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A. Arkhipova

ON THE SMOOTHNESS OF WEAK SOLUTIONS OF STRONG-NONLINEAR NONDIAGONAL ELLIPTIC SYSTEMS (THE TWO-DIMENSIONAL CASE)

Abstract. We consider a class of strong-nonlinear elliptic systems with a nondiagonal principal matrix. Weak solvability of the Dirichlet problem for such type systems was earlier proved by the author in the two-dimensional case. The solution constructed was smooth almost everywhere. Here we prove that this solution is a Hölder continuous function in the entire domain.

Dedicated to the memory of
Olga Aleksandrovna Ladyzhenskaya

In [1], the author proved the existence of a weak solution for the following Dirichlet problem:

$$\left.\begin{aligned} \frac{d}{dx_\alpha} a^k_\alpha(x,u,u_x) + b^k(x,u,u_x) = 0, \quad k=1,\dots,N, \quad x\in\Omega,\\ u\big|_{\partial\Omega} = 0, \end{aligned}\right\} \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^2$ with smooth boundary $\partial\Omega$; $u:\Omega\to\mathbb{R}^N$, $N>1$, $u=(u^1,\dots,u^N)$, $u_x=\left\{u^k_{x_\alpha}\right\}^{k\leqslant N}_{\alpha\leqslant 2}$.

It was assumed in [1] that the functions $a_\alpha=\{a^k_\alpha\}^{k\leqslant N}$, $\alpha=1,2$, and $b=\{b^k\}^{k\leqslant N}$ are differentiable on the set $\mathfrak{M}=\overline{\Omega}\times\mathbb{R}^N\times\mathbb{R}^{2N}$ and

$$|a_\alpha(x,u,z)|\leqslant\mu_1(1+|z|),\quad |b_\alpha(x,u,z)|\leqslant\mu_2(1+|z|^2), \tag{2}$$

Moreover, we postulated also the natural behavior of all the derivatives and the ellipticity condition in the strong form

$$\frac{\partial a^k_\alpha(x,u,z)}{\partial z^l_\beta}\xi^k_\alpha\xi^l_\beta\geqslant\nu|\xi|^2,\quad \xi\in\mathbb{R}^{2N};\quad \left\|\frac{\partial a}{\partial z}(x,u,z)\right\|\leqslant\mu_1. \tag{3}$$

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Here, the constants μ_1, μ_2, and ν are fixed arbitrarily.

The second condition in (2) describes the quadratic (limit) growth in the gradient of the function b.

The existence of a weak solution $u \in \overset{\circ}{W}{}^1_2(\Omega)$ of (1) (in the sense of distributions) was proved in [1] under the additional condition

$$a_\alpha(x,u,z)z_\alpha + b(x,u,z)u \geqslant \nu_0|z|^2 - \mu_0, \quad (x,u,z) \in \mathfrak{M}, \tag{4}$$

where ν_0, $\mu_0 = \text{const} > 0$.

The solution u is $C^{2+\alpha}$-smooth almost everywhere in $\overline{\Omega}$ with $\alpha \in (0,1)$, and the singular set σ consists of at most finitely many points (see [1, Theorem 1 and Remark 1]). This solvability result is a simple consequence of the theorem on the *quasireverse* Hölder inequalities proved by the author in [2].

Analyzing the proof of Theorem 1 in [1], one can deduce the existence of a weak solution of (1) in the class $\overset{\circ}{W}{}^1_2(\Omega) \cap C^\alpha(\overline{\Omega} \setminus \sigma)$ under less restrictive assumptions on the data than (3). More precisely, it is enough to require that condition (2), (4) and the inequality

$$\big(a_\alpha(x,u,z) - a_\alpha(x,u,\zeta)\big)(z_\alpha - \zeta_\alpha) \geqslant 0, \ \ (x,u) \in \overline{\Omega}\times\mathbb{R}^N, \ z, \zeta \in \mathbb{R}^{2N}, \tag{5}$$

hold. In this case, we do not assume the existence of the derivatives of a_α and b.

Conditions (2), (4), and (5) are close to assumptions in [3], where J. Freshe proved the existence of a solution of problem (1) in the class $\overset{\circ}{W}{}^1_2(\Omega) \cap C^\alpha(\overline{\Omega})$, $\alpha \in (0,1)$.

Note that the coerciveness condition was required in [3] in a slightly stronger form than in (4).

Remark 1. We can allow the functions a_α and b to have a power growth in the argument "u," as it was done in [3]. This is not essential if the coerciveness property of the operator is preserved.

Remark 2. It is easy to see that the conditions $a_\alpha(x,u,z)z_\alpha \geqslant \nu|z|^2 - \mu_3$ and the so-called "one-sided condition" $b(x,u,z)u \geqslant -\nu_*|z|^2 - \mu_3$, $\nu - \nu_* > 0$, provide the validity of assumption (4).

Paper [1] had already been published when the author proved that a weak solution constructed in [1] under conditions (2), (4), and (5) is a Hölder function in the entire domain $\overline{\Omega}$, i.e., $\sigma = \varnothing$. The aim of this note to explain this fact.

The method of approximations in [1] and in the present paper seriously differs from that in [3]. At the same time, we essentially use J. Freshe's idea of the estimation of local energy norms of approximations with the help of the inequality

$$\frac{1}{r^2} \int\limits_{\Omega_r(x^0)} |v|^2 dx \leqslant K |\ln r| \, \|v\|^2_{W_2^1(\Omega)} \quad \text{for} \quad v \in \overset{\circ}{W}{}^1_2(\Omega), \; n = 2, \tag{6}$$

$\Omega_r(x^0) = \Omega \cap B_r(x^0)$, $x^0 \in \overline{\Omega}$, $r \leqslant \frac{1}{2}$, $K = K(\Omega) = \text{const} > 0$ (see [3, Lemma 2.4)].

In this note, we accept all the notation from [1] and prove the following result.

Theorem. *Let Ω be a bounded domain in $\mathbb{R}^n$ with Lipschitz boundary $\partial\Omega$. Let the functions $a_\alpha(x,u,z)$ and $b(x,u,z)$ be defined on $\mathfrak{M} = \overline{\Omega} \times \mathbb{R}^N \times \mathbb{R}^{2N}$ and let they be measurable in x and continuous in (u,z) on $\mathfrak{M}$. Let conditions (2), (4), and (5) hold. Then a solution $u \in \overset{\circ}{W}{}^1_2(\Omega) \cap C^\alpha(\overline{\Omega})$ of problem (1) with a certain $\alpha \in (0,1)$ exists.*

To prove the theorem, we consider the same approximations of (1) as in [1]:

$$\left.\begin{aligned} -\frac{d}{dx_\alpha}\Big(a^k_\alpha(x,u,u_x) + \varepsilon u^k_{x_\alpha}(1+|u_x|^2)^{\frac{m}{2}-1}\Big) + b^k(x,u,u_x) = 0, \\ k = 1,\dots,N, \quad x \in \Omega; \quad u\big|_{\partial\Omega} = 0, \quad \varepsilon \in (0,1]. \end{aligned}\right\} \tag{7}$$

We shall choose the parameter $m > 2$ below.

By the Leray–Lions theory, conditions (2), (4), and (5) guarantee the existence of a solution $u^\varepsilon \in \overset{\circ}{W}{}^1_m(\Omega)$ of (7) for any fixed $\varepsilon \in (0,1]$.

By the embedding theorem, $u^\varepsilon \in C^\beta(\overline{\Omega})$, where $\beta = 1 - \frac{2}{m} > 0$. From condition (2), it follows that

$$\|u^\varepsilon_x\|^2_{L_2(\Omega)} + \varepsilon \|u^\varepsilon_x\|^m_{L_m(\Omega)} \leqslant \mathbf{e}_0, \tag{8}$$

where the constant $\mathbf{e}_0$ depends on the parameters ν_0, μ_0, and $|\Omega|$, but not on $\varepsilon \in (0,1]$. It follows from (8) that for a sequence $\{\overline{\varepsilon}\} \to 0$, $u^{\overline{\varepsilon}}_x \rightharpoonup u_x$ in $L_2(\Omega)$, $u^{\overline{\varepsilon}} \to u$ almost everywhere in Ω, $\overline{\varepsilon}^{\frac{1}{m}} u^{\overline{\varepsilon}}_x \rightharpoonup 0$ in $L_m(\Omega)$, $u \in \overset{\circ}{W}{}^1_2(\Omega)$. We shall prove that the function u is a solution of problem (1) Hölder continuous in $\overline{\Omega}$.

The following fact was proved in [1, Proposition 2]:

"There exist numbers $\theta_0 > 0$ and $p > 2$, dependent only on the parameters from (2) *and* (4)*, such that the inequality*

$$\int\limits_{\Omega_0(x^0)} \left[|u_x^{\varepsilon_i}|^2 + \varepsilon_i \Big(|u_x^{\varepsilon_i}|^2 + 1 \Big)^{\frac{m}{2}} \right] dx < \theta_0^2 \tag{9}$$

provides the estimate

$$\left\| u_x^{\varepsilon_i} \right\|_{L_p(\Omega_{\frac{R_0}{2}}(x^0))} \leqslant c_1, \tag{10}$$

where the constant c_1 does not depend on $\{\varepsilon_i\}$, and $\{\varepsilon_i\}$ is a subsequence of the sequence $\{\overline{\varepsilon}\}$."

Remark 3. The constant c_1 in (10) and the constants c and c_i in the paper may depend on the parameters ν_0, μ_0, μ_1, μ_2, and $|\Omega|$, but not on ε. In what follows, we write ε instead of $\overline{\varepsilon}$.

We note that estimate (10) implies the estimate

$$\left\| u^{\varepsilon_i} \right\|_{C^\alpha(\Omega_{\frac{R_0}{2}}(x^0))} \leqslant c_2, \quad \alpha = 1 - \frac{2}{p} > 0. \tag{11}$$

It follows from (11) that the limit function u is Hölder continuous in the vicinity of the point x^0. In this paper, we prove that for the whole sequence $\varepsilon \to 0$ and for all point $x^0 \in \overline{\Omega}$, we have the estimate

$$\int\limits_{\Omega_R(x^0)} \left(|u_x^\varepsilon|^2 + \varepsilon \Big(1 + \left| u_x^\varepsilon \right|^2 \Big)^{\frac{m}{2}} \right) dx \leqslant \tau(R) \tag{12}$$

with a certain function $\tau(R)$, where $\tau(R) \to 0$ as $R \to 0$, and $\tau(R)$ does not depend on x^0 and ε.

Obviously, inequality (12) ensures the validity of condition (9) and the theorem.

First we note that the following global estimate holds for u^ε:

$$\int\limits_{\Omega} |u^\varepsilon|^\gamma |u_x^\varepsilon|^2 \, dx \leqslant \mathbf{e}_1, \quad \gamma \in \Big(0, \frac{2\nu_0}{3\mu_1} \Big), \tag{13}$$

and the constant $\mathbf{e}_1$ does not depend on $\varepsilon \in (0, 1]$.

Indeed, the function u^ε satisfies the identity

$$\left.\begin{aligned}\int\limits_\Omega \Big[a_\alpha(x,u^\varepsilon,u^\varepsilon_x)h_{x_\alpha} + \varepsilon u^\varepsilon_{x_\alpha}\big(1+|u^\varepsilon_x|^2\big)^{\frac{m}{2}-1}h_{x_\alpha} + \\ + b(x,u^\varepsilon,u^\varepsilon_x)h\Big]\,dx = 0, \quad \forall\, h \in \overset{\circ}{W}{}^1_m(\Omega).\end{aligned}\right\} \tag{14}$$

From (14) with $h = u^\varepsilon|u^\varepsilon|^\gamma$ and $\gamma \in \Big(0, \dfrac{2}{3}\,\dfrac{\nu_0}{\mu_1}\Big)$, we derive the inequality

$$\Big(\nu_0 - \frac{3}{2}\gamma\mu_1\Big)\int\limits_\Omega |u^\varepsilon_x|^2|u^\varepsilon|^\gamma\,dx \leqslant c(\gamma,\mu_0,\mu_1)\int\limits_\Omega (|u^\varepsilon|^2+1)\,dx.$$

Together with (8), it yields estimate (13).

To derive (12), we define the function G by the relation

$$G^2(x) = 1 + \nu_0|u^\varepsilon_x(x)|^2 + \varepsilon|u^\varepsilon_x(x)|^m, \quad x \in \overline{\Omega}, \tag{15}$$

and henceforth we fix the number m by the relation

$$m = 2 + \gamma, \tag{16}$$

where γ is the parameter from (13).

Following the proof of Lemma 2.5 in [3], we fix a number $\omega > 0$ and sequences of radii $R_j = \frac{1}{2^j}$ and numbers θ_j satisfying the relation

$$(1+\theta_j)^{-1} = \Big(1 - \frac{1}{j}\Big)^\omega, \quad j \in \mathfrak{N}. \tag{17}$$

For a fixed j, we put $R = R_j$, $2R = R_{j-1}$, and $\theta = \theta_j$ and prove the following proposition.

Proposition. *The function G satisfies the inequality*

$$\int\limits_{\Omega_R(x^0)} G^2 dx \leqslant \frac{1}{1+\theta}\int\limits_{\Omega_{2R}(x^0)} G^2\,dx + C_3 J(R,\theta), \tag{18}$$

where $x^0 \in \partial\Omega$ or $x^0 \in \Omega$ and $2R \leqslant \operatorname{dist}\{x^0,\partial\Omega\}$. Here $J(R,\theta) = \theta^2\big|\ln R\big|^{\frac{2}{m}} + R^2 + \theta^{\frac{m}{2}+1}|\ln R|$ if $\varepsilon \leqslant \Big(\dfrac{R}{\sqrt{\theta}}\Big)^{m-2}$, and $J(R,\theta) = R^2$ if $\varepsilon > \Big(\dfrac{R}{\sqrt{\theta}}\Big)^{m-2}$ and $\theta \leqslant \theta_*$ for a number θ_* dependent only on the data.

Proof of the proposition. First, let $\varepsilon \leqslant \left(\frac{R}{\sqrt{\theta}}\right)^{m-2}$ and $x^0 \in \overline{\Omega}$ be fixed arbitrarily. From (14) with $h = u^\varepsilon \zeta^2$, where ζ is a cut-off function for the ball $B_{2R}(x^0)$, $\zeta = 1$ in $B_R(x^0)$, we obtain

$$\begin{aligned} \int\limits_{\Omega_{2R}} G^2\zeta^2 dx \leqslant 2\varepsilon \int\limits_{\Omega_{2R}} |u_x|\big(1+|u_x|^2\big)^{\frac{m}{2}-1}|u|\zeta|\zeta_x|\,dx + \\ + \int\limits_{\Omega_{2R}} |a_\alpha|\;|u|\,2\zeta|\zeta_x|dx + (\mu_0+1)\pi R^2 \equiv j_1 + j_2 + j_3. \end{aligned} \tag{19}$$

In (19) and below, we write u and Ω_r instead of u^ε and $\Omega_r(x^0)$, respectively.

We denote $T_{2R} = \Omega_{2R}\backslash\Omega_R$ and estimate the integrals j_1 and j_2 in (19).

By the Cauchy inequality, we find that

$$\left.\begin{aligned} j_1 \leqslant \frac{\varepsilon}{\theta}\int\limits_{T_{2R}} |u_x|^m\,dx + \frac{c(m)\varepsilon}{R^m}\theta^{m-1}\int\limits_{T_{2R}} |u|^m\,dx + \frac{c\varepsilon}{R}\int\limits_{T_{2R}} |u|\,dx; \\ j_2 \leqslant \frac{\nu_0}{\theta}\int\limits_{T_{2R}} |u_x|^2\,dx + c(\nu_0,\mu_1)\frac{\theta}{R^2}\int\limits_{T_{2R}} |u|^2\,dx. \end{aligned}\right\} \tag{20}$$

Now, we put $v = |u^\varepsilon|^{\frac{m}{2}}$ in (6) to assert that

$$\left.\begin{aligned} \frac{1}{R^2}\int\limits_{\Omega_{2R}(x^0)} |u^\varepsilon|^m dx \leqslant c|\ln R|\int\limits_{\Omega}\Big(|u^\varepsilon|^m + |u^\varepsilon|^{m-2}|u^\varepsilon_x|^2\Big)\,dx \leqslant \\ \underset{(8),(13)}{\leqslant} c_4|\ln R|. \end{aligned}\right\} \tag{21}$$

By the Hölder inequality, from (21) we have

$$\begin{aligned} \frac{1}{R^2}\int\limits_{\Omega_{2R}(x^0)} |u^\varepsilon|^2\,dx \leqslant c_5|\ln R|^{\frac{2}{m}}; \\ \frac{\varepsilon}{R}\int\limits_{\Omega_{2R}(x^0)} |u^\varepsilon|\,dx \leqslant c_6\varepsilon R|\ln R|^{\frac{1}{m}}. \end{aligned} \tag{22}$$

From (19), (20), (21), and (22), we deduce the inequality

$$\left.\begin{aligned}&\int\limits_{\Omega_R} G^2 dx \leqslant \frac{1}{\theta}\int\limits_{T_{2R}} G^2 dx+\\&\qquad + c_7\biggl(\frac{\varepsilon\theta^{m-1}}{R^{m-2}}|\ln R| + \theta|\ln R|^{\frac{2}{m}} + \varepsilon R|\ln R|^{\frac{1}{m}} + R^2\biggr).\end{aligned}\right\} \qquad (23)$$

By the standard "hole-filling" technique, from (23) we obtain estimate (18) under restriction $\varepsilon \leqslant \Bigl(\frac{R}{\sqrt{\theta}}\Bigr)^{m-2}$.

Let now $\varepsilon > \Bigl(\frac{R}{\sqrt{\theta}}\Bigr)^{m-2}$ and $x^0 \in \partial\Omega$ or $x^0 \in \Omega$ and $2R \leqslant \operatorname{dist}(x^0, \partial\Omega)$. In this case, we put $h = (u - l_R)\zeta^m$, where $l_R = 0$ if $x^0 \in \partial\Omega$, and $l_R = \fint\limits_{T_{2R}} u\,dx$ if $x^0 \in \Omega$ and $2R \leqslant \operatorname{dist}(x^0, \partial\Omega)$; ζ is the same cut function as before. In both cases, the function h is admissible for identity (14), and the following inequality is valid:

$$\begin{aligned}&\int\limits_{\Omega_{2R}} \Bigl(a_\alpha u_{x_\alpha} + \varepsilon|u_x|^2(1+|u_x|^2)\Bigr)^{\frac{m-2}{2}}\zeta^m\,dx \leqslant\\&\leqslant m\mu_1\int\limits_{T_{2R}}(1+|u_x|)|u-l_R|\zeta^{m-1}|\zeta_x|dx + \int\limits_{\omega_{2R}}|b|\,|u-l_R|\zeta^m\,dx+\\&+\varepsilon c(m)\int\limits_{T_{2R}}(1+|u_x|^{m-1})\zeta^{m-1}\,|u-l_R|\,|\zeta_x|\,dx. \qquad (24)\end{aligned}$$

We estimate the integral with the function $|b|$ in (24) in the way

$$\begin{aligned}\mathcal{L}_R &= \int\limits_{\Omega_{2R}}|u_x|^2|u-l_R|\zeta^m\,dx \leqslant\\&\leqslant \biggl(\int\limits_{\Omega_{2R}}|u_x|^m\zeta^m\,dx\biggr)^{\frac{2}{m}}\biggl(\int\limits_{\Omega_{2R}}|u-l_R|^{\frac{m}{m-2}}\,dx\biggr)^{\frac{m-2}{m}} \leqslant\\&\underset{(*)}{\leqslant} c\biggl(\frac{\varepsilon}{4}\int\limits_{\Omega_{2R}}|u_x|^m\zeta^m\,dx\biggr)^{\frac{2}{m}}\biggl(\int\limits_{\Omega_{2R}}|u_x|^2\,dx\biggr)^{\frac{1}{2}}\frac{R^{2(1-\frac{2}{m})}}{\varepsilon^{\frac{2}{m}}} \leqslant\end{aligned}$$

$$\underset{(**)}{\leqslant} c\theta^{\frac{m-2}{m}} \Big(\frac{\varepsilon}{4} \int\limits_{\Omega_{2R}} |u_x|^m \zeta^m \, dx\Big)^{\frac{2}{m}} \Big(\int\limits_{\Omega_{2R}} |u_x|^2 \, dx\Big)^{\frac{1}{2}} \leqslant$$

$$\leqslant \frac{\varepsilon}{4} \int\limits_{\Omega_{2R}} |u_x|^m \zeta^m \, dx + c\theta \mathbf{e}_0^{\frac{4-m}{2(m-2)}} \int\limits_{\Omega_{2R}} |u_x|^2 \, dx.$$

Here the inequality $(*)$ is valid by the embedding theorem $W_2^1(\Omega_{2R}) \hookrightarrow L_{\frac{m}{m-2}}(\Omega_{2R})$, $n = 2$, and the inequality $(**)$ holds, because $\varepsilon > \Big(\frac{R}{\sqrt{\theta}}\Big)^{m-2}$.

Now, from (24) it follows that

$$\int\limits_{\Omega_R} G^2 \, dx \leqslant c_8 \int\limits_{T_{2R}} G^2 \, dx + c_9\theta \int\limits_{\Omega_{2R}} G^2 \, dx + c_{10}R^2. \tag{25}$$

Now we fix a number θ satisfying the restriction

$$\theta \leqslant \theta_* = \min\Big\{\frac{1}{2c_9}; \frac{1}{2c_8+1}\Big\}, \tag{26}$$

and note that in this case

$$\frac{c_8 + \frac{1}{2}}{c_8+1} \leqslant \frac{1}{1+\theta}.$$

By the "hole–filling" technique, we derive (18) from (25) under condition (26) and the restriction $\varepsilon > \Big(\frac{R}{\sqrt{\theta}}\Big)^{m-2}$. The proposition is proved. □

Proof of the theorem. Let the location of a point $x^0 \in \overline{\Omega}$ be the same as it is indicated in the proposition. Recalling the notation of R and θ, we note that definition (17) yields the estimate $\theta = \theta_j \leqslant \frac{2\omega}{j}$ for $j \geqslant 2$. Moreover, we consider

$$j \geqslant j_0 = \max\Big\{2, \frac{2\omega}{\theta_*}\Big\}$$

to satisfy the restriction $\theta_j \leqslant \theta_*$ (θ_* is fixed in the proposition).

From (18) and (17), we obtain the inequality

$$\int\limits_{\Omega_{R_j}} G^2 \, dx \leqslant$$

$$\leqslant \Big(\frac{j-1}{j}\Big)^{\omega} \int\limits_{\Omega_{R_{j-1}}} G^2 \, dx + c_3\Big(\theta_j^2 |\ln R_j|^{\frac{2}{m}} + R_j^2 + \theta_j^{\frac{m}{2}+1} |\ln R_j|\Big) \tag{27}$$

under any relation between ε and $\left(\dfrac{R_j}{\sqrt{\theta_j}}\right)^{m-2}$.

Since $|\ln R_j| = j \ln 2$, we have

$$j^\omega \int\limits_{\Omega_{R_j}} G^2\,dx \leqslant (j-1)^\omega \int\limits_{\Omega_{R_{j-1}}} G^2\,dx + c_{11}\Big(j^{\omega+\frac{2}{m}-2} + j^{\omega+1-\frac{m}{2}-1} + \frac{j^\omega}{4^j}\Big).$$

From the last inequality, it follows that

$$j^\omega \int\limits_{\Omega_{R_j}} G^2\,dx \leqslant$$

$$\leqslant (j_0-1)^\omega \int\limits_{\Omega_{R_{j_0-1}}} G^2\,dx + c_{12}\sum_{i=j_0}^{j}\left(\frac{i^\omega}{4^i} + \frac{1}{i^{2-\omega-\frac{2}{m}}} + \frac{1}{i^{\frac{m}{2}-\omega}}\right). \quad (28)$$

Now fix $\omega \in \left(0, \dfrac{m-2}{m}\right)$ to ensure the convergence of the series and (28) derive the estimate

$$\int\limits_{\Omega_{R_j}(x_0)} G^2\,dx \leqslant \frac{c_{13}}{|\ln R_j|^\omega}. \quad (29)$$

It is easy to see that the estimate

$$\int\limits_{\Omega_{R_j}(x_0)} G^2\,dx \leqslant \frac{c_{13}}{|\ln 2R_j|^\omega}, \quad j \geqslant j_0 + 1$$

is valid for an arbitrary location of x^0 in $\overline{\Omega}$. Obviously, this implies estimate (12). The theorem is proved. □

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С.-Петербургский
государственный
университет
e-mail:arina@aa1101.spb.edu

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