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BINARY MIXTURE CONVECTION IN A HORIZONTAL CHANNEL UNDER THE SORET EFFECT ACTION

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Abstract. The mathematical model describing stationary flow of binary liquid mixture is under consideration. The exact solution of the equations of convective heat and mass transfer is constructed for the special form of the functions of temperature and concentration. All possible statements of boundary value problems are studied in order to analyse thermal diffusion separation of binary mixture in a long horizontal channel with rigid walls. It is discovered that only one of the statements (at inhomogeneous heating of both rigid walls) leads to feasible results. With the help of the constructed solution for the mentioned boundary value problem the effect of layer thickness, given flow rate and action of gravity force on the separation process, is analysed.

1. Introduction

Convective motion of liquids is a common phenomenon in nature and the basic mechanism of heat and mass transfer. It should be noted that the convection generated by heat changes only differs from convection induced by joint temperature and concentration inhomogeneities. Firstly, concentration gradient leads to appearance of one more component of the convective force. Secondly, a competition of dissipative effects (heat conductivity and diffusion) can affect the motion of liquid. And finally, there are reciprocal Dufour and Soret effects (diffusive thermal conductivity and thermodiffusion, correspondingly) in mixtures which influence heat and mass transfer. According to [1] the Dufour effect is weak in liquid mixtures. At the same time, due to the Soret effect, the change of temperature on some degrees gives variations in concentration by a few percent. That is why the Soret effect is taken into account in the mathematical model of convective and molecular mass transfer while the action of the Dufour effect is usually neglected [1]. The above listed features of the flows of binary mixtures give a reason to assume that the description of such flows forms a separate class of problems in fluid mechanics. The analysis of

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the class allows to understand behavior of binary mixtures and differences between flows of two-component mixture and a pure heat-conducting fluid.

Mathematical model of convective heat and mass transfer in a binary mixture contains the Navier–Stokes equations in the framework of Oberbeck-Boussinesque approximation. It means that the liquid is assumed to be incompressible and the changes of density are taken into account in the term corresponding to buoyancy force only. Furthermore, the density depends on temperature and concentration with respect to linear law. Distribution of temperature and changes of concentration are described by means of equations corresponding to Fourier's and Fick's laws respectively [2], temperature and concentration deviations from constant equilibrium values are assumed to be moderate. The Soret effect is taken into account in the mass transfer equation. The obtained system of convective heat and mass transfer equations is nonlinear in common case. The construction of its exact solution is an interesting problem from the mathematical point of view. It is also impossible to underestimate the role of exact solutions for describing the processes occurring in binary mixtures. The solution in the closed form can provide analysis for the whole complex of parameters of the problem simultaneously whereas the numerical solution is found at given values of all parameters. At the same time, it is clear that the exact solution can be constructed for the simplified problem statements only, while the solution of the problems in two or three dimensional space is possible by means of numerical methods only.

Further we consider the reduced system of the governing equations. The simplifications are described as follows. We deal with unidirectional flows, only the horizontal velocity component is nonzero and depends on the vertical coordinate y only. The fields of pressure, temperature and concentration are the functions of x and y coordinates. Moreover, dependence of temperature and concentration on x is linear. Assumption on such a form of these functions arose due to papers of Ostroumov [3] and Birikh [4]. These authors suggested to describe convective flows in the central part of a long narrow channel by means of linear temperature function with respect to the longitudinal coordinate. It allowed them to find the solution of corresponding system of equations in the closed form. Similar reasoning is used in papers [5,6] at derivation of solution describing advective flow in a horizontal layer. If we consider the governing system in the assumption about unidirectional motion without suggestion on the form of temperature and concentration functions, then the following fact can be revealed. If the concentration and temperature functions are polynomials, then they can be linear, quadratic and cubic functions with respect to x. This is proven in papers [7,8], where compatibility of the described system is investigated. It is shown that the equations of unidirectional stationary and non-stationary flow are solvable if the functions of temperature and concentration are the linear ones with respect to x.

Assuming temperature and concentration are linear functions of x and dealing with unidirectional stationary flow, we can integrate the governing equations in the closed form. In order to describe the possible modes of binary mixture flow, the boundary conditions should be posed. We consider the flow arising in the central part of long narrow channel (see Fig. 1). One of the application of discussed solution and described geometry is modelling of thermal diffusion separation at the design of the set-up for the measurements of thermal diffusion coefficient [9, 10]. The set-up can be heated form up and below or both walls can be thermally insulated. Sometimes, it is more convenient to heat either upper wall or lower one only. Such modes of heating correspond to four types of boundary conditions for the function of temperature. All of them are studied in the present paper. The boundary conditions for all desired functions are described in the next section in details.

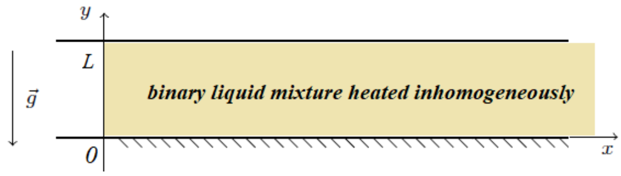


FIGURE 1. The geometry of long narrow channel filled by binary mixture. The rigid walls of channel $y = 0$ and $y = L$ are heated or thermally insulated.

It is necessary to note that in earlier paper of the author [11] the exact solution for the heat and mass transfer equations with Dirichlet boundary conditions for the temperature function is presented without details. Influence of inhomogeneous heating of the both walls on the thermal diffusion separation is analysed. The investigation of action of other factors on the flow is as important as effect of thermal load of walls. In the present paper, we study the influence of flow rate through a cross section of the layer, action of gravity force and change of the layer width on the distribution of components in ethanol-water mixture. We show that the constructed exact solution catches plausible loss of the near-wall effects at the growth of the layer width and intensification of the motion at the increase of the flow rate through the cross section of the layer. Investigation of the gravity force action is useful due to the above described experiment since measurements of thermal diffusion coefficients are being carried out both on the Earth and weightlessness conditions. We show that the constructed solution predicts influence of gravity change on the concentration deviations correctly.

The paper is organized as follows. At first, the governing equations of heat and mass transfer are introduced and their exact solution is constructed if the temperature and concentration functions are linear with respect to longitudinal coordinate. Then, four types of boundary conditions of the temperature function are studied for the description of motion of the mixture in a long horizontal layer. In Section 3, the posed problem is solved for every boundary conditions type, all characteristics of the flow are found. It is revealed that the most interesting case occurs if the inhomogeneous heating of the rigid walls is used as boundary condition for the temperature function. The exact solution constructed for this type of boundary conditions is utilized for further analysis of heat and mass transfer in the ethanol-water mixture in Section 4. Features of the thermal diffusion separation are described. Influence of layer thickness, flow rate and gravity action on the flow is analysed.

2. Governing equations. Exact solution construction

We consider the geometry of the problem shown in Fig. 1. A binary mixture occupies the horizontal layer between two rigid walls which can be heated or thermally insulated. The layer width L is assumed to be much shorter then its length in the direction of x-axis. The gravity acceleration vector $\mathbf{g}$ has the coordinates $(0, -g)$. We assume that the velocity vector $\boldsymbol{u}$ has the horizontal component $u(y)$ only, the functions of temperature T, concentration C and the modified pressure up to hydrostatic term p^* depend on both x and y. All characteristics of the motion are stationary ones.

In view of the listed assumptions about the desired functions the Navier–Stokes equations complemented by the heat and mass transfer equations take the form [12]

$$\begin{aligned} &\nu u_{yy} = \frac{1}{\rho_0} p^*_x, && g(\beta_1 T + \beta_2 C) = \frac{1}{\rho_0} p^*_y, \\ &u T_x = \chi (T_{xx} + T_{yy}), && u C_x = D(C_{xx} + C_{yy}) + D^\theta (T_{xx} + T_{yy}), \end{aligned} \tag{2.1}$$

where ρ_0 is some reference value of the density of the liquid, ν is the kinematic viscosity coefficient, χ is the thermal diffusivity coefficient, D is the coefficient of diffusion, β_1 and β_2 are the coefficients of thermal and concentration expansion respectively. The last term in the equation for concentration corresponds to taking into account the thermal diffusion (Soret) effect, parameter D^θ is the thermal diffusion coefficient [13]. If $D^\theta < 0$, then the thermal diffusion is called normal, the gradients of temperature and concentration are co-directional. If $D^\theta > 0$, then we deal with abnormal thermal diffusion effect and gradients of temperature and concentration have the opposite directions.

Using the following dimensionless variables

$$\hat{x} = \frac{x}{L}, \qquad \hat{y} = \frac{y}{L}, \qquad \hat{u} = \frac{\nu}{g\beta_1 \Delta T L^2} u,$$
$$\hat{p^*} = \frac{1}{\rho_0 g \beta_1 \Delta T L} p^*, \qquad \hat{T} = \frac{1}{\Delta T} T, \qquad \hat{C} = \frac{\beta_2}{\beta_1 \Delta T} C,$$

we rewrite equations (2.1) in the following form

$$\begin{gathered} u_{yy} = p_x, \quad T + C = p_y, \\ \mathrm{Gr}\, u\, T_x = \frac{1}{\mathrm{Pr}} (T_{xx} + T_{yy}), \\ \mathrm{Gr}\, u\, C_x = \frac{1}{\mathrm{Sc}} \big[C_{xx} + C_{yy} - \psi (T_{xx} + T_{yy}) \big], \end{gathered} \tag{2.2}$$

the symbols hat and asterisk are omitted, $\mathrm{Gr} = g\beta_1 \Delta T L^3/\nu^2$ is the Grashof number, $\mathrm{Pr} = \nu/\chi$ is the Prandtl number, $\mathrm{Sc} = \nu/D$ is the Schmidt number, $\psi = -\beta_2 D^\theta/(\beta_1 D)$ is the separation ratio. The values ΔT and L are taken as the representative temperature and the scale of the width of the layer respectively.

Equations (2.2) are nonlinear in the general case. The construction of exact solutions of such a system is interesting not only as possibility to describe physical phenomenon but also from mathematical point of view.

We consider the following representation for the functions of temperature and concentration

$$T(x,y) = (a_1 y + a_2)x + B(y), \quad C(x,y) = (n_1 y + n_2)x + K(y), \tag{2.3}$$

where a_i, n_i, $i = 1,2$, are constants, $B(y)$, $K(y)$ are the smooth functions to be found further.

If we differentiate the first equation of the system (2.2) with respect to y and the second equation from (2.2) with respect to x, compare the obtained expressions and use ansatz (2.3), the following equation for the function of velocity $u(y)$ can be derived $u_{yyy} = (a_1+n_1)y + a_2 + n_2$. Integrating the latter three times, the function u can be found as

$$u = \frac{(a_1+n_1)y^4}{24} + \frac{(a_2+n_2)y^3}{6} + \frac{u_1 y^2}{2} + u_2 y + u_3 \tag{2.4}$$

with arbitrary constants u_i, $i = 1,2,3$.

The third and fourth equations from (2.2) give relationships for the functions $B(y)$ and $K(y)$

$$\frac{1}{\mathrm{Pr}} B_{yy} = \mathrm{Gr} u (a_1 y + a_2), \quad \frac{1}{\mathrm{Sc}}(K_{yy} - \psi B_{yy}) = \mathrm{Gr} u (n_1 y + n_2). \tag{2.5}$$

Substituting the function u from (2.4) into equations (2.5) and integrating both of them twice with respect to y, we have

$$B = \mathrm{Gr}\mathrm{Pr}\Big[\frac{U_A^1 y^7}{42} + \frac{U_A^2 y^6}{30} + \frac{U_A^3 y^5}{20} + \frac{U_A^4 y^4}{12} + \frac{U_A^5 y^3}{6} + \frac{U_A^6 y^2}{2}\Big] + b_1 y + b_2, \tag{2.6}$$

$$\begin{aligned} K = \mathrm{Gr}\Big[&(\mathrm{Sc}U_N^1 + \psi\mathrm{Pr}U_A^1)\frac{y^7}{42} + (\mathrm{Sc}U_N^2 + \psi\mathrm{Pr}U_A^2)\frac{y^6}{30} + (\mathrm{Sc}U_N^3 + \psi\mathrm{Pr}U_A^3)\frac{y^5}{20} \\ &+ (\mathrm{Sc}U_N^4 + \psi\mathrm{Pr}U_A^4)\frac{y^4}{12} + (\mathrm{Sc}U_N^5 + \psi\mathrm{Pr}U_A^5)\frac{y^3}{6} \\ &+ (\mathrm{Sc}U_N^6 + \psi\mathrm{Pr}U_A^6)\frac{y^2}{2}\Big] + k_1 y + k_2. \end{aligned} \tag{2.7}$$

where U_N^i, U_A^i, $i = 1,\dots,6$, consist of multiplications of constants a_j, n_j, u_r, $j = 1,2$, $r = 1,2,3$. The expressions for U_N^i, U_A^i, $i = 1,\dots,6$ are the following

$$\begin{aligned} &U_A^1 = \frac{(a_1+n_1)a_1}{24}, && U_A^2 = \frac{(a_2+n_2)a_1}{6} + \frac{(a_1+n_1)a_2}{24}, \\ &U_A^3 = \frac{u_1 a_1}{2} + \frac{(a_2+n_2)a_2}{6}, && U_A^4 = \frac{u_1 a_2}{2} + u_2 a_1, \\ &U_A^5 = u_3 a_1 + u_2 a_2, && U_A^6 = u_3 a_2; \\ &U_N^1 = \frac{(a_1+n_1)n_1}{24}, && U_N^2 = \frac{(a_2+n_2)n_1}{6} + \frac{(a_1+n_1)n_2}{24}, \\ &U_N^3 = \frac{u_1 n_1}{2} + \frac{(a_2+n_2)n_2}{6}, && U_N^4 = \frac{u_1 n_2}{2} + u_2 n_1, \\ &U_N^5 = u_3 n_1 + u_2 n_2, && U_N^6 = u_3 n_2. \end{aligned} \tag{2.8}$$

Constants b_j, k_j, $j = 1, 2$, are arbitrary.

The pressure in the mixture can be obtained with respect to the formula

$$p(x, y) = \Big\{\frac{(a_1 + n_1)y^2}{2} + (a_2 + n_2)y + u_1\Big\}x + \int_0^y [B(z) + K(z)]dz + p_0 \quad (2.9)$$

with arbitrary constant p_0.

Expressions (2.3), where $B(y)$ and $K(y)$ are from (2.6) and (2.7), together with (2.4) and (2.9) present the exact solution of equations (2.2).

3. Different statements of the boundary value problems for heat and mass transfer in a horizontal layer and their realization for the constructed solution

Let us discuss probable boundary conditions for the description of heat and mass transfer in the horizontal layer shown in Fig. 1 by means of the constructed solution (2.3), (2.4), (2.6)–(2.9). We consider the motion of binary mixture in a long horizontal channel with rigid walls $y = 0$ and $y = 1$. Influence of different aspects (layer width, gravity force, given flow rate and thermodiffusion) on the process of binary mixtures separation is analysed in the next Section.

We start with the boundary condition for the velocity function u. Primarily, the no-slip conditions $u(0) = u(1) = 0$ should be fulfilled. The first of them leads to vanishing the constant u_3 in formula (2.4). Furthermore, we assume that the constant flow rate q is defined by the formula

$$\int_0^1 u(y)dy = q = \text{const}. \quad (3.1)$$

The nondimensional value q is connected with its dimensional analog V by means of the relationship $q = V/(\nu\rho_0\mathrm{Gr})$. Further, we test four types of boundary conditions for the temperature function. We suppose that temperature distribution is posed on both walls of the channel. As the temperature has the form (2.3) inside the channel, the same form for the temperature should be used in the boundary condition on the walls. The second type of the conditions for the temperature is assumption about thermal insulation of the boundaries. And the third and fourth types are the mixed conditions. In this case, one wall is considered thermally insulated and the other wall is heated with respect to law similar to given in (2.3).

For the function of concentration, a condition of absence of mass flux through the both rigid walls is used. As the Soret effect is taken into account in the study, the mass flux is driven by concentration and temperature gradients. The condition contains two summands related to diffusion and thermal diffusion contribution into flux and has the form

$$\frac{\partial C}{\partial y} - \psi\frac{\partial T}{\partial y} = 0 \quad \text{at} \quad y = 0, \ y = 1. \quad (3.2)$$

The solution of boundary value problem for the function C (the last equation in (2.2) and boundary conditions (3.2)) is defined up to a constant. It is necessary to give additional condition for the function of concentration. We use requirement of the average concentration preservation in the form

$$\int_0^1 C(x=0,y)dy \overset{\text{due to (2.3)}}{=} \int_0^1 K(y)dy = C_0. \tag{3.3}$$

Further, we consider all boundary conditions in details.

I. Both rigid walls are heated according to linear law as defined by the form of the temperature function, i.e.

$$T(x,0) = A_1x + B_1, \quad T(x,1) = A_2x + B_2, \tag{3.4}$$

where A_i, B_i, $i = 1,2$, are given constants. Substituting $y = 0$ and $y = 1$ into law for the temperature function in (2.3), taking into account the expression for the function $B(y)$ from (2.6) and comparing the obtained formula with the both boundary conditions in (3.4), we obtain

$$\begin{gathered} a_1 = A_2 - A_1, \quad a_2 = A_1, \\ b_1 = B_2 - B_1 - \mathrm{GrPr}\Big[\frac{U_A^1}{42} + \frac{U_A^2}{30} + \frac{U_A^3}{20} + \frac{U_A^4}{12} + \frac{U_A^5}{6}\Big], \quad b_2 = B_1. \end{gathered} \tag{3.5}$$

From the conditions for the concentration function (3.2) the following relationships are derived

$$n_1 = \psi a_1, \quad K'(0) - \psi B'(0) = 0, \quad K'(1) - \psi B'(1) = 0. \tag{3.6}$$

Using formulas (2.6) and (2.7), from the second equation in (3.6) we can find that

$$k_1 = \psi b_1. \tag{3.7}$$

Conditions $u(1) = 0$, (3.1) and the last equation in (3.6) are used for the construction of the system of linear equations for the constants u_1, u_2 and n_2. The solution of the obtained system is

$$\begin{aligned} n_2 &= \frac{\psi(A_1 - A_2)}{2(\psi(A_1 - A_2) + 720q)}[A_1(\psi - 1) - A_2(\psi + 1) + 720q], \\ u_1 &= \frac{3\psi(A_1 - A_2)}{20} - \frac{(7A_1 + 3A_2)}{20} - \frac{n_2}{2} - 12q, \\ u_2 &= \frac{\psi(A_1 - A_2)}{30} + \frac{3A_1 + 2A_2}{60} + \frac{n_2}{12} + 6q. \end{aligned} \tag{3.8}$$

The last constant to be found is k_2. It is expressed from equality (3.3) in the following way

$$\begin{aligned} k_2 = C_0 - \frac{k_1}{2} - \mathrm{Gr}\Big[&\frac{\mathrm{Sc}U_N^1 + \psi\mathrm{Pr}U_A^1}{336} + \frac{\mathrm{Sc}U_N^2 + \psi\mathrm{Pr}U_A^2}{210} \\ &+ \frac{\mathrm{Sc}U_N^3 + \psi\mathrm{Pr}U_A^3}{120} + \frac{\mathrm{Sc}U_N^4 + \psi\mathrm{Pr}U_A^4}{60} + \frac{\mathrm{Sc}U_N^5 + \psi\mathrm{Pr}U_A^5}{24}\Big]. \end{aligned} \tag{3.9}$$

The formulas for U_A^i, U_N^i, $i = 1, \dots, 5$, in (3.5) and (3.9) are given in (2.8). The found values of a_j, b_j, u_j, $j = 1,2$, from (3.5) and (3.8) should be substituted into expressions (2.8). In such a way all constants included into the functions u, T and C are defined. The function p can be found from formula (2.9) and the problem is completely solved for this type of boundary conditions for the temperature function.

REMARK 3.1. It is interesting to note that if $A_1 = A_2$ and $q = 0$ simultaneously, the solution of the posed boundary value problem does not exist. The denominator in the expression for n_2 in (3.8) vanishes in this case. If $q \neq 0$ and $A_1 = A_2$, then $a_1 = n_1 = n_2 = 0$, the formulas for the functions u, T, C and p are essentially simplier then derived above. Furthermore, the function C does not depend on the x-variable. It means that the concentration field is homogeneous in all sections $y =$const.

II. Both rigid walls are thermally insulated ones. It implies that the condition $\partial T/\partial y = 0$ is fulfilled at $y = 0$ and $y = 1$. Then, the conditions of absence of mass fluxes (3.2) are transformed into $\partial C/\partial y = 0$ at $y = 0$ and $y = 1$. We find the following constants: $a_1 = a_2 = n_1 = n_2 = b_1 = k_1 = 0$, $u_1 = -12q$, $u_2 = 6q$, $k_2 = C_0$, and b_2 remains arbitrary. Such a way the solution of the posed problem is found up to two constants (b_2 and p_0)

$$u = 6q(y - y^2), \quad T = b_2, \quad C = C_0, \quad p = (b_2 + C_0)y + p_0.$$

This solution is of no interest from the physical point of view because temperature and concentration fields do not change.

III. The upper rigid wall is thermally insulated, the lower wall is heated with respect to the linear law, i.e. the following relationships are valid

$$T = A_1 x + B_1 \quad \text{at} \quad y = 0,$$
$$\frac{\partial T}{\partial y} = 0 \quad \text{at} \quad y = 1.$$

Here A_1, B_1 are the given constants. The full condition of zero mass flux (3.2) is fulfilled at $y = 0$. At $y = 1$ the derivative of the function C vanishes, i.e. (3.2) transforms to the condition $\partial C/\partial y = 0$. We find integration constants in the following way: from equation $T(x, 0) = 0$ constants $a_2 = A_1$ and $b_2 = B_1$ are defined; from equations $\partial T/\partial y(x, 1) = 0$ and $\partial C/\partial y(x, 1) = 0$ constants $a_1 = 0$, $n_1 = 0$, $b_1 = -A_1 q\mathrm{GrPr}$ and $k_1 = -A_1\psi q\mathrm{GrPr}$; from the full condition for the concentration (3.2) at $y = 0$ we have equation $K'(0) - \psi B'(0) = 0$. Using this equation together with condition $u(1) = 0$ and the expression for the flow rate (3.1), constants n_2, u_1 and u_2 are written as follows

$$n_2 = 0, \quad u_1 = -\frac{A_1}{2} - 12q, \quad u_2 = \frac{A_1}{12} + 6q.$$

The remain constant k_2 is expressed from integral condition (3.3)

$$k_2 = C_0 - A_1\mathrm{GrPr}\psi\Big(\frac{A_1}{1440} - \frac{7q}{20}\Big).$$

The solution can be constructed with respect to formulas (2.3), (2.4), (2.6)–(2.9), using the found constants listed above. It should be noted that in this case the function C does not depend on x-variable.

IV. The lower rigid wall is thermally insulated, the upper wall is heated with respect to the linear law, i.e. the following relationships are valid

$$\frac{\partial T}{\partial y} = 0 \quad \text{at} \quad y = 0, \qquad T = A_2 x + B_2 \quad \text{at} \quad y = 1.$$

Here A_2, B_2 are the given constants. The full condition of zero mass flux (3.2) is fulfilled at $y = 1$. At $y = 0$ the derivative of the function C vanishes, i.e. $\partial C/\partial y = 0$. The process of finding integration constants is very similar to case **III**. Here, we give the constants without details

$$a_1 = n_1 = n_2 = b_1 = k_1 = 0, \quad a_2 = A_2, \quad b_2 = B_2 - A_2\mathrm{GrPr}\Big(\frac{A_2}{720} - \frac{q}{2}\Big),$$

$$u_1 = -\frac{A_2}{2} - 12q, \quad u_2 = \frac{A_2}{12} + 6q, \quad k_2 = C_0 + A_2\mathrm{GrPr}\psi\Big(\frac{A_2}{1440} - \frac{3q}{20}\Big).$$

It can be seen that in this case the function C does not depend on x-variable as well as in case **III**.

Thereby, in this section, all possible boundary value problem statements for the description of heat and mass transfer in the horizontal layer are analysed and exact solutions in the closed forms are constructed. The solution from item **I** is used in the next section for further study of heat and mass transfer and thermal diffusion separation in the layer between two rigid walls. The choice of this solution for further application is conditioned by dependence of the concentration field on both spatial coordinates.

4. The motion of binary mixture between the rigid walls

In this section, we demonstrate the use of the solution constructed for binary mixture in channel with rigid walls heated with respect to linear law (it corresponds to item **I** from Section 3). The ethanol-water mixture with concentration of ethanol 70 % is utilized for the analysis. The physical parameters of the mixture are given in Table 1. The data are found in [14–16].

TABLE 1. Physical parameters of water-ethanol mixture (70 % of ethanol)

ν, m^2/s	$2.345 \cdot 10^{-6}$
χ, m^2/s	$0.843 \cdot 10^{-7}$
D, m^2/s	$4.481 \cdot 10^{-10}$
D^θ , $m^2/(K{\cdot}s)$	$-0.386 \cdot 10^{-12}$
ρ_0, kg/m^3	863.4
β_1, 1/K	$0.994 \cdot 10^{-3}$
β_2	0.277
C_0	0.7

For the plotting the velocity profile with respect to formula (2.4) with constants from (3.8) we need to have values A_j, $j = 1, 2$, and flow rate q. The constants B_j, $j = 1, 2$ are not included into function u. They are used for the calculation of temperature and concentration functions. The following parameters are used for visualization of the solution

$$T|_{y=0} = A_{1\dim}x + B_{1\dim} = 15x + 20^\circ\mathrm{C},$$
$$T|_{y=L} = A_{2\dim}x + B_{2\dim} = x + 10^\circ\mathrm{C} \tag{4.1}$$

in the dimensional variables. The upper wall $y = 1$ is more heated in this case. The values of $A_{j\dim}$, $B_{j\dim}$, $j = 1, 2$, are recalculated in the dimensionless variables with respect to relationships

$$A_j = \frac{A_{j\dim}L}{\triangle T}, \quad B_j = \frac{B_{j\dim}}{\triangle T},$$

where temperature scale is $\triangle T = 20^\circ$C.

We proceed with study of influence of three factors on the binary mixture separation in the layer: layer width L, value of the flow rate V and the action of the mass force gravity.

4.1. Influence of the channel geometry on the motion. We consider three values of the layer width $L = 0.001, 0.003, 0.009$ m. The flow rate in the dimensional variables is 10^{-6}kg/(m·s) and the acceleration of the gravity is $g = 9.8$ m/s^2. The velocity profiles depending on the layer width are shown in Fig. 2a: curve 1 corresponds to $L = 0.001$ m, curve 2 is constructed for $L = 0.003$ m, and curve 3 matches up $L = 0.009$ m. Further growth of the layer width (beginning from L =0.014 m) leads to loss of physical meaning of the problem. The changes of the concentration are out of the interval $(0, 1)$. Curve 1 in Fig. 2a looks like the Poiseuille profile. An increase in the layer thickness conduces to a decrease in absolute value of velocity and the appearance of reverse flow zones, the function of the velocity changes a sign (curves *2,3*). The graphs of the velocity function are consistent with the pressure gradient profiles shown in Fig. 2d: the fluid moves from an area of higher pressure to an area where the pressure is lower (in the case of $L = 0.001$ m $p_x < 0$ $\forall y \in [0, 1]$). When the layer thickness is growing up, the viscous effects become weaker. It explains the increase of the velocity intensity close to the walls and its decrease in the center of the layer, where the maximal pressure gradient is observed. In this case, the ethanol component carried by the convective flow accumulates in the near-wall region in the vicinity of the line $x = 0$ (Fig. 4c).

We observe a rise of temperature gradients along the layer width. The dimensionless temperature changes from $T = 0.5$ to $T \approx 1.01$ (Fig. 3a), to $T \approx 1.03$ (Fig. 3b), to $T \approx 1.09$ (Fig. 3c). The concentration drop grows up also at the layer expansion. The concentration changes from $C \approx 0.639$ till $C \approx 0.741$ (Fig. 4a), from $C \approx 0.618$ till $C \approx 0.740$ (Fig. 4b), from $C \approx 0.577$ till $C \approx 0.760$ (Fig. 4c) with respect to the growth of layer thickness. Thus, the greatest inhomogeneities of temperature field and in the ethanol distribution take place in the layer with the maximal thickness, i.e. $L = 0.009$ m.

4.2. Influence of flow rate on the motion characteristics. We put the layer width as $L = 0.003$ m. The influence of changes of the flow rate q on the velocity, temperature and concentration is under study for the same mixture of ethanol (70 %) and water (30 %). We use the following values of flow rate

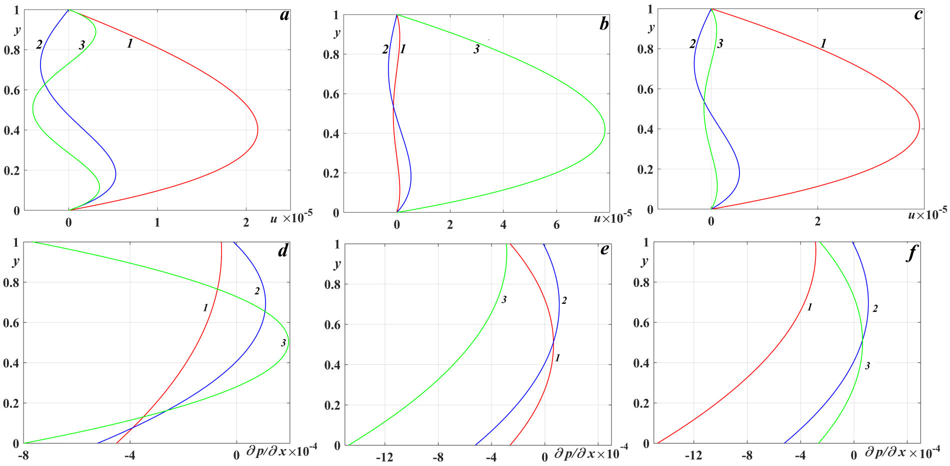


FIGURE 2. The velocity profile u (a, b, c) and the pressure gradient p_x (d, e, f) depending on the layer thickness (a, d): curve 1 corresponds to $L = 0.001$ m, curve 2 – 0.003 m, curve 3 – 0.009 m; depending on flow rate (b, e): curve 1 corresponds to $V = 0$ kg/(m·s), curve 2 – $V = 10^{-6}$ kg/(m·s), curve 3 – $V = 10^{-4}$ kg/(m·s) (curve 3 is scaled, see explanation in section 4.2); depending on influence of gravity force acceleration (c, f): curve 1 corresponds to microgravity action $g = g_0 \cdot 10^{-2}$ (curve 1 is scaled, see explanation in section 4.3), curve 2 is for terrestrial gravity and $g = g_0 = 9.8\,\mathrm{m/s^2}$, curve 3 – for hypergravity $g = g_0 \cdot 10^2$

$V = 0, 10^{-6}, 10^{-4}$ kg/(m·s) in the dimensional variables. The profiles of velocity and pressure gradient depending on V are shown in Fig. 2b,e. Intensification of the motion is observed when the value of V increases. The greatest value of the flow rate corresponds to the most essential changes of the velocity function. We pay attention that curve 3 in Fig. 2b is presented for a half-values of real velocity function for more clear visualization. In this case, the velocity profile is close to the Poiseuille profile, zones of inverse motion are absent for the value of $V = 10^{-4}$ kg/(m·s). The viscous effects in the near-wall region influence weakly on motion at big flow rate values. And vice versa, at zero or small value of flow rate ($V = 10^{-6}$ kg/(m·s)) the essential effect of viscosity in near-wall regions can be observed. It explains the behaviour of curves 1, 2, 3 in Fig. 2b. In the same manner as the analysis of the layer thickness influences the flow, the conclusion is justified that the velocity distribution depending on the flow rate of the mixture is consistent with the behaviour of the pressure gradient (Fig. 2e). Minimal velocity values are achieved at maximal flow pressure gradients (curves 1, 2 in Fig. 2b,e).

Further, the temperature and concentration fields depending on the flow rate are considered. If we substitute the found integration constants into formula for the temperature function $T = (a_1y+a_2)x+B(y)$, where $B(y)$ is presented in (2.6), then

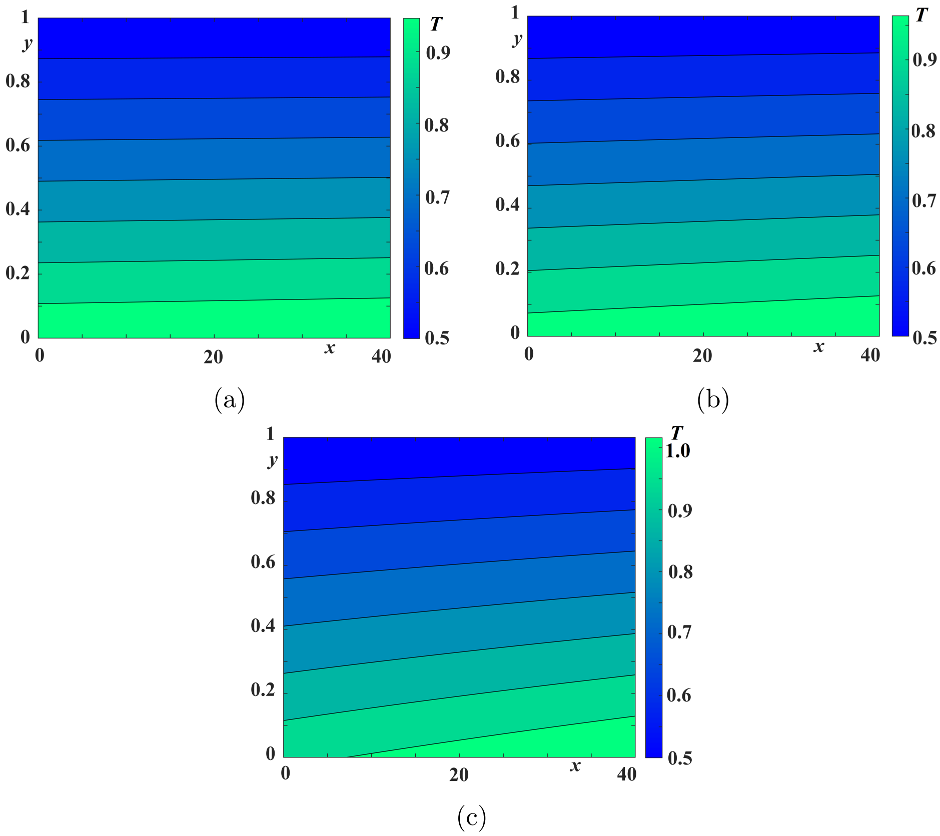
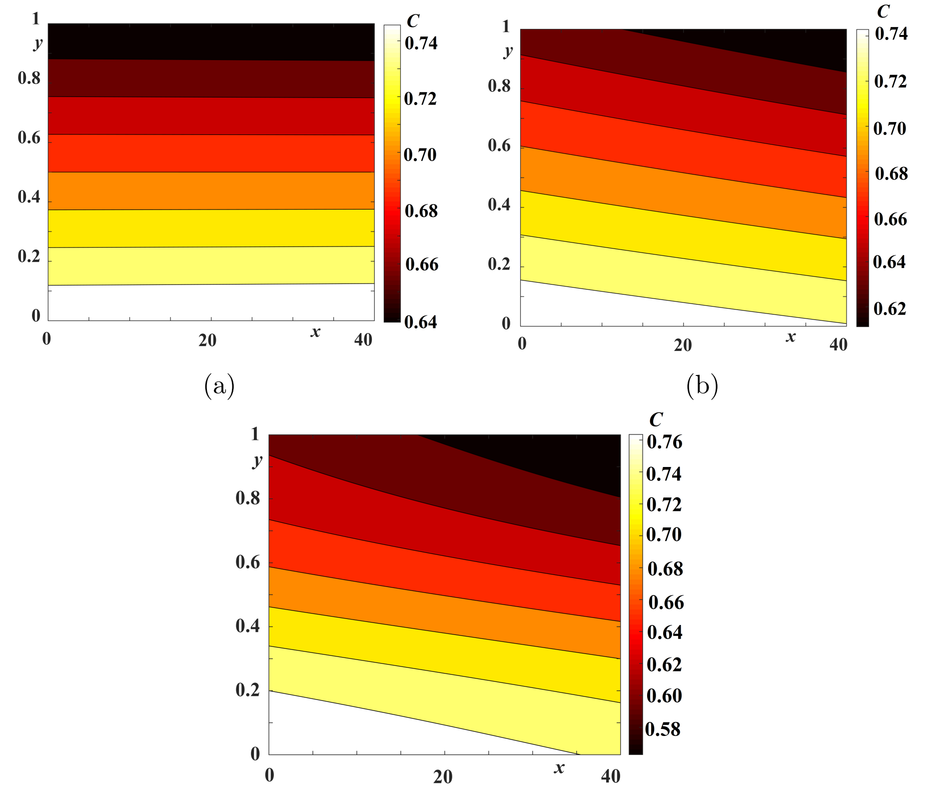


FIGURE 3. Field of temperature depending on layer width: (a) are for $L = 0.001\,\text{m}$; (b) are for $L = 0.003\,\text{m}$; (c) are for $L = 0.009\,\text{m}$

we observe that the value of V is contained in expression for the function B only in terms which give small contribution into changes of the function T. It means that flow rate influences heat distribution weakly. In the dimensionless variables, the temperature changes in the interval from 0.5 till 1.1125 for all values V used.

The influence of flow rate values on reconstruction of the concentration field is estimated below. If the flow rate vanishes then there are two mechanisms affecting the flow. They are temperature drop on the walls and thermal diffusivity. This is easy to check looking at formulas (2.3), (2.4), (2.6) and (2.7) taking into account the constants from formulas (3.6)–(3.9). In this case, it can be observed that drops of the concentration are the greatest among the considered values of V (see the second column in Table 2). The concentration changes from 0.614 till 0.760 at $V = 0$. The thermodiffusion effect prevails in such configuration. When the value of V increases up to $10^{-6}\,\text{kg}/(\text{m}\cdot\text{s})$ the competition of mechanical effects and diffusion transition occurs in flow. The variations of the concentration are from 0.630 till 0.760 for this value of flow rate (see the third column in Table 2). At last,



(c)

FIGURE 4. Field of concentration depending on layer width: (a) are for $L = 0.001\,\mathrm{m}$; (b) are for $L = 0.003\,\mathrm{m}$; (c) are for $L = 0.009\,\mathrm{m}$

for the greatest value of $V = 10^{-4}\,\mathrm{kg/(m{\cdot}s)}$ strong mechanical effects suppress the weaker thermodiffusion effect. The smallest drops of ethanol concentration (see the fourth column in Table 2) take place in this case.

TABLE 2. Changes of concentration at increasing flow rate

V, kg/(m · s)	0	10^{-6}	10^{-4}
$\triangle C$, %	14.6	13.0	12.5

4.3. Influence of gravity action on the binary mixture motion. The layer width L and flow rate V are equal to $0.003\,\mathrm{m}$ and $10^{-6}\,\mathrm{kg/(m{\cdot}s)}$ respectively. We analyse the gravity action on the mixture motion for $g = g_0{\cdot}10^{-2}$ (microgravity), for $g = g_0 \cdot 10^2$ (hypergravity) and $g = g_0 = 9.8\,\mathrm{m/s^2}$. The latter corresponds to the value of acceleration of gravity force in the Earth conditions. The profiles of velocity and pressure gradients depending on g are shown in Fig. 2c,f. The greatest values of velocity function appear at microgravity. Curve 1 in Fig. 2c corresponds to half-values of real velocity function for better visualization. The velocity graph is similar to the Poiseuille profile in this case. There are the zones with inverse flow in the terrestrial and hypergravity conditions. The velocity changes a sign and becomes less with respect to absolute value in comparison with the velocity at microgravity (curves 2, 3 in Fig. 2c). As in the cases studied before in Sections 4.1 and 4.2 the greatest values of pressure gradient correspond to the smallest values of the velocity. The effect of viscosity of liquid can be observed near the walls of the channel.

TABLE 3. Changes of concentration in dependence of gravity force action

g, m/(s^2)	$9.8 \cdot 10^{-2}$	9.8	$9.8 \cdot 10^{2}$
$\triangle C$, %	12.7	13.0	14.8

If we consider concentration as functions of gravity force acceleration, it allows to trace that inhomogeneity of the concentration increases at growth of g. The difference between maximum and minimum of concentration is presented in Table 3. It is necessary to note that the solution predicts that the influence of gravity changes on temperature function is weaker than on concentration. The reason is also in the form of the function T, where factor g is in the function B only.

In conclusion, some important remarks should be formulated concerning the analysed solution.

REMARK 4.1. In all configurations studied above, the normal thermal diffusion effect is preserved: ethanol accumulates near the more heated wall $y = 0$ as a lighter component of the mixture (its density is less than the water density). This result is in agreement with experimental works (e.g., [17]), where it is proved that the mixture of ethanol-water demonstrates normal thermal diffusion effect at the concentration of the ethanol of more than 30 %.

REMARK 4.2. It should be noted that if in all above examples we set the separation ratio $\psi = 0$, i.e. if we do not take into account the thermal diffusion effect, the concentration field does not change, the liquid remains homogeneous for all other changing parameters of the problem. This derivation follows from the substitution of $\psi = 0$ into formulas (2.7) and then into (2.3) and simple calculation, which leads to $C(x, y) \equiv C_0$.

5. Conclusion

Mathematical modelling of the binary mixture flows is more complicated task than the same problem concerning one-component liquid motions. Additional unknown function of concentration appears in equations of heat and mass transfer. It should be connected with other unknown functions (velocity, temperature, pressure). It also influences solution of the problem and behaviour of the flow at whole.

In this paper, we neatly construct the exact solution of equations of convective heat and mass transfer, provide the analysis of different types of boundary conditions for the temperature functions and treat the influence of gravity force, flow rate in cross section and layer width on process of thermal diffusion separation of ethanol-water mixture in a long horizontal layer.

The basic conclusions are formulated further. When the separation ratio ψ does not vanish, the heterogeneity of the concentration field is more pronounced at the large layer thickness, zero flow rate and in the conditions of hypergravity action. Apparently, these conditions seem to be optimal for controlling the separation of the mixture filling the horizontal layer into components. Thereby, the constructed solution correctly predicts all flow characteristics and their changes depending on the processing parameters. The obtained solution replenishes the database of exact solutions suitable for mathematical modelling of heat and mass transfer processes and can be useful for more clear understanding of the thermal diffusion separation in mixtures at different regimes of flow.

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КОНВЕКЦИЈА БИНАРНЕ СМЕШЕ У ХОРИЗОНТАЛНОМ КАНАЛУ ПОД ДЕЈСТВОМ СОРЕОВОГ ЕФЕКТА

Резиме. Разматра се математички модел који описује стационарно струјање бинарне течне смеше. За посебан облик функција температуре и концентрације конструисано је тачно решење једначина конвективног преноса топлоте и масе. Проучавају се све могуће формулације граничних проблема у циљу анализе термичке дифузионе сепарације бинарне смеше у дугом хоризонталном каналу са крутим зидовима. Утврђено је да само једна од формулација (при нехомогеном загревању оба крута зида) доводи до изводљивих резултата. Уз помоћ конструисаног решења за поменути гранични проблем анализиран је ефекат дебљине слоја, дате су брзине протока и дејства силе гравитације на процес сепарације.

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