k there exists a unique $X(\Omega)$-optimal error functional $l_{j,opt}^{(k)}(x)$ (see, e.g., [1]). The extremal function $u_{j,opt}^{(k)}(x)$ of $l_{j,opt}^{(k)}(x)$ belongs to $X(\Omega)$; and the value of $u_{j,opt}^{(k)}(x)$ at each node $\tilde{x}_i^{(m)}$ of (1) equals 0. If $j=1$ and $k=1$ then we assume that $u_{j,opt}^{(k)}(x)\equiv 1$. Let $u_{j,opt}^{(k)}(x)$ and $u_{j_1,opt}^{(k_1)}(x)$ be the distinct functions for $j\neq j_1$ or $k\neq k_1$. Then $a_j^k=u_{j,opt}^{(k)}(\tilde{x}_j^{(k)})\neq 0$ for all j and k. Hence, we can define the following functions $h_j^{(k)}(x)=u_{j,opt}^{(k)}(x)/a_j^k$. The set $H_0=\{h_j^{(k)}(x)\mid k=0,1,2,\ldots,j=1,2,\ldots,\sigma(k)\}$ is called a Δ-hierarchical system (see, e.g., [2]).

Theorem. *There exists a Hilbert space $X_1(\Omega)$ such that the Δ-hierarchical system H_0 is an orthogonal basis of $X_1(\Omega)$. The space $X(\Omega)$ is dense in $X_1(\Omega)$ with respect to the norm of $X_1(\Omega)$.*

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Vershik A.M.

(Steklov Mathematical Institute, St.Pertersburg)

Poisson-Furstenberg boundary of the braid group

The problem of the description of Poisson boundary of the groups has a long history, but almost nothing is known about explicit form of the boundary. A satisfactory answer exists for few numbers of the groups. The point is that we need for special (stable) normal form of the elements of groups with respect to given set of generators. The braid group looks till now as a hopeless example in this sense because among many of the known normal forms we can't find a stable one. In the papers by Mazur-Kaimanovich was proved that Poisson boundary coincides with Thurston boundary for mapping class groups but it does not mean that the question about explicit form was solved. In our paper with A.MALUTIN we suggested a new approach to the problem which leads in the end of ends to complete solution. Namely in order to find a stable normal form we use the geometric ideas together with well-known Artin's representation of the braid group. More details. Let braid group B_n acts as group of automorphisms of the fundamental group of punched disk (=free group F_{n+1}). This action on the system of free generators $\{u_1, u_2, \dots u_{n+1}\}$ of the group F_{n+1} could be express as follow: if $g \in B_n$ и $\pi : B_n \to Aut F_{n+1}$ - Artin representation, then

$$\pi(g)(u_k) = h_{g,i}^{-1} \cdot u_i \cdot h_{g,i},$$

where $h_{g,i} \in B_n$, $i = s_g(k)$, and s_g is an element of symmetric group, which corresponds to the element $g \in B_n$ under canonical map from the group B_n onto symmetric group S_n. The studying of the random walk on the orbits of this action allows us to assert the following main fact about stability of the elements $h_{g,i}$. Theorem Let $\{\sigma_1, \dots \sigma_n\}$ be the standard generators of the braid group B_n and μ is a uniform measure on its and inverse elements. For almost all trajectories

$$\{g_0 = e, g_1, \dots g_m, \dots\}$$

of the random walk on the group B_n with respect to the measureμ the *left* action on the group F_{n+1} of that sequence of $g_m, m = 1, \dots$ generates n sequences of the elements $\{h_{g_m,i}\}, m = 1 \dots; i = 1, 2 \dots n$ of the group F_{n+1}, which are *stable* e.g for arbitrary natural k there is some $N(k)$ such that for all $m > N(k)$ and all $i = 1, \dots n$ the first k symbols of elements $h_{g_m,i}$ does not change. This theorem allows to define stable normal form of the elements of the group B_n. A geometric interpretation of the stabilization is very transparent and very closed to some kind of the approximation of Thurston's laminations by the cycles of fundamental group.

Veselov A.P.

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Lacunae and quantum integrable systems

Consider a hyperbolic equation of the form

$$(\partial_0^2 - \partial_1^2 - \dots - \partial_N^2 + u(x))\phi(x) = 0,$$

where the potential u is some function of $x = (x_0, x_1, \dots, x_N)$. J. Hadamard [1] raised the question when such an equation has fundamental solution located on the characteristic cone, i.e. the whole interior of this cone is the lacuna of the equation. The last property is known as the Huygens principle (in the narrow Hadamard's sense). In the talk the latest progress in this direction due to O.Chalykh, M.Feigin and the speaker will be discussed. The approach is based on the modern theory of the quantum integrable systems of Calogero-Moser type.

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Videman J.H.

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Steady Flows of Jeffrey-Hamel Type from the Half-Plane into an Infinite Channel

Let $\Omega \subset \mathbb{R}^2$ denote an unbounded plane domain, symmetric with respect to x_1-axis in Cartesian coordinates, i.e.

$$\Omega = \{x \ : \ (x_1, -x_2) \in \Omega\}.$$

Assume that outside the circle $\mathbb{B}_R = \{x \ : \ |x| < R\}$, Ω coincides with the union of the semi-strip

$$\Pi_- = \{x : x_1 < 0, |x_2| < 1\}$$

and the right half-plane

$$K := \mathbb{R}^2_+ = \{x : x_1 > 0\}$$

and suppose, for simplicity, that the boundary $\partial\Omega$ consists of two smooth curves. In Ω, let us consider the Navier-Stokes problem

$$\begin{cases} -\nu \Delta_x v(x) + (v(x) \cdot \nabla_x) v(x) + \nabla_x p(x) = f(x) & x \in \Omega, \\ -\nabla_x \cdot v(x) = g(x), & x \in \Omega, \\ v(x) = h(x), & x \in \partial\Omega, \end{cases} \qquad (0.1)$$

where the dot " $\cdot$ " denotes the scalar product in $\mathbb{R}^2$, $\nabla_x = \text{grad}$, $\nabla_x \cdot = \text{div}$ and $\Delta_x = \nabla_x \cdot \nabla_x$ is the Laplacian. Moreover, $v = (v_1, v_2)$, $f = (f_1, f_2)$, $h = (h_1, h_2)$, and v stands for the fluid velocity, p is the pressure field and ν the constant viscosity of the fluid. Let us also introduce the (linear) Stokes problem

$$\begin{cases} -\nu\Delta_x v(x) + \nabla_x p(x) = f(x), & x \in \Omega, \\ -\nabla_x \cdot v(x) = g(x), & x \in \Omega, \\ v(x) = h(x), & x \in \partial\Omega \end{cases} \tag{0.2}$$

and, to simplify the notation, denote by u its solution (v, p) and write it in a one-line form

$$\mathrm{S}(\nabla_x)u = (f, g) \quad \text{in } \Omega, \qquad v = h \quad \text{on } \partial\Omega.$$

The analogous form of the Navier-Stokes problem (0.1) reads as follows

$$\mathrm{S}(\nabla_x)u + \mathrm{N}(v, v) = (f, g) \quad \text{in } \Omega, \qquad v = h \quad \text{on } \partial\Omega.$$

where $\mathrm{N}(v, v) = ((v(x) \cdot \nabla_x)v(x), 0)$. Let $\widetilde{\Omega}$ stand for a small perturbation of the symmetric domain Ω, i.e. $\widetilde{\Omega} = \kappa(\Omega)$ and $\kappa : \mathbb{R}^2 \to \mathbb{R}^2$ is a diffeomorphism such that $\kappa = \mathbb{I}$ outside the circle $\mathbb{B}_R$ and the norm of $\kappa - \mathbb{I}$ is small (in other words, κ is almost equal to the identity $\mathbb{I}$ everywhere in Ω). The change of variables

$$\widetilde{\Omega} \ni \widetilde{x} \longmapsto x = \kappa^{-1}(\widetilde{x}) \in \Omega \ , \tag{0.3}$$

turns the Navier-Stokes problem in $\widetilde{\Omega}$ into a perturbed problem in Ω

$$\mathrm{S}(\nabla_x)u + \mathrm{N}(v, v) + \mathrm{P}(u) = (f, g) \quad \text{in } \Omega, \qquad v = h \quad \text{on } \partial\Omega \ , \tag{0.4}$$

where P is a non-linear operator resulting from the change of variables (0.3), with $\mathrm{P}(0) = 0$. We decompose the right-hand side (f, g, h) in (0.4) into symmetric and anti-symmetric parts

$$(f, g, h) = (f^s, g^s, h^s) + (f^a, g^a, h^a)$$

and, by imposing different smallness restrictions on (f^s, g^s, h^s) and (f^a, g^a, h^a), look for a solution to problem (0.4) with prescribed flux $\Phi \in \mathbb{R}$ in the outlet to infinity K. We consider problem (0.4) as a perturbation of the following linear problem

$$\mathrm{S}(\nabla_x)u^1 + \mathrm{N}(v^s, v^1) + \mathrm{N}(v^1, v^s) = (f^1, g^1) \quad \text{in } \Omega, \qquad v^1 = h^1 \quad \text{on } \partial\Omega \ , \tag{0.5}$$

where $u^s = (v^s, p^s)$ denotes the symmetric solution to the Navier-Stokes problem in Ω, i.e. a small perturbation of the sum

$$v^{os} + \Phi v^f \ ,$$

with $u^{os} = (v^{os}, p^{os})$ being a solution to problem (0.2) with the symmetric right-hand side (f^s, g^s, h^s) and where $u^f = (v^f, p^f)$ stands for the solution to the homogeneous Stokes problem driving the unit flux. The distinguishing feature of the nonlinear problem (0.4) is that the convective term $N(v, v)$ has the same asymptotic behaviour as the linear ones, which makes it impossible to consider the problem (0.4) as a perturbation of the linear problem (0.2) in usual weighted Sobolev spaces. Hence, we employ the technique of weighted spaces with detached asymptotics developed in [1], see also [2]. Assuming that the the parameters ρ_s and ρ_a which characterize the symmetric and anti-symmetric parts of the problem data are small and satisfy $\rho_a = o(\rho_s)$, we prove that provided the flow is directed into the strip-like outlet ($\Phi < 0$), there exists a unique small solution taking the Jeffrey-Hamel asymptotic form $r^{-1}V(\varphi)$ in K ((r, φ) denote the polar coordinates in $\mathbb{R}^2$). Previously such solutions have been found only under symmetry assumptions. This is a joint work with S. Nazarov (St. Petersburg) and A. Sequeira (Lisbon).

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Vinogradov A.M.

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New cohomological methods in pde's and secondary calculus

The talk is designed to be an introduction to basic ideas and results of a new theory which plays the same role with respect to partial differential equations as affine algebraic geometry does with respect to algeraic ones. On the corresponding geometric objects, called diffieties, a kind of differential calculus (SECONDARY CALCULUS) can be developped. It deals with homotopy classes of specific differential complexes over them. The usual "differential mathematics" appears to be the zero-dimensional case of secondary calculus and any natural concept of "classical mathematics" has a "secondary" analogue. The SECONDARIZATION PROBLEM is, therefore, to define and to COMPUTE these analogues, i.e. cohomologies of suitable (Spencer-like) complexes over diffieties. This problem may be viewed as a mathematical paraphrase of the general quantisation problem and an analogue of the Bohr correspondence principle guides its solution by starting from PRIMARY CALCULUS, which is a "logical skeleton" of the standard naive

CALCULUS. The objects like ghosts, antifields, etc, emerged last decades in QFT, turn out to be specific secondary tensors. In this perspective the theory of (non-linear) partial differential equations is seen as a kind of "quantum mathematics" with new insights and unexpected perspectives.

Vinokurov V.A.
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The formula of trace for potential containing δ-functions

First boundary problem on segment $[0,\ell]$ for differential equation $y'' + (\lambda + q(x))y =$
$= 0$ is considered. It reduces to the boundary problem, consisting of integral equation

$$z(x) = x - \int_0^\ell \nu(x-t)z(t)\,d(\sigma(t)+\lambda t), \quad x \in [0,\ell] \tag{0.1}$$

and boundary condition

$$z(\ell) = 0. \tag{0.2}$$

Here $\sigma(x) = \int_0^x q(x)\,dx$ – is a given function, function $\nu(\xi) \equiv \begin{cases} 0, & \xi < 0, \\ \xi, & \xi \geqslant 0 \end{cases}$ and it is required to find such $\lambda \in \mathbf{C}$ and such function $z \in C[0,\ell]$, that the relations (0.1, 0.2) take place. If function $\sigma(x)$ is equal to zero, then $\lambda_{n,0} \equiv \left(\frac{n\pi}{\ell}\right)^2$ are eigenvalues of an confluent problem, $y_{n,0}(x) \equiv \sqrt{\frac{2}{\ell}}\sin\left(\frac{n\pi x}{\ell}\right)$ are normalized eigenfunctions of an confluent problem, $n \in \mathbf{N}$. Let us introduce linear subspace $BV_c[0,\ell] \subset BV[0,\ell]$, consisting of all functions with bounded variation $\sigma(x)$, continuous from right in any point $x \in [0,\ell]$ and continuous in points $x = 0$ and $x = \ell$. Each limited variation function $\sigma \in BV_c[0,\ell]$ can have on $[0,\ell]$ no more than countable number of points of discontinuity $x_i \in]0,\ell[$, $i \in I$, inside the interval $]0,\ell[$, in which exist the limit from right $\sigma(x_i+0)$ and the limit from left $\sigma(x_i-0)$ and is defined the value of saltus $c_i \equiv \sigma(x_i+0) - \sigma(x_i-0)$. Due to the boundedness of full variation of function σ the series $\sum\limits_{i\in I}|c_i|$ converges. Takes place the following formula for regularized trace.

Theorem 1 *Let the function $\sigma \in BV_c[0,\ell]$ and $\{x_i\}_{i\in I} \subset]0,\ell[$ is a set of its discontinuity points, and c_i is a corresponding value of its saltus in the discontinuity* *point x_i, $i \in I$, then the following series converge and takes place the equality*

$$\sum_{n=1}^{\infty}(\lambda_n(\sigma) - (\lambda_{n,0} - \int_0^{\ell} y_{n,0}^2(x)\, d\sigma(x))) = -\frac{1}{8}\sum_{i\in I} c_i^2 .$$

Note that in case of continuous function $\sigma(x)$ the set $I = \varnothing$ is empty and it is assumed by definition that $\sum_{i\in\varnothing} c_i^2 = 0$.

Corollary 1 *The function $\sigma(x)$ of a class $BV_c[0,\ell]$ is continuous on segment $[0,\ell]$ then and only then, when*

$$\sum_{n=1}^{\infty}(\lambda_n(\sigma) - (\lambda_{n,0} - \int_0^{\ell} y_{n,0}^2(x)\, d\sigma(x))) = 0 .$$

The full text and proof of presented results can be found at "http://vinokur.narod.ru/spectrum".

Vishik M.I.

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Trajectory and global attractors for the 3D Navier–Stokes system

To study the behaviour of solutions to the main boundary value problem for the 3D Navier–Stokes system the *trajectory attractor* $\mathfrak{A}$ is constructed. It is not known yet whether any weak solution of this problem is unique. The trajectory attractor consists of a class of bounded in H weak solutions (trajectories) of the Navier–Stokes system defined on a positive time semiaxis $\mathbb{R}_+$, which admit prolongations on the entire time axis $\mathbb{R}$ as bounded in H weak solutions of this system. Any bonded in $L_\infty(\mathbb{R}_+; H)$ family of solutions of the Navier–Stokes system tends to the trajectory attractor $\mathfrak{A}$ as $h \to +\infty$ in the metric of the space $C([h, h+T]; H^{-\delta})$ for every $T > 0$. Here $\delta > 0$ and δ is arbitrary small. The solutions $u(x,t), x \in \Omega, t \geqslant 0$, of the Navier–Stokes system belonging to $\mathfrak{A}$ are continuous functions of time t with values in $H^{-\delta}$ $(u(\cdot,t) \in C(\mathbb{R}_+; H^{-\delta}))$. Therefore there exists the restriction $\mathfrak{A}|_{t=0} = \mathfrak{A}(0)$ of the trajectory attractor $\mathfrak{A}$ for $t = 0$. The set

$$\mathcal{A} := \mathfrak{A}|_{t=0} = \mathfrak{A}(0)$$

is said to be the *global attractor* of the 3D Navier–Stokes system. The set $\mathcal{A}$ is bounded in H and compact in $H^{-\delta}$ for any $\delta > 0$. Moreover $\mathcal{A}$ has properties analogues to the properties of global attractors of evolution equations for which the uniqueness theorem of the corresponding Cauchy problem holds. The trajectory and global attractors have been constructed for the 3D Navier–Stokes system with external force of the form $g(x)$ or $g(x,t)$, $x \in \Omega \Subset \mathbb{R}^3$, $t \geqslant 0$. We have proved that the trajectory attractor $\mathfrak{A}_m$ and the global attractor $\mathcal{A}_m$ of the Galerkin approximation system of order m tends as $m \to \infty$ to the trajectory attractor $\mathfrak{A}$ and to the global attractor $\mathcal{A}$ of the 3D Navier–Stokes system, respectively. We have studied some questions of averaging of these attractors for the 3D Navier–Stokes systems having rapidly oscillating external forces of the form $g(x, x/\varepsilon)$ or $g(x,t,t/\varepsilon)$ as $\varepsilon \to 0+$. The similar theorems have been proved for some other dissipative equations and systems of mathematical physics. All these results were obtained in the collaboration with V.V.Chepyzhov.

Владимиров А.А.
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О накоплении собственных значений дифференциальных оператор-функций

Пусть на конечном интервале $(\sigma,\tau) \subset \mathbb{R}$ определена оператор-функция $L(\lambda)$, чьи значения порождены в пространстве $L_2[0,1]$ дифференциальным выражением

$$l_\lambda(y)(x) = (-1)^n \left(\frac{y^{(n)}(x)}{p_0(\lambda,x)}\right)^{(n)} + \sum_{k=1}^{n}(-1)^{n-k}\big(p_k(\lambda,x)y^{(n-k)}(x)\big)^{(n-k)}$$

и достаточно общими самосопряжёнными краевыми условиями, также зависящими от спектрального параметра λ. Коэффициент $p_0(\lambda,x)$ предполагается положительным, и все коэффициенты $p_k(\lambda,x)$, $k = 0,\ldots,n$ предполагаются суммируемыми по x на отрезке $[0,1]$ при любом фиксированном $\lambda \in (\sigma,\tau)$, то есть значения оператор-функции $L(\lambda)$ регулярны (см. [1]).

Для оператор-функции указанного вида могут быть установлены некоторые весьма общие достаточные условия накопления (и, наоборот, ненакопления) её спектра к правому концу её интервала определения. Для оператор-функции, порождённой дифференциальным выражением

$$l_\lambda(y)(x) = \big(p(\lambda,x)y''(x)\big)'' + q(\lambda,x)y(x)$$

и самосопряжёнными краевыми условиями, в которых коэффициенты при квазипроизводных (см. [1]) не зависят от λ, эти условия могут быть конкретизированы, например, в такой форме:

Теорема. *Пусть функции $p(\lambda, x) > 0$ и $q(\lambda, x)$ определены и непрерывны при всех $\lambda \in (\sigma, \tau]$, $x \in [0,1]$, за исключением случая $\lambda = \tau$, $x = x_0 \in (0,1)$. Пусть для некоторого действительного γ существуют конечные пределы*

$$\hat{p} := \lim_{x \to x_0} \frac{p(\tau, x)}{|x - x_0|^{\gamma}}, \quad \hat{q} := \lim_{x \to x_0} \frac{q(\tau, x)}{|x - x_0|^{\gamma - 4}}.$$

Тогда, если справедливо неравенство

$$\hat{q} < -\frac{1}{16}(\gamma - 1)^2(\gamma - 3)^2 \hat{p},$$

то собственные значения оператор-функции накапливаются к точке τ. Если же справедливо неравенство

$$\hat{q} > -\frac{1}{16}(\gamma - 1)^2(\gamma - 3)^2 \hat{p},$$

причём при любом $x \in [0,1]$ функция $q(\lambda, x)$ убывает по λ, а функция $p(\lambda, x)$ невозрастает, то собственные значения оператор-функции не накапливаются к точке τ.

В частности, рассмотрим задачу

$$(-(1 + \lambda\alpha(x))y''(x))'' - (\lambda\beta(x) + \lambda^2\rho(x))\, y(x) = 0,$$

$$y(0) = y'(0) = y''(1) = y'''(1) = 0,$$

где функции $\alpha(x) > 0$, $\beta(x)$ и $\rho(x)$ предполагаются непрерывными на отрезке $[0,1]$, причём $\alpha(x)$ имеет на этом отрезке единственную точку своего минимума $x_0 \in (0,1)$, для которой $\alpha(x) \in C^4(x_0)$ и $\alpha'(x_0) = \alpha''(x_0) = \alpha'''(x_0) = 0$. При выполнении условия

$$\beta(x_0) - \frac{\rho(x_0)}{\alpha(x_0)} < -\frac{3}{128}\alpha^{(IV)}(x_0)$$

собственные значения этой задачи будут накапливаться снизу к точке $-(\alpha(x_0))^{-1}$. При выполнении аналогичного условия с противоположным знаком неравенства и при некоторых дополнительных условиях на функции $\alpha(x)$, $\beta(x)$ и $\rho(x)$ такое накопление не будет иметь места.

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Бета-функции локальных полей характеристики нуль; применения к струнным амплитудам

Из анализа известна формула

$$\int_{(0,1)^n} \delta(1-\sum_{i=1}^n x_i)\prod_{i=1}^n x_i^{\alpha_i-1}dx_i = \frac{\prod_{i=1}^n \Gamma(\alpha_i)}{\Gamma(\sum_{i=1}^n \alpha_i)} =$$

$$\mathrm{B}^{(n)}(\alpha_1,\alpha_2,\dots,\alpha_n), \quad \mathrm{Re}\,\alpha_i>0, i=1,2,\dots,n=1,2,\dots, \tag{1}$$

так что $\mathrm{B}_1^{(1)}=1, \mathrm{B}^{(2)}=\mathrm{B}$. Здесь Γ и B – гамма- и бета-функции Эйлера. Формула (1) обобщается на локальные поля K характеристики нуль. (Все такие поля хорошо известны: это – $\mathbb{R},\mathbb{C},\mathbb{Q}_p$ и их конечные алгебраические расширения $\mathbb{Q}_p(\epsilon)$. Наряду с бета-функцией поля K

$$\mathrm{B}_{\mathrm{K}}(\alpha,\theta;\beta,\theta';\gamma,\theta'') = C_2\int_{\mathrm{K}}\theta(x)|x|^{\alpha-1}\theta'(1-x)|1-x|^{\beta-1}dx =$$

$$\Gamma_{\mathrm{K}}(\alpha;\theta)\Gamma_{\mathrm{K}}(\beta;\theta')\Gamma_{\mathrm{K}}(\gamma;\theta''), \quad \alpha+\beta+\gamma=1, \theta\theta'\theta''=1$$

для мультипликативных характеров θ и θ' поля K (здесь $\Gamma_{\mathrm{K}}(\alpha;\theta)$ – гамма-функция поля K для характера θ), вводится новая последовательность бета-функций

$$\mathrm{B}_{\mathrm{K}}^{(n)}(\alpha_1,\theta_1;\alpha_2,\theta_2;\dots;\alpha_n,\theta_n) =$$

$$C_n\int_{\mathrm{K}^n}\delta(1-\sum_{i=1}^n x_i)\prod_{i=1}^n\theta_i(x_i)|x_i|^{\alpha_i-1}dx_i, \quad n=1,2,\dots, \tag{2}$$

где dx – (нормированная) мера Хаара поля K и $|x|$ – нормирование на K. Нормировочная постоянная C_n в (2) вычисляется в явном виде для указанных полей K и выбирается с таким расчетом, чтобы были справедливы следующие формулы: при четном $n=2,4,\dots$

$$\mathrm{B}_K^{(n)}(\alpha_1,\theta_1;\alpha_2,\theta_2;\dots;\alpha_{n+1},\theta_{n+1}) =$$

$$\Gamma_{\mathrm{K}}(\alpha_1;\theta_1)\Gamma_{\mathrm{K}}(\alpha_1;\theta_2)\dots\Gamma_{\mathrm{K}}(\alpha_{n+1};\theta_{n+1}),$$

$$\alpha_1+\alpha_2+\dots+\alpha_{n+1}=1, \theta_1\theta_2\dots\theta_{n+1}=1; \tag{3}$$

при нечетном n=1,3, ...

$$\mathrm{B}_K^{(n)}(\alpha_1,\theta_1;\alpha_2,\theta_2;\dots;\alpha_n,\theta_n) = \Gamma_{\mathrm{K}}(\alpha_1;\theta_1)\Gamma_{\mathrm{K}}(\alpha_1;\theta_2)\dots\Gamma_{\mathrm{K}}(\alpha_n;\theta_n)$$

$$\alpha_1 + \alpha_2 + \cdots + \alpha_n = 1, \theta_1\theta_2 \ldots \theta_n = 1. \tag{4}$$

В частности,

$$\mathrm{B}^1_{\mathrm{K}} = C_1 = \Gamma_{\mathrm{K}}(0;\theta_1), \quad \mathrm{B}^2_{\mathrm{K}} = \mathrm{B}_{\mathrm{K}}.$$

Эти формулы позволяет применить адельные формулы в случае, если соответствующий характер на группе иделей $A^\times_{\mathrm{K}}$ тривиален на $\mathrm{K}^\times$ [1,2]. Физические применения: четырех-точечные древесные струнные амплитуды выражаются через бета-функции $\mathrm{B}^{(2)}_{\mathrm{K}} = \mathrm{B}_{\mathrm{K}}$, а суперструнные амплитуды – через бета-функции $\mathrm{B}^{(3)}_{\mathrm{K}}$ на соответствующих многообразиях (3) или (4) и при надлежащем выборе поля K и характеров θ_i [2]. Дано новое доказательство формулы, связывающую четырех-точечную древесную амплитуду для замкнутой струны (амплитуду Вирасоро) через произведение двух амплитуд для открытых струн (классические амплитуды Венециано). Работа выполнена частично при финансовой поддержке РФФИ (проект 00-15-96073).

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Vlasov V.V.[25]

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Spectral problems, arising in the theory of differential equations with delay

We study the spectral problems naturally arising in the theory of functional-differential equations (FDE) including Riesz basisness of exponential solutions.

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We also research the asymptotic behaviour of the solutions of FDE (see [1]–[5]). We consider an initial-value problem

$$\sum_{j=0}^{n}\left(B_j u(t-h_j)+D_j\frac{du}{dt}(t-h_j)\right)+\int_0^h K(s)u(t-s)\,ds=0,\quad t>0,\tag{1}$$

$$u(t)=y(t),\quad t\in[-h,0),\quad u(+0)=\varphi_0=y(-0).\tag{2}$$

Here B_j, D_j are constant complex $(m\times m)$-matrices, h_j are real numbers such that $0=h_0<h_1<\cdots<h_n=h$; the elements of the matrix-valued function $K(s)$ belong to the space $L_2(0,h)$. Denote by $\mathcal{L}(\lambda)$ a matrix-valued function

$$\mathcal{L}(\lambda)=\sum_{j=0}^{n}(B_j+\lambda D_j)\exp(-\lambda h_j)+\int_0^h K(s)\exp(-\lambda s)\,ds,\tag{3}$$

by $l(\lambda)=\det\mathcal{L}(\lambda)$, by ν_q the multiplicities for the roots λ_q of the function $l(\lambda)$, by $y_{q,j,s}(t)$ the exponential solutions (see [1]–[5]) of the equation (1). Denote by $W^p_{2,v}((-h,0),\mathbb{C}^m)$ the subspace of the Sobolev space $W^p_2((-h,0),\mathbb{C}^m)$, $p\in\mathbb{N}$, satisfying the following conditions

$$\sum_{j=0}^{n}(B_j u^{(k)}+D_j u^{(k+1)}(-h_j))+$$

$$+\int_0^h K(s)u^{(k)}(-s)\,ds=0,\quad k=0,1,\dots,p-2;\ p\geqslant 2.$$

Theorem.

Let $\det D_0\neq 0$, $\det D_n\neq 0$, $\inf_{\lambda_p\neq\lambda_q}|\lambda_p-\lambda_q|>0$, *and* $y(t)\in W^p_{2,v}((-h,0),\mathbb{C}^m)$.

(a) *Every solution* $u(t)\in W^p_2((-h,T),\mathbb{C}^m)$, $T>0$, *of the problem* (1), (2) *satisfies the inequality*

$$\|u\|_{W^p_2(t-h,t)}\leqslant d\exp(\varkappa t)(t+1)^{N-1}\|y\|_{W^p_2(-h,0)},\quad t\geqslant 0,\tag{4}$$

where $\varkappa=\sup\operatorname{Re}\lambda_q$, $N=\max\nu_q$ *and constant* d *is independent of all* $y(t)$.

(b) *The system of subspaces* $\{V_{\lambda_q}\}$, *where* $\{V_{\lambda_q}\}$ *is the span of all exponential solutions* $y_{q,j,s}(t)$, *corresponding to* λ_q, *forms a Riesz basis of subspaces of the space* $W^p_{2,v}((-h,0),\mathbb{C}^m)$.

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An Analytic-Numerical Method for Solving BVPs for the Laplace equation in Domains with Cones or Polyhedral Corners

We present a new analytic–numerical method for solving boundary value problems (BVPs) for the Laplace equation in 3D complex–shaped domains with cones of arbitrary base, in particular, with trihedral corners. This method is a generalization (in some sence) of the multipole method, which is an analytic–numerical method designed in previous works of the authors for solving BVPs for some elliptic equations in 2D and 3D domains of complex shape.

The specific feature of the method proposed here is the use of a fundamentally new system of basic functions, which reflect adequatelly the structure of the solution near a cone (polyhedral corner). More precisely, our basic functions identically satisfy the Laplace equation in the initial domain, meet the appropriate boundary conditions on the surface of the cone (polyhedral corner), and possess good approximation properties. An important point is that our basic functions are expressed in explicit analytic form in terms of special functions. In this work there have been constructed the basic functions for the trihedral corner, whose all three dihedral angles are $3\pi/2$. In particular, we obtained all the exponents of the singularities at corner point with more high accuracy then known results.

Due to those features the method possesses high efficiency. It provides precise computation of the solution and its derivatives up to the surface of the cone (polyhedral corner), in other words, up to such singularities as edges and vertex of the polyhedral corner. An important advantage of our method is that it yields the value of intensity factors in the vertex and on the edges simultaneously with the solution itself.

It is worth to be stressed that, in contrast to FEM, our basic functions are not local, but global. They are defined in the whole domain, and their linear combinations approximate the BVP solution in the whole domain also. That is why our method does not need any mesh at all.

By means of this method it has been found a numerical solution to the specific mixed BVP for the Laplace equation in the Fichera corner; the latter is a typical domain of complex shape with trihedral corner, considered in many works. There have been obtained numerical values of the solution and its gradient near the singularities with relative error less then 10^{-5} by the use of only 30 basic functions. The values of intensity factors near the vertex and near the edges have been obtained also with the same accuracy. By means of this solution it has been also obtained the precise value of capacity of the 3D domain, which is a cube with cuted out concentered cube of a less size.

This work was supported by the Russian Foundation for Basic Research (project 01–01–00341).

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Newton's polygon in the theory of boundary value problems

On a smooth manifold Ω with boundary we consider a boundary value problem

$$(A(x,D)-\lambda I)u(x)=f(x), \quad x\in\Omega, \quad B(x',D)u(x')=g(x'), \quad x'\in\partial\Omega.$$

Here $A(x,D)$ is a $N\times N$ matrix PDO, elliptic in the sense of Douglis-Nirenberg with orders of elements $\leqslant s_i+t_j$. We suppose that $2r_j := s_j+t_j$ is a strictly decreasing sequence and $r_N>0$. As it follows from conditions below, r_j are integers. We pose $R_j := r_1+\dots+r_j$. Boundary conditions are given by a rectangular $R_N\times N$ matrix of PDO. The orders of elements $\leqslant m_j+t_k$, and the sequence $m_1, m_2, \dots$ is increasing. Moreover we suppose that $m_{R_j} < m_{1+R_j}$ for $j=1,\dots,N-1$. If $\kappa_1, \kappa_2 = 1,\dots,N$ are integers, then by $A(\kappa_1,\kappa_2)(\xi)$ we denote the rectangular matrix containing the elements of the matrix A belonging to lines $1,\dots,\kappa_1$ and the colomns $1,\dots,\kappa_2$. The same notation in the case of matrix B. By $A^0, B^0, \dots$ we denote principal parts of corresponding operators. Pose $E_\kappa = \mathrm{diag}(0,\dots,0,1)$. We suppose that following conditions are satisfied. (i). A is N-elliptic with parameter. It means that on the complex plane there exists a sector L with the vertex at origin such that for each $\kappa=1,\dots,N$ and $x\in\Omega$

$$\det(A^0(\kappa,\kappa)(x,\xi)-\lambda E_\kappa)\neq 0 \quad \text{for } \xi\neq 0 \text{ and } \lambda\in L\,.$$

Now we formulate the analog of Shapiro-Lopatinskii condition for the above problem. For simplicity of notation we suppose that Ω is the half-space $x_n > 0$,

$x' \in \mathbb{R}^{n-1}$ and operators A and B coincide with their principal parts. We freeze the coefficients at some point of the plane $x_n = 0$. For $\kappa = 1, \dots, N$ we pose $u^\kappa := (u_1, \dots, u_\kappa)$. (ii). For each $\kappa = 1, \dots, N$ for $|\xi| \neq 0$ and $\lambda \in L$ the ODE problem

$$(A(\kappa,\kappa)(\xi', D_n) - \lambda I_\kappa)V^\kappa(x_n) = 0, x_n > 0, B(R_\kappa,\kappa)(\xi', D_n)V^\kappa(0) = G_\kappa \in \mathbb{C}^{R_\kappa}$$

has a unique stable (exponentially decreasing) solution. (iii). For each $\kappa = 1, \dots, N$ and $\lambda \in L$, $|\lambda| = 1$ the system

$$(A(0, D_n) - \lambda E_\kappa)V^\kappa(x_n) = 0, \quad x_n > 0,$$

with boundary conditions

$$\sum_{k=1}^{\kappa} B_{jk}(0, D_n)V_k(0) = g_j, \quad j = r_{\kappa-1} + 1, \dots, r_\kappa$$

has a unique stable solution. Condition (ii) can be rewritten as a problem with small parameter $1/\lambda$ at highest derivatives in the last equation of the system. Condition (iii) permits to apply to this problem the Vishik-Lyusternik method of boundary layer and construct the formal asymptotic solution of the problem. Condition (i) is deaply connected with the Newton polygon of $\det(A(x,\xi) - \lambda I)$ considered as the polynomial in ξ, λ. It is possible to construct functional spaces with λ-dependent norms and realize the above boundary value problem as a bounded operator in these space. Then (i), (ii), (iii) are necessary and sufficient conditions for the existence of the bounded inverse operator for sufficiently large $|\lambda|$. For the proof of this result the methods of the Newton polygon and the boundary layer are constantly used. All the results were obtained jointly with Robert Denk.

Volkov Ye.A.

(Steklov Mathematical Institute)

Exponentially convergent numerical-analitic block method for solving boundary value problems for the Laplace equation on poligonals

Under analitic mixed boundary conditions on poligonal sides, the method proposed has the following properties. For a given accuracy $\varepsilon > 0$, the obtained system of linear algebraic equations is of order $O(|\ln \varepsilon|)$, is stable, can be solved with the cost $O(|\ln^3 \varepsilon|$, and takes $O(|\ln \varepsilon|)$ computer memory. A value of approximate solution at a point is computed for $O(\ln^2 \varepsilon)$ ariphmetic operations, and the number of operations may be reduced up to $O(|\ln\varepsilon|)$ when boundary conditions are chosen in the form of polinomials. In the case of nonanalitic boundary conditions, a value of solution of the Dirichlet boundary value problem at each point with an accuracy $\varepsilon > 0$ may be found for a number of operations that coincides by order with the necessary one (due to N.Bakhvalov) for computation of a definite integral on a segment with the same accuracy and the same smoothness of integrand and boundary conditions. Generalizations and examples of solution of boundary value problems and conforming maps with singularities with the accuracy 10^{-10} and lesser are considered.

Volosov K.A.
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Invariant properties of the ansatz of the Hirota method for Quasilinear Parabolic equations

In the first part of the report propose the new invariant properties of the ansatz of the Hirota method which have been discovered recently. This invariant properties allows one to construct the made classes of solutions for a certain class the dissipative equations classified by degrees of homogeneity. This algorithm is similar to the method of "dressing" the solutions of integrable equations. The made classes of solutions for some equations are constructed. The program of calculation of solutions by methods of computer algebra is created. For a construction of more composite solutions the original circuit(scheme) is offered which we call " with a property of zero denominators ". It is grounded on detected in [1] the fact bound with a set of equations obtained after a substitution ansatz in an input equation. At sequential calculation (in any order) flexons of functions included in ansatz it is necessary to find such which has nontrivial denominator. The process of calculation of derivatives stops. An obtained denominator we call it " almost with invariant ". Let's equate it to zero. We express one of functions and is corrected ansatz. Further process is iterated. In the second part of the report the example of modification is reduced (logarifmic ratio) lr-conversions. Such conversion for the first time is entered in [2], [3] p. 229 for solutions of some selected class quasilinear the dissipative equations

$$Z_\delta - (K(\delta, Z, Z_\xi))Z_\xi)_\xi = 0, \tag{0.1}$$

where $Z = Z(\xi, \delta)$ the required function, ξ, δ (nonlinear) coordinate and time. Is proved, that the solutions are enumerated (are put in correspondence) in solutions linear the equations and back,where $u = u(x,t)$ the function, x, t (linear) coordinate and time ,and $\xi = \xi(x,t), \delta = \delta(x,t)$. For a construction of precise solution, it is necessary sequentially to solve some equations. Then to find solution of the auxiliary nonlinear equation. It is possible to consider concrete solution and to clarify what solution the linear equation to it corresponds. Then to construct in "neighbourhood" *it precise solution in the parametric form. And selecting various* sort of Ir-conversion one solution of the quasilinear equation is mapped in solutions various the linear equations. Being returned back is possible to build various sorts of precise solutions in parametric form (anyway locally). The details will be given at the talk. The author is grateful to V. G. Danilov and S. Yu. Dobrohotov for constant attention to his work and useful discussions and to V. P. Maslov, and A. D. Polynin for constructive advice.

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Nonlinear Quantum Computations and NP-complete Problems

Standard *linear* models of quantum computing probably do not allow the effective solution of NP-complete problems. One has to use new *nonlinear* models of computation to solve such problems. Schrodinger equation is the linear equation but the dynamics of an entangled state is described by a nonlinear equation. In the talk a general framework will be presented which includes classical and quantum computing schemes with linear as well as with nonlinear gates. An approach to solution of NP-complete problems by using nonlinear models of quantum computation is proposed in a joint work with M. Ohya. The approach is based on a new model of computation which combines the ordinary quantum computer and chaotic dynamics amplifier. We consider the satisfiability problem and argue that the problem can be solved in polynomial time if one uses the new model of computation. Related approach is given by atomic quantum computer where one can build nonlinear quantum gates.

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Realizations of the Virasoro moduli in the Fock–Krein spaces of the rigorous Coulomb gas formalism

In the framework of spaces with indefinite metrics, a rigorous formulation for 2D minimal conformal theories and systems described by the Coulomb gas formalism is presented [1], which is free of cut-offs and technical restrictions of any kind. The starting base is a rigorous formulation [2,3] of the model of 2D free massless scalar field in the Fock–Krein space over the one-particle space of the Pontryagin type Π_1. This formulation is complemented with central physical objects of our formalism, the normally ordered square of the current and the corresponding stress tensor. Problems of correct definitions of these objects as well as the complete field algebra of the theory are solved. In the next stage, which is the main one, we construct the representations of the Virasoro algebra in the Fock–Krein space of the theory, by means of giving correct definitions to the Laurent expansions of the stress tensor. It is found that there are specific obstacles preventing from the correct definition of the generator of special conformal transformations. Similar obstacles were found earlier in various kinds of conformally invariant systems. Due to these obstacles, the structure of the Virasoro moduli can only be defined in terms of quadratic forms. It is proved, however, that the obstacles can be removed by means of a special regularization, so that finally we are able to construct the Virasoro moduli in the full operator sense.

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The topological analysis of the two centre problem on the 2-sphere

For the first time the problem of investigation of dynamical systems in spaces of constant curvature was formulated by N.I. Lobachevsky. He generalized Newton's attraction law for spaces of negative curvature. The generalized problem of two centers (the motion of a material point in the field generated by two fixed centers) on the 3-sphere was studied in the paper [1]. The bifurcation set in the plane of integrals of motion was constructed and the classification of the domains of possible motion was given. The topological analysis of dynamical systems is of great interest because, as a result, we obtain obvious and compact description of the motion of systems studied. In the present work the two-centre problem on the 2-sphere is studied from the topological point of view. The Fomenko–Zieschang invariants, which completely describe the topology of Liouville foliations of isoenergy 3-manifolds Q^3, are constructed. All kinds of motion (regular motions and limit motions corresponding to bifurcations of Liouville tori) on the configurational space are described. The connection between Fomenko–Zieschang invariants (marked molecules) and different types of motion are investigated. The two-centre problem is completely integrable in the sense of Liouville. The reduction to quadratures can be made by the standard method of separation of variables. But it turns out that this problem has very nontrivial topological properties. For example, there are some new "molecules" (Fomenko–Zieschang invariants) in this case, which did not appear in integrable cases investigated by many authors earlier. The connection between integrable systems describing some problems of celestial mechanics on spaces of constant curvature is investigated as well. It is shown that those problems are transformed one to another as the curvature varies. In particular, it is proved that Kepler problem and the two centers problem on spaces of nonzero constant curvature λ turn to the corresponding classical problems on the plane.

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Differential equations in queueing systems theory

We consider models of large queueing systems with dynamic routing based on the principal of balanced load. The asymptotical approach to large queueing systems (when the size of the model is growing) brings in the boundary-value problems for differential-difference or integro-differential equation. Here some of these problems are presented. In the simplest case a differential-difference equation of the form

$$\dot{u}(t,n) = u(t,n+1) - u(t,n) + \lambda(u^2(t,n-1) - u^2(t,n)), \quad n = 1,2,\ldots$$

is considered. Its generalization is a system for $u_i(t,n)$, $1 \leqslant i \leqslant J$:

$$\dot{u}_i(t,n) = u_i(t,n+1) - u_i(t,n) + \tag{1}$$

$$+\sum_{j=1}^{J}\Big(\lambda_{ij} + \sum_{s=1}^{J} p_{s,ij} u_s(t,1)\Big)\Big[u_i(t,n-1) - u_i(t,n)\Big]\Big[u_j(t,n-1) + u_j(t,n)\Big],$$

$$u_i(t,0) \equiv 1, \ \lim_{n\to\infty} u_i(t,n) = 0, \tag{2}$$

$$u_i(0,n) = g_i(n), \ \ 1 \geqslant g(n) \geqslant g(n+1) \geqslant 0, \tag{3}$$

$$\lambda_{ij} \geqslant 0, \ p_{s,ij} \geqslant 0, \ \sum_{i,j} p_{s,ij} \leqslant 1.$$

An other generalization:

$$\dot{u}_i(t,n) = L[u_i(t,n+1) - u_i(t,n)] +$$

$$+\sum_{j=1}^{J}\lambda_{ij}\sum_{l=1}^{L}\Big[u_i(t,n-l) - u_i(t,n-l+1)\Big]\Big[u_j(t,n-l) + u_j(t,n-l+1)\Big], \tag{4}$$

$$u_i(t,n) \equiv 1, n \leqslant 0, \ \lim_{n\to\infty} u_i(t,n) = 0. \tag{5}$$

An integro-differential equations:

$$\frac{\partial u_i(t,x)}{\partial t} = \frac{\partial u_i(t,x)}{\partial x} - \sum_{j=1}^{J}\lambda_{ij}\int_{x-1}^{x}\frac{\partial u_i(t,s)}{\partial s}u_j(t,s)ds, \tag{6}$$

$$u_i(t,x) \equiv 1, x \leqslant 0, \ \lim_{x\to\infty} u_i(t,x) = 0, \ \ u_i(t,x) = g_i(x), \ 1 \geqslant g(x) \geqslant 0. \tag{7}$$

The existence and uniqueness of the stationary solution to (1),(2) and to (4),(5) and the convergence of solutions (1)-(3) and (4),(5),(3) to the corresponding stationary solutions is investigated. Similar problems for (6),(7) are considered. Sufficient (in some cases necessary and sufficient) conditions for the existence of 'proper' stationary solutions are presented.

Walther H.-O.
(Universitaet Giessen, Mathematisches Institut)

Stable and hyperbolic periodic orbits of delay differential equations

The lecture begins with a brief survey of known results on existence, uniqueness, and stability of periodic orbits of autonomous delay differential equations (DDEs), and on their role in global attractors. Then present work with A.L. Skubachevsky on Floquet multipliers of periodic solutions to DDEs is discussed. In case of single delay equations of the form

$$\dot{x}(t) = -\mu x(t) + f(x(t-1))$$

and periodic solutions with rational period we obtain a characteristic equation for their Floquet multipliers. This generalizes older work for special periodic solutions with period 4. We derive hyperbolicity criteria and apply them to obtain stable and unstable hyperbolic periodic solutions. Here one problem involved is to guarantee existence of periodic solutions with prescribed rational period. The nonlinearities $f : \mathbb{R} \to \mathbb{R}$ considered in the applications of the hyperbolicity criteria are constant outside a (non-small) neighbourhood of 0, model positive and negative feedback, and occur, e.g., in neural network theory. Finally another method is explained which yields contracting Poincaré return maps, and thereby attractive periodic orbits. It applies to a class of equations as above, with f smooth but in a certain sense close to the step function $x \mapsto -a\,\mathrm{sign}\,(x)$, not necessarily constant on any nontrivial interval. The approach also leads to stable and attractive periodic solutions of differential equations with state-dependent delay, where lack of smoothness precludes Floquet multipliers.

Wanner G.
(University of Geneva)

ROCK Methods for Large Stiff Systems

Numerical methods for ordinary differential equations, and — via the method of lines — also for certain initial problems for partial differential equations, can be classified mainly into two categories: *explicit* methods and *implicit* methods. For both classes of methods an impressive literature with great results has been accumulated during the 20th Century and excellent computer codes are now available. But much less attention has been drawn to a class of methods *between* these two great blocks; such methods are called *stabilized* or *Chebyshev* methods. Inside the class of Chebyshev methods we can roughly distinguish between two approaches: Chebyshev methods by *composition* (Lebedev-Finogenov 1976, Lebedev-Medovikov 1995) and Chebyshev methods based on *three-term recurrence relations* (Van der Houwen-Sommeijer 1980, Sommeijer-Shampine-Verwer 1998). The subject of this talk is to give an overwiev of recent work of A. ABDULLE and A. MEDOVIKOV which combines both types of methods in a new class, called ROCK methods (Runge-Kutta-Orthogonal-Chebyshev methods), and which preserve the advantages of both of the above described types. The construction proceeds in three steps:

- construct polynomials which produce as much stability as possible by respecting the desired order conditions for linear problems;

- embed slight variations of these polynomials into a sequence of orthogonal polynomials with respect to a certain weight function; the corresponding three-term recursion allows the stable implementation of a Runge-Kutta scheme for arbitrary nonlinear systems;

- compose this scheme with a "finishing" Runge-Kutta scheme and assure 4th order, as well as an embedded 3rd order method for step-size control, for arbitrary nonlinear systems by applying the operations of the Butcher Group.

We conclude the talk by some numerical experiments. The corresponding codes are available since some time on the Web pages of the Numerical Analysis Group in Geneva, and have already encountered much interest from many researchers.
Abstract: Chebyshev methods for the integration of large stiff systems have a long tradition in Russia (Lebedev, Medovikov). Combining these ideas with the Butcher Group from Runge-Kutta theory and with orthogonality relations and three term recurrence relations (A. Medovikov, Van der Houwen and Sommeijer), A. Abdulle has recently developped 'Orthogonal Runge-Kutta Chebyshev (ROCK) methods' of order 4 with excellent numerical properties. The talk wants to decribe these results.

Freudenberg W., Ohya M.
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Watanabe N.
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On Quantum Logical Gate on Fock space

In usual computer, there exists an upper bound of computational speed because of irreversibility of logical gate. In order to avoid this demerit, Fredkin and Toffoli [3] proposed a conservative logical gate. Based on their work, Milburn [4] constructed a physical model of reversible quantum logical gate with beam splittings and a Kerr medium. This model is called FTM (Fredkin - Toffoli - Milburn gate) in this paper. This FTM gate was described by the quantum channel and the efficiency of information transmission of the FTM gate was discussed in [10]. FTM gate is using a photon number state as an input state for control gate. The photon number state might be difficult to realize physically. In this paper, we introduced a new device on symmetric Fock space in order to avoid this difficulty. In Section 1, we briefly review quantum channels and beam splittings. In Section 2, we explain the quantum channel for FTM gate In Section 3, we intruduced a new device on symmetric Fock space and discuss the truth table for our gate. The details will be given at the talk.

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On high order boundary integral equation methods

The aim of our contribution is an efficient method which is also parallelizable providing the following features:

1. The computation of boundary displacements and boundary tractions with high accuracy either on the whole boundary curve ($n = 2$) or boundary surface ($n = 3$) or on an a-priori chosen subregion of the boundary and
2. the computation of displacements, strains and stresses near and up to the boundary with high accuracy.

Our method is based on the decomposition idea applied to rather different parts of the boundary integral equation method:
a) the decomposition of the boundary curve or boundary surface into an overlapping geometric partition,
b) the decomposition of the trial space for the desired quantities into a regular coarse grid space and local spaces on fine grids; and
c) the additive decomposition of the boundary integral operators involved into a standard principal part and a smoothing remainder. As an example of our approach we consider the Lamé equations of linearized isotropic homogeneous elasticity. Near to the boundary we use Hadamard's natural local coordinates in normal and tangential directions of the boundary, respectively. To exemplify our method, we consider the Dirichlet problem where we have to solve a boundary integral equation for the conormal derivative, e.g. the integral equation of the first kind,

$$V\vec{\psi} = \vec{F} := (\tfrac{1}{2}I + K)\vec{\varphi} \quad \text{on } \Gamma . \tag{0.1}$$

In a first step, this equation is solved on the whole boundary where we use coarse grid trial functions on a regular grid associated with the mesh width H. In order to obtain high order convergence, we use the method proposed in [1], [2], where we approximate simultaneously tangential derivatives of $\vec{\psi}$ on a subdomain $\Gamma_0 \subset \Gamma$, similar as in the partition of the unity method. In addition, we split the operator into a simplified principal part V_0 and a remainder that has a kernel which is less singular. Then we solve the local boundary integral equation for $\widetilde{u} \in \widetilde{H}^{-\frac{1}{2}}(\Gamma_0)$ on the fine grid where the simplified operator, i.e.

$$\langle V_0\, \widetilde{u}, \widetilde{v}\rangle = \langle V_0 \omega u_H, \widetilde{v}\rangle - \langle \omega(V u_H - F), \widetilde{v}\rangle \quad \text{for all} \quad \widetilde{v} \in \widetilde{H}^{-\frac{1}{2}}(\Gamma_0) \tag{0.2}$$

defines the corresponding local influence matrix whereas the modified right-hand side contains all the global information. Now, we use simultaneous approximation of tangential derivatives and recovery for improving the efficiency of the local method. For more general boundary conditions, we employ hybrid boundary element approximations of the Steklov-Poincaré operator [3]

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Hyperbolic Field Equations for Porous Bodies

The subject of the talk concerns a multicomponent continuous modeling of poroelastic materials. It is assumed that the set of fields describing the behaviour of such a system consists of the following functions defined on the reference configuration of the solid component, and on an interval of time: the motion of the skeleton: $\mathbf{f}^S$, the velocity fields of A fluid components: $\acute{\mathbf{x}}^\alpha, \alpha = 1 \ldots A$, the partial mass densities of the skeleton, and of the fluids $\rho^S, \rho^\alpha, \alpha = 1 \ldots A$, the porosity n, and the absolute temperature T common for all components. In the case of nonadiabatic processes an additional field of the bulk heat flux $\mathbf{Q}_{intr}$ is introduced. For these fields we propose the set of hyperbolic field equations following from partial balance laws of mass, and momentum, from the bulk energy balance, and from additional balance equations for the porosity and the heat flux. In the talk we present two aspects of such a model. The first one contains the evaluation of the second law of thermodynamics, and, in particular, a specification of constitutive laws for systems whose processes do not deviate much from the thermodynamical equilibrium. This problem of thermodynamical admissibility is solved by means of Lagrange multipliers. Incidentally this method yields the representation of the set of field equations in mean fields, identical with Lagrange multipliers, for which the system is symmetric. The second aspect is connected with a presence of interfaces in porous materials. We distinguish two classes of such surfaces. The first one describes a propagation of strong discontinuities (e.g. shock waves or combustion fronts), and it is not material with respect to any of the components. The second one is material with respect to the skeleton, and it models an interface between two different systems. We present admissible boundary conditions on such interfaces, and discuss the problem of their consistency with general dynamic compatibility conditions following from field equations in their weak formulation. We show that it is neccessary to introduce surface sources on such interfaces. This means that they cannot be ideal. This yields the problem of a physical interpretation of the temperature which is in general not measurable. It may suffer a jump on the interface. Solely under the condition of a small deviation from the thermodynamical equilibrium which, in turn yields the continuity of chemical potentials, the absolute temperature is a physically meaningful quantity, and we are able to formulate an effective model of processes with heat conduction. The general model is illustrated with two simple examples: a fully linear model in which we show the existence of $A+2$ modes of propagation of weak discontinuity waves, and a weakly nonlinear model in which an equilibrium porosity is not constant but it may rather depend on partial mass densities. We demonstrate the existence of two small parameters in such a model which yield a possibility of construction of asymptotic solutions. We qoute a first result of such a construction of soliton-like solutions.

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Estimates for eigenfunctions of elliptic operators with respect to the spectral parameter

The talk deals with the derivation of uniform estimates for the moduli of the L_2-normalized eigenfunctions $u_n(x)$ of an elliptic operator

$$Lu \equiv \sum_{i,j=1}^{N} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial u}{\partial x_j}\right) + a(x)u \quad (N \geqslant 2). \tag{0.1}$$

We assume that the coefficients in (0.1) are measurable and, moreover, satisfy the conditions

$$\alpha^{-1}|\xi|^2 \leqslant \sum_{i,j=1}^{N} a_{ij}(x)\xi_i\xi_j \leqslant \alpha|\xi|^2, \tag{0.2}$$

where $\alpha > 0, a_{ij} = a_{ji}$, and $|a(x)| \leqslant a_0$ (here α and a_0 are constants).

Theorem 1. *The L_2-normalized eigenfunctions of the spectral boundary value problem*

$$\sum_{i,j=1}^{N} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial u}{\partial x_j}\right) + a(x)u + \lambda u = 0, \quad u|_{\partial D} = 0$$

with measurable coefficients satisfying conditions (0.2) in a domain $D \subset \mathbb{R}^N$ (the boundary of D obeys the exterior cone condition) satisfy the estimates

$$\sup_{x\in D} |u_n(x)| \leqslant C\lambda_n^{N/4}, \tag{0.3}$$

where the constant C depends only on the numbers α, a_0, and N. The attainability of estimate (0.3) is expressed by the following theorem.

Theorem 2. *There exists an elliptic operator with almost everywhere continuous coefficients in a closed N-dimensional domain $\overline{K}_N$ such that countably many L_2-normalized eigenfunctions of this operator satisfy the estimates*

$$\max_{x\in\overline{K}_N} |u_n(x)| \geqslant C\lambda_n^{N/4},$$

where the constant C is independent of n.

Yosifian G.A.
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Some homogenization problems for variational inequalities in elasticity

Iosifovich Yu.N.
(Belarus State University)

Non local boundary value problems for the parabolic partial equations

Yurko V.A.
(Saratov State University)

Singular Non-Selfadjoint Differential Operators with a Discontinuty in an Interior Point

Consider the boundary value problem $\mathcal{L}$ of the form

$$\ell y := -y'' + q(x)y = \lambda y, \quad x > 0, \tag{1}$$

$$y'(0) - hy(0) = 0, \quad [y, y']^T(a+0) = A[y, y']^T(a-0)$$

with the jump condition in an interior point $a > 0$, where $A = [a_{jk}]_{j,k=1,2}$ is a transition matrix, $\det A \neq 0$, $q(x), h$ and a_{jk} are complex, and $(1+x)q(x) \in L(0,\infty)$. Let $b_\pm := (a_{11} \pm a_{22})/2$, and let for definiteness $|b_-| > |b_+| > 0$, $a_{12} = 0$. In this case, in contrast to the classical Sturm-Liouville operators, the discrete spectrum is unbounded, and there are new qualitative effects in the investigation of direct and inverse problems of spectral analysis. Let $\lambda = \rho^2$, $\operatorname{Im}\rho \geqslant 0$. Denote $\Delta(\rho) := e'(0,\rho) - he(0,\rho)$, where $e(x,\rho)$ is the solution of (1) satisfying the jump condition and $\lim_{x\to\infty} e(x,\rho)\exp(-i\rho x) = 1$. For sufficiently large $|\rho|$, the function $\Delta(\rho)$ has simple zeros of the form

$$\rho_k = \pi(k+\theta)/a + O(k^{-1}),\ |k| \to \infty,\ \theta := (2\pi)^{-1}(-i\ln|b_+/b_-| + \arg(-b_+/b_-)). \tag{2}$$

Denote

$$\Lambda' = \{\lambda = \rho^2 : \operatorname{Im}\rho > 0,\ \Delta(\rho) = 0\},$$
$$\Lambda'' = \{\lambda = \rho^2 : \operatorname{Im}\rho = 0,\ \rho \neq 0,\ \Delta(\rho) = 0\},$$
$$\Lambda = \Lambda' \cup \Lambda'',$$
$$M(\lambda) := e(0,\rho)/\Delta(\rho),$$
$$V(\lambda) = (2\pi i)^{-1}(M^-(\lambda) - M^+(\lambda)), \qquad \lambda > 0,$$

where $M^{\pm}(\lambda) := \lim_{z\to 0,\, Re\, z>0} M(\lambda \pm iz)$. For brevity, let $\mathcal{L}$ have simple spectrum, i.e. all zeros of $\Delta(\rho)$ are simple, have no finite limit points, and $\rho M(\lambda) = O(1)$ as $\rho \to 0$. Let $S := (\{V(\lambda)\}_{\lambda>0}, \{\lambda_k, \alpha_k\}_{\lambda_k\in\Lambda})$, $\lambda_k = \rho_k^2$ be the spectral data of $\mathcal{L}$, where

$$M_k = e(0,\rho_k)\Big(\Big(\frac{d}{d\lambda}\Delta(\rho)\Big)_{\rho=\rho_k}\Big)^{-1}, \quad \alpha_k = \begin{cases} M_k & \text{for } \rho_k \in \Lambda', \\ M_k/2 & \text{for } \rho_k \in \Lambda'', \end{cases}$$

Theorem 1. *The spectral data S have the following properties:*

(i_1) $\rho_k \neq \rho_s$ *for* $k \neq s$; *if* $\rho_k \in \Lambda''$, *then* $-\rho_k \notin \Lambda''$;

(i_2) *as* $|k| \to \infty$, *(2) is valid;*

(i_3) $\alpha_k \neq 0$, *and* $\alpha_k = 2/a + O(k^{-1})$ *as* $|k| \to \infty$;

(i_4) *the function* $V(\lambda)$ *is continuously differentiable for* $\{\lambda > 0\} \setminus \Lambda''$; *for* $\lambda_k \in \Lambda''$ *there exist the finite limits* $V_k := \lim_{\lambda\to\lambda_k} (\lambda - \lambda_k)V(\lambda) \neq 0$, *and* $V_k = i\pi^{-1}\alpha_k \operatorname{sign}\rho_k$;

(i_5) $\rho V(\lambda) = O(1)$ *as* $\lambda \to 0$, *and* $V(\lambda) = V_0(\lambda) + O(\lambda^{-1})$, *as* $\lambda \to +\infty$, *where* $V_0(\lambda) := \det A\Big(\pi\rho\Delta_0(\rho)\Delta_0(-\rho)\Big)^{-1}$, $\rho > 0$, $\Delta_0(\rho) := b_+ + b_- \exp(2i\rho a)$. The inverse problem consists in recovering $\mathcal{L}$ from the given spectral data S. Let us formulate the uniqueness theorem for the solution of this inverse porblem.

Theorem 2. *The specification of the spectral data S uniquely determines $\mathcal{L}$.* Using the method of spectral mappings [1], one can also obtain an algorithm for the solution of the inverse problem considered, along with necessary and sufficient conditions of its solvability.

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Moment functions of solutions for the carry equation with stochastic coefficients

We consider the Cauchy problem

$$\frac{\partial u(t,x)}{\partial t} = \varepsilon_1(t)\frac{\partial u(t,x)}{\partial x} + \varepsilon_2(t)\frac{\partial^2 u(t,x)}{\partial x^2} + f(t,x), \qquad (1)$$

$$u(t_0, x) = u_0(x). \qquad (2)$$

Here u the unknown function, $f : [t_0, t] \times R \to R, u_0 : R \to R$ stochastic processes, the prozess u_o is independent with stochastic coefficients $\varepsilon_1, \varepsilon_2, f$, and the last be given with characteristic functional $\varphi(v_1(\cdot), v_2(\cdot), w(\cdot))$, and $v_1(\cdot), v_2(\cdot), w(\cdot)$ is integrable functions. Let $\chi(t_0, t, \cdot)$ is the characteristic function for $[t_0, t]$, that is $\chi(t_0, t, s) = 1$ for $s \in [t_0, t]$ and $\chi(t_0, t, s) = 0$ for $s \notin [t_0, t]$. We found the formulas for first and second moment functions of the solution $u(t, x)$.

Theorem.

$$Mu(t,x) = Mu_0(x) * F_\xi^{-1}[\varphi(-\xi\chi(t_0,t,\cdot), i\xi^2\chi(t_0,t,\cdot), 0)] -$$

$$-i \int_{t_0}^{t} F_\xi^{-1}[F_x[\frac{\delta}{\delta w(\tau,x)}\varphi(-\xi\chi(\tau,t,\cdot), i\xi^2\chi(\tau,t,\cdot), 0)]]d\tau$$

is the mean value for the generalized solution of Cauchy problem (1), (2). Here $F_x[g]$ the Fourer transformation on variable x, F_ξ^{-1} the inverse Fourer transformation, $*$ the convolution symbol and $\delta/\delta w(\tau, x)$ the variational derivative [1, 2].

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About convergence of the Trotter product formula for quasi-sectorial contractions

We extend the Chernoff theory of approximation of contraction semigroups à la Trotter. It is shown that the Trotter-Neveu-Kato convergence theorem holds in operator norm for a family of uniformly m-sectorial generators in a Hilbert space. Then we obtain a Chernoff-type approximation theorem for quasi-sectorial contractions on a Hilbert space in the operator norm. Necessary and sufficient conditions are given for the operator-norm convergence of Trotter-type product formulae.

The main Theorem [1]:

Theorem. Let $\{\Phi(s)\}_{s\geqslant 0}$ be a family of quasi-sectorial contractions on a Hilbert space $\mathcal{H}$. Let there exist $0 < \alpha < \pi/2$ such that its numerical range

$\Theta(\Phi(s)) \subseteq D_\alpha = \{z \in C : |z| \leqslant \sin\alpha\} \bigcup \{z \in C : |\arg(1-z)| \leqslant \alpha$ and $|z - 1| \leqslant \cos\alpha\}$, for all $s \geqslant 0$. Let $X(s) = (I - \Phi(s))/s$, and let X_0 be a closed operator with non-empty resolvent set, defined in a closed subspace $\mathcal{H}_0 \subseteq \mathcal{H}$. Then the family $\{X(s)\}_{s>0}$ converges in the uniform resolvent sense to X_0 as $s \to +0$ if and only if

$$\lim_{n\to\infty} \left\|\Phi(t/n)^n - e^{-tX_0}P_0\right\| = 0,\ t > 0.$$

Here P_0 denotes the orthogonal projection onto $\mathcal{H}_0$.

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Formal Operators and Modern Group Analysis

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Singularities in control systems

The classification of simple singularities of the contact of an embedding with a nested system of submanifolds in the target space obtainedin [1-3] provides various applications in control systems, for example, when constraints have the form of inequalities. To find the critical values (in particular, extrema) of a function $h : \mathbb{R}^n \to \mathbb{R}$ subjected to equality constraints $g = 0$ (where $g : \mathbb{R}^n \to \mathbb{R}^k$, $k < n$) and an inequality $f \geqslant 0$, (where $f : \mathbb{R}^n \to \mathbb{R}$) one should distinguish the critical values of h on the total space $\mathbb{R}^n$, critical values of the restriction of h to the regular part of $g = 0$, values of h on the critical locus of the variety $g = 0$, critical values of h on the regular part of the variety $f = 0, g = 0$ and values of h on the critical locus of the restriction of f to the regular part of $g = 0$. If all the entries depend on parameters Λ then the critical values described above form, generally speaking, a reducible hypersurface Σ in the product $\Lambda \times \mathbb{R}$ of the parameter space and the set of values of h. Generically Σ is an open subset of the bifurcation diagram of the flag contact singularity of the embedding

$$\mathbb{R}^n \to \mathbb{R}^k \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}^n,\quad (x) \mapsto (g(x), h(x), f(x), x))$$

and the flag

$$\{0\} \times \{0\} \times \{0\} \times \mathbb{R}^n \subset \{0\} \times \{0\} \times \{\mathbb{R}\} \times \mathbb{R}^n \subset$$

$$\{0\} \times \{\mathbb{R}\} \times \{\mathbb{R}\} \times \mathbb{R}^n \subset \{\mathbb{R}^k\} \times \{\mathbb{R}\} \times \{\mathbb{R}\} \times \mathbb{R}^n$$

with three elements. This statement and results of [1-3] imply the classification of simple singularities of Σ. Supported by RFBI 99010147 grant.

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To the theory of the equations of the mixed parabolic-hyperbolic type with the distributed and concentrated delay

The development of the problems of a supersonic flow of a surface, covered with a material with thermal memory, results in a solution of initial-boundary value problems for the equation

$$L\,u(x,y) = H(x)\left[\alpha(x)\,u(x-\tau,y) + \int_0^\tau R(t)\,u(x-t,y)\,dt\right], \qquad (1)$$

$L \equiv H(-x)\,\partial^2/\partial x^2 - \partial^2/\partial y^2 + H(x)\,\partial/\partial x$, $0 < \tau \equiv \text{const}$, $H(\xi)$ – Heaviside function, $0 < \alpha(x)$, $R(x)$ – limited functions, in area $D = D^+ \cup D^-$, where $D^+ = \{(x,y) : x > 0, 0 < y < \pi\}$ and

$$D^- = \{(x,y) : -x < y < \pi + x, -\pi/2 < x < 0\}$$

–parabolic and hyperbolic parts of D, and

$$D^+ = \bigcup_{k=0}^{+\infty} D_k^+,\ D_k^+ = \{(x,y) : k\tau \leqslant x \leqslant (k+1)\tau, 0 < y < \pi\}.$$

Problem A. *To find in area D solution $u(x,y)$ of equation (1) from a class $C(\overline{D^+\cup D^-})\cap C^1\left((\overline{D^+\cup D^-})\setminus\ \partial(\overline{D^+\cup D^-})\right)\cap C^2(D^+\cup D^-)$, satisfying to conditions*

$$u(x,0)=\varphi_1(x),\ u(x,\pi)=\varphi_2(x),\ 0\leqslant x<+\infty;$$

$$u(-y,y)=\psi(y),\ 0\leqslant y\leqslant \pi/2;\ \ u(x,y)=f(x,y),\ (x,y)\in\overline{D^+_{(-1)}},$$

where $\varphi_i(x)$ $(i=1,2)$, $\psi(y)$, $f(x,y)$ – continuous, rather smooth functions, and $\psi(0)=\varphi_1(0)$, $\varphi_i(+\infty)=0$ $(i=1,2)$, $f(x,y)=\begin{cases} u^-(x,y),\ (x,y)\in\overline{D^-}, \\ g(x,y),\ (x,y)\in\overline{D^+_{(-1)}\setminus D^-}, \end{cases}$ *and $u^-(x,y)=\omega(y+x)-\psi((y+x)/2)+\psi((y-x)/2)$ – solution of the problem A in D^-, in which $\omega(y)=\int\limits_0^\pi G(y,t)\,\psi(t/2)\,dt$, and $G(y,t)$ – Green function of a boundary value problem $\omega''(y)-\omega'(y)=-\psi'(y/2)$, $\omega(0)=\omega(\pi)=0$, $0<y<\pi$,* and $\psi(y)\in C^{(1,\lambda)}(0,\pi/2)$, $0<\lambda<1$. The proof of uniqueness of a solution of the problem A is conducted with the help of method of auxiliary functions. The solution of the problem A in area D^+ is found in the form

$$u(x,y)=\sum_{n=1}^{+\infty}A_n\delta(x,n)\sin ny,\ \ (x,y)\in\overline{D^+},$$

where $A_n=\dfrac{2}{\pi}\int\limits_0^\pi \omega(t)\sin nt\,dt$, and $\delta(x,n)$ – solution of the equation

$$\delta'(x,n)+n^2\delta(x,n)-\int\limits_0^x R(x-t)\,\delta(t,n)\,dt=f(x)\equiv$$

$$\equiv\alpha(x)\,\delta(x-\tau,n)+\int\limits_{x-\tau}^0 R(x-t)\,\delta(t,n)\,dt,\ \ x>0,$$

under condition of $\delta(0,n)=1$.

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Spatial and dynamical chaos generated by reaction-diffusion equations in unbounded domains

We consider the following quasilinear reaction-diffusion system in an unbounded domain $\Omega \subset \mathbb{R}^n$:

$$\partial_t u = a\Delta_x u - (\vec{L}, \nabla_x)u - f(u) + g(x), \quad u\big|_{\partial\Omega} = 0, \quad u\big|_{t=0} = u_0, \tag{1}$$

where $u = (u^1, \cdots, u^k)$ is an unknown vector-valued function, a is a given diffusion matrix, $(\vec{L}, \nabla_x)u := \sum_{i=1}^n L_i(x)\partial_{x_i}u$, $\vec{L}(x)$ is a given vector-valued function, $f(u)$ and $g(x)$ are given nonlinear interaction function and external force respectively.

It is proved that under certain assumptions on f, a and $\vec{L}$, the equation (1) generates a dynamical system $S_t : \Phi_b \to \Phi_b$, $t \geqslant 0$, in the corresponding phase space $u_0 \in \Phi_b$, which possesses a global attractor $\mathcal{A}$ in it.

Recall, that in contrast to the case of bounded domains Ω, in unbounded domains the Hausdorff and fractal dimension of the attractor $\mathcal{A}$ is usually infinite (see [1], [2]), consequently (following to [2], [3]) in order to obtain quantitative and qualitative information about the attractor it is natural to study it's Kolmogorov's ε-entropy. In the present paper we give sharp upper and lower bounds for the ε-entropy for the restriction $\mathcal{A}\big|_{B^R_{x_0}}$ of the attractor $\mathcal{A}$ to an arbitrary ball $B^R_{x_0}$ of radius R, centered in x_0.

Moreover, we give more detailed investigation of the attractor $\mathcal{A}$ in the spatially homogeneous case: $\Omega = \mathbb{R}^n$, $\vec{L} = const$, $g = const$. In this case the spatio-temporal chaos on the attractor of (1) can be described in terms of embeddings of multidimensional Bernulli schemes with continual number of symbols $\omega \in [0, 1]$ to the spatio-temporal dynamics.

Particularly, we prove that under the natural assumptions on the equation (1) the associated dynamical system S_t restricted to the attractor $\mathcal{A}$ has an infinite topological entropy and that every finite dimensional dynamics can be obtained up to a homeomorphism restricting the semigroup S_t to the corresponding invariant subset of the attractor.

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Cohomology of the Lie algebra of vector fields on the line

An appropriate mathematical apparatus is developed and cohomologies of the Lie algebra of all smooth vector fields on the line with coefficients in the most important representations are presented. A generalized sequence of complexes is proposed and corresponding cohomologies are calculated.

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Свойства базисности систем собственных функций нелиненых задач типа Штурма-Лиувилля

Будет рассмотрен ряд нелинейных задач типа Штурма-Лиувилля и представлены результаты о свойствах базисности, в L_2 и других пространствах, систем собственных функций этих задач. Например, будет рассмотрена следующая задача:

$$-u'' + f(u^2)u = \lambda u, \quad u = u(x), \quad x \in (0,1), \quad u(0) = u(1) = 0, \quad u'(0) > 0,$$

$$\int_0^1 u^2(x)dx = 1.$$

Здесь все величины вещественны, λ- спектральный параметр и f- заданная функция. Результат для этой задачи состоит в том, что если $f(s)$- гладкая неубывающая функция аргумента $s \geqslant 0$, то бесконечная последовательность всех собственных функций, существование которой доказано, является базисом Бари в $L_2(0,1)$ (так что в частности она является базисом). Кроме того, будет представлен результат о возможности разложения "произвольной функции" в интеграл по собственным функциям нелинейного уравнения Шредингера на полупрямой, подобного представлению функций с помощью преобразования Фурье.

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To the theory of boundary value problems for the system of principal type on the plane

Let us consider the system of linear partial differential equations

$$\frac{\partial u}{\partial x_2} - a\frac{\partial u}{\partial x_1} = 0 \tag{1}$$

in the bounded domain $D \subseteq \mathbf{R}^2$, where $u(x)$ is an unknown l– vector-valued function and $a \in \mathbf{R}^{l\times l}$ is a constant matrix, $l \geqslant 3$. System (1) is said to be of principal (composite) type [1] if it's characteristic equation $\det(a-\nu) = 0$ has s real simple roots, $1 \leqslant s \leqslant l-2$. Let the contour $\partial D = \Gamma$ be composed of the smooth arcs Γ_j with endpoints τ_j and τ_{j+1}, $j = 1,\ldots,2s$ $(\tau_{2s+1} = \tau_1)$. We consider the following boundary value problem:

$$c_j u|_{\Gamma_j} = f_j, \quad j = 1,\ldots,2s, \tag{2}$$

where $c_j(f_j)$ is the $l_j \times l$ matrix-valued (l_j vector-valued) function on Γ_j. It is assumed that $(l-s)/2 \leqslant l_j \leqslant (l+s)/2$, $l_1+l_2+\ldots+l_{2s} = ls$. Let us denote by Γ_{jk} the characteristic line $x_1 + \nu_k x_2 = const$ containing the point τ_j, $1 \leqslant j \leqslant 2s, 1 \leqslant k \leqslant s$. The domain D is assumed admissible in the following sense: all arcs Γ_j are not tangent characteristics and $\Gamma_{jk} \cap \Gamma = \Gamma_{jk} \cap F$, $F = \{\tau_1,\ldots,\tau_{2s}\}$ for each k and j. So the set $D_k = D \setminus \cup_{j=1}^{s} \Gamma_{2j-1,k}$ has s components. Any regular solution $u(x_1,x_2)$ of a system (1) can be representented [2] in the form $u = b_1\phi^1 + 2\mathrm{Re}\, b_2\phi^2$, where ϕ^1 is a regular solution of the canonical hyperbolic system $\partial\phi^1/\partial x_2 - J_1\partial\phi^1/\partial x_1 = 0$ and ϕ^2 is the J_2 – analytic function [3]. The block 1×3-matrix $b = (b_1, b_2, \overline{b}_2)$ reduce a to the Jordan normal form $b^{-1}ab = \mathrm{diag}(J_1, J_2, \overline{J}_2)$. The solution $u(x_1,x_2)$ of system (1) is being considered in the weighted Holder space. Using this representation the problem (1), (2) is reduced to an equivalent singular integro- functional equation. The investigation on solvability of this equation is based upon the general theory of singular integro-functional operators [3]. This permits to obtain the Fredholm property and index formula of problem (1), (2). Note that the corresponding problem for one equation of high order is studied in [4]. As a consequence of general result we obtain the solvability theorem for some boundary value problems for magneto hydrodynamic system. This approach can be also applied to the system equations of high order. As example we study the problem $u_k\big|_{\Gamma_j} = f_{kj}$, $k = 1,2$, $j = 1,\ldots,4$ for the system

$$\frac{\partial^2 u_1}{\partial x_2^2} - \frac{\partial^2 u_1}{\partial x_1^2} + \frac{\partial u_3}{\partial x_1} = 0, \quad \frac{\partial^2 u_2}{\partial x_2^2} - \frac{\partial^2 u_2}{\partial x_1^2} + \frac{\partial u_3}{\partial x_2} = 0, \quad \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} = 0.$$

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Gravity as Lorentz force

The idea of the relativistic gravitation theory was proposed by Poincaré [1] : "In the paper cited Lorentz found it necessary to supplement his hypothesis so that the relativity postulate could be valid for other forces besides the electromagnetic ones. According to his idea, owing to Lorentz transformation (and therefore owing to the translational movement) all forces behave like electromagnetic. It turned out to be necessary to consider this hypothesis more attentively and to study the changes it makes in the gravity laws in particular. First of all, it enables us to suppose that the gravity forces propagate not instantly, but at the light velocity. One could think that it is enough to reject such a hypothesis, for Laplace has shown that it can not take place. But in fact the effect of this propagation is largely balanced by some other circumstance, so there is no any contradiction between the law proposed and the astronomical observations. Is it possible to find a law satisfying the condition stated by Lorentz and at the same time coming to the Newton law in all the cases when the velocities of the celestial bodies are small enough to neglect their squares (and also the products of the accelerations and the distance) with respect to the square of the velocity of light?" Poincaré found that the mathematical solution of the problem is not unique. It is easy to solve the Poincaré problem by making use of the physical reasons. The form of the Newton gravity law coincides with the form of the Coulomb law describing the interaction between two oppositely charged bodies. The relativistic form of the Coulomb law is well – known. It is the Lorentz force. Thus the relativistic form of the gravity law must be the same. We consider a simple problem of the interaction of two bodies when the mass of one body is equal to zero. It is a problem of the light propagation in the gravity field of one body. The received exact solution of this problem describes all effects predicted by the general relativity: the distortion of the light beams in the gravity field, the light motion along the closed trajectory in the gravity field, etc. If the Mercury mass is considered small, it is possible to calculate the shift of Mercury perihelion for a hundred years. It turns out to be 45". The general relativity predicts 43".

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Geometry and dynamics on flat surfaces

Dealing with Riemann surfaces of genera greater than two one usually considers the metric of constant negative curvature. However, in numerous problems of dynamics (like billiards in rational polygons, topological dynamics of measured foliations on surfaces, interval exchange transformations, etc) it is natural to consider a *flat* metric on a surface, where all curvature of the surface is collapsed to several cone-type singularities. We consider a class of such flat metrics having trivial holonomy: parallel transport of a vector along a smooth closed path leaves the vector invariant. A surface endowed with such special flat metric is called a *translation surface.* I want to present several recent results of A.Eskin, M.Kontsevich, H.Masur and myself concerning geometry and dynamics on translation surfaces. In particular I shall discuss the closed trajectories on translation surfaces, the amazing behavior of their configurations, and the quantitative aspects of the asymptotics of the number of closed geodesics of bounded length. I shall present an application of these results to the billiards in "rectangular polygones". I shall also describe the behavior of the geodesic flow in a typical flat metric. The answers to these problems are based on the study of topology, geometry, and dynamics of the moduli spaces of Abelian differentials.

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