Problems

Problem 1 Let Q - the set of quadratic residues modulo prime p. Is it true that

$$Q \sqcup \{0\} = A - A$$

for some A. In particular, when A is a multiplicative subgroup the question is also opened.

Problem 2 (N. Alon) Let Q - the set of quadratic residues modulo prime p and we have

$$A - A \subseteq Q \sqcup \{0\}.$$

Prove that

$$|A| < p^{1/2-\delta}$$

for some positive $\delta > 0$. (The known results are much more weaker!!)

Problem 3 (Roche-Newton, Zhelezov) Let

$$|\frac{A \pm A}{A \pm A}| << |A|^2.$$

Is it true that

$$|A \pm A| << |A|?$$

Problem 4 Let the set $P \subseteq A - A$ with $|P * P| \leq M|P|$. Prove that there exists a $\delta(M) > 0$ such that

$$\sum_{x \in P} |\{a_1 - a_2 = x | a_1, a_2 \in A\}| << |A|^{2-\delta(M)}.$$

Problem 5 Let the set $D = A - A$, $A \subseteq \mathbb{R}$. Prove that for any positive C there exists $k = k(C)$ such that

$$|D * \ldots * D| = |\{d_1 * \ldots * d_k | d_1, \ldots, d_k \in D\}| \geq |D|^C.$$

Problem 6 Let the set $A \subseteq \mathbb{R}$ with $|A * A| << |A|$.

Prove that for any positive C there exists $k = k(C)$ such that for all non-zero $x_i, 1 \leq i \leq k$ we have

$$|(A + x_1) * \ldots * (A + x_k)| \geq |A|^C.$$

Problem 7 Let the set $A \subseteq \mathbb{R}$ and $R[A] = \{\frac{a_1 - a}{a_2 - a} | a_2 \neq a\}$. Prove that

$$|R[A]| >> |A - A|$$

or

$$|R[A]| >> |A/A|.$$