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$\mathcal{I}^{\mathcal{K}}$ -SEQUENTIAL TOPOLOGY

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Abstract: In the literature, \mathcal{I} -convergence (or convergence in \mathcal{I}) was first introduced in [11]. Later related notions of \mathcal{I} -sequential topological space and \mathcal{I}^* -sequential topological space were introduced and studied. From the definitions it is clear that \mathcal{I}^* -sequential topological space is larger (finer) than \mathcal{I} -sequential topological space. This rises a question: is there any topology (different from discrete topology) on the topological space \mathcal{X} which is finer than \mathcal{I}^* -topological space? In this paper, we tried to find the answer to the question. We define $\mathcal{I}^{\mathcal{K}}$ -sequential topology for any ideals \mathcal{I}, \mathcal{K} and study main properties of it. First of all, some fundamental results about $\mathcal{I}^{\mathcal{K}}$ -convergence of a sequence in a topological space $(\mathcal{X}, \mathcal{T})$ are derived. After that, $\mathcal{I}^{\mathcal{K}}$ -continuity and the subspace of the $\mathcal{I}^{\mathcal{K}}$ -sequential topological space are investigated.

Keywords: Ideal convergence, $\mathcal{I}^{\mathcal{K}}$ -convergence, Sequential topology, $\mathcal{I}^{\mathcal{K}}$ -sequential topology.

1. Introduction

The notion of convergence of real or complex valued sequences was generalized using asymptotic density and was called statistical convergence by Fast [7] and Steinhaus [20] in the same year 1951, independently. After some years P. Kostyrko, T. Šalát, W. Wilczyński [11] gave a generalization of statistical convergence and called it as ideal convergence (or converges in ideal). Various fundamental properties (convergence in \mathcal{I} and \mathcal{I}^*) were investigated. Later B.K. Lahiri and P. Das in [12] discussed convergence in \mathcal{I} and in \mathcal{I}^* and investigate some additional results related to mentioned concepts [4, 8–10, 15–17].

The concept of \mathcal{I}^* -convergence of functions was extended to $\mathcal{I}^{\mathcal{K}}$ -convergence by M. Mačaj and M. Sleziak in [13] in 2011. The authors of [2, 3, 5, 6, 14] gave further properties and results about $\mathcal{I}^{\mathcal{K}}$ -convergence.

In first part of this paper we introduce $\mathcal{I}^{\mathcal{K}}$ -sequential topological (seq.-top.) space, which is a natural generalization of \mathcal{I}^* -seq.-top. space. Later we discuss the $\mathcal{I}^{\mathcal{K}}$ -continuity of the function and in last two section we write about $\mathcal{I}^{\mathcal{K}}$ -subspace and $\mathcal{I}^{\mathcal{K}}$ -connectedness. We will use further the abbreviation T.S. for a topological space.

2. Definition and preliminaries

In this part, we give some known definitions and necessary results.

Definition 1 [7, 20]. Let $\mathcal{A} \subset \mathbb{N}$, and for $m \in \mathbb{N}$ let the set

$$\mathcal{A}_m := \{x \in \mathcal{A} : x < m\}$$

and $|\mathcal{A}_m|$ stand for the cardinality of \mathcal{A}_m . Natural density of \mathcal{A} is defined by

$$\beta(\mathcal{A}) := \lim_{m \rightarrow \infty} \frac{|\mathcal{A}_m|}{m}$$

whenever the limit exists. A real sequence $\tilde{x} = (x_i)$ is said to statistically converges to x_0 if for any $\varepsilon > 0$,

$$\beta(\{n : |x_i - x_0| > \varepsilon\}) = 0$$

holds.

Definition 2 [11]. Let \mathcal{I} be any subfamily of $\mathcal{P}(\mathbb{N})$, with $\mathcal{P}(\mathbb{N})$ being the family of all subsets of \mathbb{N} . Then, \mathcal{I} is called an ideal on \mathbb{N} if the following requirements hold:

- (i) finite union of sets in \mathcal{I} is again in \mathcal{I} ;
- (ii) any subset of a set in \mathcal{I} is in \mathcal{I} .

\mathcal{I} is admissible if all singleton subsets of \mathbb{N} belong to \mathcal{I} . The ideal \mathcal{I} is non-trivial if $\mathcal{I} \neq \emptyset$ and $\mathcal{I} \neq \mathcal{P}(\mathbb{N})$. A non-trivial ideal \mathcal{I} is called proper if \mathbb{N} is not in \mathcal{I} .

The family of finite subsets of the \mathbb{N} is an admissible non-trivial ideal denoted by $\mathcal{F}in$ and the family of the subsets of \mathbb{N} with natural density zero is also an admissible non-trivial ideal denoted by \mathcal{I}_β . The set of all non-trivial admissible ideals will be denoted as NA throughout the study.

Example 1. [11] Consider the decomposition of \mathbb{N} as $\mathbb{N} = \bigcup_{j=1}^{\infty} \beta_j$ where all β_j are infinite subsets of \mathbb{N} and are mutually disjoint. Take the family

$$\mathcal{I} = \{N \subset \mathbb{N} : N \text{ intersect only finite number of } \beta_j\text{'s}\}.$$

Then, \mathcal{I} belongs to NA .

Definition 3 [19]. Assume $\mathcal{F} \subset \mathcal{P}(\mathbb{N})$. The collection \mathcal{F} is a filter on \mathbb{N} if

- (i) a finite intersection of elements of \mathcal{F} is in \mathcal{F} and
- (ii) if $\mathcal{C} \in \mathcal{F} \wedge \mathcal{C} \subseteq \mathcal{D}$, then $\mathcal{D} \in \mathcal{F}$.

If empty set is not in \mathcal{F} then \mathcal{F} is proper. If $\mathcal{I} \in NA$ then the collection

$$\mathcal{F} = \{N \subset \mathbb{N} : N^C \in \mathcal{I}\}$$

is a filter on \mathbb{N} . It is known as the \mathcal{I} -associated filter.

Definition 4 [21]. In a T.S. $(\mathcal{X}, \mathcal{T})$ a sequence $\tilde{x} = (x_i) \subset \mathcal{X}$ is called to converging in \mathcal{I} to a point $x \in \mathcal{X}$ if

$$\{i \in \mathbb{N} : x_i \in v\} \in \mathcal{F}(\mathcal{I})$$

holds for each neighborhood v of x . The point x is referred to as the ideal limit of the sequence $\tilde{x} = (x_i)$ and it is represented by $x_i \xrightarrow{\mathcal{I}} x$ (or $\mathcal{I} - \lim x_i = x$).

Remark 1.

- (i) Statistical and \mathcal{I}_β - convergence are coincide.
- (ii) Classical convergence and $\mathcal{F}in$ -convergence are coincide.

Lemma 1 [1]. Assume that $\mathcal{I}, \mathcal{I}_1$ and \mathcal{I}_2 be ideals on the set \mathbb{N} and consider a T.S. $(\mathcal{X}, \mathcal{T})$, then

1. If $\mathcal{I} \in NA$, then every convergent sequence is \mathcal{I} -convergent sequence which converges to same point.
2. If $\mathcal{I}_1 \subseteq \mathcal{I}_2$ and $(x_i) \subseteq \mathcal{X}$ is a sequence which $x_i \xrightarrow{\mathcal{I}_1} x$, then $x_i \xrightarrow{\mathcal{I}_2} x$.
3. If \mathcal{X} the Hausdorff space, then the limit of every convergent sequence is unique.

3. $\mathcal{I}^{\mathcal{K}}$ -convergence of sequence

In this part we will investigate some results related to $\mathcal{I}^{\mathcal{K}}$ -convergence of sequences which is a generalized form of \mathcal{I}^* -convergence of sequences. If we consider $\mathcal{F}in$ instead of \mathcal{K} , then we will have \mathcal{I}^* -convergence.

Definition 5 [6]. In a T.S. $(\mathcal{X}, \mathcal{T})$ a sequence $\tilde{x} = (x_i) \subset \mathcal{X}$ is called to be \mathcal{I}^* -converging to $x_0 \in \mathcal{X}$ if $\exists M \in \mathcal{F}(\mathcal{I})$ s.t. the sequence

$$y_i := \begin{cases} x_i, & i \in M, \\ x, & i \notin M \end{cases}$$

is $\mathcal{F}in$ convergent to x .

That is, for each neighborhood v of x ,

$$\{i \in \mathbb{N} : y_i \in v\} \in \mathcal{F}(\mathcal{F}in),$$

or

$$\{i \in M : y_i \notin v\} \cup \{i \in M^C : y_i \notin v\} \in \mathcal{F}in.$$

So,

$$\{i \in M : x_i \notin v\} \cup \{i \in M^C : x \notin v\} \in \mathcal{F}in.$$

This implies that

$$\{i \in M : y_i \notin v\} \in \mathcal{F}in.$$

Therefore,

$$\{i \in M : y_i \in v\} \in \mathcal{F}(\mathcal{F}in).$$

It is clear that this definition is the same as the definition given in [6]. In the definition of \mathcal{I}^* -convergence of sequence if we consider an arbitrary ideal \mathcal{K} instead of the ideal $\mathcal{F}in$ then it yields the definition of $\mathcal{I}^{\mathcal{K}}$ -convergence of a sequence. That is, $\mathcal{I}^{\mathcal{K}}$ -convergence is the generalized form of \mathcal{I}^* -convergence.

Definition 6 [13]. Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and consider a T.S. $(\mathcal{X}, \mathcal{T})$. The sequence $\tilde{x} = (x_i) \subset \mathcal{X}$ is $\mathcal{I}^{\mathcal{K}}$ -convergent to a point $x \in \mathcal{X}$ if $\exists M \in \mathcal{F}(\mathcal{I})$ s.t. the sequence

$$y_i = \begin{cases} x_i, & i \in M, \\ x, & i \notin M, \end{cases}$$

\mathcal{K} -converges to x . We represent it as $\mathcal{I}^{\mathcal{K}} - \lim(x_i) = x$ or $x_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$.

Definition 7. Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and $(\mathcal{X}, \mathcal{T})$ represent a T.S. Consider the sequences $\tilde{x} = (x_i) \subset \mathcal{X}$ and $\tilde{y} = (y_i) \subset \mathcal{X}$. Define a relation $\sim_{\mathcal{I}}$ as

$$\tilde{x} \sim_{\mathcal{I}} \tilde{y} \Leftrightarrow \{i : x_i \neq y_i\} \in \mathcal{I}.$$

The relation $\sim_{\mathcal{I}}$ is an equivalence relation. That is,

1. $\forall \tilde{x} = (x_i) \subset \mathcal{X}, \{i : x_i \neq x_i\} = \emptyset \in \mathcal{I} \Rightarrow \tilde{x} \sim_{\mathcal{I}} \tilde{x}$.
2. Let $\tilde{x} \sim_{\mathcal{I}} \tilde{y}$. Since $\{i : y_i \neq x_i\} = \{i : x_i \neq y_i\} \in \mathcal{I}$, then $\tilde{y} \sim_{\mathcal{I}} \tilde{x}$.
3. Let $\tilde{x} \sim_{\mathcal{I}} \tilde{y}$ and $\tilde{y} \sim_{\mathcal{I}} \tilde{z}$. Then, $A := \{i : x_i = y_i\} \in \mathcal{F}(\mathcal{I})$ and $B := \{i : y_i = z_i\} \in \mathcal{F}(\mathcal{I})$. So, $\{i : x_i = z_i\} = A \cap B \in \mathcal{F}(\mathcal{I})$. Hence, $\tilde{x} \sim_{\mathcal{I}} \tilde{z}$ holds.

Lemma 2. *Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and consider the T.S. $(\mathcal{X}, \mathcal{T})$ and the sequences $\tilde{x} = (x_i) \subseteq \mathcal{X}$. Assume $x_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$ for any $x \in \mathcal{X}$ and $\tilde{t} = (t_i) \subseteq \mathcal{X}$ is a sequence s.t. $\tilde{x} \sim_{\mathcal{I}} \tilde{t}$. Then, the sequence $t_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$.*

P r o o f. Let $x_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$, then $\exists M \in \mathcal{F}(\mathcal{I})$ s.t. the following sequence

$$y_i = \begin{cases} x_i, & i \in M, \\ x, & i \notin M \end{cases}$$

is \mathcal{K} -convergent to x . Since $(x_i) \sim_{\mathcal{I}} (t_i)$. So $\forall i \in M, x_i = t_i$. Therefore, the following sequence

$$y_i = \begin{cases} t_i, & i \in M, \\ x, & i \notin M \end{cases}$$

is \mathcal{K} -convergent to x which shows that $t_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$ holds. □

The Definition 7 gives the possibility that the definition of $\mathcal{I}^{\mathcal{K}}$ -convergence of a sequence can be rewritten as follows:

Definition 8. *Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and consider the T.S. $(\mathcal{X}, \mathcal{T})$. A sequence $\tilde{x} = (x_i) \subset X$ is $\mathcal{I}^{\mathcal{K}}$ -convergent to the point $x \in \mathcal{X}$ if there exist a sequence $\tilde{t} = (t_i) \subset X$ s.t. $\tilde{x} \sim_{\mathcal{I}} \tilde{t}$ and $t_i \xrightarrow{\mathcal{K}} x$ holds.*

In the following lemma we demonstrate that Definition 6 and Definition 8 are equivalent for any ideals \mathcal{I} and \mathcal{K} and for any T.S. $(\mathcal{X}, \mathcal{T})$.

Lemma 3. *Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and consider the T.S. $(\mathcal{X}, \mathcal{T})$ and $\tilde{x} = (x_i) \subset \mathcal{X}$ be a sequence. Then, $x_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$ iff $\exists \tilde{t} = (t_i) \subset \mathcal{X}$ s.t. $\tilde{x} \sim_{\mathcal{I}} \tilde{t}$ and $t_i \xrightarrow{\mathcal{K}} x$ hold.*

P r o o f. Let $x_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$ holds. Then, $\exists M \in \mathcal{F}(\mathcal{I})$ s.t. the following sequence

$$y_i = \begin{cases} x_i, & i \in M, \\ x, & i \notin M \end{cases}$$

is \mathcal{K} -convergent to x . Let us chose $(t_i) = (y_i) \forall i \in \mathbb{N}$. Then, the proof will complete if we show that $\tilde{x} \sim_{\mathcal{I}} \tilde{y}$.

Consider the fact $\{i \in \mathbb{N} : x_i = y_i\} = \{i \in M : x_i = y_i\} \in \mathcal{F}(\mathcal{I})$. Hence, $\tilde{x} \sim_{\mathcal{I}} \tilde{t}$.

Conversely, let $\tilde{x} = (x_i)$ and $\tilde{t} = (t_i)$ be sequences s.t. $\tilde{x} \sim_{\mathcal{I}} \tilde{t}$ and $t_i \xrightarrow{\mathcal{K}} x$ hold. Since $\tilde{x} \sim_{\mathcal{I}} \tilde{t}$, then

$$M = \{i \in \mathbb{N} : x_i = t_i\} \in \mathcal{F}(\mathcal{I})$$

holds. Define a sequence

$$y_i = \begin{cases} x_i, & i \in M, \\ x, & i \notin M. \end{cases}$$

Since $x_i = t_i$ hold $\forall i \in M$, then we can write

$$t_i = \begin{cases} x_i, & i \in M, \\ x, & i \notin M. \end{cases}$$

Because $\tilde{t} = (t_i)$ is \mathcal{K} -convergent to x , the sequence $\tilde{y} = (y_i)$ is also \mathcal{K} -convergent to x . Hence, the sequence $\tilde{x} = (x_i)$ is $\mathcal{I}^{\mathcal{K}}$ -convergent to the point x and this completes the proof. □

4. $\mathcal{I}^{\mathcal{K}}$ -seq.-top. space

In this section, we are going to define a new topology on the \mathcal{X} using the ideal \mathcal{I} and \mathcal{K} and investigate some properties of the new T.S. This topology will be an extended version of the \mathcal{I}^* -seq.-top. space which was discussed in [18]. If we take $\mathcal{I} = \mathcal{F}in$, then $\mathcal{I}^{\mathcal{K}}$ -seq.-top. space is coincide with \mathcal{I}^* -T.S.

Definition 9. Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and consider the T.S. $(\mathcal{X}, \mathcal{T})$. Then

1. A set $F \subseteq \mathcal{X}$ is $\mathcal{I}^{\mathcal{K}}$ -closed, if for each $(x_i) \subseteq F$ with $x_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$, then $x \in F$.
2. A set $V \subseteq \mathcal{X}$ is $\mathcal{I}^{\mathcal{K}}$ -open, if its complement V^C is $\mathcal{I}^{\mathcal{K}}$ -closed.

Remark 2. Consider the T.S. $(\mathcal{X}, \mathcal{T})$. An $O \subseteq \mathcal{X}$ is $\mathcal{I}^{\mathcal{K}}$ -open iff each sequence in $\mathcal{X} - O$ has $\mathcal{I}^{\mathcal{K}}$ -limit in $\mathcal{X} - O$.

P r o o f. The proof is evident from Definition 9. Therefore, it is omitted here. \square

Definition 10. Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and consider the T.S. $(\mathcal{X}, \mathcal{T})$. For any subset $A \subseteq \mathcal{X}$ define a set $\overline{A}^{\mathcal{I}^{\mathcal{K}}}$ (it is called $\mathcal{I}^{\mathcal{K}}$ -closure of A) by

$$\overline{A}^{\mathcal{I}^{\mathcal{K}}} := \{x \in \mathcal{X} : \exists (x_i) \subseteq A, x_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x\}.$$

It is clear that $\overline{\emptyset}^{\mathcal{I}^{\mathcal{K}}} = \emptyset$, $\overline{\mathcal{X}}^{\mathcal{I}^{\mathcal{K}}} = \mathcal{X}$, and $A \subseteq \overline{A}^{\mathcal{I}^{\mathcal{K}}}$ holds $\forall A \subseteq \mathcal{X}$.

Remark 3. A subset C of the T.S. \mathcal{X} is $\mathcal{I}^{\mathcal{K}}$ closed set iff $\overline{C}^{\mathcal{I}^{\mathcal{K}}} = C$.

P r o o f. Proof is obvious from the Definition 10. So, it is omitted here. \square

Lemma 4. Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and let $(\mathcal{X}, \mathcal{T})$ represent a T.S. For any subset $A \subseteq \mathcal{X}$, $\mathcal{I}^{\mathcal{K}}$ -closure of A is $\mathcal{I}^{\mathcal{K}}$ -closed.

P r o o f. We must show that

$$\overline{\overline{A}^{\mathcal{I}^{\mathcal{K}}}}^{\mathcal{I}^{\mathcal{K}}} = \overline{A}^{\mathcal{I}^{\mathcal{K}}}.$$

It is clear that

$$\overline{A}^{\mathcal{I}^{\mathcal{K}}} \subseteq \overline{\overline{A}^{\mathcal{I}^{\mathcal{K}}}}^{\mathcal{I}^{\mathcal{K}}}.$$

Let $x \in \overline{\overline{A}^{\mathcal{I}^{\mathcal{K}}}}^{\mathcal{I}^{\mathcal{K}}}$. Then, there exist a sequence $(x_i) \subseteq \overline{A}^{\mathcal{I}^{\mathcal{K}}}$ s.t. $x_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$ holds. Since $(x_i) \subseteq \overline{A}^{\mathcal{I}^{\mathcal{K}}}$, then there exist sequences $(x_i^n) \subseteq A$ s.t. $x_i^n \xrightarrow{\mathcal{I}^{\mathcal{K}}} x_i$. Therefore there exist the sets $M_n \in \mathcal{F}(\mathcal{I})$ s.t.

$$\{i \in M_n : x_i^n \notin v^n\} \in \mathcal{K}$$

for each neighborhood v^n of x_i . Choose m_1 the i where x_i^1 is belonging to neighborhood v^1 of x_1 , similarly m_2 the i where x_i^2 is belonging to neighborhood v^2 of x_2 . If we continue this process and take m_p the i where x_i^p is belonging to neighborhood v^n of x_p . The obtained sequence (x_{m_p}) belongs to A . The theorem will be proved if we show that $x_{m_p} \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$. Since $x_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$, so $\exists M \in \mathcal{F}(\mathcal{I})$ s.t. the sequence

$$y_i = \begin{cases} x_i, & i \in M, \\ x, & i \notin M, \end{cases} \quad y_i \xrightarrow{\mathcal{K}} x.$$

So,

$$\{i \in M : x_i \notin v\} \in \mathcal{K}$$

for each neighborhood v of x . Now,

$$\{i \in M : v^n \not\subset v\} \subseteq \{i \in M : x_i \notin v\} \in \mathcal{K}.$$

Therefore,

$$\{i \in M : v^n \not\subset v\} \in \mathcal{K}$$

and

$$\{i \in M : x_{m_p} \notin v\} \subset \{i \in M : v^n \not\subset U\} \in \mathcal{K}$$

hold. So, $x_{m_p} \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$ and $x \in \overline{A}^{\mathcal{I}^{\mathcal{K}}}$. □

Definition 11. Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and $(\mathcal{X}, \mathcal{T})$ represent a T.S. Then, for $A \subset \mathcal{X}$, $\mathcal{I}^{\mathcal{K}}$ -interior of A is defined as

$$A^{\circ \mathcal{I}^{\mathcal{K}}} := A - \overline{(\mathcal{X} - A)^{\mathcal{I}^{\mathcal{K}}}}.$$

Proposition 1. Let \mathcal{V} be a subset of T.S. \mathcal{X} , then \mathcal{V} is $\mathcal{I}^{\mathcal{K}}$ -open iff $\mathcal{V}^{\circ \mathcal{I}^{\mathcal{K}}} = \mathcal{V}$.

P r o o f. Let \mathcal{V} be an $\mathcal{I}^{\mathcal{K}}$ -open set. Then, $\mathcal{X} - \mathcal{V}$ is $\mathcal{I}^{\mathcal{K}}$ -closed set and

$$\text{cl}_{\mathcal{I}^{\mathcal{K}}}(\mathcal{X} - \mathcal{V}) = \mathcal{X} - \mathcal{V}$$

holds. So, we have

$$\mathcal{V}^{\circ \mathcal{I}^{\mathcal{K}}} = \mathcal{V} - (\mathcal{X} - \mathcal{V}) = \mathcal{V}.$$

Conversely assume that

$$\mathcal{V}^{\circ \mathcal{I}^{\mathcal{K}}} = \mathcal{V}$$

holds. From the definition of $\mathcal{I}^{\mathcal{K}}$ -interior of \mathcal{V} we have

$$\mathcal{V} = \mathcal{V} - \overline{(\mathcal{X} - \mathcal{V})^{\mathcal{I}^{\mathcal{K}}}}.$$

Hence,

$$\mathcal{V} \cap \overline{(\mathcal{X} - \mathcal{V})^{\mathcal{I}^{\mathcal{K}}}} = \emptyset.$$

Consequently

$$\overline{(\mathcal{X} - \mathcal{V})^{\mathcal{I}^{\mathcal{K}}}} \subset \mathcal{X} - \mathcal{V}.$$

Thus,

$$\overline{(\mathcal{X} - \mathcal{V})^{\mathcal{I}^{\mathcal{K}}}} = \mathcal{X} - \mathcal{V}$$

is satisfied. Therefore, $\mathcal{X} - \mathcal{V}$ is $\mathcal{I}^{\mathcal{K}}$ -closed and \mathcal{V} is $\mathcal{I}^{\mathcal{K}}$ -open. □

Definition 12 [21]. A sequence (x_i) in a T.S. \mathcal{X} is \mathcal{I} -eventually in a subset A of \mathcal{X} if

$$\{i \in \mathbb{N} : x_i \in A\} \in \mathcal{F}(\mathcal{I}).$$

Definition 13. Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and consider the T.S. $(\mathcal{X}, \mathcal{T})$. A sequence $\tilde{x} = (x_i) \subseteq \mathcal{X}$ is $\mathcal{I}^{\mathcal{K}}$ -eventually in a subset \mathcal{V} of \mathcal{X} . If there exist a sequence $\tilde{y} = (y_i) \subseteq \mathcal{X}$ s.t. $\tilde{y} \sim_{\mathcal{I}} \tilde{x}$ and \tilde{y} is \mathcal{K} -eventually in \mathcal{V} .

In the next theorem, we will provide a sequence characterization of $\mathcal{I}^{\mathcal{K}}$ -open set.

Theorem 1. Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and consider the T.S. $(\mathcal{X}, \mathcal{T})$. A subset v of \mathcal{X} is $\mathcal{I}^{\mathcal{K}}$ -open iff each $\mathcal{I}^{\mathcal{K}}$ -convergent sequence to $x_0 \in v$ is $\mathcal{I}^{\mathcal{K}}$ -eventually in v .

P r o o f. Let v is $\mathcal{I}^{\mathcal{K}}$ -open. Then, $\mathcal{X} - v$ is $\mathcal{I}^{\mathcal{K}}$ -closed and $\overline{\mathcal{X} - v}^{\mathcal{I}^{\mathcal{K}}} = \mathcal{X} - v$ holds. Let $\tilde{x} = (x_i) \subset \mathcal{X}$ be a sequence s.t. $x_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$ and $x \in v$. Then, $\exists M \in \mathcal{F}(\mathcal{I})$ s.t. the sequence

$$t_i = \begin{cases} x_i, & i \in M, \\ x, & i \notin M \end{cases}$$

is \mathcal{K} -convergent to x . Since v is a neighborhood of x , then we have

$$H = \{i \in \mathbb{N} : x_i \notin v\} \in \mathcal{K}.$$

If we choose $y_i = t_i$, then

$$\{i \in \mathbb{N} : y_i = x_i\} = \{i \in \mathbb{N} : t_i = x_i\} = M \in \mathcal{F}(\mathcal{I})$$

holds. So, $(y_i) \sim_{\mathcal{I}} (x_i)$ holds and (y_i) is eventually in v .

Conversely, let $\tilde{x} = (x_i) \subset \mathcal{X}$ is a sequence which is $\mathcal{I}^{\mathcal{K}}$ -convergent sequence to a point $x \in v$ and it is $\mathcal{I}^{\mathcal{K}}$ -eventually in v . Assume that v is not $\mathcal{I}^{\mathcal{K}}$ -open subset of \mathcal{X} . So there exists $x_0 \in \overline{\mathcal{X} - v}^{\mathcal{I}^{\mathcal{K}}}$ which $x_0 \notin \mathcal{X} - v$. This means that there exists a sequence $(x_i) \subset \mathcal{X} - v$ which is $\mathcal{I}^{\mathcal{K}}$ -convergence to $x_0 \in v$. So, (x_i) is $\mathcal{I}^{\mathcal{K}}$ -eventually in v .

Therefore, $\exists \tilde{y} = (y_i) \subset \mathcal{X}$ which $\tilde{x} \sim_{\mathcal{I}} \tilde{y}$ and \tilde{y} is \mathcal{K} -eventually in v . This implies that \tilde{y} is \mathcal{K} -eventually in v which is not in case. \square

Theorem 2. Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and consider the T.S. $(\mathcal{X}, \mathcal{T})$. A subset $\mathcal{C} \subset \mathcal{X}$ is $\mathcal{I}^{\mathcal{K}}$ -closed iff

$$\mathcal{C} = \cap \{ \mathcal{A} : \mathcal{A} \text{ is } \mathcal{I}^{\mathcal{K}}\text{-closed and } \mathcal{C} \subset \mathcal{A} \}.$$

P r o o f. Let

$$\mathcal{C} = \cap \{ \mathcal{A} : \mathcal{A} \text{ is } \mathcal{I}^{\mathcal{K}}\text{-closed and } \mathcal{C} \subset \mathcal{A} \}.$$

Let x be any element of $\mathcal{I}^{\mathcal{K}}$ -closure of \mathcal{C} . Then there exists $(x_i) \subset \mathcal{C}$ s.t. $x_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$. Let $x \notin \mathcal{C}$ so

$$x \notin \cap \{ \mathcal{A} : \mathcal{A} \text{ is } \mathcal{I}^{\mathcal{K}}\text{-closed and } \mathcal{C} \subset \mathcal{A} \}.$$

This implies that $\exists \mathcal{I}^{\mathcal{K}}$ -closed subset F of \mathcal{X} s.t. $x \notin F$, but \mathcal{C} is $\mathcal{I}^{\mathcal{K}}$ -closed and it is a subset of F , which is a contradiction.

The converse is obvious. \square

Theorem 3. Let \mathcal{I} and \mathcal{K} be ideals of \mathbb{N} and $(\mathcal{X}, \mathcal{T})$ be a T.S. A function $\text{cl}_{\mathcal{I}^{\mathcal{K}}} : \mathcal{P}(\mathcal{X}) \rightarrow \mathcal{P}(\mathcal{X})$ defined as $\text{cl}_{\mathcal{I}^{\mathcal{K}}}(A) = \overline{A}^{\mathcal{I}^{\mathcal{K}}}$ is satisfying Kuratowski closure axioms

(K1) $\text{cl}_{\mathcal{I}^{\mathcal{K}}}(\emptyset) = \emptyset$ and $\text{cl}_{\mathcal{I}^{\mathcal{K}}}(\mathcal{X}) = \mathcal{X}$,

- (K2) $A \subseteq \text{cl}_{\mathcal{I}^{\mathcal{K}}}(A) \quad \forall A \subseteq \mathcal{X}$,
 (K3) $\text{cl}_{\mathcal{I}^{\mathcal{K}}}(A) = \text{cl}_{\mathcal{I}^{\mathcal{K}}}(\text{cl}_{\mathcal{I}^{\mathcal{K}}}(A)) \quad \forall A \subseteq \mathcal{X}$,
 (K4) $\text{cl}_{\mathcal{I}^{\mathcal{K}}}(A \cup B) = \text{cl}_{\mathcal{I}^{\mathcal{K}}}(A) \cup \text{cl}_{\mathcal{I}^{\mathcal{K}}}(B) \quad \forall A, B \subseteq \mathcal{X}$.

P r o o f. (K1) and (K2) are clear from the definition of $\mathcal{I}^{\mathcal{K}}$ -closure function. By Lemma 4, $\text{cl}_{\mathcal{I}^{\mathcal{K}}}(A)$ is closed. So, $\text{cl}_{\mathcal{I}^{\mathcal{K}}}(\text{cl}_{\mathcal{I}^{\mathcal{K}}}(A)) = \text{cl}_{\mathcal{I}^{\mathcal{K}}}(A)$. Therefore, (K3) holds.

To prove (K4), let $x \in \text{cl}_{\mathcal{I}^{\mathcal{K}}}(A) \cup \text{cl}_{\mathcal{I}^{\mathcal{K}}}(B)$. Then, $x \in \text{cl}_{\mathcal{I}^{\mathcal{K}}}(A)$ or $x \in \text{cl}_{\mathcal{I}^{\mathcal{K}}}(B)$. Without lost of generality assume that $x \in \text{cl}_{\mathcal{I}^{\mathcal{K}}}(A)$. So, $\exists(x_i) \subset A$ s.t. $x_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$. Therefore, $\exists(x_i) \subset A \cup B$ s.t. $x_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$. So, $x \in \text{cl}_{\mathcal{I}^{\mathcal{K}}}(A) \cup \text{cl}_{\mathcal{I}^{\mathcal{K}}}(B)$.

Conversely, let $x \in \text{cl}_{\mathcal{I}^{\mathcal{K}}}(A \cup B)$. Then, there exist a sequence $(x_i) \subset (A \cup B)$ s.t. $x_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$. Assume that $x \notin \text{cl}_{\mathcal{I}^{\mathcal{K}}}(A)$ and $x \notin \text{cl}_{\mathcal{I}^{\mathcal{K}}}(B)$. So, neither set A nor set B contains a sequence s.t. $\mathcal{I}^{\mathcal{K}}$ -converges to the point x . Consequently, there is not any sequence in the $A \cup B$ which is convergent to x . But $x \in \text{cl}_{\mathcal{I}^{\mathcal{K}}}(A \cup B)$ which is a contradiction. Hence,

$$\text{cl}_{\mathcal{I}^{\mathcal{K}}}(A \cup B) = \text{cl}_{\mathcal{I}^{\mathcal{K}}}(A) \cup \text{cl}_{\mathcal{I}^{\mathcal{K}}}(B)$$

holds. □

Corollary 1. *A subset A of \mathcal{X} is $\mathcal{I}^{\mathcal{K}}$ -closed iff $\text{cl}_{\mathcal{I}^{\mathcal{K}}}(A) = A$ and a subset $O \subset \mathcal{X}$ is $\mathcal{I}^{\mathcal{K}}$ -open iff $\mathcal{X} - O$ is $\mathcal{I}^{\mathcal{K}}$ -closed.*

Theorem 4. *Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and consider the T.S. $(\mathcal{X}, \mathcal{T})$. Then,*

$$\mathcal{T}_{\mathcal{I}^{\mathcal{K}}} := \{A \subset X : \text{cl}_{\mathcal{I}^{\mathcal{K}}}(\mathcal{X} - A) = \mathcal{X} - A\}$$

is a topology over the set \mathcal{X} .

P r o o f. By (K1), it is clear that $\mathcal{X} \in \mathcal{T}_{\mathcal{I}^{\mathcal{K}}}$ and $\emptyset \in \mathcal{T}_{\mathcal{I}^{\mathcal{K}}}$ hold. Let $A, B \in \mathcal{T}_{\mathcal{I}^{\mathcal{K}}}$ be arbitrary sets. To prove $A \cup B \in \mathcal{T}_{\mathcal{I}^{\mathcal{K}}}$ we must to prove that

$$\mathcal{X} - A \cup B = \text{cl}_{\mathcal{I}^{\mathcal{K}}}(\mathcal{X} - A \cup B)$$

holds. By (K2), we have

$$\mathcal{X} - A \cup B \subset \text{cl}_{\mathcal{I}^{\mathcal{K}}}(\mathcal{X} - A \cup B).$$

Now, let $x \in \text{cl}_{\mathcal{I}^{\mathcal{K}}}(\mathcal{X} - A \cup B)$ be an arbitrarily element. Then, $\exists(x_i) \subset \mathcal{X} - (A \cup B)$ s.t. it is $\mathcal{I}^{\mathcal{K}}$ -convergent to x . This implies that (x_i) is not subset of $A \cup B$. So, (x_i) is neither subset of A nor subset of B . Therefore, $(x_i) \subset \mathcal{X} - A$ or $(x_i) \subset \mathcal{X} - B$ which $\mathcal{I}^{\mathcal{K}}$ -converges to point x . So, $x \in \text{cl}_{\mathcal{I}^{\mathcal{K}}}(\mathcal{X} - A)$ or $x \in \text{cl}_{\mathcal{I}^{\mathcal{K}}}(\mathcal{X} - B)$. Since $\mathcal{X} - A$ and $\mathcal{X} - B$ are closed sets, then

$$x \in (\mathcal{X} - A) \cup (\mathcal{X} - B) = \mathcal{X} - A \cup B$$

holds.

Let $\{A_i\}$ be a collection of $\mathcal{I}^{\mathcal{K}}$ -open subsets of \mathcal{X} . Then, $\text{cl}_{\mathcal{I}^{\mathcal{K}}}(\mathcal{X} - A_i) = \mathcal{X} - A_i \quad \forall i \in \mathbb{N}$. By considering (K2), we have

$$\bigcap_{i \in \mathbb{N}} (\mathcal{X} - A_i) \subseteq \text{cl}_{\mathcal{I}^{\mathcal{K}}}(\bigcap_{i \in \mathbb{N}} (\mathcal{X} - A_i)).$$

Let $x \in \text{cl}_{\mathcal{I}^{\mathcal{K}}}(\bigcap_{i \in \mathbb{N}} (\mathcal{X} - A_i))$ be an arbitrary element. Then, $\exists(x_i) \subset \bigcap_{i \in \mathbb{N}} (\mathcal{X} - A_i)$ which is $\mathcal{I}^{\mathcal{K}}$ -convergent to x . Then, $(x_i) \subset (\mathcal{X} - A_i) \quad \forall i \in \mathbb{N}$. Since $\mathcal{X} - A_i$ are closed sets, then $x \in \mathcal{X} - A_i \quad \forall i \in \mathbb{N}$. Therefore,

$$x \in \bigcap_{i \in \mathbb{N}} (\mathcal{X} - A_i).$$

Hence, the set $\mathcal{T}_{\mathcal{I}^{\mathcal{K}}}$ is a topology and $(\mathcal{X}, \mathcal{T}_{\mathcal{I}^{\mathcal{K}}})$ is a T.S. □

Definition 14. The T.S. $(\mathcal{X}, \mathcal{T}_{\mathcal{I}^{\mathcal{K}}})$ is called as $\mathcal{I}^{\mathcal{K}}$ -sequential T.S. For abbreviation we will show it by $\mathcal{I}^{\mathcal{K}}$ -seq.-top. An $\mathcal{I}^{\mathcal{K}}$ -seq.-top. $(\mathcal{X}, \mathcal{T}_{\mathcal{I}^{\mathcal{K}}})$ is said to be $\mathcal{I}^{\mathcal{K}}$ -discrete space if $\mathcal{T}_{\mathcal{I}^{\mathcal{K}}} = \mathcal{P}(\mathcal{X})$.

Theorem 5. Let $\mathcal{I}, \mathcal{K}, \mathcal{I}_1, \mathcal{K}_1, \mathcal{I}_2$ and \mathcal{K}_2 stand for ideals of \mathbb{N} and $(\mathcal{X}, \mathcal{T})$ represents a T.S. Let $\mathcal{I}_1 \subset \mathcal{I}_2$ and $\mathcal{K}_1 \subset \mathcal{K}_2$. Then,

1. $\mathcal{T}_{\mathcal{I}^{\mathcal{K}_2}} \prec \mathcal{T}_{\mathcal{I}^{\mathcal{K}_1}}$,
2. $\mathcal{T}_{\mathcal{I}_2^{\mathcal{K}}} \prec \mathcal{T}_{\mathcal{I}_1^{\mathcal{K}}}$.

P r o o f. Let v be any $\mathcal{I}^{\mathcal{K}_2}$ -open subset of \mathcal{X} . Then, $\mathcal{X} - v$ is $\mathcal{I}^{\mathcal{K}_2}$ -closed and $\text{cl}_{\mathcal{I}^{\mathcal{K}_2}}(\mathcal{X} - v) = \mathcal{X} - v$ hold. To prove v is $\mathcal{I}^{\mathcal{K}_1}$ -open subset of \mathcal{X} , we will show that

$$\text{cl}_{\mathcal{I}^{\mathcal{K}_1}}(\mathcal{X} - v) \subset \mathcal{X} - v.$$

Let $x \in \text{cl}_{\mathcal{I}^{\mathcal{K}_1}}(\mathcal{X} - v)$ be any point. Then, there exists $(x_i) \subset \mathcal{X} - v$ s.t. $x_i \xrightarrow{\mathcal{I}^{\mathcal{K}_1}} x$. Since $\mathcal{K}_1 \subset \mathcal{K}_2$, then by Proposition 3.6 in [13], $x_i \xrightarrow{\mathcal{I}^{\mathcal{K}_2}} x$. So, $x \in \text{cl}_{\mathcal{I}^{\mathcal{K}_2}}(\mathcal{X} - v)$. Therefore, $x \in \mathcal{X} - v$. Hence $\mathcal{X} - v$ is $\mathcal{I}^{\mathcal{K}_2}$ -closed set and v is $\mathcal{I}^{\mathcal{K}_2}$ -open subset of \mathcal{X} .

The second one can be proved by using the fact that if $\mathcal{I}_1 \subset \mathcal{I}_2$, then, $x_i \xrightarrow{\mathcal{I}_1^{\mathcal{K}}} x$ implies $x_i \xrightarrow{\mathcal{I}_2^{\mathcal{K}}} x$, it easily can be proved. \square

Theorem 6. Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and $(\mathcal{X}, \mathcal{T})$ represent a T.S. Then, every \mathcal{I}^* -open set is $\mathcal{I}^{\mathcal{K}}$ -open set.

P r o o f. If we take $\mathcal{K} = \mathcal{F}$ in then \mathcal{I}^* -open set will be $\mathcal{I}^{\mathcal{K}}$ -open set. \square

Theorem 7. Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and $(\mathcal{X}, \mathcal{T})$ represent a T.S. Then, every $\mathcal{I}^{\mathcal{K}}$ -open set is \mathcal{K} -open set.

P r o o f. Let v be an arbitrary $\mathcal{I}^{\mathcal{K}}$ -open subset of \mathcal{X} . Then, $\mathcal{X} - v$ is $\mathcal{I}^{\mathcal{K}}$ -closed and

$$\text{cl}_{\mathcal{I}^{\mathcal{K}}}(\mathcal{X} - v) = \mathcal{X} - v.$$

To prove v is \mathcal{K} open, it is sufficient to show that $\mathcal{X} - v$ is \mathcal{K} -closed, i.e.,

$$\mathcal{X} - v = \overline{\mathcal{X} - v}^{\mathcal{K}}.$$

It is clear that $\mathcal{X} - v \subset \overline{\mathcal{X} - v}^{\mathcal{K}}$. Let $x \in \overline{\mathcal{X} - v}^{\mathcal{K}}$ be an arbitrary element s.t. $\exists(x_i) \subset \mathcal{X} - v$ satisfying $x_i \xrightarrow{\mathcal{K}} x$.

Then, by Lemma 3.5 in [13] we have $x_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$. So, $x \in \text{cl}_{\mathcal{I}^{\mathcal{K}}}(\mathcal{X} - v) = \mathcal{X} - v$. Hence, the theorem proved. \square

Proposition 2. Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and $(\mathcal{X}, \mathcal{T})$ represent a T.S. Then, the following statements are true:

1. If $\mathcal{K} \subset \mathcal{I}$, then, each \mathcal{I} -open set is $\mathcal{I}^{\mathcal{K}}$ -open set.
2. If the space \mathcal{X} is a first countable space and the ideal \mathcal{I} has additive property with respect to \mathcal{K} (see Definition 3.10 in [13]), then, each $\mathcal{I}^{\mathcal{K}}$ -open set is \mathcal{I} -open set.
3. If $\mathcal{I} \subset \mathcal{K}$, then every \mathcal{K} -open set is $\mathcal{I}^{\mathcal{K}}$ -open.

P r o o f. The proof is obvious from Proposition 3.7 and Theorem 3.11 of [13]. \square

5. $\mathcal{I}^{\mathcal{K}}$ -continuity of functions

In this section we will define $\mathcal{I}^{\mathcal{K}}$ -continuous and sequential $\mathcal{I}^{\mathcal{K}}$ -continuous functions. We will prove that in any $\mathcal{I}^{\mathcal{K}}$ -sequential T.S. these two concepts coincide. Also, we will state some theorems that give the definition of $\mathcal{I}^{\mathcal{K}}$ -continuous function in different words and ways. At the end of this section we will see that the combination of $\mathcal{I}^{\mathcal{K}}$ -continuous functions is $\mathcal{I}^{\mathcal{K}}$ -continuous.

Definition 15. Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and $(\mathcal{X}, \mathcal{T}_{\mathcal{I}\mathcal{K}})$ $(\mathcal{Y}, \mathcal{T}'_{\mathcal{I}\mathcal{K}})$ represent $\mathcal{I}^{\mathcal{K}}$ -seq.-top. spaces. A function f , from \mathcal{X} to \mathcal{Y} is said to be

- (i) $\mathcal{I}^{\mathcal{K}}$ -continuous which provides that inverse image of any $\mathcal{I}^{\mathcal{K}}$ -open subset of \mathcal{Y} is $\mathcal{I}^{\mathcal{K}}$ -open in \mathcal{X} .
- (ii) *Sequentially $\mathcal{I}^{\mathcal{K}}$ -continuous* which provides that $f(x_i) \xrightarrow{\mathcal{I}^{\mathcal{K}}} f(x) \forall (x_i) \subset \mathcal{X}$ with $x_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$.

Theorem 8. Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and $(\mathcal{X}, \mathcal{T}_{\mathcal{I}\mathcal{K}})$ $(\mathcal{Y}, \mathcal{T}'_{\mathcal{I}\mathcal{K}})$ represent $\mathcal{I}^{\mathcal{K}}$ -seq.-top. spaces; and f , from \mathcal{X} to \mathcal{Y} be a function. Then, f is $\mathcal{I}^{\mathcal{K}}$ -continuous iff it is sequentially $\mathcal{I}^{\mathcal{K}}$ -continuous.

P r o o f. Let f be an $\mathcal{I}^{\mathcal{K}}$ -continuous function. Then, inverse image of any $\mathcal{I}^{\mathcal{K}}$ -open subset of \mathcal{Y} is $\mathcal{I}^{\mathcal{K}}$ -open subset in \mathcal{X} . Let $(x_i) \subset \mathcal{X}$ be a sequence with $x_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$. Then, there exists $M \in \mathcal{F}(\mathcal{I})$ s.t. the following sequence

$$t_i := \begin{cases} x_i, & i \in M, \\ x, & i \notin M \end{cases}$$

is \mathcal{K} -convergent to x . That is, for each neighborhood v of x we have

$$\{i \in \mathbb{N} : t_i \in v\} \in \mathcal{F}(\mathcal{K}).$$

Let \mathcal{V} be any $\mathcal{I}^{\mathcal{K}}$ -open neighborhood of $f(x)$. Then, $f^{-1}(\mathcal{V})$ is $\mathcal{I}^{\mathcal{K}}$ -open subset of \mathcal{X} which contains the point x . So, it is a neighborhood of x . Therefore,

$$\{i \in \mathbb{N} : t_i \in f^{-1}(\mathcal{V})\} \in \mathcal{F}(\mathcal{K}),$$

implies that $\{i \in \mathbb{N} : f(t_i) \in \mathcal{V}\} \in \mathcal{F}(\mathcal{K})$. Hence, the sequence

$$f(t_i) := \begin{cases} f(x_i), & i \in M, \\ f(x), & i \notin M \end{cases}$$

is \mathcal{K} -convergent to $f(x)$. So, $f(x_i) \xrightarrow{\mathcal{I}^{\mathcal{K}}} f(x)$. Hence, f is sequentially $\mathcal{I}^{\mathcal{K}}$ -continuous function.

Conversely, let the function f be sequentially $\mathcal{I}^{\mathcal{K}}$ -continuous and v is any $\mathcal{I}^{\mathcal{K}}$ -open subset of \mathcal{Y} . Assume that $f^{-1}(v)$ is not $\mathcal{I}^{\mathcal{K}}$ -open subset of \mathcal{X} . Then, $\mathcal{X} - f^{-1}(v)$ is not $\mathcal{I}^{\mathcal{K}}$ -closed subset of \mathcal{X} . So,

$$\exists (x_i) \subset \mathcal{X} - f^{-1}(v) \quad \text{s.t.} \quad x_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x \quad \text{and} \quad x \notin \mathcal{X} - f^{-1}(v),$$

i.e. $x_i \notin f^{-1}(v) \forall n$ and $x_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$ which means $x \in f^{-1}(v)$. Since f is $\mathcal{I}^{\mathcal{K}}$ -sequentially continuous function then $f(x_i) \xrightarrow{\mathcal{I}^{\mathcal{K}}} f(x)$. So, $f(x) \in v$ and $f(x_i) \notin v \forall n$. This is a contradiction. \square

Lemma 5. Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and $(\mathcal{X}, \mathcal{T}_{\mathcal{I}\mathcal{K}})$ $(\mathcal{Y}, \mathcal{T}'_{\mathcal{I}\mathcal{K}})$ represent $\mathcal{I}^{\mathcal{K}}$ -seq.-top. spaces and f , from \mathcal{X} to \mathcal{Y} be an $\mathcal{I}^{\mathcal{K}}$ -continuous function. If $(y_i) \subset \mathcal{Y}$ be a sequence s.t. $y_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} y$, then $f^{-1}(y_i) \xrightarrow{\mathcal{I}^{\mathcal{K}}} f^{-1}(y)$.

P r o o f. Let f be an $\mathcal{I}^{\mathcal{K}}$ -continuous function. Let $y_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} y$ then $\exists M \in \mathcal{F}(\mathcal{I})$ s.t. the sequence

$$s_n = \begin{cases} y_i, & i \in M, \\ y, & i \notin M \end{cases}$$

is \mathcal{K} -convergent to y . So, for each neighborhood v of \mathcal{Y} ,

$$\{i \in \mathbb{N} : y_i \in v\} \in \mathcal{F}(\mathcal{K}).$$

Since f is $\mathcal{I}^{\mathcal{K}}$ -continuous function, then inverse image of any $\mathcal{I}^{\mathcal{K}}$ -open set in \mathcal{Y} is $\mathcal{I}^{\mathcal{K}}$ -open in \mathcal{X} , $f^{-1}(v)$ is open neighborhood of x in \mathcal{X} . Then

$$\{i \in \mathbb{N} : f^{-1}(y_i) \in f^{-1}(v)\} \in \mathcal{F}(\mathcal{K}).$$

Therefore,

$$f^{-1}(s_n) = \begin{cases} f^{-1}(y_i), & i \in M, \\ f^{-1}(y), & i \notin M, \end{cases}$$

is \mathcal{K} -convergent to $f^{-1}(y)$ and hence $f^{-1}(y_i) \xrightarrow{\mathcal{I}^{\mathcal{K}}} f^{-1}(y)$. \square

Theorem 9. Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and $(\mathcal{X}, \mathcal{T}_{\mathcal{I}^{\mathcal{K}}})$ $(\mathcal{Y}, \mathcal{T}'_{\mathcal{I}^{\mathcal{K}}})$ represent $\mathcal{I}^{\mathcal{K}}$ -seq.-top. spaces. Then the function f , from \mathcal{X} to \mathcal{Y} is $\mathcal{I}^{\mathcal{K}}$ -continuous iff

$$\text{cl}_{\mathcal{I}^{\mathcal{K}}}(f^{-1}(B)) = f^{-1}(\text{cl}_{\mathcal{I}^{\mathcal{K}}}(B))$$

holds $\forall B \subset \mathcal{Y}$.

P r o o f. Assume that function f , from \mathcal{X} to \mathcal{Y} is $\mathcal{I}^{\mathcal{K}}$ -continuous function. Let

$$x \in \text{cl}_{\mathcal{I}^{\mathcal{K}}}(f^{-1}(B)).$$

Then, $\exists(x_i) \subset f^{-1}(B)$ s.t. $x_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$. Since f is $\mathcal{I}^{\mathcal{K}}$ -continuous so,

$$f(x_i) \xrightarrow{\mathcal{I}^{\mathcal{K}}} f(x).$$

In another hand $(x_i) \subset B$, so $f(x) \in \text{cl}_{\mathcal{I}^{\mathcal{K}}}(B)$ and $x \in f^{-1}(\text{cl}_{\mathcal{I}^{\mathcal{K}}}(B))$.

Now, let $x \in f^{-1}(\text{cl}_{\mathcal{I}^{\mathcal{K}}}(B))$, i.e. $f(x) \in \text{cl}_{\mathcal{I}^{\mathcal{K}}}(B)$. Therefore, $\exists(y_i) \subset B$ s.t. $x_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$. Then, by Lemma 5 there exists $(x_i) = (f^{-1}(y_i)) \subset f^{-1}(B)$ s.t. $x_i \xrightarrow{\mathcal{I}^{\mathcal{K}}} x$, where $x = f^{-1}(y)$ holds. So, $x \in \text{cl}_{\mathcal{I}^{\mathcal{K}}}(f^{-1}(B))$. Hence,

$$\text{cl}_{\mathcal{I}^{\mathcal{K}}}(f^{-1}(B)) = f^{-1}(\text{cl}_{\mathcal{I}^{\mathcal{K}}}(B)).$$

Conversely, let

$$\text{cl}_{\mathcal{I}^{\mathcal{K}}}(f^{-1}(B)) = f^{-1}(\text{cl}_{\mathcal{I}^{\mathcal{K}}}(B)), \quad \forall B \in \mathcal{P}(\mathcal{Y}).$$

Let v be $\mathcal{I}^{\mathcal{K}}$ -open subset of \mathcal{Y} then

$$\text{cl}_{\mathcal{I}^{\mathcal{K}}}(\mathcal{Y} - B) = \mathcal{Y} - B.$$

Let $B = \mathcal{Y} - v$, then

$$\text{cl}_{\mathcal{I}^{\mathcal{K}}}(f^{-1}(\mathcal{Y} - v)) = f^{-1}(\text{cl}_{\mathcal{I}^{\mathcal{K}}}(\mathcal{Y} - v)) = f^{-1}(\mathcal{Y} - v).$$

This shows that $f^{-1}(\mathcal{Y} - v)$ is $\mathcal{I}^{\mathcal{K}}$ -closed. Hence, the following equality

$$f^{-1}(\mathcal{Y} - v) = \mathcal{X} - f^{-1}(v)$$

implies that $\mathcal{X} - f^{-1}(v)$ is $\mathcal{I}^{\mathcal{K}}$ -closed. Therefore $f^{-1}(v)$ is $\mathcal{I}^{\mathcal{K}}$ -open set. \square

Corollary 2. Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and $(\mathcal{X}, \mathcal{T}_{\mathcal{I}\mathcal{K}})$ $(\mathcal{Y}, \mathcal{T}'_{\mathcal{I}\mathcal{K}})$ represent $\mathcal{I}^{\mathcal{K}}$ -seq.-top. spaces. A function f , from \mathcal{X} to \mathcal{Y} is $\mathcal{I}^{\mathcal{K}}$ -continuous iff

$$\text{int}_{\mathcal{I}\mathcal{K}}(f^{-1}(B)) = f^{-1}(\text{int}_{\mathcal{I}\mathcal{K}}(B)) \quad \forall B \subset \mathcal{Y}.$$

Definition 16. Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and $(\mathcal{X}, \mathcal{T}_{\mathcal{I}\mathcal{K}})$ $(\mathcal{Y}, \mathcal{T}'_{\mathcal{I}\mathcal{K}})$ represent $\mathcal{I}^{\mathcal{K}}$ -seq.-top. spaces and f , from \mathcal{X} to \mathcal{Y} be a function. The function f is $\mathcal{I}^{\mathcal{K}}$ -continuous at a point $x \in \mathcal{X}$ if inverse image of any neighborhood of $f(x)$ is a neighborhood of x in \mathcal{X} .

Corollary 3. Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and $(\mathcal{X}, \mathcal{T}_{\mathcal{I}\mathcal{K}})$ $(\mathcal{Y}, \mathcal{T}'_{\mathcal{I}\mathcal{K}})$ represent $\mathcal{I}^{\mathcal{K}}$ -seq.-top. spaces. Then, the function f , from \mathcal{X} to \mathcal{Y} is $\mathcal{I}^{\mathcal{K}}$ -continuous iff it is $\mathcal{I}^{\mathcal{K}}$ -continuous at every point $x \in \mathcal{X}$.

Definition 17. Let \mathcal{I} and \mathcal{K} stand for the ideals of \mathbb{N} and $(\mathcal{X}, \mathcal{T}_{\mathcal{I}\mathcal{K}})$ $(\mathcal{Y}, \mathcal{T}'_{\mathcal{I}\mathcal{K}})$ represent $\mathcal{I}^{\mathcal{K}}$ -seq.-top. spaces and f , from \mathcal{X} to \mathcal{Y} be a function, f is said to be $\mathcal{I}^{\mathcal{K}}$ -closure preserving if

$$f(\text{cl}_{\mathcal{I}\mathcal{K}}(A)) = \text{cl}_{\mathcal{I}\mathcal{K}}(f(A)) \quad \forall A \subset \mathcal{X}.$$

Theorem 10. The function f , from \mathcal{X} to \mathcal{Y} is $\mathcal{I}^{\mathcal{K}}$ -continuous iff it is $\mathcal{I}^{\mathcal{K}}$ -closure preserving.

P r o o f. Let $f : \mathcal{X} \rightarrow \mathcal{Y}$ be an $\mathcal{I}^{\mathcal{K}}$ -continuous function. Then, for any subset B of \mathcal{Y}

$$\text{cl}_{\mathcal{I}\mathcal{K}}(f^{-1}(B)) = f^{-1}(\text{cl}_{\mathcal{I}\mathcal{K}}(B))$$

holds. Consider a set $A \subset \mathcal{X}$ s.t. $f(A)$ is subset of \mathcal{Y} . So,

$$\text{cl}_{\mathcal{I}\mathcal{K}}(f^{-1}(f(A))) = f^{-1}(\text{cl}_{\mathcal{I}\mathcal{K}}(f(A)))$$

holds and it implies that $f(\text{cl}_{\mathcal{I}\mathcal{K}}(A)) = \text{cl}_{\mathcal{I}\mathcal{K}}(f(A)) \quad \forall A \subset \mathcal{X}$ holds.

Conversely, let f be $\mathcal{I}^{\mathcal{K}}$ -closure preserving function, then

$$f(\text{cl}_{\mathcal{I}\mathcal{K}}(A)) = \text{cl}_{\mathcal{I}\mathcal{K}}(f(A)) \quad \forall A \subset \mathcal{X}.$$

Let v be any subset of \mathcal{Y} , then $f^{-1}(v)$ is subset of \mathcal{X} and

$$f(\text{cl}_{\mathcal{I}\mathcal{K}}(f^{-1}(v))) = \text{cl}_{\mathcal{I}\mathcal{K}}(f(f^{-1}(v))) = \text{cl}_{\mathcal{I}\mathcal{K}}(v)$$

holds. So

$$\text{cl}_{\mathcal{I}\mathcal{K}}(f^{-1}(v)) = f^{-1}(\text{cl}_{\mathcal{I}\mathcal{K}}(v))$$

and by Theorem 9 the function f is $\mathcal{I}^{\mathcal{K}}$ -continuous. □

Theorem 11. Let \mathcal{X}, \mathcal{Y} and \mathcal{Z} be $\mathcal{I}^{\mathcal{K}}$ -seq.-top. spaces. Let f , from \mathcal{X} to \mathcal{Y} and g , from \mathcal{Y} to \mathcal{Z} be $\mathcal{I}^{\mathcal{K}}$ -continuous functions. Then $g \circ f : \mathcal{X} \rightarrow \mathcal{Z}$ is $\mathcal{I}^{\mathcal{K}}$ -continuous functions.

P r o o f. Let v be any $\mathcal{I}^{\mathcal{K}}$ -open subset of \mathcal{Z} . Since g is $\mathcal{I}^{\mathcal{K}}$ -continuous function then $g^{-1}(v)$ is $\mathcal{I}^{\mathcal{K}}$ -open subset of \mathcal{Y} and because f is $\mathcal{I}^{\mathcal{K}}$ -continuous function therefore $f^{-1}(g^{-1}(v))$ is $\mathcal{I}^{\mathcal{K}}$ -open subset of \mathcal{X} hence $(g \circ f)^{-1}(v)$ is $\mathcal{I}^{\mathcal{K}}$ -open subset of \mathcal{X} . □

6. Subspace of $\mathcal{I}^{\mathcal{K}}$ -seq.-top. space

In this section subspaces of the $\mathcal{I}^{\mathcal{K}}$ -seq.-top. space and its properties under an $\mathcal{I}^{\mathcal{K}}$ -continuous function will be discussed.

Definition 18. Let $(\mathcal{X}, \mathcal{T}_{\mathcal{I}^{\mathcal{K}}})$ be an $\mathcal{I}^{\mathcal{K}}$ -seq.-top. space and $\mathcal{Y} \subset \mathcal{X}$. Then

$$C_{\mathcal{Y}} : \mathcal{P}(\mathcal{Y}) \rightarrow \mathcal{P}(\mathcal{Y}), \quad C_{\mathcal{Y}}(A) = \mathcal{Y} \cap \text{cl}_{\mathcal{I}^{\mathcal{K}}}(A)$$

is a Kuratowsky operator. Define a T.S. as $(\mathcal{Y}, \mathcal{T}_{\mathcal{I}^{\mathcal{K}}}^{\mathcal{Y}})$, where

$$\mathcal{T}_{\mathcal{I}^{\mathcal{K}}}^{\mathcal{Y}} = \{U \cap \mathcal{Y}, U \in \mathcal{T}_{\mathcal{I}^{\mathcal{K}}}\} \subset \mathcal{P}(\mathcal{Y}).$$

This T.S. is called $\mathcal{I}^{\mathcal{K}}$ -subspace of \mathcal{X} .

Lemma 6. Let \mathcal{Y} be an $\mathcal{I}^{\mathcal{K}}$ -subspace of $\mathcal{I}^{\mathcal{K}}$ -seq.-top. space \mathcal{X} . If set A is $\mathcal{I}^{\mathcal{K}}$ -open subset of \mathcal{Y} and \mathcal{Y} is an $\mathcal{I}^{\mathcal{K}}$ -subset of \mathcal{X} . Then A is $\mathcal{I}^{\mathcal{K}}$ -open subset of \mathcal{X} .

P r o o f. Let A be $\mathcal{I}^{\mathcal{K}}$ -open subset of \mathcal{Y} . Then $\exists U \in \mathcal{T}_{\mathcal{I}^{\mathcal{K}}}$ s.t. $A = \mathcal{Y} \cap U$. Since \mathcal{Y} is an $\mathcal{I}^{\mathcal{K}}$ -open subset of \mathcal{X} . Then $A \in \mathcal{T}_{\mathcal{I}^{\mathcal{K}}}$. \square

Proposition 3. Let $(\mathcal{X}, \mathcal{T}_{\mathcal{I}^{\mathcal{K}}})$ and $(\mathcal{Y}, \mathcal{T}_{\mathcal{I}^{\mathcal{K}}}^{\mathcal{Y}})$ be $\mathcal{I}^{\mathcal{K}}$ -sequential spaces, $f : \mathcal{X} \rightarrow \mathcal{Y}$ be $\mathcal{I}^{\mathcal{K}}$ -continuous function and $A \subset \mathcal{X}$ is $\mathcal{I}^{\mathcal{K}}$ -subspace of \mathcal{X} . Then $f|_A : A \rightarrow \mathcal{Y}$, the restriction f over A is $\mathcal{I}^{\mathcal{K}}$ -continuous function.

P r o o f. Let U be an $\mathcal{I}^{\mathcal{K}}$ -open subset of \mathcal{Y} . Since f is $\mathcal{I}^{\mathcal{K}}$ -continuous function then $f^{-1}(U)$ is $\mathcal{I}^{\mathcal{K}}$ -open subset of \mathcal{X} . That is $f^{-1}(U) \in \mathcal{T}_{\mathcal{I}^{\mathcal{K}}}$.

In other hand $f|_A^{-1}(U) = A \cap f^{-1}(U)$. So $f|_A^{-1}(U)$ is $\mathcal{I}^{\mathcal{K}}$ -open subset of subspace A . Hence $f|_A$ is $\mathcal{I}^{\mathcal{K}}$ -continuous function. \square

Lemma 7. If A is $\mathcal{I}^{\mathcal{K}}$ -subspace of $\mathcal{I}^{\mathcal{K}}$ -sequential T.S. \mathcal{X} . Then the inclusion map $j : A \rightarrow \mathcal{X}$ is $\mathcal{I}^{\mathcal{K}}$ -continuous.

P r o o f. If U is $\mathcal{I}^{\mathcal{K}}$ -open in \mathcal{X} then $j^{-1}(U) = U \cap A$ is $\mathcal{I}^{\mathcal{K}}$ -open in subspace \mathcal{Y} hence j is $\mathcal{I}^{\mathcal{K}}$ -continuous. \square

Proposition 4. Let $(\mathcal{X}, \mathcal{T}_{\mathcal{I}^{\mathcal{K}}})$ and $(\mathcal{Y}, \mathcal{T}_{\mathcal{I}^{\mathcal{K}}}^{\mathcal{Y}})$ be $\mathcal{I}^{\mathcal{K}}$ -sequential spaces, $B \subset \mathcal{Y}$ be subspace of \mathcal{Y} and $f : \mathcal{X} \rightarrow B$ be $\mathcal{I}^{\mathcal{K}}$ -continuous function. Then, $h : \mathcal{X} \rightarrow \mathcal{Y}$ obtained by expanding the range of f is $\mathcal{I}^{\mathcal{K}}$ -continuous.

P r o o f. To show $h : \mathcal{X} \rightarrow \mathcal{Y}$ is $\mathcal{I}^{\mathcal{K}}$ -continuous function, if B as subspace of \mathcal{Y} then note that h is the composition of the map $f : \mathcal{X} \rightarrow B$ and $j : B \rightarrow \mathcal{Y}$. \square

7. Conclusion

In this article we defined the notion of $\mathcal{I}^{\mathcal{K}}$ -closed (resp. $\mathcal{I}^{\mathcal{K}}$ -open) set in a T.S. $(\mathcal{X}, \mathcal{T})$ and established some important results concerning this notion. Furthermore, we defined the $\mathcal{I}^{\mathcal{K}}$ -seq.-top., which is a generalized form of the \mathcal{I}^* -sequential space. We also talked about $\mathcal{I}^{\mathcal{K}}$ -continuity of functions and saw that in $\mathcal{I}^{\mathcal{K}}$ -seq.-top. space the notion of continuity and sequential continuity are the same. And in the last section of the paper, subspace of $\mathcal{I}^{\mathcal{K}}$ -sequential space have been studied and some important results established.

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