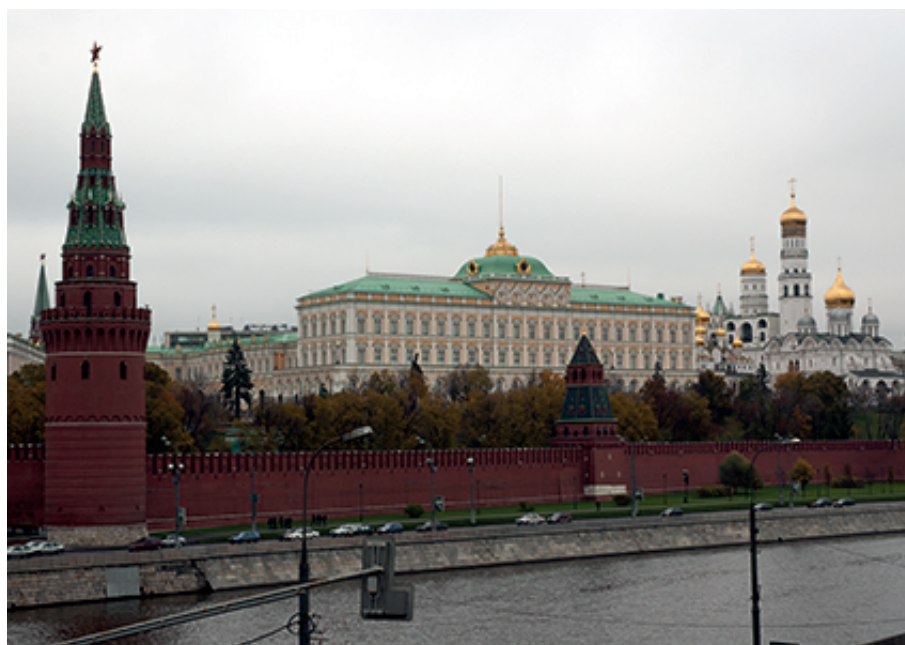


The third Russian-Chinese conference  
on complex analysis, algebra,  
algebraic geometry and mathematical physics

Book of abstracts



May 10–16, 2016  
Steklov Mathematical Institute of RAS, Moscow

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## Abstracts

**Anna Chugainova** (*Steklov Mathematical Institute of RAS*)

***Stability of stationary solutions of the generalized KdV-Burgers equation***

---

The stability of discontinuities representing solutions of a model generalized KdV–Burgers equation with a nonmonotone potential is analyzed. The spectral (linear) stability of the structure of special discontinuities was previously studied. Here the spectral stability of nonspecial discontinuities is investigated. The structure of a nonspecial discontinuity represents a phase curve joining two special points: a saddle (the state ahead of the discontinuity) and a focus or node (the state behind the discontinuity). The set of nonspecial discontinuities is examined depending on the dispersion and dissipation parameters. A set of stable nonspecial discontinuities is found.

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**Alexey Davydov** (*Lomonosov Moscow State University, Vladimir State University*)

***Theory of normal forms of linear second order mixed type PDEs on the plane***

---

This theory originates from the known treatise by F. Tricomi (1923), in which normal form for such an equation was proposed near a typical point of type change line, at which the characteristic direction is not tangent to the line. The correct proof for this form was given a decade later by M. Cibrario.

In general, the creation of the theory of normal forms for generic linear second order mixed type partial equations on the plane was completed in the late twentieth century.

These results and recent achievements of this theory are discussed in the talk.

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**Sergey Gorchinskiy** (*Steklov Mathematical Institute of RAS*)

***Categorical measures for varieties with finite group actions***

---

The talk is based on a common work with D. Bergh, M. Larsen, and V. Lunts. Given a variety with a finite group action, we compare categorical measures of the corresponding quotient stack and the extended quotient. Using weak factorization for orbifolds, we show that for a wide range of cases, these two measures coincide, which implies, in particular, a conjecture of Galkin and Shinder on categorical and motivic zeta-functions of varieties. We provide examples showing that in general, these two measures are not equal.

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**Mikhail Katanaev** (*Steklov Mathematical Institute of RAS*)

***On homogeneous and isotropic universe***

---

We give a simple example of spacetime metric, illustrating that homogeneity and isotropy of space slices at all moments of time is not obligatory lifted to a full system of six Killing vector fields in spacetime, thus it cannot be interpreted as a symmetry of a four dimensional metric. The metric depends on two arbitrary and independent functions of time. One of these functions is the usual scale factor. The second function cannot be removed by coordinate transformations. We prove that it must be equal to zero, if the metric satisfies Einstein's equations and the matter energy momentum tensor is homogeneous and isotropic. A new, equivalent, definition of homogeneous and isotropic spacetime is given.

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**Sergei Kozyrev** (*Steklov Mathematical Institute of RAS*)

***Dark states in quantum photosynthesis***

---

A model of quantum photosynthesis with degeneracy in the light-harvesting system is discussed. We consider interaction of excitons in chromophores with light and phonons (vibrations of environment). These interactions have dipole form but are different (are related to non-parallel vectors of “bright” states). We show that this leads to excitation of non-decaying “dark” states. We discuss relation of this model to the known from spectroscopical experiments phenomenon of existence of photonic echo in quantum photosynthesis.

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**Viktor Kulikov** (*Steklov Mathematical Institute of RAS*)

***On some actions of the symmetric group  $\mathbb{S}_4$  on  $K3$  surfaces***

---

In the talk, I’ll consider the actions of the symmetric group  $\mathbb{S}_4$  on  $K3$  surfaces  $X$  having the following property:

- (\*) there exists an equivariant bi-rational contraction  $\bar{c} : X \rightarrow \bar{X}$  to a  $K3$  surface  $\bar{X}$  with  $ADE$ -singularities such that  $\bar{X}/\mathbb{S}_4 \simeq \mathbb{P}^2$ .

I’ll show that up to equivariant deformations there exist exactly 15 such actions and these actions can be realized as the actions of the Galois group on the Galois normal closures of the dualizing coverings of the projective plane associated with rational quartics having no singularities of types  $A_4$ ,  $A_6$  and  $E_6$ .

\* \* \*

**Jinsong Liu** (*Institute of Mathematics of CAS*)

***Deformation of circle patterns and its applications***

---

Given a circle pattern on the Riemann sphere  $\bar{\mathbb{C}}$ , in this talk we prove that its quasiconformal deformation space can be naturally identified with the product of the Teichmüller spaces of its interstices. By using the intersection number technique, together with Teichmüller theory of circle packings, we provides a rigidity result of the Midscribability Theorem. Furthermore, by using these methods, we shall investigate the stability of some inscribable graphs.

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**Tuen Wai Ng** (*University of Hong Kong*)

***Fermat functional equations revisited***

---

The problem of the existence of transcendental meromorphic or entire solutions for the Fermat functional equation  $f^n + g^n + h^n = 1$  was first studied by Walter Hayman in 1984. It is known that meromorphic (entire) solutions exist for  $n \leq 6$  ( $n \leq 5$ ) and no meromorphic (entire) solution exists when  $n \geq 9$  ( $n \geq 7$ ). In this talk we will revisit this problem from a more geometric view point. This is a joint work with Sai-Kee Yeung.

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**Anatol Odziejewicz** (*University of Bialystok*)

***Groupoids, algebroids and Poisson structures related to  $W^*$ -algebras***

---

In this talk we present complex Banach-Lie groupoids, algebroids and fiber-wise linear complex Banach sub Poisson structures defined canonically by the structure of a  $W^*$ -algebra  $\mathfrak{M}$ . In particular we show that these structures are arranged in the short exact sequence of complex Banach sub Poisson  $\mathcal{VB}$ -groupoids with the groupoid  $\mathcal{G}(\mathfrak{M}) \rightrightarrows \mathcal{L}(\mathfrak{M})$  of partially invertible

elements of  $\mathfrak{M}$  as the side groupoid. The case of the  $W^*$ -algebra of bounded operators on a separable Hilbert space is considered as an application.

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**Denis Osipov** (*Steklov Mathematical Institute of RAS, National University of Science and Technology "MISIS"*)

***Distribution spaces of two-dimensional local fields and representations of the discrete Heisenberg group***

---

The talk is based on joint paper with A.N. Parshin: arXiv:1510.02423.

The discrete Heisenberg group  $\text{Heis}(3, \mathbb{Z})$  is the group of integer upper-triangular  $3 \times 3$  matrices with units on the diagonal. This group is the simplest non-Abelian nilpotent group of class 2. The classical theory of unitary representations of locally compact groups in a Hilbert space does not work smoothly in this case. But in our case of the group  $\text{Heis}(3, \mathbb{Z})$ , if we change the category of representation spaces and consider, instead of the Hilbert spaces, vector spaces of countable dimension and without any topology, then the situation will be much better. For irreducible nonunitary representations, the new approach yields a moduli space which is a complex manifold.

We consider a two-dimensional local field  $K$  that is isomorphic to the field of iterated Laurent series  $\mathbb{F}_q((u))((t))$ . This field naturally appears from a flag of subvarieties on an algebraic surface over a finite field  $\mathbb{F}_q$ : a point and an irreducible curve on this surface, via localization and completion procedure. D.V. Osipov and A.N. Parshin have previously constructed the infinite-dimensional  $\mathbb{C}$ -vector space of distributions  $\mathcal{D}'_{\mathcal{O}}(K)$  on  $K$  (note that  $K$  is not locally compact group).

We construct an explicit family of pair-wise non-isomorphic irreducible infinite-dimensional representations of  $\text{Heis}(3, \mathbb{Z})$  inside of  $\mathcal{D}'_{\mathcal{O}}(K)$  such that this family is parametrized by points of an elliptic curve  $\mathbb{C}^*/q^{\mathbb{Z}}$ . To calculate the traces on these representations, we consider an action of the extended discrete Heisenberg group, which is isomorphic to  $\text{Heis}(3, \mathbb{Z}) \rtimes \mathbb{Z}$  and it is a discrete nilpotent group of class 3. The traces which we obtain are classical Jacobi theta functions.

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**Vladimir Popov** (*Steklov Mathematical Institute of RAS*)

***On the equations defining affine algebraic groups***

---

For the coordinate algebras of abelian varieties, the problem of finding a presentation by generators and relations canonically determined by the group structure has been explored and solved by D. Mumford. Since every connected algebraic group is an extension of a connected linear algebraic group by an abelian variety, the analogous problem for connected affine algebraic group naturally arises. The talk is intended to describing its solution based on solving two problems posed by D. E. Flath and J. Towber in 1992. From the standpoint of this theory, the usual naive presentation of  $SL(n)$  as a hypersurface  $\det = 1$  in an  $n^2$ -dimensional affine space is adequate only for  $n = 2$ : the canonical presentation defines  $SL(3)$  as the intersection of 2 homogeneous and 2 inhomogeneous quadrics in a 12-dimensional affine space,  $SL(4)$  as the intersection of 20 homogeneous and 3 inhomogeneous quadrics in a 28-dimensional affine space, etc.

\* \* \*

**Alexey Shchuplev** (*Siberian Federal University*)

***McMullen's formula and a multidimensional analog of the Weierstrass  $\zeta$ -function***

---

In 1899 G. Pick found a simple formula relating the area  $\text{Area}(P)$  of a plane polygon  $P$  with vertices in integer points with the number  $I$  of its interior points and on its boundary  $B$ :

$$\text{Area}(P) = I + \frac{B}{2} - 1.$$

However, this formula can not be simply extended even to three-dimensional case as Reeve's example demonstrates. Instead of one simple formula there exist several formulas, obtained by combinatorial or number-theoretical methods, or by methods of algebraic geometry. One such formula due to P. McMullen says that for an integer polyhedron with centrally-symmetric facets its volume is equal to the sum of all solid angles at each its integer point of the polyhedron.

A multidimensional analog of the Weierstrass  $\zeta$ -function

$$\zeta(z) = \eta(z) + \sum_{\gamma \in \mathbb{Z}^{2n} \setminus \{0\}} \left( \eta(z - \gamma) + \eta(\gamma) + \sum_{i=1}^n \left( \frac{\partial \eta}{\partial z_i}(\gamma) z_i + \frac{\partial \eta}{\partial \bar{z}_i}(\gamma) \bar{z}_i \right) \right),$$

where  $\eta(z)$  is the Bochner-Martinelli differential form allows to prove this statement analytically.

\* \* \*

**Sergey Suetin** (*Steklov Mathematical Institute of RAS*)

***The Problem of Analytic Continuation and Zero Distribution of Hermite–Padé Polynomials***

---

During the talk we will discuss the classical problem of analytic continuation of a multivalued analytic function which is given by a germ. The approach under discussion is based on the notion of Hermite–Padé polynomials and closely connected with Nuttall's conjecture about the special partition of a three-sheeted Riemann surface.

The talk is based on the results that obtained recently in collaboration with A. V. Komlov, N. G. Kruzhilin, and R. V. Palvelev.

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**Xiaotao Sun** (*Institute of Mathematics of CAS*)

***Étale fundamental group and  $D$ -modules in characteristic  $p > 0$***

---

I will report a joint work with H. Esnault (Stratified bundles and étale fundamental group, Ann. Sc. Norm. Super. Pisa Cl. Sci. (5) Vol. XIII (2014), 795-812), in which we prove the last two statements of a conjecture raised by D. Gieseker on 1975 (Ann. Scuola Norm. Sup. Pisa Cl. Sci. (2) 4 (1975), 1-31).

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**Dmitry Treschev** (*Steklov Mathematical Institute of RAS*)

***On the inclusion of a diffeomorphism into a flow***

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Given a smooth self-map of a compact manifold we discuss the following questions. Is it possible to present this diffeo as a time-one map for a smooth, in general, time-periodic vector field. When this field can be made time independent or almost time independent?

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**Anton Trushechkin** (*Steklov Mathematical Institute of RAS*)

***Perturbative treatment of inter-site couplings in the local description of open quantum networks***

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The problem of construction of a quantum master equation for a system of sites weakly coupled to each other and to one or more reservoirs (open quantum network) is considered. Microscopic derivation of a quantum master equation requires a diagonalization of the Hamiltonian of the network, which can be a difficult task. When the inter-site couplings are weak, the local approach, which neglects the influence of the inter-site couplings on the system-reservoir couplings, is often used. Recently, some doubts were cast to the consistency of the local approach. We develop a systematic perturbation expansion to derive corrections to the local approach and establish its range of validity. Using this extension of the local approach, we derive an expression for the heat flux for a particular model and show that it does not violate the second law of thermodynamics. The talk is based on the joint work with Prof. I.V. Volovich published in EPL 113:3 (2016), 3000; arXiv: 1509.05754.

\* \* \*

**August Tsikh** (*Siberian Federal University*)

***New versions of the Cauchy-Kovalevskaya theorem and the Weierstrass preparation theorem***

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In the traditional formulation of the Cauchy-Kovalevsky theorem one assumes solvability with respect to maximal pure derivative, say  $\partial^m u / \partial x_n^m$ , where  $m$  is the order of the differential equation. For linear differential equations with analytic coefficients this means that the point  $(0, \dots, 0, m) \in \mathbb{Z}^n \subset \mathbb{R}^n$  is a vertex of the Newton polytope of the characteristic polynomial  $P(x_0, \xi)$  for the equation

$$P(x, D)u = f, \text{ where } x = (x_1, \dots, x_n), D = \left( \frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_n} \right).$$

Hörmander gave a version of the Cauchy-Kovalevsky theorem where solvability with respect to some arbitrary derivative  $D^\beta u$ , with  $|\beta| = \beta_1 + \dots + \beta_n$  equal to  $m$ , was assumed. However, in this case one had to require the coefficients of the other highest order derivatives to be small in the point  $x_0$ . Moreover, the initial values should be given not just on one coordinate plane, but on a union, or cross, of several such planes. Such initial values correspond to a Cauchy-Goursat problem.

It was pointed out in a paper by Palamodov (1968) that the Cauchy-Kovalevsky theorem is intimately related with the Weierstrass preparation and division theorems.

In the talk I will tell how to relax the Hörmander condition for a Cauchy-Goursat problem for an equation in two variables with constant coefficients. The solvability conditions are obtained in terms of an amoeba of a characteristic equation. In parallel to this we formulate the corresponding version of the Weierstrass preparation theorem.

\* \* \*

**Igor Volovich** (*Steklov Mathematical Institute of RAS*)

***Anti-de Sitter/conformal field theory correspondence, black holes and quantum biology***

---

There are successful applications of the holographic anti-de Sitter/conformal field theory (AdS/CFT) correspondence to high energy and condensed matter physics. We apply the holographic approach to photosynthesis that is an important example of nontrivial quantum phenomena relevant for life which is being studied in the emerging field of quantum biology.

We use the holographic approach to evaluate the time dependence of entanglement entropy and quantum mutual information in the Fenna-Matthews-Olson (FMO) light-harvesting complex in bacteria during the transfer of an excitation from an antenna to a reaction center. We show that the time evolution of the mutual information simulating the Lindblad open quantum systems master equation for the FMO complex in some cases can be obtained by means of a dual gravity describing black hole formation in the AdS-Vaidya spacetime.

\* \* \*

**Yuefei Wang** (*Institute of Mathematics of CAS*)  
***Dynamics of Newton maps and complexity***

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We shall talk about recent progress on the global dynamics of Newton maps, rigidity of stable rational maps and the critical singularities of polynomials.

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**Siye Wu** (*National Tsing Hua University*)  
***Quantisation of Hitchin's moduli space***

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In this talk, I will construct prequantum line bundles on Hitchin's moduli spaces of orientable and non-orientable surfaces and study the geometric quantisation and quantisation via branes by complexification of the moduli spaces.

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**Nanhua Xi** (*Institute of Mathematics of CAS*)  
***Some infinite dimensional representations of reductive groups with Frobenius maps***

---

We construct certain irreducible infinite dimensional representations of algebraic groups with Frobenius maps. In particular, a few classical results of Steinberg and Deligne and Lusztig on complex representations of finite groups of Lie type are extended to reductive algebraic groups with Frobenius maps.

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**Xiaoping Xu** (*Institute of Mathematics of CAS*)  
***Generalizations of the classical theorem on harmonic polynomials***

---

Classical harmonic analysis says that the spaces of homogeneous harmonic polynomials (solutions of Laplace equation) are irreducible modules of the corresponding orthogonal Lie group (algebra) and the whole polynomial algebra is a free module over the invariant polynomials generated by harmonic polynomials. Algebraically, this gives an  $(sl(2, \mathbb{R}), o(n, \mathbb{R}))$  Howe duality. In this talk, we will represent various generalizations of the above theorem. In our noncanonical generalizations, the constant-coefficient Laplace equation changes to variable-coefficient partial differential equations.

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**Chung Chun Yang** (*Nanjing University*)  
***Value distribution theory and its applications to functional equations***

---

In the talk, we first give a brief introduction of the Nevanlinna's value distribution theory and then illustrate its applications regarding meromorphic solutions of certain types of functional and differential equations. Moreover, some (old and new) conjectures are posed for further studies.

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**Xiao Zhang** (*Institute of Mathematics of CAS*)

***Noncommutative Differential Geometry via Deformation Quantization and Applications in Gravity***

---

Noncommutative metrics and curvatures are discussed in frame of deformation quantization, and the noncommutative Einstein field equations are also proposed. The deformation of classical pp-wave is shown to be the exact solution of vacuum field equations and the deformation of classical Schwarzschild solution is shown to be the quantum black hole solution not depending on the time. Some physical implications of these solutions are also discussed. The talk is based on the joint works with Chaichian, Tureanu, W. Sun, D. Wang, N. Xie, R.B. Zhang.

\* \* \*

**Viktor Zharinov** (*Steklov Mathematical Institute of RAS*)

***On the Backlund transformation***

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The general definition of the Backlund transformation of the systems of nonlinear PDE's will be presented. Their main properties will be discussed and a number of relevant examples will be presented.

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**Xiangyu Zhou** (*Institute of Mathematics of CAS*)

***Recent progress on multiplier ideal sheaves and optimal  $L^2$  extensions***

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In this talk, we'll present our recent proof of Demailly's strong openness conjecture about multiplier ideal sheaves, solution of optimal  $L^2$  extension problem, and their applications.

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**Alexander Znamenskiy** (*Siberian Federal University*)

***A refinement of the Kovalevskaya theorem on analytic solvability of a Cauchy problem***

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Let

$$P = z_n^m + \sum_{\alpha \in A} a_\alpha z^\alpha \tag{1}$$

be a polynomial where  $A \subset \mathbb{Z}_{\geq 0}^{n-1} \times \{0, 1, \dots, m-1\}$  is a finite set of exponents. Consider a differential equation

$$P(\mathcal{D})y = f \tag{2}$$

with  $f = \sum_{k \in \mathbb{Z}_{\geq 0}^n} b_k x^k$  given as a power series. Note that  $\frac{\partial^m}{\partial x_n^m}$  is the highest derivative in  $x_n$  but not necessarily the highest derivative in the equation. Consider a Cauchy problem for (2) with initial conditions

$$\frac{\partial^k y}{\partial x^k}(x', 0) = y_k(x'), \quad k = 0, \dots, m-1, \tag{3}$$

where  $x' = (x_1, \dots, x_{n-1})$ .

*Theorem. If the right hand side  $f$  of (2) is an entire function of exponential type then the Cauchy problem (2), (3) has a unique analytic solution.*

A strict condition on  $f$  is dictated by the fact that in the proof we apply the Borel transform to  $f$ . The condition that  $f$  is an entire function of exponential type is crucial for the domain of convergence of this transform to be non-empty.

Note that relaxation of the condition on  $A$  in (1) reflects in stricter conditions on the right hand side as demonstrated by the well-known example by Kovalevskaya about the heat transfer equation.

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