Алгебра и анализ
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To my sister

ON ISOTROPY FIELDS OF Spin(18)-TORSORS

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It is shown that any 18-dimensional nondegenerate quadratic form of trivial discriminant and Clifford invariant acquires Witt index at least 5 over some finite ground field extension of degree not divisible by 2^4. On the basis of previous research, a general formula is also established for all possible similar statements for forms of arbitrary dimension.

§1. Introduction

Given a split semisimple algebraic group G and a parabolic subgroup $P \subset G$, a G-torsor E over an extension of the ground field of G is said to be *P-isotropic* if the quotient variety E/P has a rational point, cf. [12]. As an example, for a Borel subgroup $B \subset G$, "B-isotropic" means the same as "trivial".

Let E be a *generic G-torsor*, i.e., the generic fiber of the quotient morphism

$$\mathrm{GL}(N) \to \mathrm{GL}(N)/G$$

given by an embedding of G into the general linear group $\mathrm{GL}(N)$ for some $N \geqslant 1$. It is interesting to know the g.c.d. of degrees of finite extensions L of its ground field that are *P-isotropy fields* of E, i.e., the torsor E_L is P-isotropic. This number is the *index* of the variety E/P, defined (for any algebraic variety) as the g.c.d. of degrees of its closed points. For instance, for $P = B$ what we get is the well-studied *torsion index* of the group G (see [13, Theorem 1.1]).

Our motivation to consider a generic G-torsor E relies on the fact that any G-torsor E' over an extension of the ground field of G is a specialization of E. For that reason, the index of E'/P is a divisor of the index of E/P (see [9, Theorem 6.4]).

Ключевые слова: quadratic forms over fields, affine algebraic groups, spin groups, projective homogeneous varieties, Chow rings.

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