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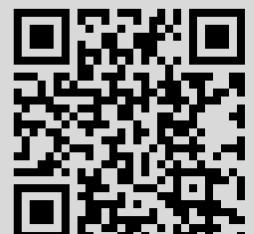
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NOTE ON SUPER $(a, 1)$ - P_3 -ANTIMAGIC TOTAL LABELING OF STAR S_n

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Abstract: Let $G = (V, E)$ be a simple graph and H be a subgraph of G . Then G admits an H -covering, if every edge in $E(G)$ belongs to at least one subgraph of G that is isomorphic to H . An $(a, d) - H$ -antimagic total labeling of G is bijection $f : V(G) \cup E(G) \rightarrow \{1, 2, 3, \dots, |V(G)| + |E(G)|\}$ such that for all subgraphs H' of G isomorphic to H , the H' weights $w(H') = \sum_{v \in V(H')} f(v) + \sum_{e \in E(H')} f(e)$ constitute an arithmetic progression $\{a, a + d, a + 2d, \dots, a + (n - 1)d\}$, where a and d are positive integers and n is the number of subgraphs of G isomorphic to H . The labeling f is called a super $(a, d) - H$ -antimagic total labeling if $f(V(G)) = \{1, 2, 3, \dots, |V(G)|\}$. In [5], David Laurence and Kathiresan posed a problem that characterizes the super $(a, 1) - P_3$ -antimagic total labeling of Star S_n , where $n = 6, 7, 8, 9$. In this paper, we completely solved this problem.

Keywords: H -covering, Super $(a, d) - H$ -antimagic, Star.

1. Introduction

Let $G = (V(G), E(G))$ and $H = (V(H), E(H))$ be simple and finite graphs. Let $|V(G)| = v_G$, $|E(G)| = e_G$, $|V(H)| = v_H$ and $|E(H)| = e_H$. An edge covering of G is a family of different subgraphs $H_1, H_2, H_3, \dots, H_k$ such that any edge of $E(G)$ belongs to at least one of the subgraphs H_j , $1 \leq j \leq k$. If the H_j 's are isomorphic to a given graph H , then G admits an H -covering. Gutierrez and Lladó [2] defined H -magic labeling, which is a generalization of Kotzig and Rosa's edge magic total labeling [4]. A bijection $f : V(G) \cup E(G) \rightarrow \{1, 2, 3, \dots, v_G + e_G\}$ is called an H -magic labeling of G if there exists a positive integer k such that each subgraph H' of G isomorphic to H satisfies

$$w(H') = \sum_{v \in V(H')} f(v) + \sum_{e \in E(H')} f(e) = k.$$

In this case, they say that G is H -magic. When $f(V(G)) = \{1, 2, 3, \dots, v_G\}$, we say that G is H -super magic. On the other hand, Inayah et al. [3] introduced $(a, d) - H$ -antimagic total labeling of G which is defined as a bijection $f : V(G) \cup E(G) \rightarrow \{1, 2, 3, \dots, v_G + e_G\}$ such that for all subgraphs H' of G isomorphic to H , the set of H' -weights

$$w(H') = \sum_{v \in V(H')} f(v) + \sum_{e \in E(H')} f(e)$$

constitutes an arithmetic progression $a, a + d, a + 2d, \dots, a + (n - 1)d$, where a and d are some positive integers and n is the number of subgraphs isomorphic to H . In this case, they say that G is $(a, d) - H$ -antimagic. If $f(V(G)) = \{1, 2, 3, \dots, v_G\}$, they say that f is a super $(a, d) - H$ -antimagic total labeling and G is super $(a, d) - H$ -antimagic. This labeling is a more general case of super (a, d) -edge-antimagic total labelings. If $H \cong K_2$, then we say that super $(a, d) - H$ -antimagic

labelings, which is also called super (a, d) -edge-antimagic total labelings and have been introduced in [6]. They studied some basic properties of such labeling and also proved the following theorem.

Theorem 1 [3]. *If G has a super $(a, d) - H$ -antimagic total labeling and t is the number of subgraphs of G isomorphic to H , then G has a super $(a', d) - H$ -antimagic total labeling, where $a' = [(v_G + 1)v_H + (2v_G + e_G + 1)e_H] - a - (t - 1)d$.*

Several authors are studied antimagic type labeling of graphs see [1]. In 2015, Laurence and Kathiresan [5] obtained an upper bound of d for any graph G , and they investigated the existence of super $(a, d) - P_3$ -antimagic total labeling of star graph S_n . First, they observed that S_n admits a P_h -covering for $h = 2, 3$, and the star S_n contains

$$t = \binom{n}{h-1}$$

subgraphs P_h , $h = 2, 3$, which is denoted by P_h^j , $1 \leq j \leq h$. In 2005, Sugeng et al. [7] investigated the case $h = 2$ using super (a, d) -edge-antimagic total labeling. In 2015, the case of $h = 3$ was investigated by Laurence and Kathiresan [5]. Here they observed that if the star S_n , $n \geq 3$ admits a super $(a, d) - P_3$ -antimagic total labeling then $d \in \{0, 1, 2\}$. Now, they proved the star S_n , $n \geq 3$ has super $(4n + 7, 0) - P_3$ -antimagic total labeling and S_n , $n \geq 3$ admits a super $(a, 2) - P_3$ -antimagic total labeling if and only if $n = 3$. Also, they proved the following theorems and posed a problem.

Theorem 2 [5]. *If the star S_n , $n \geq 3$ has super $(a, 1) - P_3$ -antimagic total labeling, then $3 \leq n \leq 9$. Moreover, the star S_n admits a super $(a, 1) - P_3$ -antimagic total labeling, where $a = 19$, for $n = 3$ and $a = 21$, for $n = 4$.*

Theorem 3 [5]. *For $n = 5$, the star S_n has no super $(a, 1) - P_3$ -antimagic total labeling.*

Problem 1. [5] *For each n , $6 \leq n \leq 9$ characterize the super $(a, 1) - P_3$ -antimagic total labeling for the star S_n .*

In this paper, we present the complete solution to the above problem.

2. Main Results

Let $S_n \cong K_{1,n}$, $n \geq 1$ be the star graph and let v_0 be the central vertex and let v_i , $1 \leq i \leq n$ be its adjacent vertices. Thus S_n has $n + 1$ vertices and n edges.

Theorem 4. *The star S_6 has no super $(a, 1) - P_3$ -antimagic total labeling.*

P r o o f. Let $V(S_6) = \{v_0, v_1, v_2, v_3, v_4, v_5, v_6\}$ and $E(S_6) = \{v_0v_1, v_0v_2, v_0v_3, v_0v_4, v_0v_5, v_0v_6\}$ be the vertex set and the edge set of Star S_6 . Suppose there exists a super $(a, 1) - P_3$ -antimagic total labeling $f : V \cup E \rightarrow \{1, 2, 3, \dots, 13\}$ for S_6 and let v_0 be the central vertex of S_6 . In the computation of P_3 — weights the label of the central vertex v_0 , $f(v_0)$ is used 15 times and label of other vertices and edges say i are used 5 times each. Therefore,

$$10f(v_0) + 5 \sum_{i=1}^{13} (i) = \frac{15}{2}[2a + 14],$$

which implies $a = (70 + 2f(v_0))/3$. Since $1 \leq f(v_0) \leq 7$, it follows that $a = 24$ if $f(v_0) = 1$, $a = 26$ if $f(v_0) = 4$ and $a = 28$ if $f(v_0) = 7$.

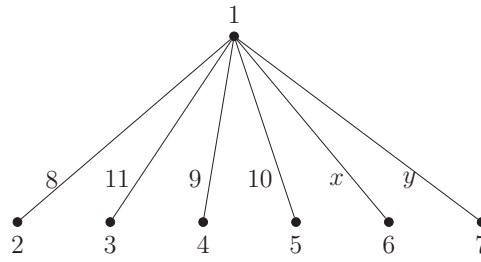


Figure 1. There is no possible to obtain P_3 -weight 27.

Case (i): $f(v_0) = 1$. Then $a = 24$ and the P_3 — weights of S_6 are given by $W = \{24, 25, \dots, 38\}$. Now, the P_3 — weight 24 is getting exactly two possible 5 elements sum $(1, 2, 4, 8, 9)$ or $(1, 2, 3, 8, 10)$ and hence the label of edges $e_1 = v_0v_1$ and $e_2 = v_0v_3$ or v_0v_2 is $f(e_1) = 8$ and $f(e_2) = 9$ or 10 .

Subcase (i): $f(e_2 = v_0v_3) = 9$. Then $a = 24$ and hence the label of the vertices and edges are $f(v_0) = 1, f(v_1) = 2, f(v_3) = 4, f(e_1 = v_0v_1) = 8$ and $f(e_2 = v_0v_3) = 9$. Now, the P_3 — weight 25 is getting exactly one possible 5 elements sum $(1, 2, 3, 8, 11)$ and hence the label of an edge $e_3 = v_0v_2$ is $f(e_3) = 11$. Also, the P_3 — weight 26 is getting exactly one possible 5 elements sum $(1, 2, 5, 8, 10)$ and hence the label of an edge $e_4 = v_0v_4$ is $f(e_4) = 10$.

Let $x = v_0v_5$ and $y = v_0v_6$ be two edges of S_6 (see Fig. 1). Clearly, the label of the edges x and y is $f(x) = 12$ or 13 and $f(y) = 13$ or 12 . If $f(x) = 12$ then $f(y) = 13$ and hence there is no P_3 — weight 27. Also, if $f(x) = 13$ then $f(y) = 12$ and hence there is no P_3 — weight 27, which is a contradiction.

A similar contradiction arises, if the edges $e_1 = v_0v_1$ and $e_2 = v_0v_2$ with $f(e_1) = 9$ and $f(e_2) = 8$ for the P_3 — weight 24 is used to getting the P_3 — weight 27.

Subcase (ii): $f(e_2 = v_0v_2) = 10$. Then $a = 24$ and hence the label of the vertices and edges of P_3 — weight 24 is $f(v_0) = 1, f(v_1) = 2, f(v_2) = 3, f(e_1 = v_0v_1) = 8$ and $f(e_2 = v_0v_2) = 10$. Now, the P_3 — weight 25 is getting exactly one possible 5 elements sum $(1, 2, 5, 8, 9)$ and hence the label of an edge $e_3 = v_0v_4$ is $f(e_3) = 9$. Also, the P_3 — weight 26 is getting exactly one possible 5 elements sum $(1, 2, 4, 8, 11)$ and hence the label of an edge $e_4 = v_0v_3$ is $f(e_4) = 11$. Let $x = v_0v_5$ and $y = v_0v_6$ be two edges of S_6 (see Fig. 2). Clearly, the label of the edges x and y is $f(x) = 12$ or 13 and $f(y) = 13$ or 12 . If $f(x) = 12$ then $f(y) = 13$ and hence there is no P_3 — weight 27. Also, If $f(x) = 13$ then $f(y) = 12$ and hence there is no P_3 — weight 27, which is a contradiction.

A similar contradiction arises, if the edges $e_1 = v_0v_1$ and $e_2 = v_0v_2$ with $f(e_1) = 10$ and $f(e_2) = 8$ for the P_3 — weight 24 is used to getting the P_3 — weight 27.

Case (ii): $f(v_0) = 7$. Then $a = 28$. Now, if f is a super $(28, 1) - P_3$ -antimagic total labeling of S_6 , then by Theorem 1 [3], \bar{f} is a super $(24, 1) - P_3$ -antimagic total labeling, which does not exist by Case (i).

Case (iii): $f(v_0) = 4$. Then $a = 26$ and hence the P_3 — weights of S_6 are given by $W = \{26, 27, \dots, 40\}$. Now, the P_3 — weight 26 is getting exactly four possible 5 elements sum such as $(4, 1, 2, 8, 11)$, $(4, 1, 2, 9, 10)$, $(4, 2, 3, 8, 9)$ and $(4, 1, 3, 8, 10)$ and hence the edges $e_1 = v_0v_1$ or v_0v_2 and $e_2 = v_0v_2$ or v_0v_3 with $f(e_1) = 8$ or 9 and $f(e_2) = 9$ or 10 or 11 .

Subcase (i): $f(e_1 = v_0v_1) = 8$ and $f(e_2 = v_0v_2) = 11$. Then $a = 26$ and hence the label of the vertices and edges of P_3 — weight 26 is $f(v_0) = 4, f(v_1) = 1, f(v_2) = 2, f(e_1 = v_0v_1) = 8$ and

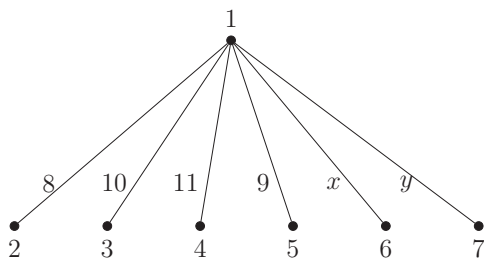


Figure 2. The possible edge labels x and y are obtain P_3 -weight 27.

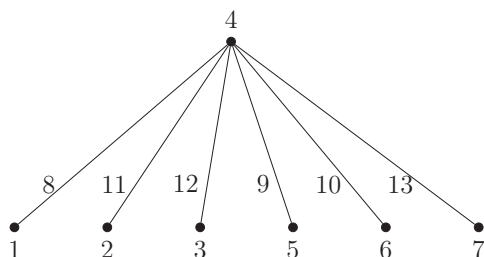


Figure 3. There is no possible to obtain P_3 -weight 30.

$f(e_2 = v_0v_2) = 11$. Now, the P_3 — weight 27,28 and 29 are getting exactly one possible 5 elements sum $(4, 1, 5, 8, 9), (4, 1, 3, 8, 12)$ and $(4, 1, 6, 8, 10)$. Hence the label of the edges $e_3 = v_0v_3$, $e_4 = v_0v_4$, $e_5 = v_0v_5$ and $e_6 = v_0v_6$ is $f(e_3) = 12, f(e_4) = 9, f(e_5) = 10$ and $f(e_6) = 13$. From Fig. 3, there is no P_3 — weight is 30, which is a contradiction.

A similar contradiction arises, if the edges e_1 and e_2 with $f(e_1 = v_0v_1) = 11$ and $f(e_2 = v_0v_2) = 8$ for P_3 — weight 26 are used to getting the P_3 — weight 33, for more details see Fig. 4.

Subcase (ii): $f(e_1 = v_0v_1) = 9$ and $f(e_2 = v_0v_2) = 10$. Then $a = 26$ and hence the label of the vertices and edges of P_3 — weight 26 is $f(v_0) = 4, f(v_1) = 1, f(v_2) = 2, f(e_1 = v_0v_1) = 9$ and $f(e_2 = v_0v_2) = 10$. Now, the P_3 — weight 27 is getting exactly two possibles 5 elements sum such as $(4, 2, 3, 10, 8)$, $(4, 1, 5, 9, 8)$ and hence the label of the edges $e_3 = v_0v_3$ or v_0v_4 is $f(e_3) = 8$. If an edge $e_3 = v_0v_3$ with $f(e_3) = 8$ then we get the P_3 — weight as sum of 5 elements $(4, 1, 3, 9, 8)$ is 25, which is a contradiction. If an edge $e_3 = v_0v_4$ with $f(e_3) = 8$ then we get the P_3 — weights from 28 to 32 are getting exactly one possible 5 elements sum such as $(4, 1, 3, 9, 11), (4, 2, 5, 10, 8), (4, 2, 3, 10, 11), (4, 3, 5, 11, 8)$ and $(4, 1, 6, 9, 12)$. From Fig. 5, there is no P_3 — weight 33, which is a contradiction.

A similar contradiction arises, if the edges $e_1 = v_0v_1$ and $e_2 = v_0v_2$ with $f(e_1 = v_0v_1) = 10$ and $f(e_2 = v_0v_2) = 9$ for the P_3 — weight 26 is used to getting the P_3 — weight 27, which is a contradiction.

Subcase (iii): $f(e_1 = v_0v_2) = 8$ and $f(e_2 = v_0v_3) = 9$. Then $a = 26$ and hence the label of the vertices and edges of P_3 — weight 26 is $f(v_0) = 4, f(v_2) = 2, f(v_3) = 3, f(e_1 = v_0v_2) = 8$ and $f(e_2 = v_0v_3) = 9$. Now, the P_3 — weight 27 is getting exactly one possible 5 elements sum $(4, 1, 3, 9, 10)$ and hence the label of an edge $e_3 = v_0v_1$ is $f(e_3) = 10$. Thus, we get a P_3 — weight

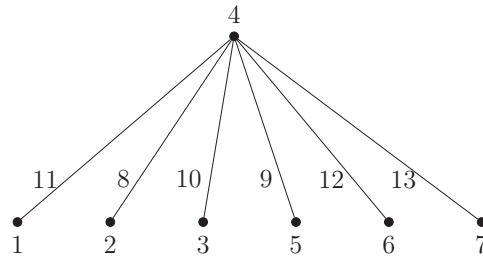


Figure 4. The possible edge label is obtain to P_3 -weight 33.

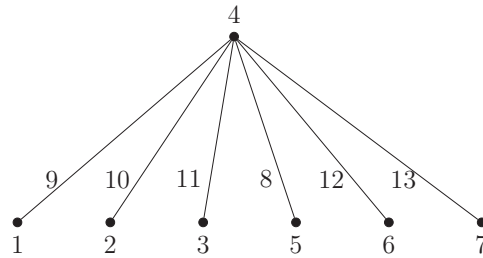


Figure 5. There is no possible to obtain P_3 -weight 33.

as sum of 5 elements $(4, 1, 2, 10, 8)$ is 25, which is a contradiction.

A similar contradiction arises, if the edges $e_1 = v_0v_2$ and $e_2 = v_0v_3$ with $f(e_1 = v_0v_2) = 9$ and $f(e_2 = v_0v_3) = 8$ for the P_3 — weight 26. The P_3 — weight 27 is getting exactly one possible 5 elements sum $(4, 1, 2, 11, 9)$ and hence the label of an edge $f(e_3 = v_0v_1) = 11$. Thus, we get the $P_3 = (v_0, v_1, v_3, e_3 = v_0v_1, e_2 = v_0v_3)$ with weight $(4 + 1 + 3 + 11 + 8)$ is 27, which is a contradiction.

Subcase (iv): $f(e_1 = v_0v_1) = 8$ and $f(e_2 = v_0v_3) = 10$. Then $a = 26$ and hence the label of the vertices and edges of P_3 — weight 26 is $f(v_0) = 4, f(v_1) = 1, f(v_3) = 3, f(e_1 = v_0v_1) = 8$ and $f(e_2 = v_0v_3) = 10$. Now, the P_3 — weight 27 is getting exactly two possibles 5 elements sum such as $(4, 1, 2, 8, 12), (4, 1, 5, 8, 9)$ and hence the label of the edges $e_3 = v_0v_2$ or v_0v_4 is $f(e_3) = 12$ or 9. If an edge $e_3 = v_0v_2$ with $f(e_3) = 12$ then the P_3 — weights 28 and 29 are getting exactly one possible 5 elements sum $(4, 1, 6, 8, 9)$ and $(4, 1, 5, 8, 11)$. From Fig. 6, there is no P_3 — weight 30, which is a contradiction. If an edge $e_4 = v_0v_4$ with $f(e_4) = 9$ then the P_3 — weight 28 is getting exactly one possible 5 elements sum $(4, 1, 2, 8, 13)$ and hence the label of an edge $e_5 = v_0v_2$ is $f(e_5) = 13$. From Fig. 7, there is no P_3 — weight 29 when $x = 11$ or 12 and $y = 12$ or 11, which is a contradiction.

A similar contradiction arises, if the edges $e_1 = v_0v_1$ and $e_2 = v_0v_3$ with $f(e_1 = v_0v_1) = 10$ and $f(e_2 = v_0v_3) = 8$ for the P_3 — weight 26 are used to getting the P_3 — weight 27, which is a contradiction. \square

Theorem 5. *The star S_7 has no super $(a, 1) - P_3$ -antimagic total labeling.*

P r o o f. Let $V(S_7) = \{v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and $E(S_7) = \{v_0v_1, v_0v_2, v_0v_3, v_0v_4, v_0v_5, v_0v_6, v_0v_7\}$ be the vertex and edge set of star S_7 . Suppose there exists a super $(a, 1) - P_3$ -antimagic total labeling $f : V \cup E \rightarrow \{1, 2, 3, \dots, 15\}$ for S_7 and let v_0 be the central vertex of S_7 . In the

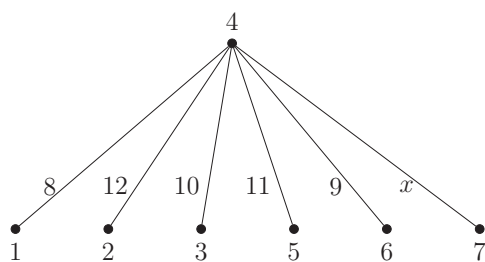


Figure 6. There is no possible to obtain P_3 -weight 30.

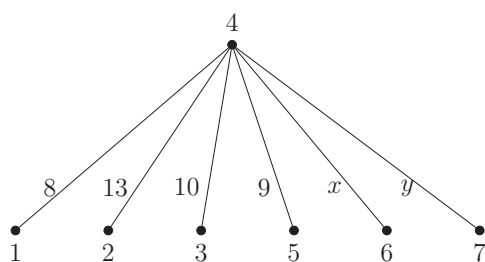


Figure 7. There is no possible to obtain P_3 -weight 29.

computation of P_3 — weights the label of the central vertex v_0 , $f(v_0)$ is used 21 times and label of other vertices and edges say i are used 6 times each. Therefore,

$$15f(v_0) + 6 \sum_{i=1}^{15} (i) = \frac{21}{2}[2a + 20],$$

which implies that we get

$$a = \frac{15f(v_0) + 510}{21}.$$

Since $1 \leq f(v_0) \leq 8$, we have only two values a such as $a = 25$ if $f(v_0) = 1$ and $a = 30$ if $f(v_0) = 8$.

Case (i): $f(v_0) = 1$. Then $a = 25$ and the P_3 — weights of S_7 is given by $W = \{25, 26, \dots, 45\}$. Now, the P_3 — weight 25 is getting exactly one possible 5 elements sum $(1, 2, 3, 9, 10)$ and hence the label of edges $e_1 = v_0v_1$ and $e_2 = v_0v_2$ is $f(e_1) = 9$ and $f(e_2) = 10$. Since the minimum possible sum of vertices labels for P_3 — weight is 7, it follows that there is no P_3 — weight 26, which is a contradiction. A similar contradiction arises, if the edges $e_1 = v_0v_1$ and $e_2 = v_0v_2$ with $f(e_1) = 10$ and $f(e_2) = 9$ for the P_3 — weight 25 is used to getting the P_3 — weight 27.

Case (ii): $f(v_0) = 8$. Then $a = 30$. Now, if f is a super $(30, 1) - P_3$ -antimagic total labeling of S_6 , then by Theorem 1 [3], \bar{f} is a super $(25, 1) - P_3$ -antimagic total labeling, which does not exist by Case (i). □

Theorem 6. *The star S_8 has no super $(a, 1) - P_3$ -antimagic total labeling.*

P r o o f. Let $V(S_8) = \{v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ and $E(S_8) = \{v_0v_1, v_0v_2, v_0v_3, v_0v_4, v_0v_5, v_0v_6, v_0v_7, v_0v_8\}$ be the vertex and edge set of star S_8 . Suppose there exists a super $(a, 1) - P_3$ -antimagic total labeling $f : V \cup E \rightarrow \{1, 2, 3, \dots, 17\}$ for S_8 and let v_0 be the central vertex of S_8 .

In the computation of P_3 — weights the label of the central vertex v_0 , $f(v_0)$ is used 28 times and label of other vertices and edges say i are used 7 times each. Therefore,

$$21f(v_0) + 7 \sum_{i=1}^{17} (i) = \frac{28}{2}[2a + 27],$$

which implies that we get

$$a = \frac{21f(v_0) + 693}{28}.$$

Since $1 \leq f(v_0) \leq 9$, we have only two values a such as $a = 27$, if $f(v_0) = 3$ and $a = 30$, if $f(v_0) = 7$.

Case (i): $f(v_0) = 3$. Then $a = 27$ and the P_3 — weights of S_8 is given by $W = \{27, 28, \dots, 54\}$. Now, the P_3 — weight 27 is getting exactly one possible 5 elements sum $(3, 1, 2, 10, 11)$ and hence the label of edges $e_1 = v_0v_1$ and $e_2 = v_0v_2$ is $f(e_1) = 10$ and $f(e_2) = 11$. Since the minimum possible sum of vertices labels for P_3 — weight is 8, it follows that there is no P_3 — weight 29, which is a contradiction. A similar contradiction arises, if the edges $e_1 = v_0v_1$ and $e_2 = v_0v_2$ with $f(e_1) = 11$ and $f(e_2) = 10$ for the P_3 — weight 27 is used to getting the P_3 — weight 29.

Case (ii) $f(v_0) = 7$ Then $a = 30$. Now, if f is a super $(30, 1) - P_3$ -antimagic total labeling of S_6 , then by Theorem 1 [3], \bar{f} is a super $(27, 1) - P_3$ -antimagic total labeling, which does not exist by Case (i). \square

Theorem 7. *The star S_9 has no super $(a, 1) - P_3$ -antimagic total labeling.*

P r o o f. Let $V(S_9) = \{v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$ be the vertex set of star S_9 . Suppose there exists a super $(a, 1) - P_3$ -antimagic total labeling $f : V \cup E \rightarrow \{1, 2, 3, \dots, 19\}$ for S_9 and let v_0 be the central vertex of S_9 . In the computation of P_3 — weights the label of the central vertex v_0 , $f(v_0)$ is used 36 times and label of other vertices and edges say i are used 8 times each. Therefore,

$$28f(v_0) + 8 \sum_{i=1}^{19} (i) = \frac{36}{2}[2a + 35],$$

which implies that we get

$$a = \frac{14f(v_0) + 445}{18}.$$

Since $1 \leq f(v_0) \leq 10$, we have that a is not an integer, which is a contradiction. \square

From Theorem 2-3 [5], Theorem 4-7, we get the following result.

Theorem 8. *The star $S_n, n \geq 3$ admits a super $(a, 1) - P_3$ -antimagic total labeling if and only if $n = 3$ and 4.*

3. Conclusion and Scope

In [5], they investigated the existence of super (a, d) - P_3 -antimagic total labeling of star S_n and posed the Problem 1 [5]. This paper proved the star S_n has no super $(a, 1)$ - P_3 -antimagic total labeling, where $n = 6, 7, 8, 9$. Therefore, we have entirely solved Problem 1 [5].

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