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QUASI-CLASSICAL LIMIT OF KP HIERARCHY,
W-SYMMETRIES AND FREE FERMIONS

Dispersionless versions of integrable hierarchies (KP, Toda, etc.) [1], like the ordinary hierarchies, have many applications in low dimensional integrable field theories. For example, the dispersionless KP hierarchy, as well as more general hierarchies of Whitham-type [2, 3], possesses many solutions that are related to topological conformal field theory [4, 5]. The dispersionless Toda hierarchy arises in continuous (or large- N) limit of the ordinary Toda field theory [6–8], and also related to 4D self-dual gravity [9] via dimensional reduction.

In previous work on the dispersionless KP and Toda hierarchies [10], we stressed analogy of these hierarchies with self-dual gravity. A main result, which we believe reflects the integrable nature of these hierarchies most clearly, is the existence of infinitely many symmetries with the structure of a W -infinity algebra, $w_{1+\infty}$ [11]. Most ideas and tools used therein are borrowed from a construction of similar $\mathcal{L}w_{1+\infty}$ (the loop algebra of $w_{1+\infty}$) symmetries of self-dual gravity in earlier papers [12, 13].

This approach, in its nature, tells us nothing on how the $w_{1+\infty}$ symmetries of a dispersionless hierarchy are related to $W_{1+\infty}$ symmetries of the original hierarchy, i.e., symmetries with the structure of another typical W -infinity algebra, $W_{1+\infty}$ [11]. These $W_{1+\infty}$ symmetries are known to play a key role in 2D quantum and topological gravity [14–18] as “ W -constraints.” Actually, their $w_{1+\infty}$ versions also emerge in similar models [2, 10]. The two types of W -infinity symmetries should be related more directly!

In this paper, we reorganize our previous work along with new results from the point of view of quasi-classical limit. Dispersionless limit, in fact, can also be understood as quasi-classical limit, as already well recognized in 2D gravity [17]. For illustration, we mostly deal with the dispersionless KP hierarchy, and derive its Lax formalism, tau function, W -infinity symmetries, and general solutions from their counterparts in the KP hierarchy in the limit of $\hbar \rightarrow 0$.

The dispersionless KP hierarchy (which we called SDiff(2) KP hier-

archy in the previous paper [10]) has a Lax-type representation [1-3]

$$\frac{\partial \mathcal{L}}{\partial t_n} = \{\mathcal{B}_n, \mathcal{L}\}, \quad \mathcal{B}_n \stackrel{\text{def}}{=} (\mathcal{L}^n)_{\geq 0}, \quad n = 1, 2, \dots, \quad (1)$$

where \mathcal{L} is a Laurent series in the "momentum" k (conjugate variable of $x = t_1$) of the form

$$\mathcal{L} = k + \sum_{n=1}^{\infty} u_{n+1}(t) k^{-n}, \quad (2)$$

" $(\)_{\geq 0}$ " means the projection onto a polynomial in k dropping negative powers, and " $\{ \ , \ }$ " the Poisson bracket in 2D "phase space" (k, x) ,

$$\{A(k, x), B(k, x)\} = \frac{\partial A(k, x)}{\partial k} \frac{\partial B(k, x)}{\partial x} - \frac{\partial A(k, x)}{\partial x} \frac{\partial B(k, x)}{\partial k}. \quad (3)$$

This obviously imitates the Lax representation of the KP hierarchy, replacing pseudo-differential operators (in x) and their commutators by Laurent series (in k) and Poisson brackets. We argued in the previous work [10] that this Lax representation should be extended to a larger system. The extended Lax representation consists of \mathcal{L} and another Laurent series \mathcal{M} of the form

$$\mathcal{M} = \sum_{n=1}^{\infty} n t_n \mathcal{L}^{n-1} + \sum_{n=1}^{\infty} v_{n+1}(t) \mathcal{L}^{-n-1} \quad (4)$$

that satisfies the same Lax-type equations and the canonical Poisson relation:

$$\frac{\partial \mathcal{M}}{\partial t_n} = \{\mathcal{B}_n, \mathcal{M}\}, \quad n = 1, 2, \dots, \quad (5)$$

$$\{\mathcal{L}, \mathcal{M}\} = 1. \quad (6)$$

Actually, the above three sets of equations for \mathcal{L} and \mathcal{M} can be cast into a single exterior differential equation,

$$d\mathcal{L} \wedge d\mathcal{M} = \sum_{n=1}^{\infty} d\mathcal{B}_n \wedge dt_n, \quad (7)$$

which is very similar to a 2-form equation that lies in the heart of integrability of self-dual gravity [12, 13]; our previous construction of $w_{1+\infty}$ is based on this fact. An analogue of the KP tau function, which we here write $\tau_{\text{dKP}}(t)$, is also introduced in our previous work as:

$$\frac{\partial \log \tau_{\text{dKP}}}{\partial t_n} = v_{n+1}, \quad n = 1, 2, \dots \quad (8)$$

We now introduce a "Planck constant" \hbar into the ordinary setting of the KP hierarchy [19–21]. As in the dispersionless case, it is crucial to extend the ordinary Lax formalism by adding a second Lax operator M [22, 16–18]. The extended Lax representation of the KP hierarchy, in the presence of \hbar , is given by

$$\begin{aligned} \hbar \frac{\partial L}{\partial t_n} &= [B_n, L], \quad B_n = (L^n)_{\geq 0}, \\ \hbar \frac{\partial M}{\partial t_n} &= [B_n, M], \quad n = 1, 2, \dots, \\ [L, M] &= \hbar, \end{aligned} \tag{9}$$

where L and M , counterparts of \mathcal{L} and \mathcal{M} , are pseudo-differential operators (in $x = t_1$),

$$\begin{aligned} L &= \hbar \partial + \sum_{n=1}^{\infty} u_{n+1}(\hbar, t) (\hbar \partial)^{-n}, \quad \partial = \partial / \partial x, \\ M &= \sum_{n=1}^{\infty} n t_n L^{n-1} + \sum_{n=1}^{\infty} v_{n+1}(\hbar, t) L^{-n-1}, \end{aligned} \tag{10}$$

and " $(\)_{\geq 0}$ " now stands for the projection onto a differential operator dropping negative powers of ∂ . The coefficients $u_n(\hbar, t)$ and $v_n(\hbar, t)$ are assumed to have such an asymptotic form as $u_n(\hbar, t) = u_n(t) + O(\hbar)$, $v_n(\hbar, t) = v_n(t) + O(\hbar)$ in the limit of $\hbar \rightarrow 0$. Accordingly the Baker-Akhiezer function and the tau function, too, are to depend on \hbar . The Baker-Akhiezer function $\Psi(\hbar, t, \lambda)$, where λ is the so called "spectral parameter," is by definition a function (or a formal Laurent series of λ) with the asymptotic form

$$\Psi(\hbar, t, \lambda) = (1 + O(\lambda^{-1})) \exp(\hbar^{-1} \sum_{n=1}^{\infty} t_n \lambda^n) \quad (\lambda \rightarrow \infty, \hbar \rightarrow 0) \tag{11}$$

that satisfies the linear equations

$$\lambda \Psi = L \Psi, \quad \hbar \frac{\partial \Psi}{\partial \lambda} = M \Psi, \quad \hbar \frac{\partial \Psi}{\partial t_n} = B_n \Psi. \tag{12}$$

The tau function $\tau(\hbar, t)$ is by definition a function that reproduces the Baker-Akhiezer function as

$$\begin{aligned} \Psi(\hbar, t) &= \frac{\tau(\hbar, t - \hbar \epsilon(\lambda^{-1}))}{\tau(\hbar, t)} \exp(\hbar^{-1} \sum_{n=1}^{\infty} t_n \lambda^n), \\ \epsilon(\lambda^{-1}) &= \left(\frac{1}{\lambda}, \frac{1}{2\lambda^2}, \frac{1}{3\lambda^3}, \dots \right). \end{aligned} \tag{13}$$

The dispersionless KP hierarchy now emerges in the limit of $\hbar \rightarrow 0$ as follows. Following the prescription of Kodama and Gibbons [1], we rewrite the Baker-Akhiezer function into the WKB form:

$$\Psi(\hbar, t, \lambda) = \exp[\hbar^{-1}S(t, \lambda) + O(\hbar^0)] \quad (\hbar \rightarrow 0). \quad (14)$$

The “phase function” $S(t, \lambda)$ then satisfies a set of Hamilton-Jacobi (or eikonal) equations that follow from the linear equations of the Baker-Akhiezer function. Those arising from the second and third linear equations can be cast into a single 1-form equation:

$$dS(t, \lambda) = \mathcal{M}(t, \lambda)d\lambda + \sum_{n=1}^{\infty} \mathcal{B}_n(t, \lambda)dt_n, \quad (15)$$

where

$$\begin{aligned} \mathcal{M}(t, \lambda) &= \sum_{n=1}^{\infty} nt_n \lambda^{n-1} + \sum_{n=1}^{\infty} v_{n+1}(t) \lambda^{-n-1}, \\ \mathcal{B}_n(t, \lambda) &= \left(\frac{\partial S(t, \lambda)}{\partial x} \right)^n + \sum_{i=0}^{n-2} b_{n,i}(t) \left(\frac{\partial S(t, \lambda)}{\partial x} \right)^i, \end{aligned} \quad (16)$$

and the coefficients $b_{n,i}(t)$, like $u_n(t)$ and $v_n(t)$, are the leading (\hbar^0) term in \hbar -expansion of the coefficients $b_{n,i}(\hbar, t)$ of $B_n = (\hbar\partial)^n + \sum_{i=0}^{n-2} b_{n,i}(\hbar, t)(\hbar\partial)^i$. The Hamilton-Jacobi equation for $\lambda\Psi = L\Psi$ is given by

$$\lambda = \frac{\partial S(t, \lambda)}{\partial x} + \sum_{n=1}^{\infty} u_{n+1}(t) \left(\frac{\partial S(t, \lambda)}{\partial x} \right)^{-n}. \quad (17)$$

If we make change of the independent variables $(t, \lambda) \rightarrow (t, k)$ by

$$k = \frac{\partial S(t, \lambda)}{\partial x} \quad (x = t_1), \quad (18)$$

λ becomes a function of (t, k) as $\lambda = \mathcal{L}(t, k) = k + \sum_{n=1}^{\infty} u_{n+1}(t)k^{-n}$, and we can define $\mathcal{M} = \mathcal{M}(t, \mathcal{L})$ and $\mathcal{B}_n = \mathcal{B}_n(t, \mathcal{L})$ in a desired form. These functions \mathcal{L} , \mathcal{M} , and \mathcal{B}_n satisfy 2-form equation (7) (hence the dispersionless KP hierarchy) as an immediate consequence of 1-form equation (15).

One can easily show that the tau function $\tau(\hbar, t)$ has to behave as

$$\tau(\hbar, t) = \exp[\hbar^{-2}F(t) + O(\hbar^{-1})] \quad (\hbar \rightarrow 0) \quad (19)$$

so as to reproduce the WKB asymptotic form of the Baker-Akhiezer function. Here $F(t)$ is a function that satisfies the equations

$$\frac{\partial F(t)}{\partial t_n} = v_{n+1}(t), \quad n = 1, 2, \dots, \quad (20)$$

therefore can be identified with $\log \tau_{\text{dKP}}$. This relation resembles the relation between the partition function and the free energy of matrix models in large- N limit ($\hbar \sim 1/N$) [23]. Because of this, $F(t)$ may be called the “free energy” of the dispersionless KP hierarchy.

We now turn to the issue of W -infinity symmetries. Basic notations and tools are mostly borrowed from the work of Date et al. exploiting free fermions and vertex operators [21]. We first modify their bosonized vertex operator $Z(\tilde{\lambda}, \lambda)$ by rescaling $t_n \rightarrow \hbar^{-1}t_n$, $\partial/\partial t_n \rightarrow \hbar\partial/\partial t_n$ as:

$$Z(\hbar, \tilde{\lambda}, \lambda) = \frac{\exp[\hbar^{-1}t(\tilde{\lambda}) - \hbar^{-1}t(\lambda)] \exp[-\hbar\tilde{\partial}(\tilde{\lambda}^{-1}) + \hbar\tilde{\partial}(\lambda^{-1})] - 1}{\tilde{\lambda} - \lambda},$$

$$t(\lambda) = \sum_{n=1}^{\infty} t_n \lambda^n, \quad \tilde{\partial}(\lambda^{-1}) = \sum_{n=1}^{\infty} \lambda^{-n} \frac{1}{n} \frac{\partial}{\partial t_n}. \quad (21)$$

This rescaled vertex operator gives a 2-parameter family of infinitesimal symmetries of the KP hierarchy with \hbar inserted, acting on the tau function as $\tau \rightarrow \tau + \epsilon Z(\hbar, \tilde{\lambda}, \lambda)\tau$. A set of generators of $W_{1+\infty}$ symmetries, $W_n^{(\ell)}(\hbar)$ ($\ell \geq 1$, $n \in \mathbf{Z}$), are given by

$$W_n^{(\ell)}(\hbar, \lambda) = \sum_{n=-\infty}^{\infty} W_n^{(\ell)}(\hbar) \lambda^{-n-\ell} = \left(\frac{\partial}{\partial \tilde{\lambda}} \right)^{\ell-1} Z(\hbar, \tilde{\lambda}, \lambda)|_{\tilde{\lambda}=\lambda}. \quad (22)$$

Each of $W_n^{(\ell)}(\hbar)$ is a differential operator of finite order. For example,

$$W_n^{(1)}(\hbar) = \hbar\partial/\partial t_n, \quad W_{-n}^{(1)}(\hbar) = nt_n/\hbar, \quad n = 1, 2, \dots,$$

$$W_0^{(1)}(\hbar) = 0, \quad (23)$$

and they form, along with the identity operator 1, a $U(1)$ -current (or Heisenberg) algebra.

These $W_{1+\infty}$ symmetries give rise to $w_{1+\infty}$ symmetries of the dispersionless KP hierarchy as follows. Let us define $w_n^{(\ell)}F$ by the limit

$$w_n^{(\ell)}F = \lim_{\hbar \rightarrow 0} \tau(\hbar, t)^{-1} \hbar^\ell W_n^{(\ell)}(\hbar) \tau(\hbar, t). \quad (24)$$

Simple calculations show that this limit does exist and is given by

$$w_n^{(\ell)}F = \frac{1}{\ell} \text{Res} \mathcal{M}^\ell \mathcal{L}^{n+\ell-1} d\mathcal{L}, \quad (25)$$

where "Res" means the coefficient of dk/k in a Laurent series of k (or, equivalently, the coefficients of $d\mathcal{L}/\mathcal{L}$ in a Laurent series of \mathcal{L}). These are nothing but the generators of $w_{1+\infty}$ symmetries constructed in our previous paper [10], which act on $F(t)$ as $F \rightarrow F + \epsilon w_n^{(\ell)} F$.

To translate these observations into the fermionic language, let us recall that the tau function of the ordinary ($\hbar = 1$) KP hierarchy can be written, in general, as a vacuum expectation value,

$$\tau(t) = \langle 0 | e^{H(t)} g | 0 \rangle. \quad (26)$$

Here the Fock vacuum states $\langle 0 |$ and $| 0 \rangle$ are annihilated by half of Fourier modes of the Date-Jimbo-Kashiwara-Miwa free fermion fields

$$\psi(\lambda) = \sum_{n=-\infty}^{\infty} \psi_n \lambda^n, \quad \psi^*(\lambda) = \sum_{n=-\infty}^{\infty} \psi_n^* \lambda^{-n-1} \quad (27)$$

with the anti-commutation relations

$$[\psi_i, \psi_j]_+ = [\psi_i^*, \psi_j^*]_+ = 0, \quad [\psi_i, \psi_j^*]_+ = \delta_{ij}. \quad (28)$$

as

$$\begin{aligned} \psi_n | 0 \rangle &= 0 \quad (n \leq -1), & \psi_n^* | 0 \rangle &= 0 \quad (n \geq 0), \\ \langle 0 | \psi_n &= 0 \quad (n \geq 0), & \langle 0 | \psi_n^* &= 0 \quad (n \leq -1). \end{aligned} \quad (29)$$

The generator $H(t)$ of the time evolutions is given by

$$H(t) = \sum_{n=1}^{\infty} t_n H_n, \quad H_n = \sum_{i=-\infty}^{\infty} : \psi_i \psi_{i+n}^* :. \quad (30)$$

Finally, g is a Clifford operator written, in general, as

$$g = \text{const. exp} \int \int : A(\tilde{\lambda}, \lambda) \psi(\tilde{\lambda}) \psi^*(\lambda) : d\tilde{\lambda} d\lambda. \quad (31)$$

If the Planck constant \hbar comes into the game, the above expression of the tau function should change as:

$$\tau(\hbar, t) = \langle 0 | e^{H(t)/\hbar} g(\hbar) | 0 \rangle. \quad (32)$$

The Clifford operator $g(\hbar)$ has to be suitably chosen so as to ensure the correct asymptotic form of the tau function in the limit of $\hbar \rightarrow 0$.

An ansatz for $g(\hbar)$ can be inferred as follows. Let us first note that $W_{1+\infty}$ symmetries are implemented by inserting a fermion bilinear form in front of the Clifford operator $g(\hbar)$. This is due to the basic relation

$$Z(\hbar, \tilde{\lambda}, \lambda) \tau(\hbar, t) = \langle 0 | e^{H(t)/\hbar} : \psi(\tilde{\lambda}) \psi^*(\lambda) : g(\hbar) | 0 \rangle \quad (33)$$

pointed by Date et al. in the case of $\hbar = 1$. For example, insertion of

$$\mathcal{O}_n^{(\ell)}(\hbar) = \oint : \lambda^{n+\ell-1} \left(\hbar \frac{\partial}{\partial \lambda} \right)^{\ell-1} \psi(\lambda) \cdot \psi^*(\lambda) : \frac{d\lambda}{2\pi i}, \quad (34)$$

where the integral is along a circle $|\lambda| = \text{const.}$, corresponds to the action of $\hbar^{-1} W_n^{(\ell)}(\hbar)$ on $\tau(\hbar, t)$, from which we can derive $w_n^{(\ell)F}$ as described above. This can be generalized to fermion bilinear forms of the form

$$\mathcal{O}_A(\hbar) = \oint : A \left(\lambda, \hbar \frac{\partial}{\partial \lambda} \right) \psi(\lambda) \cdot \psi^*(\lambda) : \frac{d\lambda}{2\pi i}. \quad (35)$$

They generate $W_{1+\infty}$ symmetries $\hbar^{-1} W_A(\hbar)$ that contract to $w_{1+\infty}$ symmetries $w_A^F (= -\delta_A F$ in the notation of our previous work [10]) of the dispersionless KP hierarchy. If one can exponentiate (i.e., integrate) these infinitesimal symmetries, finite symmetries thus obtained on the space of solutions should generate nontrivial solutions out of the trivial solution $\tau(\hbar, t) = 1$. This is indeed achieved by inserting a Clifford operator exponentiating a fermion bilinear form $\mathcal{O}_A(\hbar)$; $g(\hbar)$ should be such a Clifford operator. Actually, to ensure correct asymptotic behavior of $\tau(\hbar, t)$ in the limit of $\hbar \rightarrow 0$, the exponent should be $\hbar^{-1} \mathcal{O}_A(\hbar)$ rather than $\mathcal{O}_A(\hbar)$, just like $\hbar^{-1} H(t)$ in the boost operator of time evolutions. We are thus led to the following ansatz:

$$g(\hbar) = \exp \hbar^{-1} \mathcal{O}_A(\hbar). \quad (36)$$

The validity of this ansatz has been checked by several different methods. Further, these solutions, depending on an arbitrary function $A(\lambda, \mu)$, actually give general solutions of the dispersionless KP hierarchy. This is a consequence of our previous construction of $w_{1+\infty}$ symmetries based on a kind of Riemann-Hilbert problem [10].

We have thus seen several aspects of dispersionless hierarchies from the standpoint of quasi-classical limit, taking the dispersionless KP hierarchy as an example. These results can be extended to other hierarchies, such as the modified KP and Toda hierarchies. A new characteristic of these two hierarchies is the presence of a discrete variable, say $\nu \in \mathbf{Z}$,

other than continuous ones like t_n 's. In quasi-classical limit, such a variable has to be rescaled as $\hbar\nu = s$, s being a continuous variable.

Our results in this and previous papers suggest a deep link of dispersionless hierarchies with recent various approaches to $c = 1$ matrix models [24] and their "quantum fluid" picture in quasi-classical limit [25]. In particular, our field theoretical tools are technically very close to the free fermion formalism of Das et al., though dealing with apparently different models. Elucidating these possible relations is an intriguing issue, which might lead to a similar approach to 4D self-dual gravity.

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