Национальная академия наук Беларуси
Труды Института математики. 2014. Том 22. № 2. С. 96–108

УДК 511.42

ON THE SIZE OF p-ADIC CYLINDER FOR WHICH THE REGULAR SYSTEM OF ALGEBRAIC NUMBERS CAN BE CONSTRUCTED

N. V. Budarina, M. V. Lamchanovskaya

Khabarovsk Division of Institute for Applied Mathematics,
Far Eastern Branch of the Russian Academy of Science,
Institute of Mathematics, The National Belarus Academy of Sciences
E-mail address: buda77@mail.ru, lammv@mail.ru
Поступила 01.12.2014

On the relation between a factorization of a polynomial resultant and the frequency of its occurrence. A lower bound is obtained for the number of polynomial pairs of a given degree and bounded heights such that their resultants are divisible by a fixed prime number.

1. Introduction. Y. Bugeaud in [1] stated the problem of the length of the interval depends on the height of algebraic numbers, which form the regular system (see [2]) on this interval. The natural number $H(\alpha)$ denotes the height of an algebraic number α, which is the absolute height of the minimal polynomial of α over $\mathbb{Z}$. In this paper, we address to Y. Bugeaud's problem for the p-adic numbers with $n = 3$.

Throughout $c_1 = c_1(n)$, $c_2 = c_2(n)$, $\dots$ are constants depending only on n. The Haar measure of a measurable set $S \subset \mathbb{Q}_p$ is denoted by $\mu(S)$. For positive integer Q define the set of polynomials

$$\mathcal{P}_3(Q) = \{P \in \mathbb{Z}[x] : \ \deg P = 3, \ H(P) \le Q\}. \tag{1}$$

Denote by $\mathcal{A}_{3,p}$ the set of algebraic numbers of degree three lying in $\mathbb{Z}_p$.

Theorem 1. *Let K be a finite disk in K_0. Then there exists positive constants C_1, c_4 and a positive number $T_0 = c_4\mu(K)$ such that for any $T \ge T_0$ there exists numbers $\alpha_1, \dots, \alpha_t \in \mathcal{A}_{3,p} \cap K$ such that*

$$\begin{gathered} H(\alpha_i) \le T^{1/4} \quad (1 \le i \le t), \\ |\alpha_i - \alpha_j|_p \ge T^{-1} \quad (1 \le i < j \le t), \\ t \ge C_1 T \mu(K). \end{gathered} \tag{2}$$

Note that from Theorem 1 it follows that the set $\mathcal{A}_{3,p}$ with the function $N(\alpha) = H^4(\alpha)$ form a regular system in K_0.

Let $\delta_0 \in \mathbb{R}^+$. Denote by $\bar{\mathcal{L}}_3 = \bar{\mathcal{L}}_3(Q, \delta_0, K)$ the set of $w \in K$ for which the system of the inequalities

$$|P(w)|_p < Q^{-4}, \quad |P'(w)|_p < \delta_0 \tag{3}$$

has a solution in polynomials $P \in \mathcal{P}_3(Q)$. The proof of Theorem 1 is based on the following metric result.

Theorem 2. *For any real number* s*, where* $0 < s < 1$*, there exists a constant* δ_0*, which satisfies the following property. For any disk* K *in* K_0 *there exists a sufficiently large number* $Q_0 = Q_0(K)$ *such that for*

$$\mu(K) > c_5 Q_0^{-1}$$

and sufficiently large constant c_5*, which does not depend on* Q_0*, and for all* $Q > Q_0$

$$\mu(\bar{\mathcal{L}}_3) < s\mu(K) \tag{4}$$

holds.

2. Proof of Theorem 2. Let α_1, α_2 and α_3 be the roots of the polynomial $P \in \mathcal{P}_3(Q)$ in $\mathbb{Q}_p^*$. Define the sets

$$S(\alpha_i) = \{w \in \mathbb{Q}_p : \ |w - \alpha_i|_p = \min_{1 \le j \le 3} |w - \alpha_j|_p\}, \quad i = 1, 2, 3.$$

Lemma 1. *Let* $w \in S(\alpha_i)$*, then*

$$|w - \alpha_i|_p \le |P(w)|_p |P'(w)|_p^{-1}, \quad |w - \alpha_i|_p \le |P(w)|_p |P'(\alpha_i)|_p^{-1}. \tag{5}$$

Lemma 1 is proven in [3]. Further assume that $i = 1$.

Define the subset $\tilde{\text{L}}_3$ of the set $\bar{\mathcal{L}}_3$ containing $w \in K$ for which there exists polynomial $P \in \mathcal{P}_3(Q)$ such that the system

$$|P(w)|_p < Q^{-4}, \quad pQ^{-2} < |P'(w)|_p < \delta_0 \tag{6}$$

holds.

Denote by $\sigma_0(P)$ the set of solutions w of the system (6) for a fixed polynomial $P \in \mathcal{P}_3(Q)$. Then we have

$$\tilde{\text{L}}_3 = \bigcup_{P \in \mathcal{P}_3(Q)} \sigma_0(P).$$

Let $P \in \mathcal{P}_3(Q)$ and $w \in \sigma_0(P) \cap S(\alpha_1)$ where $P(\alpha_1) = 0$. By the Taylor's formula

$$P'(w) = P'(\alpha_1) + P''(\alpha_1)(w - \alpha_1) + P''(\alpha_1)(w - \alpha_1)^2/2.$$

Using $|w - \alpha_1|_p < Q^{-4}|P'(w)|_p^{-1}$ from Lemma 1 and estimating each term gives

$$|P'(\alpha_1)|_p = |P'(w)|_p.$$

Therefore, the set $\sigma_0(P) \cap S(\alpha_1)$ is contained in $\sigma(P)$ which is defined by

$$|w - \alpha_1|_p < Q^{-4}|P'(\alpha_1)|_p^{-1}. \tag{7}$$

Further to obtain the measure of $\tilde{\text{L}}_3$ it is necessary to consider 5 cases depending on the value of $|P'(\alpha_1)|_p$ in the range (pQ^{-2}, δ_0).

Let $0 < v < 1$. Define the subset $\mathcal{L}_{31}$ of the set $\tilde{\mathcal{L}}_3$ for which there exists at least one polynomial $P \in \mathcal{P}_3(Q)$ satisfying (6) and the inequality

$$Q^{-v} < |P'(\alpha_1)|_p < \delta_0, \tag{8}$$

where α_1 is the closest root to w of P.

Proposition 1. *For sufficiently small* δ_0 *and sufficiently large* Q *we have*

$$\mu(\mathcal{L}_{31}) < 2^{-4}s\mu(K).$$

Proof. For a polynomial $P \in \mathcal{P}_3(Q)$ define the cylinder

$$\sigma_1(P) := \{w \in S(\alpha_1) \cap K : \ |w - \alpha_1|_p < c_6 Q^{-2} |P'(\alpha_1)|_p^{-1}\}. \tag{9}$$

From (7) and (9) we get

$$\mu(\sigma(P)) < c_6^{-1} Q^{-2} \mu(\sigma_1(P)). \tag{10}$$

Note that from (8) it follows that $\mu(\sigma_1(P)) < c_6 Q^{-2+v}$ and $\mu(\sigma_1(P)) < \mu(K)$ for $Q > Q_0$ and $v < 1$.

Decompose the polynomial P into Taylor series on the cylinder $\sigma_1(P)$ so that

$$P(w) = P'(\alpha_1)(w - \alpha_1) + 1/2P''(\alpha_1)(w - \alpha_1)^2 + 1/6P'''(\alpha_1)(w - \alpha_1)^3.$$

Using (8) and (9), estimate each term of the decomposition to obtain

$$|P(w)|_p < c_6 Q^{-2}, \quad w \in \sigma_1(P), \tag{11}$$

for $Q > Q_0$ and $v < 1$.

Fix the vector $b_1 = (a_3, a_2)$ which consists of the coefficients of the polynomial

$$P(x) = \sum_{i=0}^{3} a_i x^i \in \mathcal{P}_3(Q).$$

Let the subclass of polynomials $P \in \mathcal{P}_3(Q)$ with the same vector b_1 be denoted by $\mathcal{P}_3(Q, b_1)$. The cylinders $\sigma_1(P)$ divide into two classes using Sprindzuk's method of essential and inessential domains [3]. The cylinders $\sigma_1(P)$ is called *inessential* if there is a polynomial $\bar{P} \in \mathcal{P}_3(Q, b_1)$ (with $P \neq \bar{P}$), such that

$$\mu(\sigma_1(P) \cap \sigma_1(\bar{P})) \geq 1/2\mu(\sigma_1(P)), \tag{12}$$

and *essential* otherwise.

First, the essential cylinders $\sigma_1(P)$ are investigated. By definition

$$\sum_{P \in \mathcal{P}_3(Q, b_1)} \mu(\sigma_1(P)) \leq \mu(K).$$

Using the last estimate, (10) and the fact that the number of different vectors b_1 does not exceed $(2Q+1)^2$, it follows that

$$\sum_{b_1} \sum_{P \in \mathcal{P}_3(Q, b_1)} \mu(\sigma(P)) < 2^3 c_6^{-1} Q^2 Q^{-2} \mu(K) = 2^3 c_6^{-1} \mu(K). \tag{13}$$

Second, we consider the inessential cylinders $\sigma_1(P)$. Let

$$\sigma_1(P, \bar{P}) = \sigma_1(P) \cap \sigma_1(\bar{P}),$$

where $P, \bar{P} \in \mathcal{P}_3(Q, b_1)$ and $P \neq \bar{P}$. Then on the set $\sigma_1(P, \bar{P})$ with the measure at least $1/2\mu(\sigma_1(P))$ for the polynomials P and $\bar{P}$ the inequality (11) holds. Now consider the new polynomial $R(w) = P(w) - \bar{P}(w)$ which is a linear polynomial since the polynomials P and $\bar{P}$ have the same coefficients a_3 and a_2. Thus,

$$|R(w)|_p = |aw - b|_p < c_6 Q^{-2}, \quad \max(|a|, |b|) < 2Q, \quad w \in \sigma_1(P, \bar{P}). \tag{14}$$

Assume that $a > 0$. Develop P' as a Taylor series in the neighborhood $\sigma_1(P)$ of α_1 to obtain

$$|P'(w)|_p < \delta_0 \tag{15}$$

for $v < 2$ and $Q > Q_0$. Therefore, $|a|_p = |P'(w) - \bar{P}'(w)|_p < \delta_0$ and using (14) we obtain the estimate $|b|_p < \delta_0$. Let $a = p^\beta a_1$ and $b = p^\beta b_1$, where $(a_1, p) = 1$, $p^{-\beta} < \delta_0$, $b_1 \in \mathbb{Z}$. Thus, we can rewrite (14) in the form

$$|a_1 w - b_1|_p < p^\beta c_6 Q^{-2}, \quad |a_1| < p^{-\beta} Q. \tag{16}$$

Now the measure of $w \in K$ for which the system (16) holds is estimated. For fixed a_1 and b_1 the first inequality in (16) holds for points $w \in K$ from the cylinder

$$|w - b_1/a_1|_p < p^\beta |a_1|_p^{-1} c_6 Q^{-2} = p^\beta c_6 Q^{-2}. \tag{17}$$

Then we need to sum the last estimate over all a_1 and b_1 such that $b_1/a_1 \in K$, where $|a_1| < p^{-\beta} Q$. For fixed a_1 denote by $M_K(a_1)$ the number of such points b_1. For $M_K(a_1)$ the following formula holds:

$$M_K(a_1) \le \begin{cases} a_1 \mu(K) + 1 \le 2a_1 \mu(K) & \text{if} \quad a_1 \ge \mu(K)^{-1}, \\ 1 & \text{if} \quad a_1 < \mu(K)^{-1}. \end{cases} \tag{18}$$

Let $a_1 \ge \mu(K)^{-1}$ and we use the first estimate in (18). Using $p^{-\beta} < \delta_0$, we obtain

$$\sum_{1 \le a_1 \le p^{-\beta} Q} \ \sum_{b_1 :\ b_1/a_1 \in K} p^\beta c_6 Q^{-2} < 2p^{-\beta} c_6 \mu(K) < 2\delta_0 c_6 \mu(K). \tag{19}$$

Let $a_1 < \mu(K)^{-1}$ and we use the second estimate in (18). Summing over a_1 and b_1 we get

$$\sum_{1 \le a_1 \le p^{-\beta} Q} \ \sum_{b_1 :\ b_1/a_1 \in K} p^\beta c_6 Q^{-2} < c_6 Q^{-1} < 2^{-5} s \mu(K) \tag{20}$$

for $c_5 \ge 2^5 s^{-1} c_6$.

Therefore, for the measure of the set $\mathcal{L}_{31}(Q)$ the bounds, obtained for both essential and inessential cylinders from (13), (19) and (20), can be rewritten as which we rewrite in the form

$$\mu(\mathcal{L}_{31}) \le (2^3 c_6^{-1} + 2c_6 \delta_0 + 2^{-5} s) \mu(K). \tag{21}$$

Choosing $c_6 = 2^9 s^{-1}$ and $\delta_0 = 2^{-16} s^2$. The bound (21) has the form $\mu(\mathcal{L}_{31}(Q)) < 2^{-4} s \mu(K)$. This completes the proof of Proposition 1. □

For some $c_7 > 0$ define the subset $\mathcal{L}_{32}$ of the set $\tilde{\mathcal{L}}_3$, containing the $w \in K$, for which there exists at least one polynomial $P \in \mathcal{P}_3(Q)$ satisfying (6) and the inequality

$$c_7 Q^{-1} < |P'(\alpha_1)|_p \le Q^{-v},$$

where α_1 is the closest root to w of P.

Proposition 2. *For $c_7 = 2^4 s^{-1/2}$ and sufficiently large Q we have $\mu(\mathcal{L}_{32}) < 2^{-4} s \mu(K)$.*

Proof. The proof of the Proposition 2 is closely related to the proof of Proposition 1. As before, for $P \in \mathcal{P}_3(Q)$ and some positive constant c_8 (which will be specified later) we consider the cylinder $\sigma(P)$ and define the cylinder

$$\sigma_2(P) := \{w \in S(\alpha_1) \cap K :\ |w - \alpha_1|_p < c_8 Q^{-2} |P'(\alpha_1)|_p^{-1}\}. \tag{22}$$

It is clear that

$$\mu(\sigma(P)) < c_8^{-1} Q^{-2} \mu(\sigma_2(P)). \tag{23}$$

The definition of $\mathcal{L}_{32}$ gives us that $\mu(\sigma_2(P)) < \mu(K)$ for $c_5 \geq c_8 c_7^{-1}$. Develop P and P' as a Taylor series on $\sigma_2(P)$ to obtain

$$|P(w)|_p < c_8 Q^{-2}, \tag{24}$$

$$|P'(w)|_p = |P'(\alpha_1)|_p, \tag{25}$$

for $c_8 \leq c_7^2$. Further consider the essential and inessential cylinders $\sigma_2(P)$. In the case of the essential cylinders we have

$$\sum_{P \in \mathcal{P}_3(Q, b_1)} \mu(\sigma_2(P)) \leq \mu(K),$$

$$\sum_{b_1} \sum_{P \in \mathcal{P}_3(Q, b_1)} \mu(\sigma(P)) < 2^3 c_8^{-1} \mu(K). \tag{26}$$

In the case of the inessential cylinders for the polynomial $T(w) = P(w) - \bar{P}(w) = kw - d$, $P, \bar{P} \in \mathcal{P}_3(Q)$, $P \neq \bar{P}$, on the intersection $\sigma_2(P, \bar{P}) = \sigma_2(P) \cap \sigma_2(\bar{P})$ the inequality

$$|kw - d|_p < c_8 Q^{-2} \tag{27}$$

holds. Therefore,

$$|k|_p = |P'(w) - \bar{P}'(w)|_p < Q^{-v}$$

and using (14) we obtain the estimate $|d|_p < Q^{-v}$. Let $k = p^\eta k_1$ and $d = p^\eta d_1$, where $(k_1, p) = 1$, $p^{-\eta} < Q^{-v}$, $d_1 \in \mathbb{Z}$. Thus, we can rewrite (27) in the form

$$|w - d_1/k_1|_p < p^\eta c_8 Q^{-2}, \quad |k_1| < p^{-\eta} Q. \tag{28}$$

Now we estimate the measure of $w \in K$ for which the system (28) holds. For fixed k_1 and d_1 the first inequality in (28) holds for the points $w \in K$ from the cylinder

$$|w - d_1/k_1|_p < p^\eta c_8 Q^{-2}. \tag{29}$$

Then it is necessary to sum the last estimate over k_1 and d_1 such that d_1/k_1 belongs to K and $|k_1| < p^{-\eta} Q$. For fixed k_1 denote by $M_K(k_1)$ the number of such points d_1. For $M_K(k_1)$ the following formula holds:

$$M_K(k_1) \leq \begin{cases} k_1 \mu(K) + 1 \leq 2 k_1 \mu(K) & \text{if} \quad k_1 \geq \mu(K)^{-1}, \\ 1 & \text{if} \quad k_1 < \mu(K)^{-1}. \end{cases} \tag{30}$$

Let $k_1 \geq \mu(K)^{-1}$ and we use the first estimate in (30). Using $p^{-\eta} < Q^{-v}$ and summing over k_1 and d_1 we get

$$\sum_{1 \leq k_1 \leq p^{-\eta} Q} \; \sum_{d_1 \colon \, d_1/k_1 \in K} p^\eta c_8 Q^{-2} < 2 c_8 Q^{-v} \mu(K) < 2^{-6} s \mu(K) \tag{31}$$

for sufficiently large Q.

Let $k_1 < \mu(K)^{-1}$ and we use the second estimate in (30). Summing over k_1 and d_1 we get

$$\sum_{1 \leq k_1 \leq p^{-\eta} Q} \; \sum_{d_1 \colon \, d_1/k_1 \in K} p^\eta c_8 Q^{-2} < c_8 Q^{-1} < 2^{-6} s \mu(K) \tag{32}$$

for $c_5 \geq 2^6 s^{-1} c_8$. From this and (26), (31) it follows that

$$\mu(\mathcal{L}_{32}) < (2^3 c_8^{-1} + 2^{-6} s + 2^{-6} s)\mu(K). \tag{33}$$

Choose $c_8 = 2^8 s^{-1}$ and $c_7 = 2^4 s^{-1/2}$. This completes the proof of Proposition 2. □

Denote by $\mathcal{L}_{33} \subset \tilde{\mathcal{L}}_3$ the set of $w \in K$, for which there exists at least one polynomial $P \in \mathcal{P}_3(Q)$ satisfying (6) and the inequality

$$2^{-3} Q^{-1} < |P'(\alpha_1)|_p \leq 2^4 s^{-1/2} Q^{-1},$$

where α_1 is the closest root to w of P.

Proposition 3. *For sufficiently large Q we have $\mu(\mathcal{L}_{33}) < 2^{-4} s\mu(K)$.*

Proof. For $P \in \mathcal{P}_3(Q, b_1)$ and some $c_9 > 0$ define the cylinder

$$\sigma_3(P) := \{w \in S(\alpha_1) \cap K : \ |w - \alpha_1|_p < c_{16} Q^{-2} |P'(\alpha_1)|_p^{-1}\}.$$

The definition of $\mathcal{L}_{33}$ gives us that $\mu(\sigma_3(P)) < \mu(K)$ for $c_5 \geq 2^3 c_9$. Develop P and P' as a Taylor series on $\sigma_3(P)$ to obtain

$$|P(w)|_p \leq c_{10} Q^{-2}, \quad |P'(w)|_p \leq c_{11} Q^{-1}$$

for $c_{10} = \max(c_9, 2^6 c_9^2)$ and $c_{11} = \max(2^5, 2^3 c_9)$. Then consider the essential and inessential cylinders $\sigma_3(P)$. In the case of the essential cylinders we obtain that the measure does not exceed $2^3 c_9^{-1} \mu(K)$ and for $c_9 \geq 2^8 s^{-1}$ the measure does not exceed $2^{-5} s\mu(K)$. In the case of the inessential cylinders we need to find the measure of $w \in K$ which satisfies the conditions

$$|aw - b|_p < c_{10} Q^{-2}, \quad |a|_p, |b|_p < c_{11} Q^{-1}. \tag{34}$$

Let $a = p^\lambda a_2$ and $b = p^\lambda b_2$, where $(a_2, p) = 1$, $p^{-\lambda} < c_{11} Q^{-1}$, $b_2 \in \mathbb{Z}$. Now we estimate the measure of $w \in K$ for which the inequality $|w - b_2/a_2|_p < c_{10} p^\lambda Q^{-2}$ holds. Similarly as in (18) we consider two cases. Let $a_2 \geq \mu(K)^{-1}$. Using $p^{-\lambda} < c_{11} Q^{-1}$ and summing over a_2 and b_2 such that $b_2/a_2 \in K$ we get

$$\sum_{1 \leq a_2 \leq p^{-\lambda} Q} \ \sum_{b_2 :\ b_2/a_2 \in K} c_{10} p^\lambda Q^{-2} < 2 c_{10} p^{-\lambda} \mu(K) < 2 c_{10} c_{11} Q^{-1} \mu(K) < 2^{-6} s\mu(K)$$

for sufficiently large Q.

Let $a_2 < \mu(K)^{-1}$. Then summing over a_2 and b_2 such that $b_2/a_2 \in K$ we get

$$\sum_{1 \leq a_2 \leq p^{-\lambda} Q} \ \sum_{b_2 :\ b_2/a_2 \in K} c_{10} p^\lambda Q^{-2} < c_{10} Q^{-1} < 2^{-6} s\mu(K)$$

for $c_5 \geq 2^6 s^{-1} c_{10}$. Thus, the measure in the case of inessential domains is at most $2^{-5} s\mu(K)$. □

For some constant $c_{12} > 0$ denote by $\mathcal{L}_{34} \subset \tilde{\mathcal{L}}_3$ the set of $w \in K$, for which there exists at least one polynomial $P \in \mathcal{P}_3(Q)$ satisfying (6) and the inequality

$$c_{12} Q^{-3/2} < |P'(\alpha_1)|_p \leq 2^{-3} Q^{-1},$$

where α_1 is the closest root to w of P.

Proposition 4. *For $c_{12} = 2^4 s^{-1/2}$ and sufficiently large Q we have $\mu(\mathcal{L}_{34}) < 2^{-4} s\mu(K)$.*

Proof. For $P \in \mathcal{P}_3(Q)$ and some $c_{13} > 1$ define the cylinder

$$\sigma_4(P) := \{w \in S(\alpha_1) \cap K : \ |w - \alpha_1|_p < c_{13} Q^{-3} |P'(\alpha_1)|_p^{-1}\}.$$

Clearly, that

$$\mu(\sigma(P)) < c_{13}^{-1} Q^{-1} \mu(\sigma_4(P)). \tag{35}$$

Fix $b_2 = (a_3)$. Let the subclass of polynomials $P \in \mathcal{P}_3(Q)$ with the same vector b_2 be denoted by $\mathcal{P}_3(Q, b_2)$. Consider again essential and inessential domains $\sigma_4(P)$, $P \in \mathcal{P}_3(Q, b_2)$.

By the definition of the essential domains, it follows that

$$\sum_{P \in \mathcal{P}_3(Q, b_2)} \mu(\sigma_4(P)) \leq \mu(K).$$

Since the number of b_2 does not exceed $(2Q+1)$ then, summing over all b_2 and using (35), gives

$$\sum_{b_2} \sum_{P \in \mathcal{P}_3(Q, b_2)} \mu(\sigma(P)) < 2^2 c_{13}^{-1} \mu(K) \leq 2^{-5} s \mu(K)$$

for $c_{13} \geq 2^7 s^{-1}$.

Now consider the inessential domains. By the Taylor expansion of $P_i(w)$ and $P_i'(w)$ on

$$\sigma_4(P_{i_1}, P_{i_2}) = \sigma_4(P_{i_1}) \cap \sigma_4(P_{i_2}), \quad P_{i_1}, P_{i_2} \in \mathcal{P}_3(Q, b_2), \quad P_{i_1} \neq P_{i_2},$$

find the upper bound of $|P_i(w)|_p$ and $|P_i'(w)|_p$, so that

$$|P_i(w)|_p < c_{13} Q^{-3}, \quad |P_i'(w)|_p = |P'(\alpha_1)|_p \quad \text{for} \quad c_{12} \geq c_{13}^{1/2}. \tag{36}$$

Since the leading coefficients of P_{i_1} and P_{i_2} are equal then

$$W(w) = P_{i_1}(w) - P_{i_2}(w) = f_2 w^2 + f_1 w + f_0$$

and, by (36),

$$|W(w)|_p < c_{13} Q^{-3}, \quad |W'(w)|_p < |P'(\alpha_1)|_p, \quad |f_i| \leq 2Q, \quad 0 \leq i \leq 2. \tag{37}$$

Then we need to consider the discriminant $D(W)$ of W and distinguish two cases: $D(W) \neq 0$ and $D(W) = 0$. In general, the representation of $D(P)$ for $P \in \mathcal{P}_n(Q)$ as a determinant leads to the upper bound

$$|D(P)| \leq 2n^{2n-1}(2n-2)! Q^{2n-2}.$$

Case 1: $D(W) \neq 0$. Let $\beta_1, \beta_2 \in \mathbb{Q}_p^*$ denote the roots of $W(w)$. Since the discriminant $D(W)$ of W satisfies

$$|D(W)|_p = |W'(\beta_1)|_p^2 < |P'(\alpha_1)|_p^2 \leq 2^{-6} Q^{-2},$$

$$|D(W)|_p \geq |D(W)|^{-1} \geq 2^{-5} Q^{-2}$$

then we have a contradiction.

Case 2: $D(W) = 0$. This implies that the polynomial W has a multiple root and has a form

$$W(w) = W_1^2(w) = (l_1 w - l_0)^2, \quad |l_1| < 2Q^{1/2}.$$

By (37) we have

$$|l_1 w - l_0|_p < c_{13}^{1/2} Q^{-3/2}. \tag{38}$$

Let $l_1 = p^\gamma l_{11}$ and $l_0 = p^\gamma l_{00}$, where $(l_{11}, p) = 1$, $l_{00} \in \mathbb{Z}$. Thus, we can rewrite (38) in the form

$$|w - l_{00}/l_{11}|_p < p^\gamma c_{13}^{1/2} Q^{-3/2}. \tag{39}$$

Then we need to sum the last estimate over all l_{11} and l_{00} such that $l_{00}/l_{11} \in K$ and $|l_{11}| < < 2p^{-\gamma}Q^{1/2}$. For a fixed l_{11} the number of such points l_{00} is equal to $M_K(l_{11})$. Summing over l_{11} and l_{00} we get

$$\sum_{1 \le l_{11} \le 2p^{-\gamma}Q^{1/2}} \sum_{l_{00}:\, l_{00}/l_{11} \in K} p^{\gamma} c_{13}^{1/2} Q^{-3/2} = \sum_{1 \le l_{11} \le 2p^{-\gamma}Q^{1/2}} p^{\gamma} c_{13}^{1/2} Q^{-3/2} M_K(l_{11}) < 2^{-6} s\mu(K) \quad (40)$$

for $Q > Q_0$ in the case when $l_{11} \ge \mu(K)^{-1}$ and for $c_5 \ge 2^7 s^{-1} c_{13}^{1/2}$ in the case when $l_{11} < \mu(K)^{-1}$. Choose $c_{12} = 2^4 s^{-1/2}$ and sum the estimates for the measure of the essential and inessential cases this concludes the proof of Proposition 4. □

Denote by $\mathcal{L}_{35} \subset \tilde{\mathcal{L}}_3$ the set of $w \in K$, for which there exists at least one polynomial $P \in \mathcal{P}_3(Q)$ satisfying (6) and the inequality

$$pQ^{-2} < |P'(\alpha_1)|_p \le 2^4 s^{-1/2} Q^{-3/2}, \quad (41)$$

where α_1 is the closest root to w of P.

Proposition 5. *For sufficiently large Q we have $\mu(\mathcal{L}_{35}) < 2^{-2} s\mu(K)$.*

Proof. Let us divide the cylinder K into smaller cylinders J_i where $\mu(J_i) = Q^{-u}$ and $u > 1$. We say the polynomial P belongs to the cylinder J_i if there exists $w \in J_i$ such that (3) and (41) hold. If there is at least one polynomial $P \in \mathcal{P}_3(Q)$ that belongs to every J_i then by Lemma 1 the measure of those w, satisfying (3) and (41), does not exceed

$$p^{-1} Q^{-2+u} \mu(K) < 2^{-4} s\mu(K)$$

for $u < 2$ and $Q > Q_0$. This is impossible when at least two irreducible polynomials belong to cylinders J_i. To see this, suppose the opposite. Assume there exists a point $w \in J_i$, for which (3) and (41) hold for two polynomials P_1 and P_2 from $\mathcal{P}_3(Q)$. By the Taylor expansion of P_1 on J_i we can estimate $|P_1(w)|_p$ from above

$$|P_1(w)|_p < 2^4 s^{-1/2} Q^{-3/2-u}$$

for $u > 3/2$. Obviously, the same estimate holds for P_2 on J_i.

Lemma 2 [4]. *Let $\delta > 0$ and $Q > Q_0(\delta)$. Further, let P_1 and P_2 be two integer polynomials of degree at most n with no common roots and $\max(H(P_1), H(P_2)) \le Q$. Let $J \subset \mathbb{R}$ be an cylinder with $\mu(J) = Q^{-\eta}$, $\eta > 0$. If there exists $\tau > 0$ such that for all $w \in J$*

$$|P_j(w)|_p < Q^{-\tau},$$

for $j = 1, 2$, then

$$\tau + 2\max(\tau - \eta, 0) < 2n + \delta. \quad (42)$$

Applying Lemma 2 for $\tau = 3/2 + u - \varepsilon$, $\varepsilon > 0$, and $\eta = u$ we get a contradiction in (42) for $u > 3/4 + \delta/2 + 3\varepsilon/2$. Choose u satisfying $1 < u < 2$.

In the case if $P \in \mathcal{P}_3(Q)$ is a reducible polynomial then $P(x) = P_1(x)P_2(x)$, where P_1 is a first degree polynomial and P_2 is a second degree polynomial or the product of two linear polynomials. First, we need to prove the following auxiliary lemmas.

Lemma 3. *Let $0 < t < 1$. Denote by $L = L(Q, K)$ the set of $w \in K$ for which the system of the inequalities*

$$|aw + b|_p < c_1 Q^{-d}, \quad \max(|a|, |b|) < c_2 Q^r$$

hold, where the parameteres $d, r \ge 0$ and constants $c_i > 0$ satisfy one of the conditions

i) $d > 1 + r$, $0 \le r \le 1$,

ii) $d = 1 + r,\ \ 0 \le r < 1,\ \ c_5 > 2c_1c_2t^{-1}$,

iii) $d = 1 + r,\ \ r = 1,\ \ c_5 > 2c_1c_2t^{-1},\ \ c_1c_2^2 < 2^{-2}t$.

Then $\mu(L) < 2t\mu(K)$. *(Here the conditions for the constant* c_5 *gives some restrictions to Theorem 2.)*

Proof. Let $a = p^k a_1,\ (a_1, p) = 1$. Then $|w + b_1/a_1|_p < p^k c_1 Q^{-d}$ and $|a_1| < 2p^{-k}c_2Q^r$. Similar as in Proposition 1 we consider two cases.

Case 1: $\#b_1 \le 2\#a_1\mu(K)$. We obtain

$$\sum_{a_1 < 2p^{-k}c_2Q^r} \ \sum_{b_1:\ b_1/a_1 \in K} p^k c_1 Q^{-d} < p^k c_1 Q^{-d}(2p^{-k}c_2Q^r)^2\mu(K) =$$

$$= 2^2 c_1 c_2^2 p^{-k} Q^{-d+2r}\mu(K) < t\mu(K)$$

for $-d + 2r < 0$ and $Q > Q_0$ or for $-d + 2r = 0$ and $2^2c_1c_2^2 < t$.

Case 2: $\#b_1 \le 1$. We get

$$\sum_{a_1 < 2p^{-k}c_2Q^r} \ \sum_{b_1:\ b_1/a_1 \in K} p^k c_1 Q^{-d} < p^k c_1 Q^{-d}(2p^{-k}c_2Q^r) < 2c_1c_2c_5^{-1}Q^{1-d+r}\mu(K) < t\mu(K)$$

for $1 - d + r < 0$ and $Q > Q_0$ or for $1 - d + r = 0$ and $2c_1c_2c_5^{-1} < t$. $\square$

Lemma 4. *Let* $0 < t < 1/2$. *Denote by* $M = M(Q, K)$ *the set of* $w \in K$ *for which the system of the inequalities*

$$|t_2(w)|_p = |b_2w^2 + b_1w + b_0|_p < c_1Q^{-d}, \quad \max_{0 \le i \le 2} |b_i| < c_2Q^r$$

hold, where the parameteres $d, r \ge 0$ *and constants* $c_i > 0$ *satisfy one of the conditions*

i) $d > 2 + r,\ \ 0 \le r \le 1$,

ii) $d = 2 + r,\ \ 0 \le r < 1,\ \ c_5 > \max\{2c_1^{1/2}c_2^{1/2}t^{-1}, 4c_2t^{-1}\}$,

iii) $d = 2 + r,\ \ r = 1,\ \ c_1 < 2^{-2}c_2^{-2}t\min\{1, 2^{-4}c_2^{-1}t\},\ \ c_5 > \max\{2c_1^{1/2}c_2^{1/2}t^{-1}, 4c_4c_2t^{-1}\}$,

where c_4 *satisfies (45). Then* $\mu(M) < 7t\mu(K)$. *(Here the conditions for the constant* c_5 *gives some restrictions to Theorem 2.)*

Proof. Let β_1 and β_2 be the roots of t_2 in $\mathbb{Q}_p^*$. Further we consider two cases.

Case 1: $|t_2'(w)|_p > c_1^{1/2}Q^{-d/2}$. It is easy to show that $|t_2'(w)|_p = |t_2'(\beta_1)|_p$ for w from the cylinder $|w - \beta_1|_p < c_1^{1/2}Q^{-d/2}$. Consider two subcases.

Subcase 1: $c_3Q^{-k} < |t_2'(\beta_1)|_p \le 1$.

For $t_2 \in \mathcal{P}_2(c_2Q^r)$ define the cylinders

$$\sigma(t_2) := \{w \in S(\beta_1) \cap K :\ |w - \beta_1|_p < |t_2(w)|_p|t_2'(\beta_1)|_p^{-1} < c_1Q^{-d}|t_2'(\beta_1)|_p^{-1}\}$$

and

$$\sigma_1(t_2) := \{w \in S(\beta_1) \cap K :\ |w - \beta_1|_p < c_4Q^{-u}|t_2'(\beta_1)|_p^{-1}\}.$$

Thus, $\sigma(t_2) \subseteq \sigma_1(t_2)$ for $u < d$ and $Q > Q_0$ or for $u = d$ and $c_4 \ge c_1$. From the definitions of $\sigma(t_2)$ and $\sigma_1(t_2)$ it follows that

$$\mu(\sigma(t_2)) < c_1c_4^{-1}Q^{u-d}\mu(\sigma_1(t_2)).$$

Fix the vector $\mathbf{b} = (b_2)$. Let the subclass of polynomials $t_2 \in \mathcal{P}_2(c_2Q^r)$ with the same vector $\mathbf{b}$ be denoted by $\mathcal{P}_{\mathbf{b}}$.

Further consider the essential and inessential cylinders $\sigma_1(t_2)$. In the case of essential cylinders $\sigma_1(t_2)$ we have

$$\sum_{\mathbf{b}} \sum_{t_2 \in \mathcal{P}_{\mathbf{b}}} \mu(\sigma(t_2)) < c_1 c_4^{-1} Q^{u-d}(2c_2 Q^r + 1)\mu(K) < 4c_1 c_2 c_4^{-1} Q^{u-d+r} \mu(K) < t\mu(K)$$

for $u - d + r < 0$ and $Q > Q_0$ or for $u - d + r = 0$ and $4c_1 c_2 c_4^{-1} < t$.

Now consider the inessential cylinders $\sigma_1(t_2)$. On $\sigma_1(t_2)$ we have

$$|t_2(w)|_p \le \max(c_4 Q^{-u}, c_4^2 Q^{-2u} c_3^{-2} Q^{2k}) = c_4 Q^{-u}$$

for $u > 2k$ and $Q > Q_0$ or for $u = 2k$ and $c_4 \le c_3^2$.

Let $R(w) = t_{2,1}(w) - t_{2,2}(w) = aw + b$, where $t_{2,1}, t_{2,2} \in \mathcal{P}_2(c_2 Q^r)$ and $t_{2,1} \ne t_{2,2}$. Then $|aw+b|_p < c_4 Q^{-u}$ and $\max(|a|,|b|) < 2c_2 Q^r$ on $\sigma_1(t_{2,1}, t_{2,2}) = \sigma_1(t_{2,1}) \cap \sigma_1(t_{2,2})$. Denote by L_1 the set of $w \in K$ for which the system of the inequalities $|aw+b|_p < c_4 Q^{-u}$ and $\max(|a|,|b|) < 2c_2 Q^r$ hold. Applying Lemma 3 to the set L_1, we obtain that $\mu(L_1) < 2t\mu(K)$ in each of the cases:

$$u > 1 + r, \quad 0 \le r \le 1,$$

$$u = 1 + r, \quad 0 \le r < 1, \quad c_5 > 4c_4 c_2 t^{-1},$$

$$u = 1 + r, \quad r = 1, \quad c_5 > 4c_4 c_2 t^{-1}, \quad c_4 c_2^2 < 2^{-4} t.$$

Subcase 2: $c_1^{1/2} Q^{-d/2} < |t_2'(\beta_1)|_p \le c_3 Q^{-k}$.

Let $D(t_2) \ne 0$. Since

$$|D(t_2)|_p = |b_2(\beta_1 - \beta_2)|_p^2 = |t_2'(\beta_1)|_p^2 \le c_3^2 Q^{-2k}$$

and

$$|D(t_2)|_p \ge |D(t_2)|^{-1} \ge 2^{-5}(c_2 Q^r)^{-2}$$

then we have a contradiction when $r < k$ and $Q > Q_0$ or for $r = k$ and $c_2 c_3 < 2^{-5/2}$. Then $D(t_2) = 0$ and $t_2(w) = (aw + b)^2$. Thus,

$$|aw + b|_p < c_1^{1/2} Q^{-d/2}, \quad \max(|a|,|b|) < c_2^{1/2} Q^{r/2}. \tag{43}$$

Denote by L_2 the set of $w \in K$ for which the system (43) holds. Applying Lemma 3, we obtain that $\mu(L_2) < 2t\mu(K)$ in each of the cases:

$$d/2 > 1 + r/2, \quad 0 \le r \le 1,$$

$$d/2 = 1 + r/2, \quad 0 \le r < 1, \quad c_5 > 2c_1^{1/2} c_2^{1/2} t^{-1}, \tag{44}$$

$$d/2 = 1 + r/2, \quad r = 1, \quad c_5 > 2c_1^{1/2} c_2^{1/2} t^{-1}, \quad c_1^{1/2} c_2 < 2^{-2} t.$$

Case 2: $|t_2'(w)|_p \le c_1^{1/2} Q^{-d/2}$. It is easy to show that $|t_2'(\beta_1)|_p \le c_1^{1/2} Q^{-d/2}$. Using the same argument as in subcase 2, we assume that $D(t_2) \ne 0$. Then we obtain two estimates $|D(t_2)|_p \ge c_1 Q^{-d}$ and $|D(t_2)| \ge 2^{-5} c_2^{-2} Q^{-2r}$ which contradicts to each other for $d > 2r$ and $Q > Q_0$. Therefore, we need to consider only the case when $D(t_2) = 0$. Then the proof coincide with the proof for the set L_2.

Conclusions. Taking into consideration the inequalities for the parameteres and constants and combining all estimates for the measure, we obtain that $\mu(M) < 7t\mu(K)$ for each of the following cases.

Case A: $d > 2 + r$ and $0 \le r \le 1$. Choose $u = 1 + r + \varepsilon$, $k = r + \varepsilon$, $\varepsilon > 0$, and $c_3 = c_4 = 1$.

Case B: $d = 2 + r,\ \ 0 \le r < 1$ and $c_5 > \max\{2c_1^{1/2}c_2^{1/2}t^{-1}, 4c_2t^{-1}\}$. Choose $u = 1 + r$, $k = r + \varepsilon$, and $c_3 = c_4 = 1$.

Case C: $d = 2 + r,\ \ r = 1,\ \ c_1 < 2^{-2}c_2^{-2}t\min\{1, 2^{-4}c_2^{-1}t\},\ \ c_5 > \max\{2c_1^{1/2}c_2^{1/2}t^{-1}, 4c_4c_2t^{-1}\}$, where c_4 satisfies (45). Choose $u = 2,\ \ k = r = 1,\ \ c_3 = c_4^{1/2}$, where c_4 satisfies

$$4c_1c_2t^{-1} < c_4 < 2^{-4}c_2^{-2}t. \tag{45}$$

□

Then, using Gelfond's lemma [1] and applying Lemma 1 and Lemma 2 to the systems

$$|P_1(w)|_p \ll Q^{-d}, \quad H(P_1) \ll Q^r,$$

and

$$|P_2(w)|_p \ll Q^{-4+d}, \quad H(P_2) \ll Q^{1-r},$$

respectively, we obtain that the ranges of $0 \le r \le 1$ and $0 \le d \le 4$ are covered by the union of the solutions of the inequalities $d \ge 1 + r$ and $d < 1 + r$. For specific choice of the constants we can obtain that the measure of the set does not exceed $3/16s\mu(K)$ in the case of irreducible polynomials. □

Define the subset $\check{\text{L}}_3$ of the set $\bar{\mathcal{L}}_3$ containing $w \in K$ for which there exists polynomial $P \in \mathcal{P}_3(Q)$ such that

$$|P(w)|_p < Q^{-4}, \quad |P'(w)|_p \le pQ^{-2}. \tag{46}$$

Define by $\sigma_*(P)$ the set of solutions to (46) for a fixed polynomial $P \in \mathcal{P}_3(Q)$. Let

$$w \in \sigma_*(P) \cap S(\alpha_1).$$

First, it is shown that the value of the derivative of P at α_1, $P(\alpha_1) = 0$, satisfies

$$|P'(\alpha_1)|_p \le pQ^{-2}. \tag{47}$$

To show this, develop P' as a Taylor series in the neighborhood of α_1 and use the estimate $|w - \alpha_1|_p < p^{-1}Q^{-2}$ from Lemma 1. Since

$$\max(|P''(\alpha_1)|_p|w - \alpha_1|_p, |2^{-1}P'''(\alpha_1)|_p|w - \alpha_1|_p^2, |P'(w)|_p) \le pQ^{-2}$$

it follows that $|P'(\alpha_1)|_p \le pQ^{-2}$.

To estimate the measure of $\check{\text{L}}_3$ two cases depending on the value of $|P'(\alpha_1)|_p$ need to be considered.

For some constant $c_{14} > 0$ denote by $\mathcal{L}_{36} \subset \check{\mathcal{L}}_3$ the set of $w \in K$, for which there exists at least one polynomial $P \in \mathcal{P}_3(Q)$ satisfying (46) and the inequality

$$|P'(\alpha_1)|_p < c_{14}Q^{-2},$$

where α_1 is the closest root to w of P.

Proposition 6. *For sufficiently small $c_{14} < 1$ and sufficiently large Q we have*

$$\mu(\mathcal{L}_{36}) < 2^{-2}s\mu(K).$$

Proof. Let $P \in \mathcal{P}_3(Q)$. Since

$$|D(P)|_p = |a_3^4(\alpha_1 - \alpha_2)^2(\alpha_1 - \alpha_3)^2(\alpha_2 - \alpha_3)^2|_p = |P'(\alpha_1)|_p^2|a_3^2|_p|\alpha_2 - \alpha_3|_p^2,$$

$$|D(P)|_p \ge |D(P)|^{-1} \ge 2^{-4}3^{-6}Q^{-4}$$

and using the estimates $|a_3|_p \le 1$, $|\alpha_2-\alpha_3|_p < c_{15}$ and $|P'(\alpha_1)|_p < c_{14}Q^{-2}$, this gives

$$2^{-4}3^{-6}Q^{-4} < c_{14}^2c_{15}^2Q^{-4},$$

which does not hold when $c_{14} < 2^{-2}3^{-3}c_{15}^{-1}$. Thus, the discriminant of P satisfies $D(P) = 0$, which implies that P has a repeated root. Following the same approach as Proposition 4 and 5, it follows that $\mu(\mathcal{L}_{36}) < 2^{-2}s\mu(K)$. $\square$

Denote by $\mathcal{L}_{37} \subset \check{\mathcal{L}}_3$ the set of $w \in K$, for which there exists at least one polynomial $P \in \mathcal{P}_3(Q)$ satisfying (46) and the inequality

$$c_{14}Q^{-2} \le |P'(\alpha_1)|_p \le pQ^{-2}.$$

where α_1 is the closest root to w of P.

Proposition 7. *For sufficiently small $c_{14} < 1$ and Q sufficiently large we have*

$$\mu(\mathcal{L}_{37}) < 2^{-2}s\mu(K).$$

Proof. Divide the cylinder K into smaller cylinders J_i' with $\mu(J_i) = Q^{-u'}$ where $u' > 1$. If there is at least one polynomial $P \in \mathcal{P}_3(Q)$ that belongs to every J_i' then by Lemma 1 the measure of those w, that satisfy the first inequality of (3), does not exceed

$$c_{14}^{-1}Q^{-2+u'}\mu(K) < 2^{-4}s\mu(K)$$

for $u' < 2$ and sufficiently large Q.

The assumption that at least two irreducible polynomials belong to the cylinder J_i' will lead to a contradiction. To show this, suppose that P_1 and P_2 belong to J_i'. Develop P_1 as a Taylor series in the neighborhood J_i' of α_1 to obtain

$$|P(w)|_p < Q^{-2u'}, \quad w \in J_i',$$

for $u' < 2$. Obviously, the same estimate holds for P_2 on J_i'. Applying Lemma 2 for $\tau = 2u'$ and $\eta = u'$, leads to a contradiction in (42) for $u' > 3/2+\delta/4$. Choose u', satisfying $3/2+\delta/4 < u' < 2$.

When the polynomials are reducible, the proof follows Proposition 5. $\square$

Additing the measures calculated in propositions 1 to 7, it follows that the measure of $\bar{Ł}_3$ satisfies (4).

3. Proof of Theorem 1. By Dirichlet's principle, it is easy to show that for any $w \in K$ the inequality $|P(w)|_p < Q^{-4}$ has a non-zero solution $P \in \mathbb{Z}[x]$, $\deg P \le 3$. Fix such a solution P. By Theorem 2 there exists a set

$$Ł_3(Q,\delta_0,K) = K \setminus \bar{Ł}_3(Q,\delta_0,K) \subset K$$

such that

$$\mu(Ł_3(Q,\delta_0,K)) \ge (1-s)\mu(K)$$

for all $Q > Q_0$, where $Q_0 > c_5\mu(K)$. Let $w \in Ł_3(Q,\delta_0,K)$. Then by Hensel's Lemma [5] there is a root $\alpha \in \mathbb{Z}_p$ of P such that

$$|w-\alpha|_p < \delta_0^{-1}Q^{-4}. \tag{48}$$

If Q is sufficiently large then $\alpha \in K$.

Choose the maximal collection $\{\alpha_1,\dots,\alpha_t\}$ of algebraic numbers in $K \cap \mathcal{A}_{3,p}$ satisfying

$$H(\alpha_i) \le Q, \quad |\alpha_i-\alpha_j|_p \ge Q^{-4}, \quad 1 \le i < j \le t.$$

Since the collection $\{\alpha_1, \ldots, \alpha_t\}$ is maximal then there exists α_i in this collection such that $|\alpha - \alpha_i|_p \leq Q^{-4}$. From this and (48) it follows that $|w - \alpha_i|_p < \delta_0^{-1} Q^{-4}$. As w is an arbitrary point of $Ł_3(Q, \delta_0, K)$ then

$$Ł_3(Q, \delta_0, K) \subset \bigcup_{i=1}^{t} \{w \in K : \ |w - \alpha_i|_p < \delta_0^{-1} Q^{-4}\}.$$

Using $\mu(Ł_3(Q, \delta_0, K)) \geq (1 - s)\mu(K)$ this gives $t \gg Q^4 \mu(K)$. Let $T_0 = Q_0^4$ then for any $T > T_0$, where $T_0 = (c_5 + 1)^4 \mu(K)^{-4}$, there exists a collection $\alpha_1, \ldots, \alpha_t \in K \cap \mathcal{A}_{3,p}$ satisfying (2).

Литература

1. *Bugeaud Y.* Approximation by algebraic numbers // Cambridge Tracts in Mathematics. 160. Cambridge, 2004.
2. *Baker A., Schmidt W.M.* Diophantine approximation and Hausdorff dimension // Proc. London Math. Soc. 1970. V. 21. P. 1–11.
3. *Sprindžuk V.* Mahler's problem in the metric theory of numbers // Amer. Math. Soc., Providence, RI. 1969. V. 25.
4. *Bernik V.I., Kalosha N.* Approximation of zero by values of integral polynomials in space $\mathbb{R} \times \mathbb{C} \times \mathbb{Q}_p$ // Vesti NAN of Belarus. Ser. fiz-mat nauk. 2004. V. 1. P. 121–123.
5. *Bernik V.I., Dodson M.M.* Metric Diophantine approximation on manifolds. Cambridge: CUP, 1999. V. 137.

N. V. Budarina, M. V. Lamchanovskaya

On the size of p-adic cylinder for which the regular system of algebraic numbers can be constructed

Summary

On the relation between a factorization of a polynomial resultant and the frequency of its occurrence. A lower bound is obtained for the number of polynomial pairs of a given degree and bounded heights such that their resultants are divisible by a fixed prime number.