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Soft Tripled Coincidence Fixed Point Theorems in Soft Fuzzy Metric Space

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Abstract. In this work, we are going to prove the fixed point theorems (FPT's) for the existence and uniqueness of soft tripled coincidence point for contractive maps in the setting of soft fuzzy metric space (SFMS). We have also given an application to our new results in finding the solution of an integral equation.

Keywords: soft set, soft tripled coincidence point, contractions, fixed point.

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Introduction

The concept of uncertainty was very well handled by Zadeh [1] by introducing the concept of fuzzy sets. Fuzzy set was then generalized to fuzzy metric space by Kramosil and Michalek [2] and George and Veeramani [3]. In case of uncertainties in data consisting of parameters, Molodstov [4] introduced soft sets. The concept of soft set was further generalized and formulated to other spaces and hence new spaces were introduced. The concept of soft set was applied to fuzzy metric space by Ferhan SBëla Erduran, Ebru Yigit, Rabia Alar and Ayten Gezici [11] and hence presenting SFMS.

The tripled point concept was given by Bernide and Borcut [9]. Further A Roldan, J Martinez-Moreno and C Roldan [10] gave FPT's for the existence and uniqueness of tripled fixed point for contractions in fuzzy metric space (FMS).

In this work we are going to extend the FPT'S given in [10] to SFMS by introducing the concept of soft tripled coincidence point.

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1. Preliminaries

In this section, we are going to discuss some basic definitions and results already present in the literature.

Definition 1.1 ([10]). *A t-norm is a map $* : [0,1] \times [0,1] \to [0,1]$ which is associative, commutative and increasing and has 1 as its identity. Further, a H-type t-norm is defined as a t-norm $*$, if for a sequence $\{n * \alpha\}$, where $n \in N$ is equicontinuous at $\alpha = 1$.*

For example: Define $* = \min$, that satisfies $\min(\iota, \kappa) \geqslant \iota\kappa$ for every $\iota, \kappa \in [0,1]$, then $*$ is a continuous H-type t-norm.

Definition 1.2 ([11]). *Consider $\bar{\Upsilon}$ an arbitrary non-empty absolute soft set, ϖ be a map from $SP(\bar{\Upsilon}) \times SP(\bar{\Upsilon}) \times (0,\infty)$ to $[0,1]$ and $*$ is a t-norm, then the 3-tuple $(\bar{\Upsilon}, \varpi, *)$ is known as SFMS if it satisfies the following assertions for all $\bar{\iota}, \bar{\kappa}, \bar{\lambda} \in \bar{\Upsilon}$ and $\varrho, \nu > 0$,*

(i) $\varpi(\bar{\iota}, \bar{\kappa}, 0) = 0$;

(ii) $\varpi(\bar{\iota}, \bar{\kappa}, \varrho) = 1$ *if and only if* $\bar{\iota} = \bar{\kappa}$;

(iii) $\varpi(\bar{\iota}, \bar{\kappa}, \varrho) = \varpi(\bar{\kappa}, \bar{\iota}, \varrho)$;

(iv) $\varpi(\bar{\iota}, \bar{\kappa}, .) : [0,\infty) \to [0,1]$ *is left continuous;*

(v) $\varpi(\bar{\iota}, \bar{\kappa}, \varrho) * \varpi(\bar{\kappa}, \bar{\lambda}, \nu) \leqslant \varpi(\bar{\iota}, \bar{\lambda}, \varrho + \nu)$.

2. Main Results

In this section, we are going to define our main fixed point theorems.

Definition 2.1. *Consider $(\bar{\Upsilon}, \varpi, *)$ be a SFMS and a function $g : \bar{\Upsilon} \to \bar{\Upsilon}$, then g is continuous at $\bar{\iota} \in \bar{\Upsilon}$ if for a sequence $\{\bar{\iota}_{e_n}\}$ in $\bar{\Upsilon}$ that converges to $\bar{\iota}$, the sequence $\{g\bar{\iota}_{e_n}\}$ is also convergent and converges to $g\bar{\iota}$.*

Remark 2.1. *For $\bar{0} \leqslant \bar{\iota} \leqslant \bar{1}$ and $0 < \alpha, \beta < \infty$, then $\alpha \leqslant \beta$ implies $\bar{\iota}^{\alpha} \geqslant \bar{\iota}^{\beta}$, i.e. for $0 < \alpha \leqslant \beta \leqslant 1$, we have $\varpi(\bar{\iota}, \bar{\kappa}, \varrho)^{\alpha} \geqslant \varpi(\bar{\iota}, \bar{\kappa}, \varrho)^{\beta} \geqslant \varpi(\bar{\iota}, \bar{\kappa}, \varrho)$.*

Definition 2.2. *Consider $G : \bar{\Upsilon} \times \bar{\Upsilon} \times \bar{\Upsilon} \to \bar{\Upsilon}$ and $h : \bar{\Upsilon} \to \bar{\Upsilon}$ be two functions, then*

(i) G and h are commuting if for every $\bar{\iota}, \bar{\kappa}, \bar{\lambda} \in \bar{\Upsilon}$, we have $hG(\bar{\iota}, \bar{\kappa}, \bar{\lambda}) = G(h\bar{\iota}, h\bar{\kappa}, h\bar{\lambda})$;

(ii) for $(\bar{\iota}, \bar{\kappa}, \bar{\lambda}) \in \bar{\Upsilon} \times \bar{\Upsilon} \times \bar{\Upsilon}$ is known as a soft tripled coincidence point of functions G and h if $G(\bar{\iota}, \bar{\kappa}, \bar{\lambda}) = h\bar{\iota}$, $G(\bar{\kappa}, \bar{\lambda}, \bar{\iota}) = h\bar{\kappa}$ and $G(\bar{\lambda}, \bar{\iota}, \bar{\kappa}) = h\bar{\lambda}$.

Now, we are going to state and prove our main FPT's.

Theorem 2.1. *Consider $*$ be a H-type norm so that $x * y \geqslant xy$. Consider $0 < \tau < 1$ and $0 \leqslant \alpha, \beta, \gamma \leqslant 1$ so that $\alpha + \beta + \gamma \leqslant 1$. Consider $(\bar{\Upsilon}, \varpi, *)$ be a complete SFMS and $G : \bar{\Upsilon} \times \bar{\Upsilon} \times \bar{\Upsilon} \to \bar{\Upsilon}$ and $h : \bar{\Upsilon} \to \bar{\Upsilon}$ be two functions so that $G(\bar{\Upsilon} \times \bar{\Upsilon} \times \bar{\Upsilon}) \subseteq h(\bar{\Upsilon})$ and h is continuous and commuting with G. Let for all $\bar{\iota}, \bar{\kappa}, \bar{\lambda}, \bar{e}, \bar{f}, \bar{g} \in \bar{\Upsilon}$ and $\varrho > 0$, (1) is satisfied,*

$$\varpi(G(\bar{\iota}, \bar{\kappa}, \bar{\lambda}), G(\bar{e}, \bar{f}, \bar{g}), \tau\varrho) \geqslant \varpi(h\bar{\iota}, h\bar{e}, \varrho)^{\alpha} * \varpi(h\bar{\kappa}, h\bar{f}, \varrho)^{\beta} * \varpi(h\bar{\lambda}, h\bar{g}, \varrho)^{\gamma}. \tag{1}$$

Then, G and h possess a unique soft tripled coincidence point. Moreover, if we consider $h^{-1} = \{\bar{\iota}\}$ and $G \equiv \bar{\iota}$ is constant, then also $(\bar{\iota}, \bar{\iota}, \bar{\iota})$ is the unique soft tripled coincidence point of G and h.

(In the proof, we will consider $\varpi(h\bar{\iota}, h\bar{\kappa}, \varrho)^o = 1$ for every $\varrho > 0$ and $\bar{\iota}, \bar{\kappa} \in \bar{\Upsilon}$).

Proof. Consider G be constant, i.e. there exists $\bar{\iota}_{e_o} \in \bar{\Upsilon}$, so that $\varpi(\bar{\iota}, \bar{\kappa}, \varrho) = \bar{\iota}_{e_o}$. As G and h are commuting, we have $h\bar{\iota}_{e_o} = hG(\bar{\iota}, \bar{\kappa}, \bar{\lambda}) = G(h\bar{\iota}, h\bar{\kappa}, h\bar{\lambda}) = \bar{\iota}_{e_o}$. Thus, $\bar{\iota}_{e_o} = h\bar{\iota}_{e_o} = G(\bar{\iota}_{e_o}, \bar{\iota}_{e_o}, \bar{\iota}_{e_o})$. Let $h^{-1}(\bar{\iota}_{e_o}) = \{\bar{\iota}_{e_o}\}$ and $(\bar{\iota}, \bar{\kappa}, \bar{\lambda}) \in \bar{\Upsilon} \times \bar{\Upsilon} \times \bar{\Upsilon}$ be another soft tripled coincidence point of G and h. Then, $h\bar{\iota} = G(\bar{\iota}, \bar{\kappa}, \bar{\lambda}) = \bar{\iota}_{e_o}$, hence $\bar{\iota} \in h^{-1}(\bar{\iota}_{e_o}) = \{\bar{\iota}_{e_o}\}$. On similar lines $\bar{\iota} = \bar{\kappa} = \bar{\lambda} = \bar{\iota}_{e_o}$ and thus G and h possess $(\bar{\iota}_{e_o}, \bar{\iota}_{e_o}, \bar{\iota}_{e_o})$ as its unique soft tripled coincidence point.
Now, consider G is not constant, then $(\alpha, \beta, \gamma) \neq (0, 0, 0)$. In this case the proof is divided into 5 steps as follows:

Step 1: Consider $\bar{\iota}_{e_o}, \bar{\kappa}_{e_o}$ and $\bar{\lambda}_{e_o}$ be any arbitrary elements of $\bar{\Upsilon}$. As $G(\bar{\Upsilon} \times \bar{\Upsilon} \times \bar{\Upsilon}) \subseteq h(\bar{\Upsilon})$, choose $\bar{\iota}_{e_1}, \bar{\kappa}_{e_1}, \bar{\lambda}_{e_1} \in \bar{\Upsilon}$ so that $h\bar{\iota}_{e_1} = G(\bar{\iota}_{e_o}, \bar{\kappa}_{e_o}, \bar{\lambda}_{e_o})$, $h\bar{\kappa}_{e_1} = G(\bar{\kappa}_{e_o}, \bar{\lambda}_{e_o}, \bar{\iota}_{e_o})$ and $h\bar{\lambda}_{e_1} = G(\bar{\lambda}_{e_o}, \bar{\iota}_{e_o}, \bar{\kappa}_{e_o})$. Choose $\bar{\iota}_{e_2}, \bar{\kappa}_{e_2}, \bar{\lambda}_{e_2} \in \bar{\Upsilon}$ so that $h\bar{\iota}_{e_2} = G(\bar{\iota}_{e_1}, \bar{\kappa}_{e_1}, \bar{\lambda}_{e_1})$, $h\bar{\kappa}_{e_2} = G(\bar{\kappa}_{e_1}, \bar{\lambda}_{e_1}, \bar{\iota}_{e_1})$ and $h\bar{\lambda}_{e_2} = G(\bar{\lambda}_{e_1}, \bar{\iota}_{e_1}, \bar{\kappa}_{e_1})$. Continuing like this, we can construct sequences $\{\bar{\iota}_{e_n}\}$, $\{\bar{\kappa}_{e_n}\}$ and $\{\bar{\lambda}_{e_n}\}$, so that for $n \geqslant 0$ we have $h\bar{\iota}_{e_{n+1}} = G(\bar{\iota}_{e_n}, \bar{\kappa}_{e_n}, \bar{\lambda}_{e_n})$, $h\bar{\kappa}_{e_{n+1}} = G(\bar{\kappa}_{e_n}, \bar{\lambda}_{e_n}, \bar{\iota}_{e_n})$ and $h\bar{\lambda}_{e_{n+1}} = G(\bar{\lambda}_{e_n}, \bar{\iota}_{e_n}, \bar{\kappa}_{e_n})$.

Step 2: Consider $\varrho \geqslant 0$, $\theta_n(\varrho) = \varpi(h\bar{\iota}_{e_n}, h\bar{\iota}_{e_{n+1}}, \varrho) * \varpi(h\bar{\kappa}_{e_n}, h\bar{\kappa}_{e_{n+1}}, \varrho) * \varpi(h\bar{\lambda}_{e_n}, h\bar{\lambda}_{e_{n+1}}, \varrho)$. As θ_n is increasing (because $\varpi(\bar{\iota}, \bar{\kappa}, \varrho)$ is an increasing function with respect to ϱ) and $\varrho - \tau\varrho \leqslant \varrho \leqslant \dfrac{\varrho}{\tau}$, thus we have

$$\theta_n(\varrho - \tau\varrho) \leqslant \theta_n(\varrho) \leqslant \theta_n\Big(\frac{\varrho}{\tau}\Big). \tag{2}$$

From (1), we have

$$\begin{aligned}\varpi(h\bar{\iota}_{e_n}, h\bar{\iota}_{e_{n+1}}, \varrho) &= \varpi(G(\bar{\iota}_{e_{n-1}}, \bar{\kappa}_{e_{n-1}}, \bar{\lambda}_{e_{n-1}}), G(\bar{\iota}_{e_n}, \bar{\kappa}_{e_n}, \bar{\lambda}_{e_n}), \varrho)\\ &\geqslant \varpi\Big(h\bar{\iota}_{e_{n-1}}, h\bar{\iota}_{e_n}, \frac{\varrho}{\tau}\Big)^{\alpha} * \varpi\Big(h\bar{\kappa}_{e_{n-1}}, h\bar{\kappa}_{e_n}, \frac{\varrho}{\tau}\Big)^{\beta} * \varpi\Big(h\bar{\lambda}_{e_{n-1}}, h\bar{\lambda}_{e_n}, \frac{\varrho}{\tau}\Big)^{\gamma};\end{aligned} \tag{3}$$

$$\begin{aligned}\varpi(h\bar{\kappa}_{e_n}, h\bar{\kappa}_{e_{n+1}}, \varrho) &= \varpi(G(\bar{\kappa}_{e_{n-1}}, \bar{\lambda}_{e_{n-1}}, \bar{\iota}_{e_{n-1}}), G(\bar{\kappa}_{e_n}, \bar{\lambda}_{e_n}, \bar{\iota}_{e_n}), \varrho)\\ &\geqslant \varpi\Big(h\bar{\kappa}_{e_{n-1}}, h\bar{\kappa}_{e_n}, \frac{\varrho}{\tau}\Big)^{\alpha} * \varpi\Big(h\bar{\lambda}_{e_{n-1}}, h\bar{\lambda}_{e_n}, \frac{\varrho}{\tau}\Big)^{\beta} * \varpi\Big(h\bar{\iota}_{e_{n-1}}, h\bar{\iota}_{e_n}, \frac{\varrho}{\tau}\Big)^{\gamma};\end{aligned} \tag{4}$$

$$\begin{aligned}\varpi(h\bar{\lambda}_{e_n}, h\bar{\lambda}_{e_{n+1}}, \varrho) &= \varpi(G(\bar{\lambda}_{e_{n-1}}, \bar{\iota}_{e_{n-1}}, \bar{\kappa}_{e_{n-1}}), G(\bar{\lambda}_{e_n}, \bar{\iota}_{e_n}, \bar{\kappa}_{e_n}), \varrho)\\ &\geqslant \varpi\Big(h\bar{\lambda}_{e_{n-1}}, h\bar{\lambda}_{e_n}, \frac{\varrho}{\tau}\Big)^{\alpha} * \varpi\Big(h\bar{\iota}_{e_{n-1}}, h\bar{\iota}_{e_n}, \frac{\varrho}{\tau}\Big)^{\beta} * \varpi\Big(h\bar{\kappa}_{e_{n-1}}, h\bar{\kappa}_{e_n}, \frac{\varrho}{\tau}\Big)^{\gamma}.\end{aligned} \tag{5}$$

Thus, we have

$$\begin{aligned}\varpi(h\bar{\iota}_{e_n}, h\bar{\iota}_{e_{n+1}}, \varrho) &\geqslant \varpi\Big(h\bar{\iota}_{e_{n-1}}, h\bar{\iota}_{e_n}, \frac{\varrho}{\tau}\Big)^{\alpha} * \varpi\Big(h\bar{\kappa}_{e_{n-1}}, h\bar{\kappa}_{e_n}, \frac{\varrho}{\tau}\Big)^{\beta} * \varpi\Big(h\bar{\lambda}_{e_{n-1}}, h\bar{\lambda}_{e_n}, \frac{\varrho}{\tau}\Big)^{\gamma}\\ &\geqslant \varpi\Big(h\bar{\iota}_{e_{n-1}}, h\bar{\iota}_{e_n}, \frac{\varrho}{\tau}\Big) * \varpi\Big(h\bar{\kappa}_{e_{n-1}}, h\bar{\kappa}_{e_n}, \frac{\varrho}{\tau}\Big) * \varpi\Big(h\bar{\lambda}_{e_{n-1}}, h\bar{\lambda}_{e_n}, \frac{\varrho}{\tau}\Big) = \theta_{n-1}\Big(\frac{\varrho}{\tau}\Big);\end{aligned}$$

$$\begin{aligned}\varpi(h\bar{\kappa}_{e_n}, h\bar{\kappa}_{e_{n+1}}, \varrho) &\geqslant \varpi\Big(h\bar{\kappa}_{e_{n-1}}, h\bar{\kappa}_{e_n}, \frac{\varrho}{\tau}\Big)^{\alpha} * \varpi\Big(h\bar{\lambda}_{e_{n-1}}, h\bar{\lambda}_{e_n}, \frac{\varrho}{\tau}\Big)^{\beta} * \varpi\Big(h\bar{\iota}_{e_{n-1}}, h\bar{\iota}_{e_n}, \frac{\varrho}{\tau}\Big)^{\gamma}\\ &\geqslant \varpi\Big(h\bar{\kappa}_{e_{n-1}}, h\bar{\kappa}_{e_n}, \frac{\varrho}{\tau}\Big) * \varpi\Big(h\bar{\lambda}_{e_{n-1}}, h\bar{\lambda}_{e_n}, \frac{\varrho}{\tau}\Big) * \varpi\Big(h\bar{\iota}_{e_{n-1}}, h\bar{\iota}_{e_n}, \frac{\varrho}{\tau}\Big) = \theta_{n-1}\Big(\frac{\varrho}{\tau}\Big);\end{aligned}$$

$$\begin{aligned}\varpi(h\bar{\lambda}_{e_n}, h\bar{\lambda}_{e_{n+1}}, \varrho) &\geqslant \varpi\Big(h\bar{\lambda}_{e_{n-1}}, h\bar{\lambda}_{e_n}, \frac{\varrho}{\tau}\Big)^{\alpha} * \varpi\Big(h\bar{\iota}_{e_{n-1}}, h\bar{\iota}_{e_n}, \frac{\varrho}{\tau}\Big)^{\beta} * \varpi\Big(h\bar{\kappa}_{e_{n-1}}, h\bar{\kappa}_{e_n}, \frac{\varrho}{\tau}\Big)^{\gamma}\\ &\geqslant \varpi\Big(h\bar{\lambda}_{e_{n-1}}, h\bar{\lambda}_{e_n}, \frac{\varrho}{\tau}\Big) * \varpi\Big(h\bar{\iota}_{e_{n-1}}, h\bar{\iota}_{e_n}, \frac{\varrho}{\tau}\Big) * \varpi\Big(h\bar{\kappa}_{e_{n-1}}, h\bar{\kappa}_{e_n}, \frac{\varrho}{\tau}\Big) = \theta_{n-1}\Big(\frac{\varrho}{\tau}\Big).\end{aligned}$$

Hence, we have

$$\varpi(h\bar{\iota}_{e_n}, h\bar{\iota}_{e_{n+1}}, \varrho), \varpi(h\bar{\kappa}_{e_n}, h\bar{\kappa}_{e_{n+1}}, \varrho), \varpi(h\bar{\lambda}_{e_n}, h\bar{\lambda}_{e_{n+1}}, \varrho) \geqslant \theta_{n-1}\Big(\frac{\varrho}{\tau}\Big) \geqslant \theta_{n-1}(\varrho). \tag{6}$$

Changing ϱ by $\varrho - \tau\varrho$ in (6), we have

$$\varpi(h\bar{\iota}_{e_n}, h\bar{\iota}_{e_{n+1}}, \varrho - \tau\varrho), \varpi(h\bar{\kappa}_{e_n}, h\bar{\kappa}_{e_{n+1}}, \varrho - \tau\varrho), \varpi(h\bar{\lambda}_{e_n}, h\bar{\lambda}_{e_{n+1}}, \varrho - \tau\varrho) \geqslant \theta_{n-1}(\varrho - \tau\varrho).$$

Therefore, we have

$$\begin{aligned}
\theta_n(\varrho) &= \varpi(h\bar{\iota}_{e_n}, h\bar{\iota}_{e_{n+1}}, \varrho) * \varpi(h\bar{\kappa}_{e_n}, h\bar{\kappa}_{e_{n+1}}, \varrho) * \varpi(h\bar{\lambda}_{e_n}, h\bar{\lambda}_{e_{n+1}}, \varrho) \\
&\geqslant \Big(\varpi\Big(h\bar{\iota}_{e_{n-1}}, h\bar{\iota}_{e_n}, \frac{\varrho}{\tau}\Big)^{\alpha} * \varpi\Big(h\bar{\kappa}_{e_{n-1}}, h\bar{\kappa}_{e_n}, \frac{\varrho}{\tau}\Big)^{\beta} * \varpi\Big(h\bar{\lambda}_{e_{n-1}}, h\bar{\lambda}_{e_n}, \frac{\varrho}{\tau}\Big)^{\gamma}\Big) \\
&* \Big(\varpi\Big(h\bar{\iota}_{e_{n-1}}, h\bar{\iota}_{e_n}, \frac{\varrho}{\tau}\Big)^{\gamma} * \varpi\Big(h\bar{\kappa}_{e_{n-1}}, h\bar{\kappa}_{e_n}, \frac{\varrho}{\tau}\Big)^{\alpha} * \varpi\Big(h\bar{\lambda}_{e_{n-1}}, h\bar{\lambda}_{e_n}, \frac{\varrho}{\tau}\Big)^{\beta}\Big) \\
&* \Big(\varpi\Big(h\bar{\iota}_{e_{n-1}}, h\bar{\iota}_{e_n}, \frac{\varrho}{\tau}\Big)^{\beta} * \varpi\Big(h\bar{\kappa}_{e_{n-1}}, h\bar{\kappa}_{e_n}, \frac{\varrho}{\tau}\Big)^{\gamma} * \varpi\Big(h\bar{\lambda}_{e_{n-1}}, h\bar{\lambda}_{e_n}, \frac{\varrho}{\tau}\Big)^{\alpha}\Big) \\
&= \Big(\varpi\Big(h\bar{\iota}_{e_{n-1}}, h\bar{\iota}_{e_n}, \frac{\varrho}{\tau}\Big)^{\alpha} * \varpi\Big(h\bar{\iota}_{e_{n-1}}, h\bar{\iota}_{e_n}, \frac{\varrho}{\tau}\Big)^{\gamma} * \varpi\Big(h\bar{\iota}_{e_{n-1}}, h\bar{\iota}_{e_n}, \frac{\varrho}{\tau}\Big)^{\beta}\Big) \\
&* \Big(\varpi\Big(h\bar{\kappa}_{e_{n-1}}, h\bar{\kappa}_{e_n}, \frac{\varrho}{\tau}\Big)^{\beta} * \varpi\Big(h\bar{\kappa}_{e_{n-1}}, h\bar{\kappa}_{e_n}, \frac{\varrho}{\tau}\Big)^{\alpha} * \varpi\Big(h\bar{\kappa}_{e_{n-1}}, h\bar{\kappa}_{e_n}, \frac{\varrho}{\tau}\Big)^{\gamma}\Big) \\
&* \Big(\varpi\Big(h\bar{\lambda}_{e_{n-1}}, h\bar{\lambda}_{e_n}, \frac{\varrho}{\tau}\Big)^{\gamma} * \varpi\Big(h\bar{\lambda}_{e_{n-1}}, h\bar{\lambda}_{e_n}, \frac{\varrho}{\tau}\Big)^{\beta} * \varpi\Big(h\bar{\lambda}_{e_{n-1}}, h\bar{\lambda}_{e_n}, \frac{\varrho}{\tau}\Big)^{\alpha}\Big) \\
&\geqslant \Big(\varpi\Big(h\bar{\iota}_{e_{n-1}}, h\bar{\iota}_{e_n}, \frac{\varrho}{\tau}\Big)^{\alpha} . \varpi\Big(h\bar{\iota}_{e_{n-1}}, h\bar{\iota}_{e_n}, \frac{\varrho}{\tau}\Big)^{\gamma} . \varpi\Big(h\bar{\iota}_{e_{n-1}}, h\bar{\iota}_{e_n}, \frac{\varrho}{\tau}\Big)^{\beta}\Big) \\
&* \Big(\varpi\Big(h\bar{\kappa}_{e_{n-1}}, h\bar{\kappa}_{e_n}, \frac{\varrho}{\tau}\Big)^{\beta} . \varpi\Big(h\bar{\kappa}_{e_{n-1}}, h\bar{\kappa}_{e_n}, \frac{\varrho}{\tau}\Big)^{\alpha} . \varpi\Big(h\bar{\kappa}_{e_{n-1}}, h\bar{\kappa}_{e_n}, \frac{\varrho}{\tau}\Big)^{\gamma}\Big) \\
&* \Big(\varpi\Big(h\bar{\lambda}_{e_{n-1}}, h\bar{\lambda}_{e_n}, \frac{\varrho}{\tau}\Big)^{\gamma} . \varpi\Big(h\bar{\lambda}_{e_{n-1}}, h\bar{\lambda}_{e_n}, \frac{\varrho}{\tau}\Big)^{\beta} . \varpi\Big(h\bar{\lambda}_{e_{n-1}}, h\bar{\lambda}_{e_n}, \frac{\varrho}{\tau}\Big)^{\alpha}\Big) \\
&= \varpi\Big(h\bar{\iota}_{e_{n-1}}, h\bar{\iota}_{e_n}, \frac{\varrho}{\tau}\Big)^{\alpha+\beta+\gamma} * \varpi\Big(h\bar{\kappa}_{e_{n-1}}, h\bar{\kappa}_{e_n}, \frac{\varrho}{\tau}\Big)^{\alpha+\beta+\gamma} * \varpi\Big(h\bar{\lambda}_{e_{n-1}}, h\bar{\lambda}_{e_n}, \frac{\varrho}{\tau}\Big)^{\alpha+\beta+\gamma} \\
&\geqslant \varpi\Big(h\bar{\iota}_{e_{n-1}}, h\bar{\iota}_{e_n}, \frac{\varrho}{\tau}\Big) * \varpi\Big(h\bar{\kappa}_{e_{n-1}}, h\bar{\kappa}_{e_n}, \frac{\varrho}{\tau}\Big) * \varpi\Big(h\bar{\lambda}_{e_{n-1}}, h\bar{\lambda}_{e_n}, \frac{\varrho}{\tau}\Big) = \theta_{n-1}\Big(\frac{\varrho}{\tau}\Big).
\end{aligned}$$

Thus, we have

$$\theta_n(\varrho) \geqslant \theta_{n-1}(\frac{\varrho}{\tau}) \geqslant \theta_{n-1}(\varrho) \geqslant \theta_{n-1}(\varrho - \tau\varrho). \tag{7}$$

Similarly, we get $\theta_n(\varrho) \geqslant \theta_{n-1}\big(\frac{\varrho}{\tau}\big) \geqslant \theta_{n-2}\big(\frac{\varrho}{\tau^2}\big) \geqslant \ldots \geqslant \theta_o\big(\frac{\varrho}{\tau^n}\big)$, where $n \geqslant 1$. Thus, we have

$$\lim_{n\to\infty} \theta_n(\varrho) \geqslant \lim_{n\to\infty} \Big(\frac{\varrho}{\tau^n}\Big) = 1 \Rightarrow \lim_{n\to\infty} \theta_n(\varrho) = 1; \tag{8}$$

and

$$\varpi(h\bar{\iota}_{e_n}, h\bar{\iota}_{e_{n+1}}, \varrho), \varpi(h\bar{\kappa}_{e_n}, h\bar{\kappa}_{e_{n+1}}, \varrho), \varpi(h\bar{\lambda}_{e_n}, h\bar{\lambda}_{e_{n+1}}, \varrho) \geqslant \theta_n(\varrho) \geqslant \theta_{n-1}(\varrho - \tau\varrho). \tag{9}$$

Claim that for every $\varrho > 0$, $n, m \geqslant 1$ (10) is satisfied,

$$\varpi(h\bar{\iota}_{e_n}, h\bar{\iota}_{e_{n+m}}, \varrho), \varpi(h\bar{\kappa}_{e_n}, h\bar{\kappa}_{e_{n+m}}, \varrho), \varpi(h\bar{\lambda}_{e_n}, h\bar{\lambda}_{e_{n+m}}, \varrho) \geqslant m * \theta_{n-1}(\varrho - \tau\varrho). \tag{10}$$

If $m = 1$ (10) holds trivially. Now, let (10) holds for some m. From (1), we have

$$\begin{aligned}
\varpi(h\bar{\iota}_{e_{n+1}}, h\bar{\iota}_{e_{n+m+1}}, \tau\varrho) &= \varpi(G(\bar{\iota}_{e_n}, \bar{\kappa}_{e_n}, \bar{\lambda}_{e_n}), G(\bar{\iota}_{e_{n+m}}, \bar{\kappa}_{e_{n+m}}, \bar{\lambda}_{e_{n+m}}), \tau\varrho) \\
&\geqslant \varpi(h\bar{\iota}_{e_n}, h\bar{\iota}_{e_{n+m}}, \varrho)^{\alpha} * \varpi(h\bar{\kappa}_{e_n}, h\bar{\kappa}_{e_{n+m}}, \varrho)^{\beta} * \varpi(h\bar{\lambda}_{e_n}, h\bar{\lambda}_{e_{n+m}}, \varrho)^{\gamma} \\
&\geqslant (m * \theta_{n-1}(\varrho - \tau\varrho))^{\alpha} * (m * \theta_{n-1}(\varrho - \tau\varrho))^{\beta} * (m * \theta_{n-1}(\varrho - \tau\varrho))^{\gamma} \\
&\geqslant (m * \theta_{n-1}(\varrho - \tau\varrho))^{\alpha} . (m * \theta_{n-1}(\varrho - \tau\varrho))^{\beta} . (m * \theta_{n-1}(\varrho - \tau\varrho))^{\gamma} \\
&= (m * \theta_{n-1}(\varrho - \tau\varrho))^{\alpha+\beta+\gamma} = m * \theta_{n-1}(\varrho - \tau\varrho).
\end{aligned}$$

Similarly, we have $\varpi(h\bar{\iota}_{e_{n+1}}, h\bar{\iota}_{e_{n+m+1}}, \tau\varrho), \varpi(h\bar{\kappa}_{e_{n+1}}, h\bar{\kappa}_{e_{n+m+1}}, \tau\varrho), \varpi(h\bar{\lambda}_{e_{n+1}}, h\bar{\lambda}_{e_{n+m+1}}, \tau\varrho) \geqslant m * \theta_{n-1}(\varrho - \tau\varrho)$. Thus, we get

$$\begin{aligned}\varpi(h\bar{\iota}_{e_n}, h\bar{\iota}_{e_{n+m+1}}, \varrho) &= \varpi(h\bar{\iota}_{e_n}, h\bar{\iota}_{e_{n+m+1}}, \varrho - \tau\varrho + \tau\varrho)\\ &\geqslant \varpi(h\bar{\iota}_{e_n}, h\bar{\iota}_{e_{n+1}}, \varrho - \tau\varrho) * \varpi(h\bar{\iota}_{e_{n+1}}, h\bar{\iota}_{e_{n+m+1}}, \tau\varrho)\\ &\geqslant \theta_{n-1}(\varrho - \tau\varrho) * (m * \theta_{n-1}(\varrho - \tau\varrho)) = (m+1) * \theta_{n-1}(\varrho - \tau\varrho).\end{aligned}$$

The similar holds for $\varpi(h\bar{\kappa}_{e_n}, h\bar{\kappa}_{e_{n+m+1}}, \varrho), \varpi(h\bar{\lambda}_{e_n}, h\bar{\lambda}_{e_{n+m+1}}, \varrho)$. Thus, (10) is satisfied.
Claim that $\{h\bar{\iota}_{e_n}\}$ is a Cauchy sequence.
Consider $\varrho > 0$ and $0 < \Sigma < 1$ be given. Then, as $*$ is a H-type t-norm, there exists $\delta \in (0, 1)$, so that $m * \alpha > 1 - \Sigma$ for every $\alpha \in (1 - \delta, 1]$. Thus $\lim_{n\to\infty} \theta_n(\varrho) = 1$, that implies the existence of $m_o \in N$ so that $\theta_n(\varrho - \tau\varrho) > 1 - \delta$ for every $n \geqslant m_o$. By (8), we get

$$\varpi(h\bar{\iota}_{e_n}, h\bar{\iota}_{e_{n+m}}, \varrho), \varpi(h\bar{\kappa}_{e_n}, h\bar{\kappa}_{e_{n+m}}, \varrho), \varpi(h\bar{\lambda}_{e_n}, h\bar{\lambda}_{e_{n+m}}, \varrho) > 1 - \Sigma.$$

Hence, $\{h\bar{\iota}_{e_n}\}$ is Cauchy. On similar lines it can be easily proved that $\{h\bar{\kappa}_{e_n}\}$ and $\{h\bar{\lambda}_{e_n}\}$ are also Cauchy sequences.
Step 3: Claim that h and G possess soft tripled coincidence point. Consider $\bar{\Upsilon}$ is complete, which implies the existence of $\bar{\iota}, \bar{\kappa}, \bar{\lambda} \in \bar{\Upsilon}$ so that $\lim_{n\to\infty} \bar{\iota}_{e_n} = \bar{\iota}$, $\lim_{n\to\infty} \bar{\kappa}_{e_n} = \bar{\kappa}$ and $\lim_{n\to\infty} \bar{\lambda}_{e_n} = \bar{\lambda}$. By the continuity of h, we have $\lim_{n\to\infty} hh\bar{\iota}_{e_n} = h\bar{\iota}$, $\lim_{n\to\infty} hh\bar{\kappa}_{e_n} = h\bar{\kappa}$ and $\lim_{n\to\infty} hh\bar{\lambda}_{e_n} = h\bar{\iota}$. As G and h are commutative, we have $hh\bar{\iota}_{e_{n+1}} = hG(\bar{\iota}_{e_n}, \bar{\kappa}_{e_n}, \bar{\lambda}_{e_n}) = G(\bar{\iota}_{e_{n+1}}, \bar{\kappa}_{e_{n+1}}, \bar{\lambda}_{e_{n+1}})$. From (1) we have

$$\begin{aligned}\varpi(hh\bar{\iota}_{e_n}, G(\bar{\iota}, \bar{\kappa}, \bar{\lambda}), \tau\varrho) &= \varpi(G(h\bar{\iota}_{e_n}, h\bar{\kappa}_{e_n}, h\bar{\lambda}_{e_n}), G(\bar{\iota}, \bar{\kappa}, \bar{\lambda}), \tau\varrho)\\ &\geqslant \varpi(hh\bar{\iota}_{e_n}, h\bar{\iota}, \varrho)^{\alpha} * \varpi(hh\bar{\kappa}_{e_n}, h\bar{\kappa}, \varrho)^{\beta} * \varpi(hh\bar{\lambda}_{e_n}, h\bar{\lambda}, \varrho)^{\gamma}\\ &\geqslant \varpi(hh\bar{\iota}_{e_n}, h\bar{\iota}, \varrho) * \varpi(hh\bar{\kappa}_{e_n}, h\bar{\kappa}, \varrho) * \varpi(hh\bar{\lambda}_{e_n}, h\bar{\lambda}, \varrho).\end{aligned}$$

Taking $n \to \infty$, we have $\lim_{n\to\infty} hh\bar{\iota}_{e_n} = G(\bar{\iota}, \bar{\kappa}, \bar{\lambda})$. Thus $G(\bar{\iota}, \bar{\kappa}, \bar{\lambda}) = h\bar{\iota}$. Similarly, we can prove that $G(\bar{\kappa}, \bar{\lambda}, \bar{\iota}) = h\bar{\kappa}$ and $G(\bar{\lambda}, \bar{\iota}, \bar{\kappa}) = h\bar{\lambda}$, therefore G and h possess $(\bar{\iota}, \bar{\kappa}, \bar{\lambda})$ as their soft tripled coincidence point.
Step 4: Claim that $\bar{\iota} = G(\bar{\lambda}, \bar{\iota}, \bar{\kappa})$, $\bar{\kappa} = G(\bar{\iota}, \bar{\kappa}, \bar{\lambda})$ and $\bar{\lambda} = G(\bar{\kappa}, \bar{\lambda}, \bar{\iota})$. From (1)

$$\begin{aligned}\varpi(h\bar{\iota}, h\bar{\kappa}_{e_{n+1}}, \tau\varrho) &= \varpi(G(\bar{\iota}, \bar{\kappa}, \bar{\lambda}), G(\bar{\kappa}_{e_n}, \bar{\lambda}_{e_n}, \bar{\iota}_{e_n}), \tau\varrho)\\ &\geqslant \varpi(h\bar{\iota}, h\bar{\kappa}_{e_n}, \varrho)^{\alpha} * \varpi(h\bar{\kappa}, h\bar{\lambda}_{e_n}, \varrho)^{\beta} * \varpi(h\bar{\lambda}, h\bar{\iota}_{e_n}, \varrho)^{\gamma};\end{aligned} \tag{11}$$

$$\begin{aligned}\varpi(h\bar{\kappa}, h\bar{\lambda}_{e_{n+1}}, \tau\varrho) &= \varpi(G(\bar{\kappa}, \bar{\lambda}, \bar{\iota}), G(\bar{\lambda}_{e_n}, \bar{\iota}_{e_n}, \bar{\kappa}_{e_n}), \tau\varrho)\\ &\geqslant \varpi(h\bar{\kappa}, h\bar{\lambda}_{e_n}, \varrho)^{\alpha} * \varpi(h\bar{\lambda}, h\bar{\iota}_{e_n}, \varrho)^{\beta} * \varpi(h\bar{\iota}, h\bar{\kappa}_{e_n}, \varrho)^{\gamma};\end{aligned} \tag{12}$$

$$\begin{aligned}\varpi(h\bar{\lambda}, h\bar{\iota}_{e_{n+1}}, \tau\varrho) &= \varpi(G(\bar{\lambda}, \bar{\iota}, \bar{\kappa}), G(\bar{\iota}_{e_n}, \bar{\kappa}_{e_n}, \bar{\lambda}_{e_n}), \tau\varrho)\\ &\geqslant \varpi(h\bar{\lambda}, h\bar{\iota}_{e_n}, \varrho)^{\alpha} * \varpi(h\bar{\iota}, h\bar{\kappa}_{e_n}, \varrho)^{\beta} * \varpi(h\bar{\kappa}, h\bar{\lambda}_{e_n}, \varrho)^{\gamma}.\end{aligned} \tag{13}$$

Consider $\Pi_n(\varrho) = \varpi(h\bar{\iota}, h\bar{\kappa}_{e_n}, \varrho) * \varpi(h\bar{\kappa}, h\bar{\lambda}_{e_n}, \varrho) * \varpi(h\bar{\lambda}, h\bar{\iota}_{e_n}, \varrho)$ for every $\varrho > 0$ and $n \geqslant 0$. Then, we have

$$\begin{aligned}\Pi_{n+1}(\tau\varrho) &= \varpi(h\bar{\iota}, h\bar{\kappa}_{e_{n+1}}, \tau\varrho) * \varpi(h\bar{\kappa}, h\bar{\lambda}_{e_{n+1}}, \tau\varrho) * \varpi(h\bar{\lambda}, h\bar{\iota}_{e_{n+1}}, \tau\varrho)\\ &\geqslant (\varpi(h\bar{\iota}, h\bar{\kappa}_{e_n}, \varrho)^{\alpha} * \varpi(h\bar{\kappa}, h\bar{\lambda}_{e_n}, \varrho)^{\beta} * \varpi(h\bar{\lambda}, h\bar{\iota}_{e_n}, \varrho)^{\gamma})\\ &* (\varpi(h\bar{\kappa}, h\bar{\lambda}_{e_n}, \varrho)^{\alpha} * \varpi(h\bar{\lambda}, h\bar{\iota}_{e_n}, \varrho)^{\beta} * \varpi(h\bar{\iota}, h\bar{\kappa}_{e_n}, \varrho)^{\gamma})\\ &* (\varpi(h\bar{\lambda}, h\bar{\iota}_{e_n}, \varrho)^{\alpha} * \varpi(h\bar{\iota}, h\bar{\kappa}_{e_n}, \varrho)^{\beta} * \varpi(h\bar{\kappa}, h\bar{\lambda}_{e_n}, \varrho)^{\gamma}) =\end{aligned}$$

$$
\begin{aligned}
&= (\varpi(h\bar{\iota}, h\bar{\kappa}_{e_n}, \varrho)^{\alpha} * \varpi(h\bar{\iota}, h\bar{\kappa}_{e_n}, \varrho)^{\gamma} * \varpi(h\bar{\iota}, h\bar{\kappa}_{e_n}, \varrho)^{\beta}) \\
&* (\varpi(h\bar{\kappa}, h\bar{\lambda}_{e_n}, \varrho)^{\beta} * \varpi(h\bar{\kappa}, h\bar{\lambda}_{e_n}, \varrho)^{\alpha} * \varpi(h\bar{\kappa}, h\bar{\lambda}_{e_n}, \varrho)^{\gamma}) \\
&* (\varpi(h\bar{\lambda}, h\bar{\iota}_{e_n}, \varrho)^{\gamma} * \varpi(h\bar{\lambda}, h\bar{\iota}_{e_n}, \varrho)^{\beta} * \varpi(h\bar{\lambda}, h\bar{\iota}_{e_n}, \varrho)^{\alpha}) \\
&\geqslant (\varpi(h\bar{\iota}, h\bar{\kappa}_{e_n}, \varrho)^{\alpha}.\varpi(h\bar{\iota}, h\bar{\kappa}_{e_n}, \varrho)^{\gamma}.\varpi(h\bar{\iota}, h\bar{\kappa}_{e_n}, \varrho)^{\beta}) \\
&* (\varpi(h\bar{\kappa}, h\bar{\lambda}_{e_n}, \varrho)^{\beta}.\varpi(h\bar{\kappa}, h\bar{\lambda}_{e_n}, \varrho)^{\alpha}.\varpi(h\bar{\kappa}, h\bar{\lambda}_{e_n}, \varrho)^{\gamma}) \\
&* (\varpi(h\bar{\lambda}, h\bar{\iota}_{e_n}, \varrho)^{\gamma}.\varpi(h\bar{\lambda}, h\bar{\iota}_{e_n}, \varrho)^{\beta}.\varpi(h\bar{\lambda}, h\bar{\iota}_{e_n}, \varrho)^{\alpha}) \\
&= \varpi(h\bar{\iota}, h\bar{\kappa}_{e_n}, \varrho)^{\alpha+\beta+\gamma} * \varpi(h\bar{\kappa}, h\bar{\lambda}_{e_n}, \varrho)^{\alpha+\beta+\gamma} * (\varpi(h\bar{\lambda}, h\bar{\iota}_{e_n}, \varrho)^{\alpha+\beta+\gamma} \\
&\geqslant \varpi(h\bar{\iota}, h\bar{\kappa}_{e_n}, \varrho) * \varpi(h\bar{\kappa}, h\bar{\lambda}_{e_n}, \varrho) * \varpi(h\bar{\lambda}, h\bar{\iota}_{e_n}, \varrho) = \Pi_n(\varrho).
\end{aligned}
$$

Hence, $\Pi_{n+1}(\tau\varrho) \geqslant \Pi_n(\varrho)$ for every $n \geqslant 0$ and $\varrho > 0$. Thus, we have (14) for every $\varrho > 0$ and $n \geqslant 1$,

$$\Pi_n(\varrho) \geqslant \Pi_{n-1}\Big(\frac{\varrho}{\tau}\Big) \geqslant \Pi_{n-2}\Big(\frac{\varrho}{\tau^2}\Big) \geqslant \ldots \geqslant \Pi_o\Big(\frac{\varrho}{\tau^n}\Big). \tag{14}$$

Therefore, we get

$$\varpi(h\bar{\iota}, h\bar{\kappa}_{e_{n+1}}, \tau\varrho) \geqslant \varpi(h\bar{\iota}, h\bar{\kappa}_{e_n}, \varrho)^{\alpha} * \varpi(h\bar{\kappa}, h\bar{\lambda}_{e_n}, \varrho)^{\beta} * \varpi(h\bar{\lambda}, h\bar{\iota}_{e_n}, \varrho)^{\gamma} \geqslant \Pi_n(\varrho) \geqslant \Pi_o\Big(\frac{\varrho}{\tau^n}\Big); \tag{15}$$

$$\varpi(h\bar{\kappa}, h\bar{\lambda}_{e_{n+1}}, \tau\varrho) \geqslant \varpi(h\bar{\kappa}, h\bar{\lambda}_{e_n}, \varrho)^{\alpha} * \varpi(h\bar{\lambda}, h\bar{\iota}_{e_n}, \varrho)^{\beta} * \varpi(h\bar{\iota}, h\bar{\kappa}_{e_n}, \varrho)^{\gamma} \geqslant \Pi_n(\varrho) \geqslant \Pi_o\Big(\frac{\varrho}{\tau^n}\Big); \tag{16}$$

$$\varpi(h\bar{\lambda}, h\bar{\iota}_{e_{n+1}}, \tau\varrho) \geqslant \varpi(h\bar{\lambda}, h\bar{\iota}_{e_n}, \varrho)^{\alpha} * \varpi(h\bar{\iota}, h\bar{\kappa}_{e_n}, \varrho)^{\beta} * \varpi(h\bar{\kappa}, h\bar{\lambda}_{e_n}, \varrho)^{\gamma} \geqslant \Pi_n(\varrho) \geqslant \Pi_o\Big(\frac{\varrho}{\tau^n}\Big). \tag{17}$$

So, $\varpi(h\bar{\iota}, h\bar{\kappa}_{e_{n+1}}, \tau\varrho), \varpi(h\bar{\kappa}, h\bar{\lambda}_{e_{n+1}}, \tau\varrho), \varpi(h\bar{\lambda}, h\bar{\iota}_{e_{n+1}}, \tau\varrho) \geqslant \Pi_o\big(\frac{\varrho}{\tau^n}\big)$ for every $\varrho > 0$ and $n \geqslant 1$. As $\lim\limits_{n\to\infty} \Pi_o\big(\frac{\varrho}{\tau^n}\big) = 1$, we have $\lim\limits_{n\to\infty} h\bar{\iota}_{e_n} = h\bar{\lambda}$ $\lim\limits_{n\to\infty} h\bar{\kappa}_{e_n} = h\bar{\iota}$ and $\lim\limits_{n\to\infty} h\bar{\lambda}_{e_n} = h\bar{\kappa}$. Thus, we get

$$
\begin{aligned}
G(\bar{\iota}, \bar{\kappa}, \bar{\lambda}) &= h\bar{\iota} = \lim_{n\to\infty} h\bar{\kappa}_{e_n} = \bar{\kappa}, \\
G(\bar{\kappa}, \bar{\lambda}, \bar{\iota}) &= h\bar{\kappa} = \lim_{n\to\infty} h\bar{\lambda}_{e_n} = \bar{\lambda}, \\
G(\bar{\lambda}, \bar{\iota}, \bar{\kappa}) &= h\bar{\lambda} = \lim_{n\to\infty} h\bar{\iota}_{e_n} = \bar{\iota}.
\end{aligned} \tag{18}
$$

Step 5: Claim that $\bar{\iota} = \bar{\kappa} = \bar{\lambda}$. Consider $\Phi(\varrho) = \varpi(\bar{\iota}, \bar{\kappa}, \varrho) * \varpi(\bar{\kappa}, \bar{\lambda}, \varrho) * \varpi(\bar{\lambda}, \bar{\iota}, \varrho)$ for every $\varrho > 0$. Then, by (1)

$$
\begin{aligned}
\varpi(\bar{\iota}, \bar{\kappa}, \tau\varrho) &= \varpi(G(\bar{\iota}, \bar{\kappa}, \bar{\lambda}), G(\bar{\kappa}, \bar{\lambda}, \bar{\iota}), \tau\varrho) \\
&\geqslant \varpi(h\bar{\iota}, h\bar{\kappa}, \varrho)^{\alpha} * \varpi(h\bar{\kappa}, h\bar{\lambda}, \varrho)^{\beta} * \varpi(h\bar{\lambda}, h\bar{\iota}, \varrho)^{\gamma} \\
&= \varpi(\bar{\kappa}, \bar{\lambda}, \varrho)^{\alpha} * \varpi(\bar{\lambda}, \bar{\iota}, \varrho)^{\beta} * \varpi(\bar{\iota}, \bar{\kappa}, \varrho)^{\gamma};
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
\varpi(\bar{\kappa}, \bar{\lambda}, \tau\varrho) &= \varpi(G(\bar{\kappa}, \bar{\lambda}, \bar{\iota}), G(\bar{\lambda}, \bar{\iota}, \bar{\kappa}), \tau\varrho) \\
&\geqslant \varpi(h\bar{\kappa}, h\bar{\lambda}, \varrho)^{\alpha} * \varpi(h\bar{\lambda}, h\bar{\iota}, \varrho)^{\beta} * \varpi(h\bar{\iota}, h\bar{\kappa}, \varrho)^{\gamma} \\
&= \varpi(\bar{\lambda}, \bar{\iota}, \varrho)^{\alpha} * \varpi(\bar{\iota}, \bar{\kappa}, \varrho)^{\beta} * \varpi(\bar{\kappa}, \bar{\lambda}, \varrho)^{\gamma};
\end{aligned} \tag{20}
$$

$$
\begin{aligned}
\varpi(\bar{\lambda}, \bar{\iota}, \tau\varrho) &= \varpi(G(\bar{\lambda}, \bar{\iota}, \bar{\kappa}), G(\bar{\iota}, \bar{\kappa}, \bar{\lambda}), \tau\varrho) \\
&\geqslant \varpi(h\bar{\lambda}, h\bar{\iota}, \varrho)^{\alpha} * \varpi(h\bar{\iota}, h\bar{\kappa}, \varrho)^{\beta} * \varpi(h\bar{\kappa}, h\bar{\lambda}, \varrho)^{\gamma} \\
&= \varpi(\bar{\iota}, \bar{\kappa}, \varrho)^{\alpha} * \varpi(\bar{\kappa}, \bar{\lambda}, \varrho)^{\beta} * \varpi(\bar{\lambda}, \bar{\iota}, \varrho)^{\gamma}.
\end{aligned} \tag{21}
$$

From (19), (20) and (21) we have

$$\begin{aligned}
\Phi(\tau\varrho) &= \varpi(\bar{\iota},\bar{\kappa},\tau\varrho) * \varpi(\bar{\kappa},\bar{\lambda},\tau\varrho) * \varpi(\bar{\lambda},\bar{\iota},\tau\varrho) \\
&\geqslant (\varpi(\bar{\kappa},\bar{\lambda},\varrho)^{\alpha} * \varpi(\bar{\lambda},\bar{\iota},\varrho)^{\beta} * \varpi(\bar{\iota},\bar{\kappa},\varrho)^{\gamma}) \\
&* (\varpi(\bar{\lambda},\bar{\iota},\varrho)^{\alpha} * \varpi(\bar{\iota},\bar{\kappa},\varrho)^{\beta} * \varpi(\bar{\kappa},\bar{\lambda},\varrho)^{\gamma}) \\
&* (\varpi(\bar{\iota},\bar{\kappa},\varrho)^{\alpha} * \varpi(\bar{\kappa},\bar{\lambda},\varrho)^{\beta} * \varpi(\bar{\lambda},\bar{\iota},\varrho)^{\gamma}) \\
&= (\varpi(\bar{\iota},\bar{\kappa},\varrho)^{\gamma} * \varpi(\bar{\iota},\bar{\kappa},\varrho)^{\beta} * \varpi(\bar{\iota},\bar{\kappa},\varrho)^{\alpha}) \\
&* (\varpi(\bar{\kappa},\bar{\lambda},\varrho)^{\alpha} * \varpi(\bar{\kappa},\bar{\lambda},\varrho)^{\gamma} * \varpi(\bar{\kappa},\bar{\lambda},\varrho)^{\beta}) \\
&* (\varpi(\bar{\lambda},\bar{\iota},\varrho)^{\beta} * \varpi(\bar{\lambda},\bar{\iota},\varrho)^{\alpha} * \varpi(\bar{\lambda},\bar{\iota},\varrho)^{\gamma}) \\
&\geqslant (\varpi(\bar{\iota},\bar{\kappa},\varrho)^{\gamma}.\varpi(\bar{\iota},\bar{\kappa},\varrho)^{\beta}.\varpi(\bar{\iota},\bar{\kappa},\varrho)^{\alpha}) \\
&* (\varpi(\bar{\kappa},\bar{\lambda},\varrho)^{\alpha}.\varpi(\bar{\kappa},\bar{\lambda},\varrho)^{\gamma}.\varpi(\bar{\kappa},\bar{\lambda},\varrho)^{\beta}) \\
&* (\varpi(\bar{\lambda},\bar{\iota},\varrho)^{\beta}.\varpi(\bar{\lambda},\bar{\iota},\varrho)^{\alpha}.\varpi(\bar{\lambda},\bar{\iota},\varrho)^{\gamma}) \\
&= \varpi(\bar{\iota},\bar{\kappa},\varrho)^{\alpha+\beta+\gamma} * \varpi(\bar{\kappa},\bar{\lambda},\varrho)^{\alpha+\beta+\gamma} * \varpi(\bar{\lambda},\bar{\iota},\varrho)^{\alpha+\beta+\gamma} \\
&\geqslant \varpi(\bar{\iota},\bar{\kappa},\varrho) * \varpi(\bar{\kappa},\bar{\lambda},\varrho) * \varpi(\bar{\lambda},\bar{\iota},\varrho) = \Phi(\varrho).
\end{aligned}$$

Thus, $\Phi(\tau\varrho) \geqslant \Phi(\varrho)$ that implies $\Phi(\varrho) \geqslant \Phi\left(\frac{\varrho}{\tau}\right) \geqslant \Phi\left(\frac{\varrho}{\tau^2}\right) \geqslant \ldots \geqslant \Phi\left(\frac{\varrho}{\tau^n}\right)$. Hence,

$$\begin{aligned}
\varpi(\bar{\iota},\bar{\kappa},\tau\varrho) &\geqslant \varpi(\bar{\kappa},\bar{\lambda},\varrho)^{\alpha} * \varpi(\bar{\lambda},\bar{\iota},\varrho)^{\beta} * \varpi(\bar{\iota},\bar{\kappa},\varrho)^{\gamma} \\
&\geqslant \varpi(\bar{\kappa},\bar{\lambda},\varrho) * \varpi(\bar{\lambda},\bar{\iota},\varrho) * \varpi(\bar{\iota},\bar{\kappa},\varrho) = \Phi(\varrho) \geqslant \Phi\left(\frac{\varrho}{\tau^n}\right);
\end{aligned}$$

$$\begin{aligned}
\varpi(\bar{\kappa},\bar{\lambda},\tau\varrho) &\geqslant \varpi(\bar{\lambda},\bar{\iota},\varrho)^{\alpha} * \varpi(\bar{\iota},\bar{\kappa},\varrho)^{\beta} * \varpi(\bar{\kappa},\bar{\lambda},\varrho)^{\gamma} \\
&\geqslant \varpi(\bar{\lambda},\bar{\iota},\varrho) * \varpi(\bar{\iota},\bar{\kappa},\varrho) * \varpi(\bar{\kappa},\bar{\lambda},\varrho) = \Phi(\varrho) \geqslant \Phi\left(\frac{\varrho}{\tau^n}\right);
\end{aligned}$$

$$\begin{aligned}
\varpi(\bar{\lambda},\bar{\iota},\tau\varrho) &\geqslant \varpi(\bar{\iota},\bar{\kappa},\varrho)^{\alpha} * \varpi(\bar{\kappa},\bar{\lambda},\varrho)^{\beta} * \varpi(\bar{\lambda},\bar{\iota},\varrho)^{\gamma} \\
&\geqslant \varpi(\bar{\iota},\bar{\kappa},\varrho) * \varpi(\bar{\kappa},\bar{\lambda},\varrho) * \varpi(\bar{\lambda},\bar{\iota},\varrho) = \Phi(\varrho) \geqslant \Phi\left(\frac{\varrho}{\tau^n}\right).
\end{aligned}$$

Taking $n \to \infty$, we get $\lim_{n\to\infty} \Phi\left(\frac{\varrho}{\tau^n}\right) = 1$, that implies $\varpi(\bar{\iota},\bar{\kappa},\tau\varrho) = \varpi(\bar{\kappa},\bar{\lambda},\tau\varrho) = \varpi(\bar{\lambda},\bar{\iota},\tau\varrho) = 1$ for every $\varrho > 0$, i.e. $\bar{\iota} = \bar{\kappa} = \bar{\lambda}$. Hence done. □

Now, we will give an example to show the validity of Theorem (2.1).

Example 2.1. *Let $\bar{\Upsilon} = R$ and $(\bar{\Upsilon},\mu)$ be a SMS under t-norm $*$ defined as $\iota * \kappa = \min\{\iota,\kappa\}$. Define SFM, $\varpi : SP(\bar{\Upsilon}) \times SP(\bar{\Upsilon}) \times (0,\infty) \to [0,1]$ as*

$$\varpi(\bar{\iota},\bar{\kappa},\varrho) = \begin{cases} e^{\frac{-\mu(\bar{\iota},\bar{\kappa})}{\varrho}} & \text{for } \varrho \neq 0 \\ 0 & \text{for } \varrho = 0 \end{cases}.$$

Consider $a, b > 0$ and $0 < \tau < 1$ so that $6a \leqslant b\tau$. Now, define $G : R\times R\times R \to R$ and $h : R \to R$ as $G(\bar{\iota},\bar{\kappa},\bar{\lambda}) = a(\bar{\iota} - \bar{\kappa})$ and $h\bar{\iota} = b\bar{\iota}$ for every $\bar{\iota},\bar{\kappa},\bar{\lambda} \in \bar{\Upsilon}$. It is easy to show that h is continuous, *G and h are commuting and* $G(R\times R\times R)=R=h(R)$. *Also,*

$$\begin{aligned}\varpi(G(\bar\iota,\bar\kappa,\bar\lambda),G(\bar e,\bar f,\bar g),\tau\varrho)&=\left(e^{|(\bar\iota-\bar e)+(\bar f-\bar\kappa)|}\right)^{\frac{a}{\tau\varrho}}\geqslant\left(e^{\frac{-2\max(|\bar\iota-\bar\alpha|,|\bar\beta-\bar\kappa|)}{\varrho}}\right)^{\frac{a}{\tau}}\\&\geqslant\left(e^{\frac{-2\max(|\bar\iota-\bar\alpha|,|\bar\beta-\bar\kappa|)}{\varrho}}\right)^{\frac{b}{6}}=\left(e^{\frac{-b}{3\varrho}}\right)^{\max(|\bar\iota-\bar\alpha|,|\bar\beta-\bar\kappa|)}\\&=\min\left(e^{\frac{-b|\bar\iota-\bar\alpha|}{3\varrho}},e^{\frac{-b|\bar\beta-\bar\kappa|}{3\varrho}}\right)=\min\left(e^{\frac{-|b\bar\iota-b\bar\alpha|}{3\varrho}},e^{\frac{-|b\bar\kappa-b\bar\beta|}{3\varrho}},e^{\frac{-|b\bar\lambda-b\bar\gamma|}{3\varrho}}\right)\\&=\min([\varpi(h\bar\iota,h\bar\alpha,\varrho)]^{\frac13},[\varpi(h\bar\kappa,h\bar\beta,\varrho)]^{\frac13},[\varpi(h\bar\lambda,h\bar\gamma,\varrho)]^{\frac13}).\end{aligned}$$

Hence, all conditions of Theorem (2.1) are satisfied that implies the existence of soft tripled coincidence point of G and h.

Now, we are going to prove our next Theorem.

Theorem 2.2. *Consider* $(\bar\Upsilon,\mu)$ *be a complete SMS and let* $G:\bar\Upsilon\times\bar\Upsilon\times\bar\Upsilon\to\bar\Upsilon$ *and* $h:\bar\Upsilon\to\bar\Upsilon$ *be two functions so that* $G(\bar\Upsilon\times\bar\Upsilon\times\bar\Upsilon)\subseteq h(\bar\Upsilon)$ *with h being continuous and commuting with G. Let G and h satisfies the following assertions for every* $\bar\iota,\bar\kappa,\bar\lambda,\bar e,\bar f,\bar g\in\bar\Upsilon$,

i. for some $0<\tau<1$, *we have* $\mu(G(\bar\iota,\bar\kappa,\bar\lambda),G(\bar e,\bar f,\bar g))\leqslant\tau max(\mu(h\bar\iota,h\bar e),\mu(h\bar\kappa,h\bar f),\mu(h\bar\lambda,h\bar g))$;

ii. for some $0<\tau<1$ *and* $0\leqslant a,b,c\leqslant\frac13$, *we have* $\mu(G(\bar\iota,\bar\kappa,\bar\lambda),G(\bar e,\bar f,\bar g))\leqslant a\mu(h\bar\iota,h\bar e)+b\mu(h\bar\kappa,h\bar f)+c\mu(h\bar\lambda,h\bar g)$;

iii. for some $0<a,b,c<1$ *and* $a+b+c<1$, *we have* $\mu(G(\bar\iota,\bar\kappa,\bar\lambda),G(\bar e,\bar f,\bar g))\leqslant a\mu(h\bar\iota,h\bar e)+b\mu(h\bar\kappa,h\bar f)+c\mu(h\bar\lambda,h\bar g)$.

Then, this implies the existence of a unique $\bar\iota\in\bar\Upsilon$ *so that* $\bar\iota=h\bar\iota=G(\bar\iota,\bar\iota,\bar\iota)$.

Proof. i. Let

$$\varpi(\bar\iota,\bar\kappa,\varrho)=\begin{cases}0 & for\ \varrho\leqslant\mu(\bar\iota,\bar\kappa)\\1 & for\ \varrho>\mu(\bar\iota,\bar\kappa)\end{cases}.$$

As $(\bar\Upsilon,\mu)$ is complete, then $(\bar\Upsilon,\varpi,\min)$ is a complete SFMS. Consider $\bar\iota,\bar\kappa,\bar\lambda,\bar e,\bar f,\bar g\in\bar\Upsilon$ be fix. Substitute $\alpha=\beta=\gamma=\frac13$ and $*=\min$ in (1). Now, if $\varpi(h\bar\iota,h\bar e,\varrho)=0$ or $\varpi(h\bar\kappa,h\bar f,\varrho)=0$ or $\varpi(h\bar\lambda,h\bar g,\varrho)=0$, then (1) holds trivially. Consider $\varpi(h\bar\iota,h\bar e,\varrho)=1$ or $\varpi(h\bar\kappa,h\bar f,\varrho)=1$ or $\varpi(h\bar\lambda,h\bar g,\varrho)=1$ that implies $\mu(\bar\iota,\bar e)<\varrho$, $\mu(\bar\kappa,\bar f)<\varrho$ and $\mu(h\bar\lambda,h\bar g)<\varrho$. Thus, $\varrho>\max(\mu(h\bar\iota,h\bar e),\mu(h\bar\kappa,h\bar f),\mu(h\bar\lambda,h\bar g))$ and $\tau\varrho>\max(\mu(h\bar\iota,h\bar e),\mu(h\bar\kappa,h\bar f),\mu(h\bar\lambda,h\bar g))\geqslant\mu(G(\bar\iota,\bar\kappa,\bar\lambda),G(\bar e,\bar f,\bar g))$. Therefore, $\varpi(G(\bar\iota,\bar\kappa,\bar\lambda),G(\bar e,\bar f,\bar g),\tau\varrho)=1$.

(ii) Now, we have

$$\begin{aligned}\mu(G(\bar\iota,\bar\kappa,\bar\lambda),G(\bar e,\bar f,\bar g))&\leqslant\tau(a\mu(h\bar\iota,h\bar e)+b\mu(h\bar\kappa,h\bar f)+c\mu(h\bar\lambda,h\bar g))\\&<\tau\Big(\frac13\mu(h\bar\iota,h\bar e)+\frac13\mu(h\bar\kappa,h\bar f)+\frac13\mu(h\bar\lambda,h\bar g)\Big)\\&=\frac{\tau}{3}(\mu(h\bar\iota,h\bar e)+\mu(h\bar\kappa,h\bar f)+\mu(h\bar\lambda,h\bar g))\\&\leqslant\frac{\tau}{3}\times3\max(\mu(h\bar\iota,h\bar e),\mu(h\bar\kappa,h\bar f),\mu(h\bar\lambda,h\bar g)).\end{aligned}$$

(iii) Let $\tau=a+b+c<1$, then

$$
\begin{aligned}
\mu(G(\bar{\iota},\bar{\kappa},\bar{\lambda}),G(\bar{e},\bar{f},\bar{g})) &\leqslant a\mu(h\bar{\iota},h\bar{e})+b\mu(h\bar{\kappa},h\bar{f})+c\mu(h\bar{\lambda},h\bar{g})\\
&\leqslant a\max(\mu(h\bar{\iota},h\bar{e}),\mu(h\bar{\kappa},h\bar{f}),\mu(h\bar{\lambda},h\bar{g}))\\
&+b\max(\mu(h\bar{\iota},h\bar{e}),\mu(h\bar{\kappa},h\bar{f}),\mu(h\bar{\lambda},h\bar{g}))\\
&+c\max(\mu(h\bar{\iota},h\bar{e}),\mu(h\bar{\kappa},h\bar{f}),\mu(h\bar{\lambda},h\bar{g}))\\
&=(a+b+c)max(\mu(h\bar{\iota},h\bar{e}),\mu(h\bar{\kappa},h\bar{f}),\mu(h\bar{\lambda},h\bar{g}))\\
&=\tau\max(\mu(h\bar{\iota},h\bar{e}),\mu(h\bar{\kappa},h\bar{f}),\mu(h\bar{\lambda},h\bar{g})).
\end{aligned}
$$
□

Following is an example to show the validity of Theorem (2.2).

Example 2.2. *Consider* $\bar{\Upsilon}=R$, $\mu(\bar{\iota},\bar{\kappa})=\mid\bar{\iota}-\bar{\kappa}\mid$ *for every* $\bar{\iota},\bar{\kappa}\in\bar{\Upsilon}$ *and* $p,q,r,s\in R$ *be so that* $L>\mid p\mid+\mid q\mid+\mid r\mid$. *Define maps* $G:\bar{\Upsilon}\times\bar{\Upsilon}\times\bar{\Upsilon}\to\bar{\Upsilon}$ *and* $h:\bar{\Upsilon}\to\bar{\Upsilon}$ *as* $G(\bar{\iota},\bar{\kappa},\bar{\lambda})=\dfrac{p\bar{\iota}+q\bar{\kappa}+r\bar{\lambda}+s}{L}$ *and* $h\bar{\iota}=\bar{\iota}$ *for every* $\bar{\iota},\bar{\kappa},\bar{\lambda}\in R$. *Then, it is trivial that part (iii) of Theorem (2.2) is satisfied and* G *and* h *possess* $(\bar{\iota}_{e_o},\bar{\iota}_{e_o},\bar{\iota}_{e_o})$ *as their unique soft tripled coincidence point, where* $\bar{\iota}_{e_o}=\dfrac{s}{(L-p-q-r)}$.

3. Application

Now, we are giving an application to our newly developed results in finding the solution of an Integral system:
Consider $\bar{p},\bar{q}\in R$ where $\bar{p}<\bar{q}$ and $\eth=[\bar{p},\bar{q}]$. Let $\bar{\Upsilon}=\Phi^1(\eth)$, where $\mu(M,N)=\int_{\eth}\mid M(\varrho)-N(\varrho)\mid d\varrho$, where $\int_{\eth}$ is the Lebesgue integral.
Then, $(\Phi^1(\eth),\mu)$ is a complete SMS. Consider $\tau,\eta_1,\eta_2,\eta_3\in R$ and $H:R\times R\times R\to R$ a map defined as follows, for every $(\bar{\iota}_{e_1},\bar{\iota}_{e_2},\bar{\iota}_{e_3}),(\bar{\kappa}_{e_1},\bar{\kappa}_{e_2},\bar{\kappa}_{e_3})\in R\times R\times R$;

$$
\mid H(\bar{\iota}_{e_1},\bar{\iota}_{e_2},\bar{\iota}_{e_3})-H(\bar{\kappa}_{e_1},\bar{\kappa}_{e_2},\bar{\kappa}_{e_3})\mid\leqslant\tau\sum_{i=1}^{3}\eta_i\mid\bar{\iota}_{e_i}-\bar{\kappa}_{e_i}\mid.
$$

Claim that for $\Delta\in R$, there exists $M_1,M_2,M_3\in\Phi^1(\eth)$ so that (22) holds for every $\bar{\iota}\in\eth$, $j=1,2,3$,

$$
M_j(\bar{\iota})=\Delta+\int_{[\bar{p},\bar{\iota}]}H(\eta_j(\varrho),\eta_{j+1}(\varrho),\eta_{j+2}(\varrho))d\varrho. \tag{22}
$$

Now, for every $M_1,M_2,M_3\in\Phi^1(\eth)$ and $\bar{\iota}\in\eth$, define

$$
G(M_1,M_2,M_3)(\bar{\iota})=\Delta+\int_{[\bar{p},\bar{\iota}]}H(M_1(\varrho),M_2(\varrho),M_3(\varrho))d\varrho.
$$

Hence, $G(M_1,M_2,M_3)\in\Phi^1(\eth)$ so that $G:\Phi^1(\eth)\times\Phi^1(\eth)\times\Phi^1(\eth)\to\Phi^1(\eth)$ is well defined. And also we have,

$$
\begin{aligned}
\mu(G(M_1,M_2,M_3),G(N_1,N_2,N_3)) &= \int_{\eth}\mid G(M_1,M_2,M_3)(\bar{\iota})-G(N_1,N_2,N_3)(\bar{\iota})\mid d\bar{\iota}\leqslant\\
&\leqslant\int_{\eth}\left(\int_{[\bar{p},\bar{\iota}]}\mid H(M_1(\varrho),M_2(\varrho),M_3(\varrho))-H(N_1(\varrho),N_2(\varrho),N_3(\varrho))\mid d\varrho\right)d\bar{\iota}\leqslant\\
&\leqslant\int_{\eth}\left(\int_{[\bar{p},\bar{\iota}]}\tau\sum_{j=1}^{3}\eta_j\mid M_j(\varrho)-N_j(\varrho)\mid d\varrho\right)d\bar{\iota}\leqslant
\end{aligned}
$$

$$\leqslant \tau \sum_{j=1}^{3} \eta_j \int_{\eth} \left(\int_{\eth} \mid M_j(\varrho) - N_j(\varrho) \mid d\varrho \right) d\bar{\iota} =$$
$$= \tau \sum_{j=1}^{3} \eta_j \int_{\eth} \mu(M_j, N_j) d\bar{\iota} = \tau(\bar{q} - \bar{p}) \sum_{j=1}^{3} \eta_j \mu(M_j, N_j).$$

Let $\Lambda = \tau(\bar{q} - \bar{p})(\eta_1 + \eta_2 + \eta_3) < 1$, then G satisfies (1) for every $M \in \Phi^1(\eth)$. Then, integral equation (22) possess a unique solution of the form (M_o, M_o, M_o), where $M_o \in \Phi^1(\eth)$ i.e.

$$M_o(\bar{\iota}) = \Delta + \int_{[\bar{p}, \bar{\iota}]} H(M_o(\varrho), M_o(\varrho), M_o(\varrho)) d\varrho.$$

Conclusion

In this work, we have given new FPT's for the existence and uniqueness of soft tripled coincidence point in SFMS. To support our new results, we have given examples along with an application to show the existence of a solution to a Lebesgue Integral system. These new results can also be extended and formulated to other new spaces together with the development of other new results.

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Мягкие тройные теоремы о фиксированной точке совпадения в мягком нечетком метрическом пространстве

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Аннотация. В этой работе мы собираемся доказать теоремы о неподвижной точке (FPT) для существования и единственности мягкой тройной точки совпадения для сжимающих отображений в условиях мягкого нечеткого метрического пространства (SFMS). Мы также дали приложение нашим новым результатам по нахождению решения интегрального уравнения.

Ключевые слова: мягкое множество, мягкое тройное совпадение, контракции, фиксированная точка.